Chapter 3

1 A Model of Minimum Wage Effects on Labor Market Careers

Minimum wage changes can have complex effects on the labor market experiences of individuals through decisions to participate in the labor market, human capital investment activity, the frequency and duration of unemployment spells, the accepted wage distribution, and a myriad of other phenomena. The model we develop will abstract from many of these issues, but in the conclusion and at selective points in the empirical analysis we will comment on how some of our theoretical and empirical analysis would change when we bring these phenomena into the picture, and how the policy implications drawn from our analysis could be skewed by their omission.

The model we will use to theoretically and empirically investigate minimum wage effects on labor market experiences views individuals and firms operating in stochastic environments that are governed by probabilistic laws that do not change over time. This is quite an abstraction from reality, but for purposes of examining minimum wage effects it may not be too egregious for a number of reasons. Because the majority of individuals paid the minimum wage are young, as we saw in the previous chapter, the major impact of minimum wage laws on labor market outcomes is likely to be concentrated in the first few years of labor market activity.\(^1\) Over such a relatively short period, the probabilistic structure of the labor market is not likely to change dramatically. Furthermore, pragmatically speaking, the data to which we have access

\(^1\)This will be the case as long as the minimum wage is relatively low in comparison with the median compensation level, for instance. As the minimum wage is raised, we can expect that the proportion of workers directly impacted by it, and the average age of workers paid the minimum wage, will steadily increase.
(drawn from the Current Population Survey) are essentially static. To estimate a dynamic model using such data requires us to assume that the labor market environment does not change over time. In particular, under the assumption of stationarity (i.e., constancy of the probabilistic laws governing the economy) the rational behavior posited for all labor market participants and firms can be characterized in terms of decision rules that are time-invariant. If we were to allow the model to be nonstationarity, we would be forced to specify all of the conditions that individuals and firms have faced in the past and will face in the future in order to describe their decisions at any point in time. Clearly we don’t have access to data which would allow us to undertake such a modeling effort, nor would such a complex theoretical analysis be particularly enlightening from a conceptual point of view.

Why attempt to build such a model in the first place? The point which we will make repeatedly throughout this monograph is that it is necessary to have a relatively complete, equilibrium model of employment relations if we are to comprehensively summarize the impact of minimum wages on labor market outcomes and the welfare of labor market participants and firms. To make welfare valuations, it is necessary to endow each set of agents in the economy with their own set of objectives. Subject to technological and budget constraints, and given the optimizing decisions of other agents in the market, each individual or firm acts so as to maximize the value of their objective function. We will view minimum wages as constraining the actions of all labor market participants, both individuals and firms, though as we shall see the minimum wage “constraint” may, under certain circumstances, increase the welfare of labor market participants, firms, or even both. In many cases, only one side of the market will experience a welfare increase as a result of a given minimum wage increase (and yes, it could be employers), while minimum wage increases at extremely high levels will negatively impact both sides of the market.
The objective function with which we endow all labor market participants is one of expected wealth maximization. That is, individuals are assumed to care only about their (discounted) average earnings over their labor market careers; in particular, the variance of (realized) income flows does not favorably or unfavorably affect welfare. This is a strong assumption, and is made primarily for reasons of tractability. However, it is easy to show that when consumption decisions can be “decoupled” from earnings, as is the case when there exist perfect capital markets for borrowing and lending, expected wealth maximization behavior in the labor market is consistent with aversion towards consumption risk on the part of individuals. While young labor market participants may not have access to perfect capital markets, transfers between parents and children may serve the same role. There is no strong reason to expect that young labor market participants will behave in ways other than those consistent with expected wealth maximization.

Perhaps less controversially, firms will be assumed to behave so as to maximize expected profits. The model employed throughout is search theoretic. At some points in the welfare analysis we will add general equilibrium elements by assuming that the rate of contacts between searching individuals and searching employers is determined using a “matching function” setup (see, e.g., Pissarides (2000)). In this framework, firms make decisions regarding the number of vacancies to create and the contact rate is an increasing function of the number of searchers and the number of vacancies. There will be no other “general equilibrium” links between searchers and firms in our model, such as those that might occur through the public ownership of firms. The general equilibrium analysis posits an expected value of zero for all new vacancies, which occurs through the mechanism of free entry, though firms with filled jobs do earn positive profits, and the profit level is, in general, a function of the minimum wage rate. In the partial equilibrium analysis, all firms earn nonnegative profits in
equilibrium. We have thus deliberately kept our model of firm behavior and the link between firms and searchers as simple as possible subject to the restriction that it possess empirical implications broadly consistent with the CPS data that we utilize in the empirical analysis.

As is clear from the description of the model so far, many phenomena that could have significant impacts on the effect of the minimum wage on labor market outcomes have been omitted. Perhaps the first thing that comes to mind is capital goods. If firms have access to production technologies that allow them to substitute capital for labor, then substantial minimum wage increases that significantly affect the price of labor, at least at the low end of the skill distribution, may lead employers to substitute machinery for manpower. Since we have no measures of capital utilization at the firms employing individuals in our the CPS data, we have no way of directly using this information to characterize the potential degree of substitutability between capital and labor at this part of the skill distribution.

Though it is common to assume the absence of capital in search-theoretic models of the labor market, clearly it would be preferable to allow firms to make these decisions. Over the past several decades we have seen the demand for lower-skilled individuals decrease precipitously in the United States, which has given rise to a marked increase in inequality in labor market earnings. While we cannot explicitly include in the model and the empirical work a number of important factors affecting the demand for labor, our Nash bargaining formulation of the wage determination process does allow us to represent the cumulative impact of these factors in an indirect way - through the bargaining power parameter. Recently, Cahuc et al (2006) have explicitly estimated bargaining power parameters for segments of the French work force, and find that higher-skilled workers tend to have more bargaining power than less skilled workers, even after accounting for other differences in the labor market.
environments that these groups face. Of course, one problem with allowing this one characteristic to represent the plethora of omitted factors is the necessity we face of assuming that it is a “primitive” parameter, that is, that it remains the same when labor market policy (the minimum wage in our case) is even radically altered. Since many of our policy implications flow from the estimated value of the bargaining power parameter and our assumption that it is fixed, all of our findings and welfare analyses must be interpreted cautiously.

Firms may also attempt to “work around” a minimum wage if the employment contract and working conditions are multidimensional. For example, Hashimoto (1981) suggests that in response to an increase in the minimum wage, a firm may lower the investment content of jobs held by younger workers, which would then result in a flatter wage profile over the life cycle. Similar adjusts in other nonpecuniary aspects of the job, such as whether it includes an offer of employer-provided health insurance, may occur. The ability of firms to adjust their compensation packages along other dimensions will dampen the impact of the minimum wage on labor market outcomes.

1.1 Characterization of the Labor Market Career

We will think of labor market events as occurring continuously in time. By this we mean that there are no natural times, say weeks or months, at which labor market events always take place. Technically speaking, we view the labor market as a continuous-time point process, which means that at any point in time an unemployed individual can receive a job offer. Furthermore, at any point during an employment relationship the contract can be exogenously terminated. In a more general versions of the model, discussed in Appendix ??, one can allow for on-the-job search as well. In this case employed individuals at one firm may make contact with another that offers
them an employment opportunity. As a result, jobs with a given employer may end due to some exogenous separation (such as the plant closing down or the individual changing locations) or for “endogenous” reasons - the employee finds a job at which she is more productive and terminates her position at her current employer. We will mainly focus on the case of unemployed search only because it is more straightforward to analyze and because such a model can be estimated with CPS data, the largest and most representative labor market data set available to us.

Individuals begin their labor market careers as unemployed searchers, eventually locating a job paying some “acceptable” wage (we will define below what we mean by acceptable). After some period of time, randomly determined, that job will end, and she will once again enter the state of unemployment, where the job search process will be repeated in exactly the same manner as it was the first time around. This “cycling” continues unabated until the individual eventually leaves the market - which we will suppose is for an exogenous reason. In the language of stochastic process theory, the labor market career is an alternating, marked renewal process. It is alternating, because time in the labor market is spent alternatively in unemployment and employment spells. It is a renewal process, because we will be assuming that the individual’s past labor market history plays no role in determining how long she will spend in a current spell of unemployment or employment. Finally, it is a marked process because when she is employed, we will not only be concerned with how long the employment spell lasts, but also at what wage she is employed. The wage rate is the marker, or subsidiary characteristic, of interest to us over and above the timing of events.

Let us fix ideas with an example. Say that an individual begins her labor market career at time 0, an inconsequential normalization. Assuming that she will continue to participate in the market as an unemployed searcher or worker over her entire life,
her labor market career can be completely characterized by the time at which she meets prospective employers and the value of the match associated with each contact, as well as the time at which employment matches she has accepted are (exogenously) terminated. For example, since she begins her labor market career in the unemployed search state at time 0, the first ten “events” in her labor market career might be given by the values in Table 3.1.

Table 3.1

Hypothetical Early Labor Market Career

<table>
<thead>
<tr>
<th>Event Number</th>
<th>State</th>
<th>Time of Event</th>
<th>Duration Draw</th>
<th>Match Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>U</td>
<td>0.891</td>
<td>0.891</td>
<td>6.243</td>
</tr>
<tr>
<td>2</td>
<td>U</td>
<td>3.168</td>
<td>2.277</td>
<td>4.329</td>
</tr>
<tr>
<td>3</td>
<td>U</td>
<td>15.554</td>
<td>12.386</td>
<td>3.871</td>
</tr>
<tr>
<td>4</td>
<td>U</td>
<td>15.558</td>
<td>0.004</td>
<td>10.918</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>38.921</td>
<td>23.363</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>U</td>
<td>44.236</td>
<td>5.315</td>
<td>7.891</td>
</tr>
<tr>
<td>7</td>
<td>U</td>
<td>56.793</td>
<td>12.557</td>
<td>12.119</td>
</tr>
<tr>
<td>8</td>
<td>E</td>
<td>157.421</td>
<td>100.628</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>U</td>
<td>164.772</td>
<td>7.351</td>
<td>10.145</td>
</tr>
<tr>
<td>10</td>
<td>E</td>
<td>322.510</td>
<td>157.738</td>
<td>-</td>
</tr>
</tbody>
</table>

The interpretation of the figures in Table 3.1 is as follows. The individual initiated search at time 0, at which point she occupied the unemployed state, $U$. At time 0.891
she encountered her first potential employer. (Time units are arbitrary here, but it may help to think of the unit of time as the week.) When she met her first employer, the productivity of the potential match was revealed to be 6.243; this is to be thought of as the “flow” value of productivity that will be realized every moment she works for this employer. The value of this first potential match was insufficient to result in an employment contract so that the individual remained in the nonemployed state. The second potential employer was encountered at time 3.168 and the potential value of that match was 4.329, which also was deemed unacceptable. Only the fourth potential match resulted in an employment contract. This employment contract began at time 15.558 and had a flow value associated with it of 10.918. This match surplus is divided between the worker and the firm according to an idealized bargaining process which will be described in detail below. Whatever the division, it was sufficient to induce both parties to begin the employment contract, which was terminated (exogenously) at time 38.291. This caused the individual to reenter the nonemployment state, and she immediately began to search for a new acceptable employment contract. After one unsuccessful encounter at time 44.236, she found another acceptable match with a value of 12.119 at time 56.793. This match was eventually terminated at time 157.421, and the nonemployed search process was repeated.

The events described in Table 3.1 are not strictly exogenous. Some events are conditionally exogenous; for example, given the decision to begin the labor market career at time 0, the arrival of the first possible job match at time 0.891 and the total flow value of that match, 6.243, are determined strictly randomly. However, the decision to reject that possible match is a behavioral one made by the searcher and the first firm encountered. This led the individual to continue in the nonemployed state and eventually to meet her second potential match at a time which was, once again, determined randomly. Thus the labor market career is a sequence of exogenous
events followed by decisions which lead to further exogenous events [conditional on
the previous exogenous events and past decisions], and so on. The method of dynamic
programming (DP) allows us to formalize this recursive process.

The only decision explicitly made by the searcher in this simple conceptualiza-
tion of the labor market career is whether or not to accept a particular employment
contract; though the wage associated with a given match value is assumed to be
the outcome of a bargaining procedure, this procedure is largely “black-boxed,” and
hence not strictly behavioral.\footnote{It is possible to rationalize the Nash bargaining axiomatic solution as the outcome of a strategic
bargaining game, as was famously done in Rubenstein (1982). We won’t explicitly attempt to model
the interactions between negotiating firms and workers. For one attempt to do so in a bargaining
model similar to ours, see Cahuc et al (2006).} Under the assumption that the environment is con-
stant, the decision of whether or not to begin a particular employment match will be
made by comparing the value of the match, denoted by $\theta$, with a constant, $\theta^*$. An
employment contract will be initiated whenever a match value $\theta$ equals or exceeds
$\theta^*$.

Another decision, which is particularly relevant for young (and old) individuals, is
whether to participate (i.e., undertake search) in the labor market. For the moment,
we will ignore this decision and assume that all individuals in the population we are
considering are labor market participants. We shall return to a consideration of the
participation decision at the end of this chapter.
1.2 The Stationary Labor Market Environment

There are a number of ways to formally characterize of the stationary environment within which firms and individuals interact. Perhaps the most straightforward is in terms of the distributions of times spent in the various labor market states. We assume that there are two duration distributions, one associated with event times in the unemployment state (those are the times at which job offers are received) and the other associated with event times while employed (when the job is terminated).

The distribution of times between events while unemployed is given by $F_U(t_U)$, and it has an associated probability density function given by $f_U(t_U)$. By assuming that the labor market is “memoryless,” the implication is that the rate of receiving an offer since the last time the searcher received one is independent of the length of the interval. More formally, the instantaneous rate of receiving an offer given that the last offer received was $t_U$ ago, is

\[ h_U(t_U) = \frac{f_U(t_U)}{1 - F_U(t_U)}. \]

The function $h_U(t_U)$ is referred to as the hazard rate, and it can be thought of (loosely) as the probability of receiving an offer after waiting for a duration $t_U$.\(^3\) In our case, for the offer arrival process to be memoryless implies that the rate of receiving offers should be independent of how long it has been since the last offer was received, or

\[ h_U(t_U) = \lambda, \]

where $\lambda$ is some positive constant. A continuous distribution with a well-defined density can be equivalently represented in terms of its hazard function, and vice

\(^3\)More technically, $h_U(t_U)$ is the conditional density of duration times at $t_U$, where the conditioning event is not having received an offer prior to $t_U$. 
versa. The only continuous distribution associated with a random variable that takes positive values is the negative exponential, so that

\[ h_U(t_U) = \lambda \]

\[ \iff f_U(t_U) = \lambda \exp(-\lambda t_U) \]

\[ \Rightarrow F_U(t_U) = 1 - \exp(-\lambda t_U). \]

An individual who exits an unemployment spell immediately enters an employment spell. The employment termination process is also memoryless, which implies that the hazard rate function out of employment is given by \( h_E(t_E) = \eta \), where \( \eta \) is a positive integer and \( t_E \) is the elapsed time since the employment spell began. The constant hazard rate assumption implies a negative exponential distribution of employment durations, with

\[ f_E(t_E) = \eta \exp(-\eta t_E) \]

\[ F_E(t_E) = 1 - \exp(-\eta t_E). \]

The labor market experiences of an individual can be thought of as unfolding in the following manner. As in the example, the individual begins their labor market life in the unemployment state at time 0. They then draw a duration of time until their first job offer is received from the distribution \( F_U \). In the case of our example, this random draw resulted in a duration to first offer of 0.891. When the firm is contacted, another random draw is made, this time from the distribution of matches, \( G \). The match value drawn at the first firm contact was equal to 6.243. This job offer was rejected, and another independent draw was made from the duration distribution \( F_U \). In this case, the second draw was equal to 2.277 (Column 4), so that the event
occurred at time 3.168 from the beginning of the labor market career. The match value drawn at this contact time was 4.329, which was also rejected. All of the draws of durations until events 1 through 4 are made, independently, from the distribution $F_U$.

The fourth match value drawn during the first unemployment spell (which consists of events 1 through 4) was excepted and an employment spell began at time 38.291. At that moment in time, a draw was made from the distribution $F_E$ that determined the length of time the employment spell would last (23.363, in this case). Once that employment spell lasted, the next unemployment spell began, and it consisted of events 6 and 7. The process proceeds until the individual exits from the labor market.

We see that the labor market proceeds as a sequence of draws from the distributions $F_U$, $F_E$, and $G$. In the example of Table 3.1, the durations associated with events \{1,2,3,4,6,7,9\} were i.i.d. draws from $F_U$, while the durations associated with events \{5,8,10\} were i.i.d. draws from $F_E$. The match values associated with each contact in the unemployment state were i.i.d. draws from $G$. The endogeneity of the labor market process stems from the selection on draws made from the match distribution $G$, and the implications of this selection process for the rate of leaving unemployment and the observed accepted wage distribution. In the following section we characterize the selection rules used by searchers.

### 1.3 The Decision-Theoretic Model

The individuals in our model are posited to be taking actions so as to maximize their expected lifetime wealth. They can only maximize expected wealth due to the presence of “search frictions.” In this case, search frictions refer to the fact that
individuals do not know the location of the firms with employment vacancies, and even more importantly, do not know the identity of the firm with which they could achieve their highest productivity level. Prior to actually contacting a given firm and learning their productivity level there, all potential employers look alike to the individual.\footnote{In the search literature, this is referred to as the case of non-directed search. Directed search occurs when firms not yet contacted are not viewed as equivalent by the searcher. In this case, the individual may rank potential employers or geographic labor markets in a preferred order prior to beginning a search episode. While some form(s) of directed search probably no doubtedly occur in actual labor markets, the theoretical and empirical analyses of such models are currently at a very rudimentary stage.} In an expectational sense, then, the individual is initially indifferent with respect to the identity of the firm that is contacted.\footnote{Under the informational assumptions of this model, jobs, or more properly potential jobs, are pure search goods. Once a potential employer is contacted, both the individual and the firm will learn the total value of the match as well as the share of that value accruing to each.} Firms are only differentiated after contact.

It is important to be clear regarding our assumption as to the manner in which an individual’s productivity, $\theta$, is determined at a randomly selected firm. We assume that this productivity value is a draw from the fixed, known distribution $G$. For the moment, we do not distinguish individuals in terms of observable characteristics. One might suppose that schooling, for example, tends to make one more productive in the labor market. Within our modeling framework, a generic effect of schooling would be incorporated by conditioning the match distribution, and possibly other labor market parameters as well, on schooling. In general, let $x$ denote a characteristic or set of characteristic upon which population members are distinguished. Then the conditional matching function is $G(\theta|x)$. If $x$ was years of schooling completed, for example, we might assume that $G(\theta|x') \leq G(\theta|x)$ for all $\theta > 0$ when $x' > x$. This is a first order stochastic dominance relationship, so that the likelihood that any match draw is less or equal to $\theta$ for a highly-educated individual is no greater than it is for a less-educated individual. Of course, a logical consequence of this is that the average
match draws of the more highly-educated individual is at least as large as that of the less-educated person. Other things equal, more highly-educated individuals will receive higher wages, but at any given firm their productivity can be lower than their less-educated colleague. Given an individual’s characteristics, as summarized by $x$, match draws across alternative employers will be independently and identically distributed according to $G(\theta|x)$.

There are many other alternative formulations one could make regarding the manner in which worker-firm productivity is determined. One leading approach is to assume that individual $i$ on the supply side of the market has a time-invariant productivity-determining characteristic $a_i$, while firm $j$ on the demand side of the market has a time-invariant productivity-determining characteristic $b_j$, with the productivity of individual $i$ at firm $j$ being given by $\theta_{ij} = a_i b_j$. This is the setup utilized by Postel-Vinay and Robin (2002) and by Cahuc et al. (2006) in their analysis of labor market dynamics using matched French employer-employee data. They skillfully exploit these data and are able to recover estimates of the employee skill distribution $F(a)$ without making parametric assumptions regarding the form of the distribution. Given a distribution of firm characteristics, $Z(b)$, each individual faces their own matching distribution $Z(b) \times a$. In a number of respects, this is more general than the matching process described in the previous paragraph in that it does not require us to specify which factors, such as schooling, should be used to differentiate workers. On the other hand, it does place a number of restrictions on the distribution of match values across workers and firms that are undeniably strong.\footnote{As one example, their matching structure implies that any worker that is more productive than another worker at a given firm $b$ will be more productive at any other firm $b'$.}

At each moment in time, the individual makes decisions so as to maximize expected wealth given her current labor market state and her knowledge of the parame-
ters that characterize the labor market environment. The method used to characterize her decision is dynamic programming (DP). To illustrate the DP approach in a continuous time model like ours, we will begin by assuming that workers receive the entire value of the match. In this case, the matching distribution $G$ is identical to the wage offer distribution $G(w)$, since $w = \theta$ for any $\theta$ draw.

The value of being in any particular labor market state at a point in time consists of the sum of a “flow” value, which is the “current period return” (a period should be thought of as an instant in this case), and the discounted expected value of next period’s problem given the current state and action taken. If the current state is denoted by $s$, the current choice or action by $a$, and next period’s state is given by $s'$, then the general formulation of the problem is

$$V(s) = \max_a R(s, a) + \beta E[V(s')|s, a], \quad (1)$$

where $R(s, a)$ denotes the current period return to the action $a$ taken when the state is $s$, $\beta$ is some positive scalar, $s'$ denotes next period’s state, and $E[V(s')|s, a]$ is the conditional expectation of the value of next period’s decision problem given current state $s$ and action $a$. In computing this expectation, the conditioning on $s$ and $a$ reflects the fact that in general the probability distribution of $s'$ is not independent of the current state and the action taken ($s$ and $a$, respectively).

Consider first the value of being in state $U$. Assume that the flow value of searching is given by the constant scalar $b$. This flow value of nonemployment can reflect unemployment benefits, direct costs of search activity, and the value of other activities undertaken while the individual searches. Our strong stationarity assumption requires that $b$ be time invariant and that time spent in search does not change the parameters characterizing the labor market environment of the individual. These
are both clearly counterfactual, especially for young labor market participants. In the case of unemployment benefits, for example, there are limitations on the length of time they can be received (and they may not be received at all if an individual does not have the requisite employment history). In addition, young labor market searchers are often simultaneously investing in human capital. This investment in human capital would be expected to impact several parameters characterizing the labor market environment, such as the productivity distribution $G$ and the rates of match arrivals and dissolutions of employment contracts. Introducing human capital accumulation into the model adds another state variable, the current level of the human capital stock, considerably complicating the analysis. The fact that human capital is not directly observable makes it necessary to rely on ad hoc assumptions concerning the nature of the human capital production function and the dependence of search parameters on the human capital stock and investment activity. For these reasons we have chosen to ignore it in the present analysis.\footnote{For estimation of a version of such a model, see Bagger et al. (2006).}

Another important benefit of working under a stationarity assumption is the relative simplicity of the decision rules that are associated with optimizing behavior. In particular, the decision rule of whether to accept a job at a newly discovered job opportunity characterized by $\theta$ will only be a function of $\theta$, and, in particular, will not be a function of the duration of the unemployment spell or previously received job opportunities. This rules out the situation of an individual deciding not to accept a job at a match value $\theta$ at some arbitrary time $t$, "wishes" she would have accepted it at some time $t' > t$. The acceptance rules we derive under stationarity imply that any newly received offer will be accepted only if it is as least as large as some constant $\theta^*$. Thus if the match $\theta$ was rejected at time $t$, it would have been rejected at any
future time $t'$ as well.\footnote{Due to this feature of the decision rule, we do not have to take a position on whether searchers have access to previously rejected offers. In a nonstationary model, instead, there typically will arise situations in which previously rejected offers would be accepted if the searcher still has access to them. If so, we say that search is with recall, if not we say it is without recall. The recall option is not operative under stationary decision rules.}

Our model is set in continuous time, and therefore has no natural “periods” to distinguish the “current” from the “future.” Our strategy will be to assume that there does exist a “decision period” of duration $\varepsilon$ over which new actions are precluded.\footnote{For an excellent exposition of this approach, see Burdett and Mortensen (1978).} That is, the individual will take an action given the state $s$ and will reap the reward from this action over the period $\varepsilon$. At the conclusion of this decision period, she will take a new action given the state to which the system has then evolved. The decision period $\varepsilon$ is in the end just an artifice, for we shall define behavior and the value of the problem in the limit as $\varepsilon \to 0$.

Define the value of the unemployed search problem as

$$V_U = (1 + \rho \varepsilon)^{-1}\{b \varepsilon + \lambda \varepsilon \int \max(V_U, V_E(w)) \, dG(w) \} + (1 - \lambda \varepsilon) V_U + o(\varepsilon) \}.$$  

As currently written, (2) includes no explicit action, since we assume that the individual has already decided to participate in the labor market. We will discuss how this decision can be made endogenous below.

In terms of the correspondence between [1] and [2], note that the term in [2] that corresponds to the $\beta$ in [1] is $1/(1 + \rho \varepsilon) \equiv \beta_\varepsilon$, where $\rho$ is the discount rate. The interpretation of the term $b \varepsilon/(1 + \rho \varepsilon)$, which is the “current period return” in the state of unemployment, is as follows. Over the short period $\varepsilon$ the individual receives $b$ per instant. Thus the total amount received at the end of this period is $b \varepsilon$. However,
since this amount is “paid” at the end of the period $\varepsilon$, its beginning of period value must be appropriately discounted. Applying the discount factor, the current period return in state $U$ is $\beta_\varepsilon b\varepsilon$.

Now consider the next term. Assume that the searcher obtains exactly one job offer at wage $w$. Her choice then will be either to accept employment at that wage or to continue searching. The value of accepting a wage offer of $w$ is given by $V_E(w)$ and will be discussed shortly. Given the receipt of the offer $w$, the individual will choose the option associated with the highest value, so that the value of getting an offer of $w$ is given by $\max(V_U, V_E(w))$. This choice is the only explicit behavior in the current setup. The expected value of getting an offer is then the expectation of $\max(V_U, V_E(w))$ taken with respect to the distribution of all possible wage offers, which is given by $G(w)$ in this case. Given the receipt of an offer, the discounted expected value is $\beta_\varepsilon \int \max(V_U, V_E(w)) dG(w)$. The approximate probability of getting one offer in the short interval $\varepsilon$ is $\lambda\varepsilon$.

If no offer is received the individual will continue to search. This is true because in a stationarity environment, if a certain action was optimal at some arbitrary time when the individual faces choices in the set $C$, then the same decision will be made at any other time when the individual occupies the same state and faces the same choices.\(^{10}\) Thus the value of not receiving an offer by the end of the period is $\beta_\varepsilon V_U$, and the approximate likelihood of this event is $(1 - \lambda\varepsilon)$. In terms of the DP decomposition given in [1], the discounted expected value of future choices given search is

$$\beta_\varepsilon \lambda\varepsilon \int \max(V_U, V_E(w)) dG(w) + \beta_\varepsilon (1 - \lambda\varepsilon) V_U + \beta_\varepsilon o(\varepsilon).$$

\(^{10}\)We have already used this invariance property implicitly when we argued that given it was optimal to search at one point in time it will never be optimal to exit the labor market in the future. Obviously, if employment conditions, such as the wage offer distribution for example, were allowed to vary over time this would not be true, in general.
The term $o(\varepsilon)$ includes the value of all of the other events that could occur in the finite interval of time $\varepsilon$, such as receiving two or more offers. The likelihood of more than one event occurring is a decreasing function of the size of the interval $\varepsilon$. The term $o(\varepsilon)$ is defined through the following important property:

$$\lim_{\varepsilon \to 0} \frac{o(\varepsilon)}{\varepsilon} = 0.$$ 

This statement implies that in the limit, as the interval $\varepsilon$ becomes arbitrarily small, the likelihood that more than one event occurs in the interval goes to 0. In our simple example, this means that in any instant, the individual will, at most, receive one job offer.

Before we can analyze the problem facing the nonemployed searcher further it is necessary to examine the situation of an employed individual being paid an “instantaneous” wage of $w$. Since we have precluded on-the-job search, and since it was initially optimal to accept a wage of $w$, an employed individual who has accepted a wage of $w$ will never quit and enter the state of unemployed search. Thus, she will simply remain at her job until such time as the employment match is exogenously terminated. Formally,

$$V_E(w) = (1 + \rho \varepsilon)^{-1} \left\{ w \varepsilon + \eta \varepsilon V_U + (1 - \eta \varepsilon) V_E(w) + o(\varepsilon) \right\}, \quad (3)$$

where the current period return is now given by $\beta \varepsilon w \varepsilon$ and the expected future value of continuing to work at the job during this “period” is

$$\beta \varepsilon \eta \varepsilon V_U + \beta \varepsilon (1 - \eta \varepsilon) V_E(w) + \beta \varepsilon o(\varepsilon).$$

This term is the sum of the discounted value of being dismissed during the period and
thus ending the period in the unemployment state multiplied by the (approximate) probability of being dismissed \((\eta \varepsilon)\), the discounted value of ending the period in the same job multiplied by the probability of not being dismissed \((1 - \eta \varepsilon)\), and the discounted value of the remainder term \(o(\varepsilon)\), which reflects the value and probabilities of all other events which could occur in an interval of length \(\varepsilon\).

We determine the value of employment as follows. Multiply both sides of \([3]\) by \(1 + \rho \varepsilon\) to get
\[
V_E(w)(1 + \rho \varepsilon) = w \varepsilon + \eta \varepsilon V_U + (1 - \eta \varepsilon)V_E(w) + o(\varepsilon)
\]
\[
\Rightarrow V_E(w)(\rho + \eta)\varepsilon = w \varepsilon + \eta \varepsilon V_U + o(\varepsilon)
\]
\[
\Rightarrow V_E(w) = \frac{w + \eta V_U}{\rho + \eta} + \frac{o(\varepsilon)}{\varepsilon},
\]
where the last line is obtained after dividing both sides of the second line by \(\varepsilon\). Now taking limits, we have
\[
\lim_{\varepsilon \to 0} V_E(w) = \frac{w + \eta V_U}{\rho + \eta} + \lim_{\varepsilon \to 0} \frac{o(\varepsilon)}{\varepsilon}
\]
\[
= \frac{w + \eta V_U}{\rho + \eta}
\]
by the definition of the term \(o(\varepsilon)\).

With this definition of \(V_E(w)\), we can return to our consideration of \(V_U\). First note
that

\[
\max(V_U, V_E(w)) = \max(V_U, \frac{w + \eta V_U}{\rho + \eta})
\]

\[
= \frac{1}{\rho + \eta} \max(V_U(\rho + \eta), w + \eta V_U)
\]

\[
= \frac{\eta V_U}{\rho + \eta} + \frac{\max(\rho V_U, w)}{\rho + \eta}
\]

\[
= \frac{\eta V_U}{\rho + \eta} + \frac{\rho V_U}{\rho + \eta} + \frac{\max(0, w - \rho V_U)}{\rho + \eta}
\]

\[
= V_U + \frac{\max(0, w - \rho V_U)}{\rho + \eta}. \quad (4)
\]

This is an important result, for it shows that for a given wage offer \(w\) the option of accepting the employment match exceeds the value of the option of continuing to search when the wage offer \(w\) exceeds the scalar value \(\rho V_U\). This is an important enough result to warrant the following terminology.

**Definition 1** The reservation wage \(w^*\) is equal to \(\rho V_U\) and has the property that any wage offer \(w \geq w^*\) will be accepted and any \(w < w^*\) will be rejected.

The reservation wage \(w^*\) completely summarizes the single decision rule utilized in this simple search model. Since the value of search, \(V_U\), will depend on all of the parameters that characterize the labor market environment, so does the reservation wage. Below we shall discuss the manner in which we can solve for \(w^*\).


\[
V_U = (1 + \rho \varepsilon)^{-1} \{b \varepsilon + \lambda \varepsilon \int (V_U + \frac{\max(0, w - \rho V_U)}{\rho + \eta}) dG(w)
\]

\[
+ (1 - \lambda \varepsilon)V_U + o(\varepsilon)\}
\]

\[
= (1 + \rho \varepsilon)^{-1} (b \varepsilon + V_U + \frac{\lambda \varepsilon}{\rho + \eta} \int \max(0, w - \rho V_U) dG(w) + o(\varepsilon)\}.\]

21
Then

\[ V_U(1 + \rho \varepsilon) = b \varepsilon + V_U + \frac{\lambda \varepsilon}{\rho + \eta} \int_{\rho V_U} (w - \rho V_U) dG(w) + o(\varepsilon) \]

\[ \Rightarrow \rho V_U = b + \frac{\lambda}{\rho + \eta} \int_{\rho V_U} (w - \rho V_U) dG(w), \quad (5) \]

where the second line is obtained after dividing the first line by \( \varepsilon \) and taking limits.

Since \( w^* \equiv \rho V_U \), we can rewrite \([5]\) as

\[ w^* = b + \frac{\lambda}{\rho + \eta} \int_{w^*} (w - w^*) dG(w) \quad (6) \]

In general, the expression \([6]\) cannot be manipulated so as to yield a closed-form solution for \( w^* \). However, it is not difficult to establish that there exists a unique solution \( w^* \) to this equation which is relatively straightforward to compute. For there to be a unique reservation wage, strictly speaking, we require the distribution of wage offers to be continuously differentiable on the support\(^{11}\) of the distribution, so we assume that there exists a well-defined probability density function \( g \) everywhere on the support of the distribution.\(^{12}\)

Given the existence of the density function \( g \), when we partially differentiate both sides of \([6]\) with respect to \( w^* \), we see that the derivative of the left hand side (LHS)

\(^{11}\)Assuming that the distribution function \( G \) is everywhere differentiable, the support of the distribution is defined as the subset of the real line \( S \subseteq \mathbb{R} \) such that \( g(s) > 0 \) for all \( s \in S \). In this discussion we assume that wage offers are always strictly positive, so that \( S = \mathbb{R}_+ \), the positive real line.

\(^{12}\)If \( w \) is a discrete random variable instead, the decision rule will still possess a critical value property, but the critical value solving the problem will not be unique. Say that wages were drawn from a discrete distribution with 3 mass points, at \( w = 1, 5, \) and 10. Then say that the optimal decision was to reject wage draws equal to 1 and 5, and to accept only a wage draw of 10. In this case, the reservation wage is not unique, since any value in the interval \( w^* \in (5, 10) \) will divide the sample space into the appropriate acceptance and rejection regions.
is simply 1, while the partial derivative of the RHS is

$$\frac{\partial \text{RHS}(6)}{\partial w^*} = -\frac{\lambda}{\rho + \eta} \tilde{G}(w^*) < 0,$$

where $\tilde{G}(x)$, termed the *survivor function*, is defined as $1 - G(x)$. For now, assume that the support of the distribution $G$ is the positive real line. Then the left hand side of (6) is a linear, increasing function of $w^*$ that takes values on the interval $(-\infty, \infty)$, the right-hand side is a decreasing function of $w^*$ that takes values on the interval $(b, b + \lambda E(w)/(\rho + \eta))$. This last result follows from the fact that when the reservation wage is less than or equal to 0, the right hand side is simply $b + \lambda E(w)/(\rho + \eta)$, since all offers are accepted, while as $w^* \to \infty$, no offers are accepted and the value of the right hand side of (6) goes to $b$. Then there exists exactly one value of $w^*$ that solves (6), though that value of $w^*$ may be negative without further restrictions on the parameters.\(^{13}\)

We now illustrate the manner in which the reservation wage can be computed. We consider an example labor market characterized by the parameter values $b = -1$, $\lambda = .2$, $\rho = .005$, $\eta = .02$, and we assume that the wage offer distribution facing the individual is lognormal with parameters $\mu$ and $\sigma$ ($> 0$).\(^{14}\) For our example we have set $\mu = 1$ and $\sigma = 1$. Since the expected value of a log normally distributed random variable is given by $E(w; \mu, \sigma) = \exp(\mu + \frac{1}{2}\sigma^2)$, in this case we have $E(w) = 4.482$.

Figure 3.1 plots the LHS and RHS of [6] as a function of $w^*$. As we know, $LHS(w^*) = w^*$, while $RHS(w^*)$ is a monotone decreasing function. In the case of

\(^{13}\)See Appendix 1 for a further discussion of this point.

\(^{14}\)The log normal density is given by

$$g(w; \mu, \sigma) = \frac{1}{w\sqrt{2\pi}\sigma} \exp\{-\frac{1}{2} \left( \frac{\ln(w) - \mu}{\sigma} \right)^2 \}.$$
our example, the two lines intersect at the point $w^* = 7.439$, which is the value which completely characterizes all rational labor market behavior in this simple model. Note that $w^*$ is appreciably greater than the mean wage offer in this model. In particular, we might ask what is the probability that a randomly generated wage offer will be accepted? This probability is equal to the mass of the probability density function $g$ to the right of the point $w^*$, or $\tilde{G}(w^*; \mu, \sigma) = \tilde{G}(7.439; 1, 1) = 0.157$. Most offers, in this case, are rejected. The reason for the “choosiness” of the searcher we see in this particular example is due partially to the relatively low rate of discounting (with $\rho$ “small” the value of waiting for a good offer increases), the relatively low rate of exogenous terminations (when $\eta$ is small it is more worthwhile to wait for a better offer since it will be kept longer, on average), and the relatively high rate of offer arrivals, $\lambda$. We now turn to a short consideration of how comparative statics exercises can be conducted in this type of model.

Let us rewrite [6] in slightly different terms as

\[
\begin{align*}
  w^* & = Q(w^*; \omega), \\
  \Rightarrow 0 & = w^* - Q(w^*; \omega)
\end{align*}
\]

where $Q$ is the RHS of [6] and $\omega$ is a vector containing all of the parameters that characterize the labor market in this model. We assume that $Q$ is a differentiable function of all of the elements of $\omega$,\(^{15}\) which means that we can totally differentiate

\(^{15}\)This is only a substantive restriction on the form of the offer distribution $G$ in this case.
[7] as follows

\[ 0 = \left( 1 - \frac{\partial Q(w^*; \omega)}{\partial w^*} \right) dw^* - \frac{\partial Q(w^*; \omega)}{\partial \omega_i} d\omega_i \]

\[ \Rightarrow \frac{dw^*}{d\omega_i} = \frac{\frac{\partial Q(w^*; \omega)}{\partial \omega_i}}{1 - \frac{\partial Q(w^*; \omega)}{\partial w^*}} \quad (8) \]

where \( \omega_i \) denotes the \( i^{th} \) element of the parameter vector \( \omega \). Because \( \partial Q(x; \omega)/\partial x \) is negative, the denominator of [8] is always positive, so that

\[ \text{sgn} \left( \frac{dw^*}{d\omega_i} \right) = \text{sgn} \left( \frac{\partial Q(w^*; \omega)}{\partial \omega_i} \right), \]

where \( \text{sgn}(X) \) denotes the sign of the expression \( X \).

For example, consider the flow cost of job search, \( b \). Since \( \partial Q(x; \omega)/\partial b = 1 \), an increase in \( b \) results in an increase in the reservation wage, a result which is intuitive. Similarly, since

\[ \frac{\partial Q(x; \omega)}{\partial \lambda} = \frac{1}{\rho + \eta} \int_{w^*}^{\infty} [w - w^*] dG(w) > 0, \]

an increase in \( \lambda \) increases the reservation wage. From inspection of [6] it is obvious that \( \partial w^*/\partial \rho \) and \( \partial w^*/\partial \eta \) are both negative. It is not difficult to demonstrate that, under the lognormality assumption regarding \( G \), \( \partial w^*/\partial \mu \) and \( \partial w^*/\partial \sigma \) are both positive.

The search model we have described is simply a dynamic model of individual choice in a stationary environment. In this case choice is limited to whether to accept an offered wage when one arrives. Such a model is inadequate for studying minimum wage effects on labor market outcomes. Imagine that a minimum wage is imposed by the government, and that the imposition of this minimum wage, \( m \), has no effect.
on any other parameters of the model. Let \( w^*(\omega) \) be the original reservation wage, which by assumption was optimally chosen by the individual given her labor market environment \( \omega \). If the minimum wage is set at a value no greater than the reservation wage, i.e., \( m \leq w^*(\omega) \), then there is no effect on choices or outcomes. If \( m > w^*(\omega) \), then clearly the individual is worse off than before. The reason is that certain wage offers which were previously acceptable, those \( w \) in the interval \([w^*(\omega), m)\), are now precluded. Since the individual was free to choose a reservation wage equal to \( m \) previously but chose not to, she cannot be better off under this law. Thus in a “partial-partial” equilibrium search model the imposition of minimum wages cannot be beneficial for labor market participants.\(^{16}\)

For minimum wage laws to possibly have beneficial effects for searchers, the imposition of a minimum wage must change the search environment in some positive way from the perspective of the searcher. For minimum wages to alter the labor market environment requires, at a minimum, a partial equilibrium model of the interaction between the supply and demand sides of the market. The bargaining framework we now describe provides an acceptable context from this point of view.

### 1.4 Nash-Bargained Employment Contracts

As opposed to the model previously considered, we assume that jobs are not fundamentally differentiated by the wage they offer to a particular worker, but rather are distinguished by the productivity of the match between a particular worker and a particular firm. The flow revenue from such a match to the firm is given by \( \theta \) (we

---

\(^{16}\)One could argue that certain modifications of the objective function of the searcher could make the reservation wage choice “sub-optimal” and lead to the possibility of welfare-improving minimum wages even with a fixed labor market environment. In models of hyperbolic discounting, for example, where the individual makes seemingly time inconsistent decisions, by limiting the agent’s behavior the government could increase the long-run welfare of the agent at the expense of her “short-run self.” For the exposition of such a model, see Della Vigna and Paserman (2005).
normalize the product price to unity without loss of generality). The instantaneous profit to the firm is given by $\theta - w$, where $w$ is the instantaneous wage payment to the employee. By defining match-specific profits in this manner it is clear that we have assumed that the only factor of production is labor and that there are no other costs of employment. These assumptions are important in the derivation of the bargaining equilibrium we provide. We will consider a slightly more general model of firm behavior below.

When a searcher meets a potential employer, the (flow) value of the match is assumed to be immediately observed. If the two parties enter into an employment contract, this contract will specify a time-invariant instantaneous wage rate $w$. It is important to realize that the productivity value $\theta$ is specific to the match and not attributable to either the worker or the firm. In that sense, both have have a valid claim to it. How is it to be divided?

There are a variety of forms of “bargaining power” that we might consider in defining the surplus division problem. First is the notion that each individual should receive at least in compensation what he or she could earn from pursuing the next best option available to them. Under the assumptions that we have made about the search technology and the labor market environment, a searcher who does not receive an acceptable job offer will continue to search. Thus the value of the next best option available to a potential employee bargaining over her share of $\theta$ is $V_U$. For the moment assume that the next best option available to a potential employer has a value of $V_V$, which denotes the value of holding onto a vacancy.

Bargaining results in a division of the flow value of the output produced. Given her wage payment $w$, the individual is indifferent regarding the actual value of the match.\footnote{This statement is not true if the model is extended to allow for on-the-job search. In this case,} In other words, the value of an employment pair $\{w, \theta\}$ to the individual
is given by $V_E(w : \theta) = V_E(w)$. It then follows that the value of the surplus of the employment contact which pays $w$ to the individual is $V_E(w) - V_U$. This difference is the rent that accrues to the worker from the employment contract.

The value of the employment match to the firm requires a little more discussion. In Pissarides’s general equilibrium model of search and bargaining in a stationary environment, which we will examine in more detail below, a firm must create a job vacancy in order to search for an employee. This vacancy is costly to hold while the firm searches for an acceptable match. Since there is a large population of potential firm owners who could create a vacancy, a free entry condition (FEC) applies in which the expected value of creating a vacancy is driven to zero. We will assume that such a condition holds, so that $V_V = 0$. This value serves as the firm’s outside option in contract negotiations with the worker.

Let the firm’s value of an employment contract in which an employee has an instantaneous output level of $\theta$ and a instantaneous wage rate of $w$ be given by $V_F(\theta, w)$. Then the solution to the generalized Nash bargaining problem is given by

$$w^* (\theta; \omega, \alpha) = \arg \max_w (V_E(w) - V_U)^\alpha (V_F(\theta, w) - 0)^{1-\alpha},$$

(9)

where $\alpha \in [0, 1]$ is termed the bargaining power parameter (in this case, it measures the bargaining power of the individual, while $1 - \alpha$ is the bargaining power of the firm), and where $\omega$ contains all of the parameters describing the labor market environment with the exception of $\alpha$. Note that $V_E(w) - V_U$ measures the gain from participating in the employment contract paying a wage of $w$ with respect to the next best alternative, which is to continue searching. As we argued in the last paragraph, the next best alternative to the firm, which is continuing to hold the vacancy open, has an associated

---

future bargains struck with either one's current employer or a future employer (met during the on-going employment spell) will be a function of one's current match value.
value of 0. As a result, the surplus of the firm is simply equal to $V_F(\theta, w)$. Note that $V_F(\theta, w) > 0$ in general, since if firms are induced to create costly vacancies they must be rewarded with some positive profits when the vacancy is filled.

To determine $V_F(\theta, w)$, we will assume that firms have the same discount rate as individuals, $\rho$. If an employment contract lasts for duration $t$, the ex post value of the contract is

$$\int_0^t (\theta - w) \exp(-\rho u) \, du = \frac{(\theta - w)}{\rho} (1 - \exp(-\rho t))$$

Since the probability density function of completed employment contracts is given by $\eta \exp(-\eta t)$, the expected value of an employment contract $\{\theta, w\}$ is given by

$$V_F(\theta, w) = \mathbb{E}_t \left( \frac{(\theta - w)}{\rho} (1 - \exp(-\rho t)) \right)$$

$$= \frac{(\theta - w)}{\rho} \int (1 - \exp(-\rho t)) \eta \exp(-\eta t) \, dt$$

$$= \frac{(\theta - w)}{\rho} \int \exp(-\rho t) \eta \exp(-\eta t) \, dt$$

$$= \frac{(\theta - w)}{\rho} \left[ 1 - \frac{\eta}{\rho + \eta} \right]$$

$$= \frac{\theta - w}{\rho + \eta}.$$ 

We use this expression for $V_F(\theta, w)$ in explicitly solving the bargaining problem.

In concluding this section, it is appropriate to say a few words about $\alpha$, a parameter which will figure prominently in the theoretical, econometric, and empirical work that follows. As we stated previously, there are essentially two aspects of bargaining advantage. One is the value of the next best option available to each of the two bargainers. The “threat point” of the firm, $V_V$, has been fixed at 0 under the FEC, while the outside option of the individual has been set at $V_U$ and is an endogenously determined value. Clearly, the larger is the value of $V_U$ the higher the wage payment.
required for the individual to enter into any given employment contract.

The second aspect of bargaining advantage is the parameter $\alpha$. When $\alpha$ is equal to 1, the individual is assumed to have all of the bargaining “power” and extracts all of the surplus from the match. The case $\alpha = 1$ corresponds exactly to the simple search problem that we considered in the previous section, for in this case the matching distribution $G$ and the wage offer distribution are identical. At the other extreme, when $\alpha = 0$, firms possess all of the bargaining power. In this case, the wage payment is independent of the match value $\theta$, so that all employees are paid the same wage. If all employees are paid the same wage, then there is no motivation for further search, and all offers will be accepted. If the value of nonparticipation is fixed at 0, for example, and assuming that the instantaneous return associated with the search state $b < 0$, then it is not difficult to show that the common wage paid all workers will be

$$\hat{w} = -\frac{(\eta + \rho)}{\lambda G(\hat{w})}b.$$ 

In this case, firms will earn instantaneous profit of $\theta - \hat{w}$ on all matches $\theta \geq \hat{w}$.

In general, the bargaining power parameter $\alpha$ is not equal to 0 or 1. It then indicates the relative “strength” of the two parties in bargaining, *conditional* on their threat points. This parameter is admittedly difficult to interpret.\(^{18}\) We think of it as constituting a type of summary statistic of the labor market “position” of a particular group. For example, the match value distribution for low-skilled workers may be stochastically dominated by the match value distribution for high-skilled workers, but low-skilled workers may be at a further disadvantage due to their having little bargaining power. Their low bargaining power may derive from there being

\(^{18}\)In a celebrated paper, Rubinstein (1982) shows how the bargaining power parameter is related to the discount rates of two players trying to divide a “pie” in a setting in which they alternate making proposals regarding the division of the surplus until one is accepted.
many substitutes for them in the production process, for example. In this sense, the parameter cannot be really thought of as “primitive” since significant policy changes - such as a large increase in the minimum wage - may result in participation effects or substitution responses by firms which change the labor market “position” of the group, and hence change the bargaining power parameter. Thus comparative statics exercises and policy experiments performed with the estimates obtained from this model will only be valid locally, that is, for small changes in policy variables.\textsuperscript{19} This limitation is not unique to this parameter or this model, since any empirically-based policy prescription based on counter-factual analysis becomes more suspect as the prescription increases in distance from previously observed choices.

1.5 The Search-Bargaining Model without Minimum Wages

We are now ready to combine the search and bargaining aspects of the model. Since $V_E(w) = \frac{w + \eta V_U}{\rho + \eta}$, the individual’s surplus with respect to the alternative of continued search is

\[
V_E(w) - V_U = \frac{w + \eta V_U}{\rho + \eta} - V_U = \frac{w - \rho V_U}{\rho + \eta},
\]

\textsuperscript{19}The same argument could be made for many of the other “primitive” parameters, it must be admitted. When discussing estimates of the equilibrium model we shall provide some evidence that the rate of contacts between searchers and firms, $\lambda$, is not invariant with respect to changes in the minimum wage.
so that the solution to the bargaining problem is given by

\[
w(\theta, V_U) = \arg \max_w \left( \frac{w - \rho V_U}{\rho + \eta} \right)^{\alpha} \left( \frac{\theta - w}{\rho + \eta} \right)^{1-\alpha}
\]

\[
= \arg \max_w \left[ w - \rho V_U \right]^\alpha \left[ \theta - w \right]^{1-\alpha}
\]

\[
= \alpha \theta + (1 - \alpha) \rho V_U
\]

\[
= \alpha \theta + (1 - \alpha) \theta^*,
\]

so that the wage is a weighted average of the match value and the reservation match value, \( \theta^* \equiv \rho V_U \).

We can now compute the value of nonemployment. Instead of writing the value of employment as a function of the wage, we write it as a function of the “primitive parameters,” \( \theta \) and \( V_U \). Then rewriting [5], we have

\[
\rho V_U = b + \lambda \int_{\rho V_U} \left[ V_E(w(\theta, V_U)) - V_U \right] dG(\theta).
\]

(10)

Since

\[
V_E(w(\theta, V_U)) = \frac{\alpha \theta + (1 - \alpha) \rho V_U + \eta V_U}{\rho + \eta}
\]

\[
= \frac{\alpha \theta - \alpha \rho V_U}{\rho + \eta} + V_U,
\]

we have

\[
V_E(w(\theta, V_U)) - V_U = \frac{\alpha \theta - \alpha \rho V_U}{\rho + \eta}.
\]

(11)

Then the final (implicit) expression for the value of search is

\[
\rho V_U = b + \frac{\lambda \alpha}{\rho + \eta} \int_{\rho V_U} \left[ \theta - \rho V_U \right] dG(\theta).
\]

(12)
We see that this expression is identical to the expression for the reservation value in a model with no bargaining when $\theta$ is the payment to the individual, except for the presence of the factor $\alpha$. This is not unexpected, since when $\alpha = 1$ the entire match value is transferred to the worker, and thus search over $\theta$ is the same as search over $w$.

Now we can summarize the important properties of the model. The critical "match" value $\theta^*$ is equal to $\rho V_U^*$, which is defined by (12). Since at this match value the wage payment is equal to $w^* \equiv w(\theta^*, V_U) = \alpha \theta^* + (1 - \alpha)\theta^*$, it follows that $w^* = \theta^*$. The probability that a random encounter generates an acceptable match is given by $\tilde{G}(\theta^*)$. The rate of leaving unemployment is $\lambda \tilde{G}(\theta^*)$. As we can see from [12], since $\theta^*$ is an increasing function of $\alpha$, the likelihood of exiting a spell of unemployment is lower the larger is the worker’s bargaining power. This is the first indication we have that lower rates of leaving unemployment do not necessarily imply that the worker’s welfare level is low.

The observed wage density is a simple mapping from the matching density. Since

$$w(\theta, V_U) = \alpha \theta + (1 - \alpha)\theta^*$$

$$\Rightarrow \tilde{\theta}(w, \theta^*) = \frac{w - (1 - \alpha)\theta^*}{\alpha},$$

where $\tilde{\theta}$ is the value of the match that corresponds to an observed wage $w$ given the critical match value of $\theta^*$. The probability density function of observed wages, $f(w)$, is given by

$$f(w) = \begin{cases} \frac{\alpha^{-1}g(\tilde{\theta}(w, \theta^*)))}{\tilde{G}(\theta^*)} & w \geq \theta^* \\ 0 & w < \theta^* \end{cases}. \quad (13)$$

**An Example**
In order to fix ideas, we present a detailed example of the computation of decision rules and the characterization of equilibrium for very basic labor market environment. We have chosen to work with simple functional forms so as to make the computational steps as clear as possible. In the empirical work that we discuss in later chapters, functional forms which produce results more in line with empirical observation will be used.

We assume that the matching distribution $G(\theta)$ is uniform, with the support of the distribution equal to the interval $[0, 10]$. Thus

$$G(\theta) = \begin{cases} 
0 & \iff \theta < 0 \\
\theta/10 & \iff 0 \leq \theta \leq 10 \\
1 & \iff 10 < \theta
\end{cases}$$

and

$$g(\theta) = \begin{cases} 
0 & \iff \theta < 0 \text{ or } 10 < \theta \\
1/10 & \iff 0 \leq \theta \leq 10
\end{cases}.$$ 

We have also assumed that $\lambda = .5$, $\eta = .02$, $\rho = .01$, and $b = -1$. These values imply that on average offers arrive to unemployed searchers every two time periods (recall that in continuous time a “period” is just a normalization that we use to express frequencies of events, and that events can occur at any time), jobs last for 50 periods on average, and searchers are “strongly” forward looking (i.e., the value of $\rho$ is close to 0). We will characterize the equilibrium of the model for two values of $\alpha$, $\alpha = .3$ (low bargaining power of searchers) and $\alpha = .6$ (high bargaining power).
Under our distributional assumption regarding \( G \), \([12]\) becomes

\[
\theta^* = b + \frac{\alpha \lambda}{\rho + \eta} \int_{\theta^*}^{10} [\theta - \theta^*] \frac{1}{10} d\theta.
\]  
(14)

\[
\Rightarrow 0 = \frac{a}{20} (\theta^*)^2 - (1 + a)\theta^* + (b + 5a).
\]  
(15)

where

\[
a \equiv \frac{\alpha \lambda}{\rho + \eta}.
\]

This is a quadratic equation in \( \theta^* \), and given the parameter values we have chosen there exists a unique solution in the interval \((0, 10)\).

The labor market equilibrium in terms of equilibrium wage functions is represented in Figure 3.2. In 3.2.a we have graphed the population density of match draws, which is equal to 0.1 on the interval \([0, 10]\). Figure 3.2.b plots the wage functions corresponding to the two values of \( \alpha \) we consider. We note that the reservation match value \( \theta^* \) is considerably higher when \( \alpha = 0.6 \). The reason is intuitive. Since the searcher gets to keep more of the surplus generated by the match, it is worth holding out for a high draw. Since the exit rate from unemployment is \( \lambda \tilde{G}(\theta^*) \), this implies that unemployment spells last longer on average when \( \alpha \) is high.

The wage density corresponding to the two values of \( \alpha \) are graphed in Figures 3.2.c and 3.2.d. Because the distribution of \( \theta \) is Uniform and because the wage function is a linear mapping from \( \theta \) to \( w \), the wage distributions are both Uniform as well. The support of the wage distribution associated with \( \alpha = 0.3 \) begins to the left of the other distribution and is more concentrated. Thus low \( \alpha \) not only reduces the average accepted wage in this example, it also reduces the variance of accepted wages.
1.6 Bargaining with a Minimum Wage Constraint

The introduction of minimum wages into the search-bargaining framework is accomplished in a very straightforward manner. We assume that the labor market environment is exactly as described above, with the exception that a “side constraint” is imposed on the worker-firm bargaining problem [9]. This constraint is that any employment contract must yield a wage payment of at least \(m\) to the worker no matter what the value of \(\theta\). The minimum wage is assumed to be set by the government and applies to all potential matches. This assumption represents the U.S. case relatively well, since the minimum wage applies to virtually all employment contracts in the labor force, with a few notable exceptions.\(^{20}\)

The modified bargaining problem we develop is then represented by

\[
 w^*(\theta, m) = \arg \max_{w \geq m} (V_E(w) - V_U(m))^{\alpha} \frac{\theta - w}{\rho + \eta} \left(1 - \alpha\right),
\]

(16)

where \(V_U(m)\) is the value of search given a minimum wage of \(m\). If we define \(\rho V_U(0) = \theta_0^*\), which is the reservation match value when there is no minimum wage, it is clear that any \(m \leq \theta_0^*\) has no effect on the behavior of applicants or firms and thus would be a meaningless, or “slack,” constraint. Therefore we consider only the effects of an imposition of \(m > \theta_0^*\).

The first thing to note concerning the effect of the constraint on behavior is that, since the value of the employment contract to the firm is proportional to \(\theta - w\), no employment contract will be formed for which the match value \(\theta < m\), since in this case the firm would lose money continuously over the course of the match. If there is no opportunity cost to the firm of forming a match, then any match for which it

\(^{20}\)As we saw in Chapter 2, the principal exceptions occur in the food preparation and serving sectors, where substantially lower minimum wages apply.
doesn’t lose money will be acceptable to it. Since \( \theta^* < m \), this implies that fewer encounters between searchers and firms will result in employment contracts, which will result in an increase in the unemployment rate, other things equal. This loss of employment effect is consistent with that predicted by the simplest static competitive labor market models. Later we will investigate the extent to which this prediction is robust with respect to alterations in modeling assumptions.

The effect of the addition of the minimum wage “side constraint” on the solution to the bargaining problem is relatively intuitive. Under the “constrained” Nash-bargaining problem, in which all employment contracts must pay at least a wage of \( m \), there will exist a value of search which we denote \( V_U(m) \). If we ignore the minimum wage constraint and solve \([16]\) using \( V_U(m) \), we will get the wage offer function

\[
\tilde{w}(\theta, V_U(m)) = \alpha \theta + (1 - \alpha)\rho V_U(m).
\] (17)

Under this division of the match value, the worker would receive a wage of \( m \) when \( \theta = \hat{\theta} \), where

\[
\hat{\theta}(m, V_U(m)) = \frac{m - (1 - \alpha)\rho V_U(m)}{\alpha}.
\]

Then if \( \hat{\theta} \leq m \), all “feasible” matches would generate wage offers at least as large as \( m \). When \( \hat{\theta} > m \), this is not the case. When \( \theta \) belongs to the set \([m, \hat{\theta})\), the offer according to \([17]\) is less than \( m \). However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of \( m \) for all \( \theta \in [m, \hat{\theta}) \). Wages for acceptable \( \theta \) outside of this set [i.e., when \( \theta \geq \hat{\theta} \)] are determined according to \([17]\).

We can now consider the individual’s search problem given this wage offer function. Using the \( \epsilon \) interval formulation, the value of search under a binding minimum wage
constraint is given by

$$V_U(m) = (1 + \rho \varepsilon)^{-1} \left[ b \varepsilon + \lambda \varepsilon \left\{ \int_m^{\hat{\theta}(m,V_U(m))} \left[ \frac{m + \eta V_U(m)}{\rho + \eta} \right] dG(\theta) \right. \right.$$

$$\left. + \int_{\hat{\theta}(m,V_U(m))}^{m} \left[ \frac{\alpha \theta + (1 - \alpha)\rho V_U(m) + \eta V_U(m)}{\rho + \eta} \right] dG(\theta) + V_U(m) G(m) \right] \right.$$

$$\left. + (1 - \lambda \varepsilon) V_U(m) + o(\varepsilon) \right]$$

$$\Rightarrow (1 + \rho \varepsilon)V_U(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_m^{\hat{\theta}(m,V_U(m))} \left[ \frac{m + \eta V_U(m)}{\rho + \eta} \right] dG(\theta) \right.$$

$$\left. + \int_{\hat{\theta}(m,V_U(m))}^{m} \left[ \frac{\alpha \theta + (1 - \alpha)\rho V_U(m) + \eta V_U(m)}{\rho + \eta} \right] dG(\theta) + V_U(m) G(m) \right] \right.$$

$$\left. + (1 - \lambda \varepsilon) V_U(m) + o(\varepsilon) \right]$$

The integrand in the first integral on the right hand side of these expressions is the value of a job at a firm where the match productivity lies in the interval $[m, \hat{\theta})$, which we know pays a wage of $m$ and has a total value of $(m + \eta V_U(m))/(\rho + \eta)$. The integrand of the second integral is the value of a job when the match value is greater than $\hat{\theta}$, and $V_U(m)$ is the value of encountering a firm where the match value is less than $m$, which we know results in no job and continued search. The probability of this event is $G(m)$.

After subtracting $V_n(m)$ from both sides, we get

$$\rho \varepsilon V_n(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_m^{\hat{\theta}(m,V_n(m))} \left[ \frac{m + \eta V_n(m)}{\rho + \eta} - V_n(m) \right] dG(\theta) \right.$$

$$\left. + \int_{\hat{\theta}(m,V_n(m))}^{m} \left[ \frac{\alpha \theta + (1 - \alpha)\rho V_n(m) + \eta V_n(m)}{\rho + \eta} - V_n(m) \right] dG(\theta) \right.$$

$$\left. + G(m)(V_n(m) - V_n(m)) + o(\varepsilon) \right]$$

$$\Rightarrow \rho \varepsilon V_n(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_m^{\hat{\theta}(m,V_n(m))} \left[ \frac{m - \rho V_n(m)}{\rho + \eta} \right] dG(\theta) \right.$$

$$\left. + \int_{\hat{\theta}(m,V_n(m))}^{m} \left[ \frac{\alpha \theta + (1 - \alpha)\rho V_n(m) - \rho V_n(m)}{\rho + \eta} \right] dG(\theta) + o(\varepsilon) \right]$$

38
After dividing both sides by $\varepsilon$ and taking limits as $\varepsilon \to 0$, we arrive at

$$\rho V_{nU}(m) = b + \frac{\lambda}{\rho + \eta} \{(m - \rho V_U(m))(G(\hat{\theta}(m, V_U(m))) - G(m))$$

$$+ \alpha \int_{\hat{\theta}(m, V_U(m))} (\theta - \rho V_U(m)) dG(\theta)\}.$$

It is important to note a fundamental difference between the value $\rho V_U(m)$, which we might want to refer to as the “implicit” reservation wage in the presence of a binding minimum wage constraint, and $\rho V_n(0)$, the corresponding “explicit” reservation wage (and reservation match) value when no minimum wage constraint is binding. The value of $\rho V_U(0)$ is an acceptance value, that is, it completely characterizes the decision of whether or not an employment contract is struck. When a binding minimum wage is present, employment contacts are formed whenever $\theta \geq m$. The value of $\rho V_U(m)$ is only instrumental in determining the equilibrium wage contract and wage distribution. Put another way, when there is no binding minimum wage constraint, the smallest possible observed wage will be equal to $\rho V_U(0)$, and the distribution of wages will be continuous as long as $G$ itself is. When there is a binding minimum wage, the smallest observed wage will be given by $m$, and in this case we know that $\rho V_U(m) < m$. The observed wage distribution will consist of a mass point at the minimum wage, the size of which is given by $(G(\hat{\theta}(m, V_U(m))) - G(m))/\tilde{G}(m)$, while the distribution of wages immediately above $m$ will be continuous (once again, as long as $G$ is). In the presence of a binding minimum wage, the observed wage distribution is given by

$$f(w) = \begin{cases} \frac{\alpha^{-1}g(\hat{\theta}(w, V_U(m)))}{\tilde{G}(m)} & w > m \\ \frac{G(\hat{\theta}(m, V_U(m))) - G(m)}{G(m)} & w = m \\ 0 & w < m \end{cases}$$
An Example (Continued)

We consider the same search environment as above and determine the impact of the imposition of a minimum wage of 7 on the equilibrium wage distribution. Recall that in the previous example we computed the wage distribution under a low and high value of \( \alpha \), .3 and .6. We noted that without a minimum wage the acceptance match value was an increasing function of \( \alpha \). In our previous example the critical match value when \( \alpha = .6 \) was equal to 6.204, so that a minimum wage of 7 is binding for both values of \( \alpha \).

As noted above, when the underlying population match distribution is continuous, the imposition of a binding minimum wage results in a mixed discrete-continuous equilibrium wage distribution in which there exists a mass point at the value \( m \) with a continuous distribution of wages beginning immediately "to the right" of \( m \). To represent a random variable \( w \) that does not have a density everywhere on its support we must plot the cumulative distribution function instead of the density function (as was done in Figure 3.2). Figure 3.3.a exhibits the c.d.f. of the match distribution. The equilibrium wage function with the binding minimum wage of 7 is shown in Figure 3.3.b. In this case, since the minimum wage is binding for both values of \( \alpha \), the lower bound of the support of both distributions is the same. However, we see that for \( \alpha = .3 \), all positive wage offers are equal to the minimum. This is due to the bargaining power of searchers being low and the minimum wage being high relative to the upper bound of the support of the distribution of \( \theta \). Instead, when \( \alpha = .6 \), there is significant clustering of wages at \( m \) though most wages observed will be greater than \( m \). The critical value we have defined as \( \hat{\theta}(m) \), which is the highest value of \( \theta \) that will yield a wage payment of \( m \), is equal to 7.557 for \( \alpha = .6 \) (as opposed to 10 when \( \alpha = .3 \)).

Figures 3.3.c and 3.3.d display the wage distributions for the two values of \( \alpha \).
The wage is a degenerate random variable in the case of \( \alpha = .3 \), that is, it assumes the constant value \( m = 7 \) for all employees. In the case of \( \alpha = .6 \), while there is a considerable mass point at \( w = 7 \), about 82 percent of all wage observations are greater than 7. The conditional distribution of wages greater than 7 is Uniform.

1.7 The Labor Market Participation Decision

At this point we will consider generalizations of the basic model. The phenomena we consider are of obvious importance when attempting to evaluate the impact of a minimum wage policy on labor market outcomes, and are not intended to make the model itself more elegant or complete. In this section, we continue with the assumption of an exogenously-determined rate of contacts between searchers on both sides of the market, but now allow individuals on the supply side of the market the power to decide whether to actively participate in it.

Individuals out of the labor market have a number of activities to pursue, including leisure, home production, and investment in human capital. We assign a flow value, \( \rho V_O \), to being out of the formal labor market for every individual on the supply side of the market. We do not study the determinants of this value, but assert that it is continuously distributed in the population, with c.d.f. \( L \) and p.d.f. \( l \).

Individuals who are out of the labor force (OLF) and choose to enter must begin their labor market activity in the unemployment state. The flow value of this state, given the possibility that a binding minimum wage is in place, is given by \( \rho V_U(m) \). Define \( d = 1 \) when the individual is a labor market participant, and let \( d = 0 \) when this is not the case. Then the decision rule of an agent with an “outside option” value
of $\xi$ is

$$d = \begin{cases} 
1 & \text{if } \rho V_O \leq \rho V_U(m) \\
0 & \text{if } \rho V_O > \rho V_U(m) 
\end{cases}$$

It follows that, under the minimum wage $m$, the labor market participation rate is

$$\ell \equiv p(d = 1|m) = L(\rho V_U(m)).$$

(18)

We see immediately from (18) the manner in which the minimum wage can affect the participation rate. A minimum wage increase, in and of itself, has no direct effect on participation. Any effect on participation will be of the same sign as the impact of the minimum wage on the value of search.

1.8 Endogeneity of the Rate of Contacts

We adopt the standard set up (in the macroeconomics literature) for modeling firms’ decisions to create vacancies. At any moment in time, any firm can create a vacancy, which is a precondition for adding a worker to its staff. Think of it as setting up the workplace in advance. One rationale for why this activity has to be accomplished in advance is that a potential employee must be evaluated in this setting to determine her match value $\theta$ at that particular job. As is standard in this literature, we assume that there exists a constant returns to scale (CRS) matching technology,

$$M(\tilde{u}, v) = vq(k),$$

where $k \equiv \tilde{u}/v$, $\tilde{u}$ is the size of the set of unemployed searchers, and $v$ is the size of the set of vacancies. The contact rates differ depending on which side of the market the agent is on. On the supply side of the market the contact rate is the average
number of matches (per unit time) per unemployed searcher, or

\[ \lambda = \frac{M(\tilde{u}, v)}{\tilde{u}} = \frac{vq(k)}{\tilde{u}} = \frac{q(k)}{k}. \]

From the vacancy holder’s point of view, the contact rate is

\[ \frac{M(\tilde{u}, v)}{v} = \frac{vq(k)}{v} = q(k). \]

Assuming that there exists a population of potential (firm) entrants with an outside option value of 0, firms create vacancies until the point that expected profits are zero. Let the flow cost of creating a vacancy be given by \( \psi > 0 \). Then the expected value of creating a vacancy is given by

\[ \rho V_V = -\psi + q(k)\tilde{G}(r)(J - V_V), \]

where \( r \) denotes the acceptance match value (which is equal to the maximum of \( \rho V_U(m) \) and \( m \)), \( q(k)\tilde{G}(r) \) is the rate at which a firm fills a vacancy (\( q(k) \) is the rate at which it meets job applicants and \( \tilde{G}(r) \) is the probability that the match value drawn is greater than the lowest acceptable value, \( r \)), and \( J \) is the expected value of a filled vacancy (where the expectation is taken with respect to the distribution of acceptable matches, which are those for which \( \theta \geq r \)). By setting \( r \) to the maximum of \( \rho V_U(m) \) and \( m \), we allow for the possibility that the minimum wage \( m \) is not binding.

To close the model, we assume that firms keep creating vacancies up to the point at
which the expected value of a vacancy is 0 - this is the free entry condition (FEC) that
was discussed in the course of deriving the Nash-bargained wage contract. Imposing
\( V_F = 0 \), we have
\[
0 = -\psi + q(k)\tilde{G}(r)J. \tag{19}
\]

We can solve for the equilibrium number of vacancies given the expected value of
a filled vacancy and the size of the set of unemployed searchers using (19) and other
pieces of the model. As we shall see in Chapter 4, in the steady state the probability
that a labor market participant is unemployed is given by
\[
p(u|m, d = 1) = \frac{\eta}{\eta + \tilde{G}(r)q(k)/k}.
\]

Given the proportion of labor market participants, \( \ell \), the size of the set of unemployed
searchers (relative to the entire population) is
\[
\tilde{u} = u\ell = \frac{\eta \ell}{\eta + \tilde{G}(r)q(\tilde{u}/v)/(\tilde{u}/v)}.
\tag{20}
\]

With endogenous contact rates, a labor market equilibrium in the presence of a
minimum wage (that may or may not be binding) is characterized by the vector of val-
ues \( (\ell, u, v, \rho V_n(m)) \), which is a solely a function of the parameters \( (\rho, b, \eta, \alpha, G, Q, q, \psi) \)
and the minimum wage rate \( m \). An equilibrium, if one exists, can be constructed
by first fixing a value of the contact rate (from the searching individual’s perspec-
tive), \( \lambda \). Let \( x \equiv \rho V_n(m) \). Then given \( \lambda \), \((??)\) determines \( x(\lambda) \). The participation
rate is then determined as \( \ell(\lambda) = L(x(\lambda)) \). From (20) we have \( \tilde{u}(\lambda) = \ell(\lambda)\eta/(\eta + \lambda\tilde{G}(\max\{x(\lambda), m\})) \). Finally, \( \tilde{u}(\lambda) \) and \( J(\lambda) \) are used with (19) to determine \( v(\lambda) \).
Then let \( T(\lambda) \equiv q(\ell(\lambda)u(\lambda); \omega)/(\ell(\lambda)u(\lambda) v(\lambda)) \). There exists a unique equilibrium if and
only if there exists a unique value $\lambda^*$ such that $\lambda^* = T(\lambda^*)$. In general, without further restrictions on the parameter space and the functional forms of $q$, $G$, and $L$, there may exist no or multiple equilibria. For example, if we restrict the outside option flow value, $\xi$, to be positive for all individuals, and if we fix all other parameters aside from $b$ at some given values for which the model is well-defined\textsuperscript{21}, then there will exist sufficiently negative values of $b$ for which no one participates in the market.

Given existence, multiple equilibria can arise depending on specific properties of the distribution functions $L$ and $G$. In performing the empirical exercises and policy simulations reported below, we assumed particular functional forms for $q$, $L$, and $G$. When performing the policy simulations, in which $m$ is varied over some range of values, we have found a number of cases of nonexistence. However, when an equilibrium existed we found it to be unique in the sense that $\lim_{n \to \infty} T^n(\lambda^0) = \lim_{n \to \infty} T^n(\lambda^0) = \lambda^* \in [\lambda_0, \lambda_0^0]$, where $\lambda_0$ and $\lambda_0^0$ are small and large starting values of $\lambda$ in the iterative updating process for finding a fixed point $\lambda^*$.\textsuperscript{22}

To illustrate some of the properties of a labor market equilibrium with and without a binding minimum wage, we present the following examples. Due to the fact that the existence of an equilibrium, and its computation, is a more challenging problem in this general equilibrium setting, we have changed some of the underlying assumptions used in the partial equilibrium settings we have considered to this point.

**Example 2** Assume that $\xi$ is normally distributed with mean 5 and variance 4 in

\textsuperscript{21} By well-defined, I mean that the distribution of match values has finite expectation, the discount rate is strictly greater than 0, etc.

\textsuperscript{22} Define the function 

$$x' = \zeta T(x) + (1 - \zeta)x,$$

which takes an “old” value $x$ into a new one, $x'$, and where $\zeta$ is some number in the unit interval. If we apply this operator repeatedly, beginning with a starting value $x_0$, then after a sufficiently large number of iterations we can achieve a difference between $x'$ and $x$ that is less than some predetermined value $\varepsilon$. The statement in the text refers to the fact that any starting value of $\lambda$ in the interval $[\lambda^0, \lambda^0]$ converges to the same value $\lambda^*$ under this successive updating procedure.
the population, and that lnθ is normally distributed with mean 5 and variance 0.25. The discount rate, in “monthly” units, is .05/12, the exogenous dismissal rate (η) is 0.038, the utility flow in unemployment (b) is -20, and the flow cost of holding vacancy (ψ) is equal to 120. The matching production function M is given by

\[ M(\tilde{u}, v) = \tilde{u}^\omega v^{1-\omega}, \]  

(21)

where the Cobb-Douglas parameter is given by ω.

In Table 3.2 we present equilibrium outcomes under these assumptions regarding the environment for various combinations of the bargaining power parameter (α), the Cobb-Douglas matching function parameter (ω), and various values of the minimum wage. The equilibrium is first solved for all nine combinations of α ∈ {.2, .4, .6} and ω ∈ {.2, .5, .8} with no minimum wage constraint imposed. We then compute equilibrium outcomes for each pair of α and ω under a binding minimum wage. We set the minimum wage to 5 except in two cases when 5 would not have been a binding constraint on the Nash bargaining problem.

We begin by considering Environment 1, in which workers have very low bargaining power (0.2) and make minimal contributions to creating matches through their search effort (ω = 0.2). With no minimum wage, the unemployment rate u, which is the proportion of labor force participants who are not employed, is a relatively low 0.039. However, the participation rate is only 0.39 in this case, and the vacancy rate is equal to 0.015. The reservation match value (θ*) is 3.442.

Environment 2 is identical to the first except for the presence of a binding minimum wage of 5. When we discuss welfare measures of minimum wage impacts in the next chapter, we will see that one important indicator of welfare for individuals on the supply side of the market is the “implicit” reservation wage, ρV_n(m). Although
individuals have a relatively low value of search in Environment 1, the imposition of a minimum wage of 5 leads to an equilibrium outcome that is worse on several dimensions. First of all, the unemployment rate almost doubles in the new steady state equilibrium. Secondly, the implicit reservation wage registers a significant decline as well. Since the participation rate $l$ is a monotone increasing function of $\rho V_n(m)$, fewer individuals are in the market. With fewer individuals searching, the number of vacancies decreases as well. Taken together, this example clearly illustrates that minimum wages may not lead to improvements in equilibrium outcomes for individuals on the supply side of the market.

Sometimes minimum wages can improve the position of searchers. This is the case in Environment 5, where searchers have very little bargaining power ($\alpha = .2$) but make a large contribution to match formation ($\omega = 0.8$). The imposition of the binding minimum wage (Environment 6) results in a small increase in the unemployment rate, but large increases in the value of search and the participation rate in the economy. The vacancy creation rate is unaffected. Under the welfare metrics we shall propose in Chapter 4, the welfare of searchers is increased under this policy, in spite of the increase in the unemployment rate. The value of unemployed search is an important determinant of all of the welfare measures we will consider. Since the participation rate is an increasing function of the value of search under our simple specification of the participation decision, while the unemployment rate is not, a quick check of the impact of any minimum wage change on the welfare of searchers should involve a comparison of participation rates, not unemployment rates.

By comparing the appropriate rows, we see numerous examples in which the imposition of a minimum wage, at the arbitrary level chosen, results in increases in the value of unemployed search and the labor market participation rate. There exists one case, exhibited in Environments 15 and 16, in which the imposition of a minimum
wage of 5 results in an increase in the participation rate and a decrease in the unemployment rate due to a nonnegligible increase in vacancy creation. In Chapter 4 we will provide a discussion of the conditions under which a binding minimum wage can benefit the entire economy.

1.9 On-the-Job (OTJ) Search

A large number of jobs are terminated by the employee moving directly from her current employer to another, that is, with no intervening spell of unemployment. The model that we feature in this monograph does not allow for such transitions, and thus misses this important aspect of observed labor market behavior. Our justification is that the OTJ search model quickly becomes very complicated to work with, and that added complexity may obscure some of the basic points we wish to make. For example, allowing for worker-firm bargaining necessitates making some controversial assumptions regarding the nature of the competition between employers when “bidding” for the services of a given individual. A more pragmatic rationale for ignoring this phenomenon is that the cross-sectional nature of the CPS data all but precludes the estimation of models with complex dynamic structures, which is the case when OTJ search is added.\textsuperscript{23} While we will not feature OTJ search in this monograph, it is important for the reader to have some notion of how some of our results would be impacted if it were to be included. For this reason, we summarize the basic modeling approach taken by Flinn and Mabli (2006).

For simplicity, we will assume that the contact rates between employers and unemployed searchers are fixed at rate $\lambda_U$, while the rate of contacts between currently

\textsuperscript{23}Technically speaking, a model of OTJ search can be estimated from point sampled data like the CPS, but functional form assumptions on the matching distribution $G$ play an increasingly vital role in model identification. For this reason, in their estimation of a minimum wage model with OTJ search, Flinn and Mabli (2006) use panel data drawn from the Survey of Income and Program Participation (SIPP).
employed individuals and other, potential employers is fixed at rate $\lambda_E$.\textsuperscript{24} It is common to assume, and empirical results support the assumption, that $\lambda_U \gg \lambda_E$.\textsuperscript{25} In the absence of a binding minimum wage, there will exist a match value $\theta^*$ such that all unemployed searchers accept a match $\theta \geq \theta^*$, though this value will vary as a function of $\lambda_E$.\textsuperscript{26} When there is a binding minimum wage, all match values greater than or equal to $m$ result in employment.

An employed individual, who has a current match value of $\theta$ and wage of $w$, may leave her current match either because she obtains a better offer, or due to the match ending exogenously. If she meets a potential employer, a match draw $\theta'$ is drawn from the distribution $G$. The modeling assumptions used in Flinn and Mabli produce efficient mobility in the sense that the most productive match of the pair ($\theta, \theta'$) is always accepted. In this case, conditional on a current match value, $\theta$, the rate of exit from the job is simply given by

$$h(\theta) = \eta + \lambda_e \tilde{G}(\theta),$$

that is, the total hazard rate out of a job with a match value of $\theta$ is simply the sum of the independent destination-specific hazard rates, with $\eta$ the rate of transiting into unemployment and $\lambda_e \tilde{G}(\theta)$ the rate of leaving for a better job. The latter hazard rate is the product of the rate of making contacts with a new firm when employed and the

\textsuperscript{24}Contact rates between firms and unemployed and employed searchers are determined endogenously in Flinn and Mabli (2006).

\textsuperscript{25}In their seminal model of OTJ search in a partial-partial equilibrium setting, Burdett and Mortensen (1978) allow individuals to choose search activity, labor supply, and leisure in a continuous-time framework. Instantaneous utility is defined over leisure and consumption, and rates of offer arrivals are determined endogenously. In their framework, time devoted to search is a decreasing function of the current wage, thus producing lower offer arrival rates for employed workers than unemployed searchers.

\textsuperscript{26}We have already examined the case of $\lambda_E = 0$, which is no OTJ search. In general, the value of $\theta^*$ is a decreasing function of $\lambda_E$. This is relatively intuitive, since the opportunity cost of accepting a low match value is reduced when the individual is not precluded from receiving better offers while employed.

49
probability that the alternative match value exceeds the current one. Obviously, the higher is one’s current match value, the lower is the total hazard $h(\theta)$. It also follows that the likelihood that an exogenous separation was the cause of an employee leaving a match of type $\theta$ is an increasing function of $\theta$.

While the labor market dynamics are straightforward to describe, the evolution of wages is not. The bargaining model with competing employers has been considered by Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006), though a minimum wage constraint of the type we consider was not present in any of these papers. We follow Dey and Flinn, as do Flinn and Mabli (2006), in their approach to the bargaining problem, since the matching structure they used is the same as ours.

We assume the following information and bargaining structure. When an individual, currently paid a wage $w$ which is less than or equal to her current firm-specific productivity $\theta$, meets a new firm, the value of her productivity there is immediately revealed to be $\theta'$. This productivity value is observed by both the new firm and the searcher. At this point negotiations begin, and a series of credible offers are communicated between the two firms and the individual. Negotiations continue until one of the firms drops out of the bidding. This final offer is used as the individual’s outside option in further negotiations with the winning firm, with the final wage being determined using a Nash bargaining rule. Assuming that the new match value $\theta' > \theta$, under these assumptions the new firm will employ the worker, and her wage payment will be

$$w(\theta', \theta) = \arg \max_w (V_E(w, \theta') - V_E(\theta, \theta))^\alpha V_F(w, \theta')^{1-\alpha}.$$ 

The function $V_E(w, \theta)$ denotes the value of a job with match value $\theta$ with a wage $w$, as before. The outside option to the searcher under our bargaining structure is
$V_E(\theta, \theta)$, since the firm at which the match value is $\theta$ bids the entire surplus of the match (i.e., $w = \theta$) as a wage in a futile attempt to retain the worker.\footnote{Recall that firms earn an expected value of 0 on any unfilled vacancy. The firm is willing to give away increasing portions of its surplus since any nonnegative remaining surplus puts it in as good as a position as it would have with an unfilled vacancy.}

Even when a contact with a new firm does not lead to a move, an employed worker may through the process of renegotiation. As before, let the new match be $\theta'$ and the current match $\theta$, though now assume that $\theta' < \theta$. The current wage of the individual is determined from the wage function $w(\theta, \tilde{\theta})$, where $\tilde{\theta}$ is the previous best option the agent had while employed at her current firm. Then as long as $\theta' > \tilde{\theta}$, the individual will receive a wage increase, or

$$w(\theta, \theta') > w(\theta, \tilde{\theta}) \text{ for } \theta > \theta' > \tilde{\theta}.$$  \hfill (22)

The renegotiation model implies that the wage process of an individual at a given firm will be strictly increasing over the course of her tenure at a given firm, if her wage changes at all during her tenure.

However, as originally noted in Postel-Vinay and Robin (2002), the wage function need not be increasing over the course of an employment spell in this type of model. The reason is that the wage function is not necessarily monotone in the sense that, if an individual moves to match value $\theta'$ from match value $\theta$, where $\theta' > \theta$, it is possible that $w(\theta', \theta) < w(\theta, \tilde{\theta})$, where $\tilde{\theta}$ was the ‘outside option at the current match $\theta$ (so that $\theta \geq \tilde{\theta}$). Even though the individual is working at a more productive match value than she had previously (since $\theta' > \theta$), and even though her outside option match value is higher than the previous one (since $\theta > \tilde{\theta}$), her wage may actually be lower. The reason this possibility exists lies in the future bargaining advantage a high match value gives a worker. Essentially, an employment match generates two forms
of remuneration for an employee. First, and most obvious, is the wage payment she
receives. Second, is the bargaining advantage associated with a high match value.
Any future employer, encountered during the current employment spell, with whom
the agent has a match value greater than $\theta'$, will have to guarantee her a contract
value at least as large as $V_E(\theta', \theta')$. Since $V_E(x, x)$ is monotonically increasing in $x$, so
is her future payoff at alternative employers. Even without getting a match offer that
exceeds $\theta'$, she can expect to get increasingly large portions of the surplus generated
by $\theta'$ as alternative match offers arrive that are less than $\theta'$ but exceed the current
outside option.

Depending on the values of the parameters characterizing the labor market en-
vironment ($\omega$), the equilibrium wage function may exhibit nonmonotonicities of the
type just described. If individuals possess all of the bargaining power, so that $\alpha = 1$,
then $w(\theta', \theta) = \theta'$ for all $\theta' \geq \theta$, and the wage function is always monotonically in-
creasing across jobs in the same employment spell. If $\alpha = 0$, which was the case
considered by Postel-Vinay and Robin (2002), then one can potentially observe situ-
ations in which improvements in match quality are associated with wage decreases,
depending, in particular, on the shape of the matching distribution $G$, the rate of
arrival of job offers while employed, $\lambda_E$, and the exogenous separation rate, $\eta$. The
intuition for this result is that an agent with little (Nash) bargaining power derives
all of their effective bargaining power from their outside option. If the individual is
to receive relatively little of the surplus from a new match, and if the match gives the
individual a high option value when bargaining against future employers, the wage
must, of necessity, be low. In some cases, the option value associated with the new
match will be sufficiently high that the wage portion of compensation will be less
than that which was received at the job from which the individual moves. If $\lambda_E = 0$,
the case considered throughout most of the monograph, then match values have no
“option value” with regard to future bargaining, and hence increased compensation is expressed in terms of increased wages. The same hold true as $\eta$, the rate of exogenous separation, grows indefinitely large, for then the likelihood of meeting a new potential employer before entering the next unemployment spell tends to zero.

The impact of a minimum wage change on labor market and welfare outcomes in the case of OTJ search depends, as in the case of no OTJ search, on the primitive parameters that characterize the market environment. With OTJ search, minimum wage outcomes are a bit more complicated to describe. We will use Figure 3.4 to aid in the discussion.

Figure 3.4 contains a representation of a segment of an individual’s labor market history. The top portion of the figure contains the wage history that is observed, while the bottom portion contains the job history. The first spell on the time line is an (ongoing) unemployment spell. The first arrow on the bottom indicates the time at which the individual transitted from the unemployment spell into an employment spell. An employment spell is a sequence of job spells with no intervening unemployment spells. The figure contains the timing of two job spells, the second of which is on-going, and the top of the figure indicates the timing of wage changes. The wage $w_1$ is the first wage paid at job 1, and $w_3$ is the first wage paid at job 2. According to the model, wages $w_2$, $w_4$, and $w_5$, are renegotiated wages at the two job spells. Under the model, wages would only possibly be renegotiated when a new match possibility is encountered which is not better than the current match but is better than the outside option match value under which the current wage has been negotiated.

As we saw above, when there is no OTJ search, all new jobs are accepted out of the unemployment state, meaning that all wages are determined as a function of the identical outside option. This, together with the fact that the wage is the only mechanism through which an individual can be compensated, implies that wages
are nondecreasing functions of the match value. This then yields the implication that, in the presence of a binding minimum wage, only the lowest acceptable match values produce a wage equal to the minimum. In the case of OTJ search, the outside option that a minimum wage is partially determined by varies across jobs, for those jobs immediately preceded by another. It is possible for the same job match value, \( \theta \), to produce a minimum wage in some cases but not others, depending on the value of the outside option. For example, consider a very large match value \( \theta \), and another low, but acceptable, value \( \theta' \). When the outside option is equal to \( \theta \), then we know that the wage rate is simply \( \theta = w(\theta, \theta) \), no matter what the value of the primitive parameters characterizing the market or whether or not a minimum wage is present. However, we may have \( m = w(\theta, \theta'; m) \), where the new \( w \) function is one generated under the minimum wage constraint. Thus, whether a current match pays the minimum wage depends on the match value and the outside option. Low match values may not necessarily generate minimum wages, whereas high match values may. The probability that an individual is paid the minimum wage is defined over the set of \( (\theta, \theta', m) \), not simply \( (\theta; m) \), which was the case with no OTJ search. Describing this two-dimensional set (over \( \theta \) and \( \theta' \), given \( m \)) is considerably more challenging numerically.

While wages may increase or decrease when an individual moves across firms, they can only move up within the same firm. The basic logic behind this result is that a given match value \( \theta \) always generates the same future bargaining value for an employee. When wage contracts are renegotiated to give the employee a larger part of the surplus, the only way to do so is to increase the wage rate. When a binding minimum wage is considered, this implies that only the first wage at a job can be equal to the minimum, with all future wages being strictly greater than \( m \).

With these results in mind, we can now consider the possible path of wage pay-
ments in Figure 3.4. We know, for example, that $w_2 > w_1$, and that $w_5 > w_4 > w_3$. We also know, that under the implication of efficient mobility decisions, the match value at the job $j_2$, exceeds the match value at job $j_1$, or $\theta_{j_2} > \theta_{j_1}$. It also follows that the initial wage paid in job $j_1$, $w_1$, is equal to or less than $m$, so that $w_2$ is strictly greater than $m$. The first wage paid at job $j_2$, $w_3$, can be greater than, equal to, or less than $w_2$, the last wage paid at the previous job. In the case of a lower wage, it is possible that $w_3 = m$ (depending on $\theta_{j_2}, \theta_{j_1}$, and the parameters describing the labor market environment). Generally speaking, over the course of an employment spell, it is possible (under the model) to see an individual paid the minimum wage at several different points, and it is possible to observe wage decreases. However, wage decreases or several episodes of minimum wage compensation can only be observed if the individual changes employer at least once over the course of the employment spell.

As this brief description makes clear, characterizing the impact of minimum wages on labor market outcomes in considerably more challenging when $\lambda_E > 0$. Nonetheless, the same basic framework for evaluating the effect of minimum wage changes on the welfare of individuals and firms that we develop in the sequel is still applicable. Readers desiring more information on the case of OTJ search are referred to Flinn and Mabli (2006).
Table 3.1
The Beginning of a Hypothetical Labor Market Career

<table>
<thead>
<tr>
<th>Event Number</th>
<th>State</th>
<th>Time of Event</th>
<th>Match Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$U$</td>
<td>.891</td>
<td>6.243</td>
</tr>
<tr>
<td>2</td>
<td>$U$</td>
<td>3.168</td>
<td>4.329</td>
</tr>
<tr>
<td>3</td>
<td>$U$</td>
<td>15.554</td>
<td>3.871</td>
</tr>
<tr>
<td>4</td>
<td>$U$</td>
<td>15.558</td>
<td>10.918</td>
</tr>
<tr>
<td>5</td>
<td>$E$</td>
<td>38.921</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$U$</td>
<td>44.236</td>
<td>7.891</td>
</tr>
<tr>
<td>7</td>
<td>$U$</td>
<td>56.793</td>
<td>12.119</td>
</tr>
<tr>
<td>8</td>
<td>$E$</td>
<td>157.421</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$U$</td>
<td>164.772</td>
<td>10.145</td>
</tr>
<tr>
<td>10</td>
<td>$E$</td>
<td>322.510</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.2
Equilibrium Outcomes with Endogenous Contact Rates

<table>
<thead>
<tr>
<th>Environment</th>
<th>$\alpha$</th>
<th>$\omega$</th>
<th>$m$</th>
<th>$u$</th>
<th>$l$</th>
<th>$v$</th>
<th>$\rho V_n(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
<td>0.039</td>
<td>0.390</td>
<td>0.015</td>
<td>3.442</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.2</td>
<td>5</td>
<td>0.076</td>
<td>0.313</td>
<td>0.012</td>
<td>3.027</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.5</td>
<td>0</td>
<td>0.036</td>
<td>0.492</td>
<td>0.023</td>
<td>3.962</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.5</td>
<td>5</td>
<td>0.048</td>
<td>0.509</td>
<td>0.023</td>
<td>4.046</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.8</td>
<td>0</td>
<td>0.037</td>
<td>0.472</td>
<td>0.028</td>
<td>3.861</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>0.8</td>
<td>5</td>
<td>0.045</td>
<td>0.533</td>
<td>0.028</td>
<td>4.168</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>0.2</td>
<td>0</td>
<td>0.081</td>
<td>0.298</td>
<td>0.009</td>
<td>2.940</td>
</tr>
<tr>
<td>8</td>
<td>0.4</td>
<td>0.2</td>
<td>5</td>
<td>0.117</td>
<td>0.217</td>
<td>0.007</td>
<td>2.438</td>
</tr>
<tr>
<td>9</td>
<td>0.4</td>
<td>0.5</td>
<td>0</td>
<td>0.064</td>
<td>0.639</td>
<td>0.019</td>
<td>4.710</td>
</tr>
<tr>
<td>10</td>
<td>0.4</td>
<td>0.5</td>
<td>5</td>
<td>0.065</td>
<td>0.666</td>
<td>0.021</td>
<td>4.857</td>
</tr>
<tr>
<td>11</td>
<td>0.4</td>
<td>0.8</td>
<td>0</td>
<td>0.059</td>
<td>0.797</td>
<td>0.023</td>
<td>5.659</td>
</tr>
<tr>
<td>12</td>
<td>0.4</td>
<td>0.8</td>
<td>6</td>
<td>0.062</td>
<td>0.805</td>
<td>0.026</td>
<td>5.716</td>
</tr>
<tr>
<td>13</td>
<td>0.6</td>
<td>0.2</td>
<td>0</td>
<td>0.146</td>
<td>0.110</td>
<td>0.002</td>
<td>1.545</td>
</tr>
<tr>
<td>14</td>
<td>0.6</td>
<td>0.2</td>
<td>5</td>
<td>0.189</td>
<td>0.070</td>
<td>0.002</td>
<td>1.053</td>
</tr>
<tr>
<td>15</td>
<td>0.6</td>
<td>0.5</td>
<td>0</td>
<td>0.094</td>
<td>0.639</td>
<td>0.012</td>
<td>4.710</td>
</tr>
<tr>
<td>16</td>
<td>0.6</td>
<td>0.5</td>
<td>5</td>
<td>0.092</td>
<td>0.684</td>
<td>0.015</td>
<td>4.956</td>
</tr>
<tr>
<td>17</td>
<td>0.6</td>
<td>0.8</td>
<td>0</td>
<td>0.080</td>
<td>0.899</td>
<td>0.015</td>
<td>6.550</td>
</tr>
<tr>
<td>18</td>
<td>0.6</td>
<td>0.8</td>
<td>7</td>
<td>0.084</td>
<td>0.911</td>
<td>0.020</td>
<td>6.690</td>
</tr>
</tbody>
</table>