I develop and estimate an equilibrium model of the labor market in which inefficient employees are systematically eliminated from the sector of the market characterized by asymmetric information and moral hazard. Systematic selection on the distribution of productivity characteristics produces wage sequences that are increasing in tenure for employees never previously terminated even in the absence of long-term contracting between employees and individual firms. I provide sufficient conditions for there to exist a unique termination-contract type of equilibrium, and I estimate the equilibrium model using microlevel data from the National Longitudinal Survey of Youth panel.

KEY WORDS: Efficiency wages; Measurement error; Panel data; Structural estimation.

In this article I propose a new behavioral interpretation of the concavity of age–earnings profiles observed in virtually all cross-sectional and panel datasets. In the model set forth here, experienced workers are paid more than less experienced workers due to the operation of a dynamic selection process. Over time, incompetent individuals are discovered and dismissed by firms operating in a sector of the economy in which output is imperfectly observed and rents to employees are assumed to be 0. The intertemporal selection process on the skill distribution of employees in the sector with imperfectly observed output results in the wages of these workers being bid up over time. I derive conditions under which a unique labor-market equilibrium exists in which the wages paid employees in the sector in which output is imperfectly observed are consistent with their decisions regarding the supply of effort on the job.

From an empirical perspective, the model proposed here differs from others capable of generating concave age–earnings profiles in its dependence on a selection process that involves the dismissal of employees. For example, models of human-capital accumulation of either the general or specific kind (e.g., see Mincer 1974; Becker 1975) rely on assumptions concerning the production function of human capital to produce concave age–earnings profiles rather than turnover patterns. In fact, human-capital explanations of earnings growth imply little about turnover processes, other than that the probability of separation is a decreasing function of the stock of specific human capital (e.g., Oi 1962; Parsons 1972).

The empirical implications of models of worker–firm productivity matching (e.g., Johnson 1978; Jovanovic 1979; Miller 1984; Flinn 1986; McCall 1991) most closely resemble the implications for observed wage-turnover processes of the model exposited here. The prototypical matching model in which workers and firms incrementally learn the value of their match-specific productivity parameter over time and in which draws are independently and identically distributed across all worker–firm pairings generates upward-sloping age–earnings profiles. This result is due to the systematic elimination of “bad” matches so that the probability of being in a such a match declines with age. The standard matching model carries the implication that in an expectational sense productivity is constant over the course of each match, though (random) fluctuations in productivity will occur. In contrast, in my selection model productivity varies in a systematic manner within matches as employees change their effort-supply decisions. For empirical purposes, an advantage of my model is that wages are completely determined by expected productivity in each period, as opposed to most matching frameworks in which some mechanism determining how match-specific rents are allocated must be introduced (see Mortensen 1978, 1982). A disadvantage of the current formulation of the model vis-à-vis matching models is the implication that there would be no variation in wages for individuals of the same labor-market age working in the same sector of the economy.

Within the matching framework, much has been made of the fact that separations are efficient (given costless renegotiation of contracts) so that there exists no behavioral distinction between employee- and firm-initiated separations. This is in marked contrast to my model, in which only firm-initiated separations matter in the sense of changing the choice-set and future utility flows of individuals who leave a match. Although it is notoriously difficult to empirically distinguish employee- and firm-initiated separations, empirical results seem to indicate that employees reporting that they were involuntarily separated from their previous employers have lower wages on their next jobs than observationally equivalent employees reporting a voluntary separation (e.g., Bartel and Borjas 1981; Gibbons and Katz 1990). Furthermore, Ruhm (1991) and Jacobson, LaLonde, and Sullivan (1993) found that individuals experiencing an “exogenous” displacement (due to a plant closing, for example) spend slightly more time in unemployment for a few years following the separation than those who do not and also suffer substantial earnings losses that persist many years after the event. One might reasonably expect that an employee dismissed for cause would not fare better in the
labor market than one who experiences an exogenous separation, so post-separation earnings outcomes observed for exogenously separated employees may serve as an upper bound for those who experience dismissals for cause. This leads one to expect long-term effects of dismissals on earnings.

Though the model developed here is predicated on the existence of moral hazard in employment relationships, the fact that age-earnings profiles are upward sloping is not a result of long-term contracting between employees and individual firms in the primary sector of the economy [as in the models of Becker and Stigler (1974) and Lazear (1979, 1981)]. Long-term contracts that promise higher wages in the future conditional on satisfactory performance require firm compliance that is impossible to generate within the market and institutional structures assumed here. Because of this, primary-sector firms offer employees a sequence of one-period contracts conditional on satisfactory performance. I show that favorable selection (from firms' perspectives) on employee types is required to support an increasing wage equilibrium within the competitive market structure considered.

The model developed here builds on that of Shapiro and Stiglitz (1984) in several ways [as did the related models of Bulow and Summers (1986) and Albrecht and Vroman (1992)], though the focus of my analysis is substantially different. First, in my model there is no unemployment, which is central to the model of Shapiro and Stiglitz. The incentive not to shirk is provided by the chance of permanent reputation loss, whereas for Shapiro and Stiglitz the cost of dismissal was associated with the location of a new employer, the difficulty of which is indexed by the unemployment rate. These differences in the modeling of the labor market imply that that dismissal had only transitory effects in the model of Shapiro and Stiglitz (were it to occur), whereas the effects are permanent in the model I analyze.

Second, I consider the case in which employees are heterogeneous with respect to a characteristic unobservable by the firm and that enters the employee's decision rule regarding the supply of effort on the job. Shapiro and Stiglitz considered the case of homogeneous employees, as is true in most models of moral hazard applied to the labor market. The problem with such a formulation from an empirical perspective is that, generally speaking, in equilibrium no employees will shirk and hence no dismissals will be observed [an exception was shown by Flinn (1993), in which equilibrium dismissals occur even with homogeneous employees due to a stochastic production process; such a model generates dismissals and transitory effects of dismissals on earnings when firms use punishment strategies to discipline employees]. The advantage of my formulation of the problem is that in equilibrium wages and separation rates are both determined within the model, with separations occurring for labor-market participants at all (finite) experience levels with positive probability.

A model somewhat similar in spirit to the one developed here was developed by MacLeod and Malcomson (1988). Moral hazard and heterogeneous agents (whose type is private information) are prominently featured in both models. The key difference between the two formulations is the specification of the production process (continuous in effort in MacLeod and Malcomson's article and discrete here) and the information set of the employer. Even though the distribution of types is continuous, MacLeod and Malcomson derived an equilibrium in which the (representative) firm sorts employees into a finite number of ranks and periodically readjusts their positions in the hierarchy according to a fixed promotion rule that is based on output realizations. Their promotion rule results in only promotions in equilibrium; demotions (analogous to my dismissals) would not be observed. Thus, although their model is able to generate a "fuller" age-specific distribution of earnings than is ours, it yields the counterfactual implication of no dismissals in equilibrium.

The plan of the article is as follows. In Section 1, the optimization problem of employees in the primary market is described and their decision rule derived regarding the amount of effort to supply on the job given the termination contracts they face. Section 2 contains a description of the problem facing firms operating in the primary sector of the economy (in which moral hazard is present) and provides sufficient conditions for there to exist a unique equilibrium termination contract in the class of such contracts that specify that wages be increasing in tenure. In Section 3 I develop an econometric model that is used to estimate the primitive parameters that characterize the equilibrium. In Section 4, data from the National Longitudinal Survey of Youth (NLSY) are used to provide some descriptive evidence on the intertemporal relationship between wages and dismissals and to estimate the equilibrium model. The section concludes with some comparative statics exercises. Section 5 contains a brief conclusion.

1. CHARACTERIZATION OF THE PRODUCTION PROCESS AND THE EMPLOYEE'S PROBLEM

I shall consider the labor-market experiences of a cohort of participants, in which the cohort is defined in terms of its year of entry into the market (assumed exogenous). Cohort members are differentiated solely in terms of their productive ability in what I shall refer to as the "primary" sector of the economy (a completely equivalent model could be formulated in which cohort members were equally efficient in production but differed in their disutility of effort). All cohort members share the following lifetime welfare function:

$$W(\{w_s, e_s\}_{s=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^{t-1} \{m(w_t) - e_t\},$$

where $w_t$ denotes the wage payment received at time $t$, $e_t$ is the effort expended on the job at time $t$, $\beta$ is a discount factor that takes values in the open unit interval, and $m$ is a monotone increasing function. Note from the outset that both employees and employers are assumed to be risk neutral [in terms of monetary units $m(w)$] so that insurance motives will play no role in the labor-market equilibrium to be derived. Without loss of generality, I shall posit the
model assuming that \( m(x) = x \) [I shall set \( m(x) \equiv \ln(x) \) in the empirical work conducted later].

The economy consists of two sectors. In the primary sector, effort expended on the job is only imperfectly observable by the firm. In the secondary sector, effort is extracted from individuals as part of the production process (i.e., effort is not a choice variable for employees in this sector); furthermore, the level of effort extracted when employed in the secondary sector is the same for all individuals. Within the secondary sector, the onerousness of the production task is equal to the price of output, which is assumed to be time invariant. With free entry of firms, bidding for employees will produce a secondary sector wage \([w^s]\) equal to output price. Therefore, all firms in this sector will earn profits of 0 and all employees will obtain utility flows of 0 each period.

Because in the equilibrium I shall describe all primary sector employees earn rents, it follows that all cohort members and all employees will obtain utility flows of 0 each period. Only dismissal from the primary sector will lead employees to accept employment in the secondary sector, which consequently should be viewed as a punishment.

All individuals employed in the primary sector at age \( t \) have a choice of whether or not to supply an amount of effort sufficient to produce a unit of output in the period. The type \( \xi \) of a labor-market participant will be interpreted as his/her index of productive inefficiency in the primary sector. The production function for an individual of type \( \xi \) in period \( t \) is given by

\[
y(\epsilon_t; \xi) = \begin{cases} 
1 & \text{if } \epsilon_t \geq \xi \\
0 & \text{if } \epsilon_t < \xi.
\end{cases}
\]  

(2)

Thus, if an individual of type \( \xi \) supplies at least that much effort in period \( t \), one unit of output is produced with probability 1; if he/she supplies less than that much effort, a unit of output is produced with probability 0.

The labor-market choices of an individual of age \( t \) are only a function of the agent's observable labor-market history at time \( t \), which is generated in the following manner. In each period in which he/she participates in the primary labor market, he/she is monitored with a constant probability \( \pi \in (0, 1) \). If he/she is monitored in any period and found to have produced no output, he/she is assigned to the secondary sector in perpetuity. Thus, if he/she has ever been caught "shirking" in a period prior to \( t \), his/her probability of being employed in the primary sector in period \( t \) is 0.

I shall assume that employers know whether applicants have ever been dismissed "for cause." In some specifications of the econometric model to be estimated, I shall allow for the simultaneous existence of dismissals as a result of malfeasance on a given employee's part and for reasons not related to the performance of a particular employee (such as for decreases in demand for the employer's output). In this case, I assume that potential employers can distinguish between these two types of dismissal on the basis of the individual's publicly available employment history.

To simplify the form of labor-market contracts and to preclude the possibility of certain types of strategic behavior on the part of employees, I assume that the only information available to potential employers is whether an individual has ever been detected shirking. In particular, I assume that information on successfully passed monitoring events is available to no potential employer, not even the one at which the monitoring took place. This is a strong assumption and is imposed for reasons of tractability; extending the model to allow for this type of learning would be an important generalization.

I now describe the optimal sequence of effort choices under the termination contract using a dynamic programming formulation of the agent’s problem. First, note that, because effort yields disutility, no individual of type \( \xi \) will supply more effort than \( \xi \) in any period. Similarly, because the outcome of not producing a unit of output and being monitored is independent of the amount of effort (less than \( \xi \)) supplied, any individual deciding not to produce a unit of output in the primary sector will choose to supply an effort level of 0. Then the age \( t \) problem of a primary-sector participant is

\[
V_t(\xi) = \max\{w_t - \xi + \beta V_{t+1}(\xi); w_t + \beta(1 - \pi)V_{t+1}(\xi)\},
\]  

(3)

where the first argument in the max operator corresponds to the value of working and the second argument corresponds to the expected value of shirking.

It is straightforward to show that, as long as the wage sequence is bounded, the decision rules of primary sector employees will possess a critical value property in all periods. The sequence of critical values, however, is in general an exceedingly complicated function of the wage sequence. For reasons of tractability, I shall be interested in determining the form of the critical value rules for the case in which wages are monotonically increasing over the life cycle. I shall restrict attention to equilibria that exhibit this property as well.

Assuming that wages are strictly positive will serve to rule out the trivial equilibrium in which primary-sector wages and output levels are set equal to 0 each period. The condition that the wage sequence be increasing provides the opportunity to greatly simplify the computation of the critical values because an individual who is indifferent between shirking and working in period \( t \) will supply effort in all future periods due to the increased attractiveness of the sequence of primary-sector wages he/she will face. The simple form of the critical values is specified in the following proposition (I merely state most propositions and results due to space limitations; formal proofs are available from me on request):

**Proposition 1.** If primary-sector wages are positive and monotonically increasing over the life cycle, the decision rule for an agent of type \( \xi \) employed in the primary sector in period \( t \) is

\[
\text{Work if } \xi \leq \xi_t, \\
\text{Shirk if } \xi > \xi_t,
\]  

(4)

where \( \xi_t = MQ_{t+1}, M \equiv (\beta(1 - \beta)\pi)/(1 - \beta(1 - \pi)), \) and \( Q_t \equiv \sum_{s=t}^{\infty} \beta^{s-t}w_s. \)
From (4) it follows immediately that an increase in the monitoring rate increases the critical effort \( \xi_t \) in each period because \( \partial M / \partial m > 0 \). It is also not difficult to demonstrate that each critical value \( \xi_t, t \in \mathbb{N} \), is increasing in the discount factor \( \beta \).

2. FIRM BEHAVIOR AND THE DETERMINATION OF EQUILIBRIUM WAGES

In developing the effort rules used by primary-sector employees, I characterized secondary-sector firms as competitive in the input market and as price-takers in the output market. I shall also view primary-sector firms as competitive in the input market. Without loss of generality, I can think of a firm beginning its “life” with a large group of employees all of whom are beginning their labor-market careers. For simplicity, think of the new firm as hiring a continuum of such individuals. Then I shall associate this set of cohort members with the unit interval and assign subsets of cohort members the Lebesgue measure. Let there be a large but finite number of firms operating in the primary sector each period. In equilibrium, there will be no incentive for primary-sector employees to change primary-sector firms, so I can think of the set of employees of the firm in period \( t \) of its existence as the set it initially employed minus the set discovered shirking in periods 1 through \( t - 1 \). Then the objective of the representative firm can be written as

\[
W_f(\{w_\ast\}_{\mathbb{N}}) = \sum_{t=1}^{\infty} \beta^{t-1} p_t(\xi_t(\{w_\ast\}_{\mathbb{N}}) - w_t),
\]

where the representative firm’s discount factor \( \beta \) is assumed identical to that of its employees, \( \rho \) is product price, the dependence of the period \( t \) critical value \( \xi_t \) on all future wages in the primary sector has been made explicit (see Proposition 1), and where \( p_t \) is the “proportion” of the firm’s original group of employees that has not been fired by the beginning of period \( t \). Note that profits are expressed per capita in terms of the number of employees with which the firm begins its life; this is an inconsequential normalization because, in equilibrium, firms earn zero profits in every period. The expected output of a randomly selected (remaining) employee of the firm’s in period \( t \) is given by \( H_t(\xi_t) \), where \( H_t \) is the cumulative distribution function of types in period \( t \). The distribution function changes over time due to the systematic selection on types induced by termination contracts.

In building the equilibrium, I assume that (a) firms can freely enter and exit the market, (b) product price is determined exogenously, and (c) firms are unable to issue credible multiple-period contracts. Due to free entry and exit, primary-sector firms will earn zero profits in equilibrium and so are no better off than their secondary-sector colleagues. The preclusion of long-term contracts is justifiable in the absence of agents or mechanisms that could ensure firm compliance with employment contracts defined over more than one period. If multiple-period contracts are not enforceable, then the (expected) zero profit condition implied by free entry is strengthened to the condition of expected zero profit in each period of operation for each primary-sector firm.

Given that primary-sector firms earn zero profits in each period, by (5) the wage in period \( t \) is

\[
w_t = \rho H_t(\xi_t(\{w_\ast\}_{\mathbb{N}}) - w_t).
\]

It is apparent that the period \( t \) wage depends on all future wages through the critical value function \( \xi_t \). As I shall describe, the period \( t \) wage in fact depends on the entire wage sequence \( w_2, w_3, \ldots \) through the distribution function \( H_t \).

One final assumption concerns the distribution of types the firm faces. I assume that, within a cohort of individuals entering the labor market, primary-sector productive inefficiency, \( \xi_t \), is continuously distributed on \( \mathbb{R}_+ \). The cumulative distribution function of \( \xi_t \) is given by \( H_t \). \( H \) is assumed to be second-order differentiable and concave. Thus the density of \( \xi_t \) is assumed to exist and to be negatively sloped everywhere on \( \mathbb{R}_+ \). The assumption of differentiability is helpful in the proof establishing existence and uniqueness of primary-sector equilibrium and seems a practical necessity in any empirical application of the model (due to the unobservability of individual effort expenditures by primary-sector employees). The concavity assumption is strong, but many distributions commonly used in empirical analysis are contained in this class (e.g., the exponential and those created by truncating mean-zero normal, \( t \), and logistic distributions from below at 0).

Having completed a statement of the model structure, I am now ready to consider the properties of primary-sector equilibrium. I begin with the following property, which any equilibrium wage sequence must exhibit:

Result 1. In the class of monotone equilibria, every equilibrium wage sequence \( \{w_\ast\}_{\mathbb{N}} \) is convergent with limit point \( \bar{w} \in [0, \rho] \).

Recall that monotone equilibria are those in which the wage sequence in the primary sector increases in labor-market age. Now, because all employees of a given age are paid their expected revenue product, the wage paid is bounded from above by \( \rho \) under the production technology (2). Then any equilibrium wage sequence is bounded from above by \( \rho \) and from below by 0. Because the set of employees with productive inefficiency exactly equal to 0 has measure 0, the limiting wage must be strictly greater than 0.

As in any matching model (i.e., matching “competent” individuals to the primary sector), the process of selection is what produces all the interesting dynamics. Under the dismissal process described previously, a proportion \( \pi \) of all shirkers of labor-market age \( t \) are dismissed. Age-\( t \) individuals in the primary sector who shirk are those whose productive inefficiency is greater than \( \xi_t \). Thus, the upper tail of the productive inefficiency distribution is thinned each period, with the interval subject to thinning changing with the critical value \( \xi_t \). When the sequence \( \{w_\ast\}_{\mathbb{N}} \) is increasing, by (4) the sequence \( \{\xi_t\} \) is also increasing. When the sequence \( \{\xi_t\} \) is increasing, the thinning process produces a sequence of distributions that can be characterized in terms
of the initial productive inefficiency distribution \( H \) and the monitoring rate \( \pi \) as follows:

\[
H_t(\xi_t) = 1 - \tilde{H}(\xi_t) A_t(\{\xi_s\}_{s=1}^{t-1}),
\]

where

\[
A_t(\{\xi_s\}_{s=1}^{t-1}) = \left\{ 1 + \sum_{s=1}^{t-1} \frac{\pi}{1-\pi} H(\xi_s) \right\}^{-1}
\]

and \( \tilde{H} \) denotes the survivor function \( 1 - H \). To illustrate the selection phenomenon, I present Figure 1, which contains the density of productive inefficiency in the primary sector during the first four periods of the labor market. The parameters used to generate these densities are point estimates taken from the first column of Table 5, Section 4.

The sequence of distribution functions \( H_1, H_2, \ldots \) display an ordering property important in establishing the existence of equilibria over the set of increasing wage sequences.

**Result 2.** For any increasing sequence of values \( 0 < \xi_1 \leq \xi_2 \leq \cdots \leq \xi_t \) and \( s < t \), \( H_t \prec SD H_s \), where \( \prec SD \) denotes the first-order stochastic dominance operator.

From Result 2, we know that, even if the wage sequence was constant so that \( \xi_1 = \xi_2 = \ldots \), the probability of a randomly selected employee in the primary sector producing a unit of output in period \( t \) is an increasing function of \( t \); in this statistical sense, primary-sector employees become "more productive" as they gain experience. This increased productivity results in increased wage payments in a competitive labor market with spot contracts and supports the increasing wage sequences to which I am limiting my attention.

Equilibria in my model are formally specified as the fixed points of the operator

\[
T(\{w_s\}) \equiv \begin{bmatrix}
\rho H_1(\xi_1(\{w_s\}_{s=1}^{\infty})) \\
\rho H_2(\xi_2(\{w_s\}_{s=2}^{\infty})) \\
\vdots \\
\rho H_t(\xi_t(\{w_s\}_{s=t+1}^{\infty})) \\
\end{bmatrix}.
\]

Recall that, although the system appears recursive in that the critical value \( \xi_t \) is a function only of wage payments at time \( s + 1, s + 2, \ldots \), the distribution functions \( H_2, H_3, \ldots \) are functions of the wage sequence \( \{w_s\}_{s=2}^{\infty} \). For example, the distribution function of \( \xi_1 \) in the population of primary-sector employees in the second period, \( H_2 \), is a function of \( \xi_1, \pi, \) and \( H \). The probability that a primary-sector employee supplies effort in the second period is \( H_t \); but do not determine the effort supplies of period \( t \) primary-sector employees. Wages in periods \( t + 1, t + 2, \ldots \) have both compositional and direct effects in that they (partially) determine the distribution function \( H_t \) and completely characterize the effort supply decisions of period \( t \) primary-sector employees.

I first establish an important property that any solution to (8) must exhibit and that strengthens Result 1.

**Proposition 2.** For any equilibrium wage sequence, \( \tilde{w}^* = \rho \).

By inspection of the distribution function given in (7), it is clear that for any sequence of critical values the equilibrium wage in period \( t < \infty \) is strictly less than the product price; this is because for finite \( t \) a subset of positive measure of each primary-sector firm’s set of employees shirks. Only in the limit does this set of employees have measure 0; therefore, in the limit each primary-sector employee produces a unit of output with probability 1 and therefore is paid his/her expected revenue product of \( \rho \).

My main theoretical result is the following:

**Proposition 3.** There exists a unique sequence \( \{w_s^*\} \) such that \( \{w_s^*\} = T(\{w_s^*\}) \).

**Proof.** See Appendix A.

The uniqueness of the equilibrium wage sequence in the primary sector is of particular importance to our goal of empirical implementation of the model. Moreover, the fact that the operator \( T \) is differentiable in the primitive parameters of the model leads to an equilibrium wage sequence that is similarly differentiable. These differentiability properties greatly facilitate estimation of the model, as is shown in Section 3.

The unique equilibrium described in Proposition 3 is a pooling equilibrium in which only one contract is offered to all labor-market entrants. It is natural to ask whether or not separating equilibria also exist. In the context of this model, a separating equilibrium is one in which labor-
market entrants are offered different contracts that specify wage payments as a function of experience conditional on not having been dismissed previously, in which each contract is strictly preferred to all others by at least one type (ξ) of agent, and in which each contract earns zero profits each period for the primary-sector firm offering it. It is not difficult to show formally that such an equilibrium cannot exist in this model. The intuition behind the result is that, because only wage sequences differ across contracts, any contract that earns zero profits in each period and that attracts the best (i.e., most efficient) employees will pay the highest wage in every period and therefore will be preferred by employees of all types.

To this point I have merely asserted that individuals dismissed from the primary sector at any point in their labor-market careers would never be rehired by a primary-sector firm. I now demonstrate that this is an equilibrium outcome in the following specific sense. Given that a primary-sector firm has dismissed a set of employees who were determined not to have produced output in some previous period under the terms of the original termination contract, it has no incentive to offer such individuals a new termination contract of a similar form.

Proposition 4. The termination contract offered primary-sector employees is renegotiation-proof.

The basic intuition for this result is clear. Primary-sector firms make zero profits on the "best" labor-market participants—that is, those not previously dismissed. Moral-hazard considerations prevent primary-sector firms from reducing the compensation of previously dismissed employees, so primary-sector firms employing such individuals must earn negative profits. The renegotiation-proofness of these termination contracts seems to be critically dependent on the competitive-labor-markets assumption.

3. ESTIMATION OF THE WAGE-DISSMISSAL PROCESS

The stochastic process for wages and dismissals at the individual level is straightforward to construct given the equilibrium wage sequence in the primary sector, which is determined by the discount factor β, the time-invariant monitoring rate π, the heterogeneity distribution H, the product price ρ, and finally the sequence of wages paid in the secondary sector of the economy, which I have set equal to \( w_s \) in each period.

In mapping the behavioral model into an estimable econometric model, provisions must be made for measurement error in both the log wage and dismissal sequences. I assume that the log wage rate in sector \( s \) at time \( t \) is given by

\[
\ln w_t^s = \mu_t^s + \epsilon_t,
\]

where \( \mu_t^p = w_t^p \) when \( s = p \) (i.e., the wage draw is from the primary sector) and \( \mu_t^s = w_t^s \) when \( s = s \) (i.e., the wage draw is from the secondary sector) and where \( \epsilon_t \) is an iid normal random variable with mean 0 and standard deviation \( \sigma_\epsilon \).

It is a practical necessity to allow for measurement error in reported dismissals because under the model only one dismissal for malfeasance may take place, but 7% of my sample reports two or more dismissals. For this reason I cannot directly map reported dismissals into the dismissals described in my model. Although adding measurement error to continuous random variables, such as the wage rates discussed previously, is conventionally done, allowing for measurement error in discrete random variables is less standard. Because inferences drawn from the model may be quite sensitive to the manner in which this is done, I have introduced measurement error into the reported dismissal sequence in three conceptually distinct ways.

The first approach to the problem is a standard measurement error one in which the reasons for accurate or inaccurate reports of the "true" dismissal state (under the model) are not explicitly considered. Let \( d_t^* \) denote the true dismissal outcome during period \( t \), where \( d_t^* = 1 \) if a dismissal for malfeasance occurred in the interval \([t, t+1)\) and \( d_t^* = 0 \) if one did not. I have assumed that

\[
p(d_t = j | d_t^* = j) = \lambda_j \in [0,1], \quad j = \{0,1\},
\]

where \( d_t \) is the reported dismissal outcome during period \( t \), with \( d_t = 1 \) if the agent reported a dismissal occurring in the period \([t, t+1)\) and equal to 0 otherwise.

The second approach is simply to associate the first dismissal reported with a dismissal "for cause" from the primary sector. Under this interpretation, no dismissals not for cause occur in the primary sector. On the other hand, dismissals for reasons of demand fluctuations do occur in the secondary sector but have no implications for the welfare of any agent relegated to that sector (i.e., they are assumed to be able to immediately find a new job in that sector with another firm). In computing the likelihood function, only the timing of the first dismissal will enter.

The last approach explicitly allows for "exogenous" dismissals in both the primary and secondary sector. Let \( \lambda_p \) and \( \lambda_s \) denote the probabilities of exogenous primary- and secondary-sector dismissals. As before, assume that all individuals begin their labor-market careers in primary-sector jobs. While in the primary sector, the individual is monitored with probability \( \pi \) each period and dismissed for cause if found not to have produced output. Conditional on not being dismissed for cause, the employee is dismissed for reasons exogenous to his/her behavior with probability \( \lambda_p \). When the agent has been relegated to the secondary sector, he/she faces a dismissal probability equal to \( \lambda_s \) each period.

The data used to estimate the model consist of \( \ln \) wage observations for the first \( T \) years of labor-market experience for a sample of \( n \) individuals, along with the set of self-reported dismissal indicator variables \( d_1, \ldots, d_{T-1} \). The sampling frequency is one year, and I adopt the convention that the decision frequency is also one year.

With these assumptions regarding the measurement-error processes, I can now define the likelihood contribution of sample member \( i \). Because the measurement-error processes in the \( \ln \) wage and dismissal sequences are indepen-
dent, I have
\[ L_i = \sum_{d^* \in D^*} g(\ln w_i, d_i | d^*) p(d^*), \]
\[ = \sum_{d^* \in D^*} g_1(\ln w_i | d^*) g_2(d_i | d^*) p(d^*), \] (11)
where the set \( D^* \) contains all latent dismissal sequences with positive probability, \( g \) is the joint density of \( \ln \) wages and observed dismissal outcomes conditional on the true dismissal sequence, \( g_1 \) is the conditional density of \( \ln \) wages given the true dismissal sequence, and \( g_2 \) is the conditional probability function of observed dismissals given the true dismissal sequence.

In terms of the model, recall that the secondary sector of the economy is an absorbing state and that in this state no dismissal with informational value can take place. Over \( T \) periods, “true” dismissals can take place at most once. Given the fact that the critical-effort-level sequence is strictly increasing over time,
\[ p(\sum_{t=1}^{T-1} d^* = 0) = H(\xi_1) + (1 - \pi)[H(\xi_2) - H(\xi_1)] + \cdots + (1 - \pi)^{T-2} \times [H(\xi_{T-1}) - H(\xi_{T-2})] + (1 - \pi)^{T-1}[1 - H(\xi_{T-1})], \]
\[ p(d^*_i = 1) = \pi(1 - \pi)^{t-1}[1 - H(\xi_j)], j = 1, \ldots, T - 1. \] (12)
This probability distribution is determined by all the behavioral parameters of the model; that is, the critical-effort-level sequence \( \{\xi_s\} \) is a function of the parameters \( \beta, \rho, H, \) and \( \pi. \)

The distributions \( g_1 \) and \( g_2 \) are easily specified. The \( \ln \) wage sequence over the first \( T \) periods in the labor market conditional on the true dismissal sequence is
\[ g_1(\ln w_1 | \sum_{t=1}^{T-1} d^*_t = 0) = \varepsilon - T \prod_{t=1}^{T} \phi((\ln w_{it} - w_t^*)/\varepsilon), \]
\[ g_1(\ln w_t | d^*_j = 1) = \varepsilon - T \prod_{t=1}^{T} \phi((\ln w_{it} - w_t^*)/\varepsilon) \times \prod_{t=j+1}^{T} \phi((\ln w_{it} - w^s)/\varepsilon), \]
\[ j = 1, \ldots, T - 1, \] (13)
where \( \ln w_{it} \) denotes the \( \ln \) wage of sample member \( i \) in period \( t \) and \( \phi \) denotes the probability density function of a standard normal random variable.

Now consider the probability of any reported dismissal sequence given the true dismissal (for cause) sequence \( d^*. \) In estimating the model I use three alternative constructions of this conditional probability distribution, as was discussed previously. When the pure measurement-error model (10) is estimated,
\[ g_2(d_i | d^*) = \prod_{t=1}^{T} \chi[d_{it} = 1, d^*_i = 1] \lambda_1 \]
\[ + \chi[d_{it} = 0, d^*_i = 1](1 - \lambda_1) \]
\[ + \chi[d_{it} = 1, d^*_i = 0](1 - \lambda_0) \]
\[ + \chi[d_{it} = 0, d^*_i = 0] \lambda_0. \] (14a)
The second alternative, in which the first reported dismissal is assumed to be for cause, produces the following conditional reported dismissal probability distribution:
\[ g_2(d_i | d^*) = \chi[d_{i1} = 1, d^*_i = 0] \]
\[ \cdots \chi[d_{iT-1} = 0, d^*_{iT-1} = 0] \chi[d_{iT} = 1, d^*_i = 1]. \] (14b)
Note in (14b) that reported dismissal in periods following the primary sector dismissal (period \( t \) in this case) are not modeled.

The third alternative representation of the reported dismissal sequence in which primary- and secondary-sector exogenous dismissals are explicitly included is given by
\[ g_2(d_i | d^*) = \prod_{t=1}^{3} \chi[d_{it} = 1, t < t^*] (1 - \lambda_p) \chi[d_{it} = 0, t < t^*] \times \chi[d_{iT} = 1, t > t^*]. \] (14c)
where \( t^* \) denotes the period in which the dismissal “for cause” occurs (i.e., \( d^*_i = 1 \) given that one is observed during the first three periods of labor-market participation.

The log-likelihood for the sample, given independence of the measurement-error processes across individuals, is simply \( L(\theta) = \sum \ln(L_i), \) where the parameter vector \( \theta \) includes \( \beta, \pi, H, \rho, \) and \( \varepsilon. \) In proving uniqueness of any nontrivial primary-sector equilibrium, I relied heavily on the assumption of differentiability and concavity of the heterogeneity distribution function \( H. \) To ensure compliance with this assumption, I have estimated the model assuming a parametric form for \( H, \) \( \mu(\cdot, \alpha), \) in which the assumption is satisfied for all \( \alpha \in \Theta_H, \) where \( \Theta_H \) denotes the parameter space for the distribution \( H. \) In particular, the estimates reported here are obtained under the assumption that the heterogeneity distribution is half-normal so that the cumulative distribution function of \( \xi \) is given by \( H(\xi; \alpha) = 2[\Phi(\xi/\alpha) - 0.5], \) where \( \Phi \) denotes the cdf of a standard normal random variable. I also estimated versions of the model under the assumption that \( \varepsilon \) was exponentially distributed but found that estimates of parameters other than \( \alpha \) were relatively insensitive to this change.

Maximum likelihood estimates of model parameters were obtained through the use of a modified method of scoring algorithm using numerical derivatives. At each itera-
tation, current values of the behavioral parameters were used to compute the equilibrium termination contract wage sequence and corresponding critical value sequence. The algorithm used in this step of the estimation process is described in Appendix B. Given the differentiability of the equilibrium termination contract with respect to the primitive parameters, it is straightforward to demonstrate that the maximum likelihood (ML) estimator is consistent and asymptotically efficient, though the likelihood function has not been shown to be globally concave and no initial consistent estimator is available for $\theta$. I can, however, report that the algorithm converged to the same point in the parameter space when started from a variety of initial values for $\theta$.

### 4. EMPIRICAL RESULTS

As was mentioned in the introduction, the sample used in the empirical work was drawn from the NLSY, which is (for the most part) a nationally representative sample of 12,686 individuals who were 14–21 years of age in 1979. These individuals have been interviewed on an annual basis; currently, 13 waves of information are available from interviews conducted in the years 1979–1991.

In defining my subsample, several stringent criteria were imposed. To avoid problems of intermittent labor-market participation, only males were included. I attempted to minimize initial condition problems by requiring each sample member to be engaged in full-time schooling and not in the labor market in one of the years 1979–1987 and then to be employed at the time of the next four consecutive interviews; this precluded several older individuals who had left school by 1979 from sample membership. The employment condition at the time of the interviews was imposed because no rationale for unemployment is included in the model; although it might be expected to produce a nonrandom sample, the demographic profile of the final sample does not markedly change when this criterion is dropped. Only cases with complete information on wages and reasons for job separations were eligible for inclusion in the sample. Finally, only individuals from the random-sample component of the survey were included. The final sample with which I work includes 361 individuals.

In constructing the dismissal sequence, I examined the reported reason for all job changes occurring over each sample period (i.e., between successive interview dates, which were approximately one year apart). If an agent reported that any job held during the period between the interviews ended due to a “dismissal” or a “layoff,” the individual was considered to have reported a dismissal from the primary sector during that period (no more explicit reason for the job separation is ascertained). Though this definition of dismissals is admittedly somewhat arbitrary, it is probably the most accurate one available using the data at hand. We will see that the dismissal sequence defined in this way negatively impacts In wages, as one would hope to observe. Moreover, the crudeness of the definition is mitigated to some extent by the allowance for measurement error in these self-reports.

The sample is described in Table 1, where the average and the standard deviations of In hourly wages are given by period for each of the eight possible dismissal sequences. All wages were first expressed in terms of 1980 dollars. In terms of the occurrence of dismissals, more than two-thirds of this sample report no dismissal or layoff over the three-year period, though one must keep in mind that the restriction that all sample members be employed at the time of each interview almost surely leads to downward-biased estimates of population dismissal rates. One dismissal was reported by 23.8% of the sample over the entire period, 6.1% reported two, and .8% reported three. Of those individuals reporting one or two dismissals, dismissal experiences are largely concentrated in the first two years of labor-market participation.

For the group that experienced no dismissals over the sample period, average In wages increase in regular increments; in period 4, average In hourly wages are 12.9% greater than their first-period level. Across subgroups in which one or more dismissals occur, decreases in average In wage rates are much more likely to occur when a dismissal is reported than when one is not. Although there are several exceptions to this pattern, many occur in cells with very few observations. The group that reports dismissals in every period (1-1-1) seems to also begin their labor-market careers with very low wages, though not too much should be made of any result based on data for three individuals.

In terms of the mean starting wages in all other groups, the model implication that starting wages are drawn from the same distribution does not seem too objectionable.

In Table 2 I examine the effect of the individual’s history on In wages and dismissals. Columns (1)–(6) of the table report the results of regressing the individual’s In wage on his/her dismissal history and in some cases on previous In wages as well. The last six columns report the results of regressing the dismissal indicator variable on the dismissal history and in some cases also previous In wages. I have estimated the regression functions using ordinary least squares and have reported the Eicker–White heteroscedasticity-consistent standard errors that are
Joint test of history 

binary variable indicating dismissal between periods 1 and 2 in the population.

previous In wage realizations, I hope to capture individ-

ual heterogeneity in an admittedly crude way). The abso-

lute size of the coefficient is larger when the first period In wage is not included, which may indicate that the dismissal history also captures permanent individual differences in In wage levels.

Regressions of third and fourth period In wage rates on the dismissal history reveal similar patterns when I condition on the In wage history and when I do not. When I do not condition on previous In wages, dismissal in the previous period always had the strongest (negative) effect, and when I condition on previous wages, prior-period dismissal is the only dismissal coefficient significantly different from 0. Conditional on the In wage history, joint tests of the dismissal history coefficients always reject the null of no In-wage-history effect on second-period dismissals. In terms of the probability of a third-period dismissal, I find that second-period dismissals are important determinants. This remains true even when conditioning on the In wage history [col. (12)]. The In wage history is not a jointly significant determinant of a third-period dismissal.

The results in Table 2 are broadly consistent with the implications of the model in that dismissals are found to have rather large negative effects on subsequent In wage realizations (even conditional on the In wage history), but the probability of a dismissal is weakly related to previous In wage realizations and very strongly related to the dismis-

sals. In terms of the probability of a third-period dis-

missal, I find that second-period dismissals are important determinants of third-period dismissals [col. (5) of Table 2] is expli-

cable if a dismissal “for cause” in period 2 leads to relega-

tion to the secondary sector in period 3 and there exist high rates of “exogenous” dismissals in that sector. Generally speaking, the rather intricate timing patterns implied by a dynamic equilibrium model are very difficult to reproduce in an unstructured descriptive analysis. To evaluate the rea-

sonableness of the equilibrium model requires estimation of the model and assessing fit using several different metrics. I now turn to a discussion of the structural estimation exercises.

The regressions of the second-period In wage rate on the binary variable indicating dismissal between periods 1 and 2 reveal significant negative effects of this experience whether or not I condition on the period-1 In wage rate (by using previous In wage realizations, I hope to capture individual heterogeneity in an admittedly crude way). The absolute size of the coefficient is larger when the first period In wage is not included, which may indicate that the dismissal history also captures permanent individual differences in In wage levels.

Regressions of third and fourth period In wage rates on the dismissal history reveal similar patterns when I condition on the In wage history and when I do not. When I do not condition on previous In wages, dismissal in the previous period always had the strongest (negative) effect, and when I condition on previous wages, prior-period dismissal is the only dismissal coefficient significantly different from 0. Conditional on the In wage history, joint tests of the dismissal history coefficients always reject the null of no influence at conventional significance levels.

In the last six columns of Table 2, I present descriptive evidence concerning the relationship between the dismissal and In wage history and the probability of dismissal in each period using a linear probability framework. Column (7) indicates that 15% of my sample is dismissed during period 1. The results in column (8) indicate that higher first-period In wages are associated with a lower probability of first-period dismissal, though the coefficient estimate is not significantly different from 0. From column (9) we learn that individuals experiencing a dismissal in period 1 are more likely to experience a second-period dismissal, though the coefficient estimate is not much larger than its standard error. In the next column, I condition on the In wage history and find that a joint significance test would not lead to the rejection of the null of no In-wage-history effect on second-period dismissals. In terms of the probability of a third-period dismissal, I find that second-period dismissals are important determinants. This remains true even when conditioning on the In wage history [col. (12)]. The In wage history is not a jointly significant determinant of a third-period dismissal.

The results in Table 2 are broadly consistent with the implications of the model in that dismissals are found to have rather large negative effects on subsequent In wage realizations (even conditional on the In wage history), but the probability of a dismissal is weakly related to previous In wage realizations and very strongly related to the dismissal history. Although these descriptive results are suggestive, only a limited amount of information concerning the structure of the model generating the data can be inferred without imposing further structure. For example, the fact that second-period dismissals are significant determinants of third-period dismissals [col. (5) of Table 2] is expli-
cable if a dismissal “for cause” in period 2 leads to relega-
tion to the secondary sector in period 3 and there exist high rates of “exogenous” dismissals in that sector. Generally speaking, the rather intricate timing patterns implied by a dynamic equilibrium model are very difficult to reproduce in an unstructured descriptive analysis. To evaluate the rea-

sonableness of the equilibrium model requires estimation of the model and assessing fit using several different metrics. I now turn to a discussion of the structural estimation exercises.

Tables 3–5 contain the ML estimates of the structural model under the three specifications of the probability distribution of reported dismissal sequences conditional on the true “for cause” dismissal sequence. Before discussing the results, two comments are in order. First, to avoid potential numerical problems, in all estimated equilibrium models I have fixed the value of the discount factor at .95. Although it is theoretically possible to separately identify the dis-
count factor, I found that in practice it was exceedingly

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>( \ln(w_2) )</th>
<th>( \ln(w_3) )</th>
<th>( \ln(w_4) )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.941</td>
<td>.777</td>
<td>2.033</td>
<td>.427</td>
<td>2.104</td>
<td>.150</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>-1.162</td>
<td>-1.122</td>
<td>-1.133</td>
<td>-0.38</td>
<td>0.955</td>
<td>0.080</td>
</tr>
<tr>
<td>df</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Prob. under ( H_0 )</td>
<td>0.007</td>
<td>0.032</td>
<td>0.134</td>
<td>0.281</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Joint test of history of other variable |
| Test statistic: |
| df     | 10.011 | 8.791 | 4.023 | 3.823 |
| Prob. under \( H_0 \) | 0.007 | 0.032 | 0.134 | 0.281 |
difficult to do so. By performing a grid search over a variety of values of $\beta$, I found that .95 (or even a slightly larger value) was consistent with the maximization of the likelihood function for most of the specifications to be reported. Second, to determine the extent to which individuals from different schooling groups differed in terms of labor-market characteristics, I reestimated the structural model using high (COL) and low (HS) schooling subsamples. The low-schooling sample includes all individuals with 12 or fewer completed years of schooling. In Table 3, I present estimates of the model for the case in which the measurement-error process for reported dismissals is given by (14a). The estimates in column (1) are derived under the assumption that all sample members are employed in the same labor market and draw values of $\xi$ from the same distribution. The estimated monitoring rate is .866, indicating that shirkers are rapidly detected. High monitoring rates imply high growth rates early in the labor-market career and low dismissal rates after the first few years. The relatively large amount of dispersion in the distribution $H$ implies that many individuals are dismissed in the first period, in this case about 19%. This figure is close to the actual sample proportion reporting a dismissal between periods 1 and 2, which is 15%, though the estimates of the dismissal measurement-error parameters $\lambda_0$ and $\lambda_1$ indicate a substantial amount of misreporting of dismissal states, particularly with respect to the nonreporting of a “true” dismissal. The secondary sector In wage estimate of 1.649 is less than three-fourths of the limiting value of the primary sector In wage, which is 2.281.

In columns (2) and (3), I present the results of estimating the model separately for high- and low-schooling-level populations. I find the estimated monitoring rate for the high-schooling group to be much larger than for the low-schooling group. In the context of this model, this result is readily explicable. It is well known from the empirical-earnings-function literature that wage growth during the initial years of labor-market participation is an increasing function of years of schooling completed. Rapid wage growth in this model is produced by large amounts of selection (i.e., dismissals), and the monitoring rate is a key determinant of the dismissal rate. The fact that highly educated individuals are more likely to be relegated to the secondary sector is not necessarily disturbing given that the estimated “dismissal” wage is much higher for the COL sample than for the HS sample. I also note that the limiting wage for the COL group is 14% higher than the limiting wage for the HS group.

Table 4 contains estimates of the model when the first reported dismissal is assumed to coincide with the dismissal for malfeasance (14b). All subsequent reported dismissals are assumed to be generated by an unspecified dismissal mechanism that operates in the secondary sector and that has no effect on welfare outcomes while in that sector (this assumption is necessary if the critical-effort level defined in Proposition 1 is to remain valid); thus no parameters associated with the reported dismissal sequence are estimated. Under this specification of the reported dismissal sequence, the estimated monitoring rate is lower and product price and secondary sector wages are higher than in the measurement-error model estimates reported in Table 3. These estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ALL</th>
<th>HS</th>
<th>COL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>.866</td>
<td>.516</td>
<td>.828</td>
</tr>
<tr>
<td>$\alpha(H)$</td>
<td>1.745</td>
<td>1.521</td>
<td>1.662</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2.281</td>
<td>2.106</td>
<td>2.403</td>
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<tr>
<td>$w^*$</td>
<td>1.649</td>
<td>1.517</td>
<td>1.754</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>.347</td>
<td>.313</td>
<td>.354</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>.889</td>
<td>.878</td>
<td>.909</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>.233</td>
<td>.330</td>
<td>.290</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>-1,144.045</td>
<td>-523.406</td>
<td>-492.388</td>
</tr>
</tbody>
</table>

Table 4. Maximum Likelihood Estimates Under the Assumption That the First Reported Dismissal is for Malfeasance for the Entire Sample and by Schooling Group [asymptotic standard errors in brackets]

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>HS</th>
<th>COL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>.360</td>
<td>.324</td>
<td>.420</td>
</tr>
<tr>
<td>$\alpha(H)$</td>
<td>1.707</td>
<td>1.650</td>
<td>1.655</td>
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<tr>
<td>$\rho$</td>
<td>2.340</td>
<td>2.210</td>
<td>2.444</td>
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<tr>
<td>$w^*$</td>
<td>1.858</td>
<td>1.756</td>
<td>2.011</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>.418</td>
<td>.354</td>
<td>.417</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>-1,142.043</td>
<td>-515.708</td>
<td>-494.085</td>
</tr>
</tbody>
</table>

Table 5. Maximum Likelihood Estimates of Sector-Specific Exogenous Dismissal Probability Model for the Entire Sample and by Schooling Group [asymptotic standard errors in brackets]

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>COL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>.300</td>
<td>.229</td>
<td>.337</td>
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<tr>
<td>$\alpha(H)$</td>
<td>1.513</td>
<td>1.470</td>
<td>1.546</td>
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<tr>
<td>$\rho$</td>
<td>2.304</td>
<td>2.218</td>
<td>2.452</td>
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<tr>
<td>$w^*$</td>
<td>1.635</td>
<td>1.526</td>
<td>1.749</td>
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<tr>
<td>$\sigma_e$</td>
<td>.397</td>
<td>.340</td>
<td>.400</td>
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<tr>
<td>$\lambda_0$</td>
<td>.069</td>
<td>.094</td>
<td>.054</td>
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<tr>
<td>$\lambda_1$</td>
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<td>.223</td>
<td>.238</td>
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<tr>
<td>$\mathcal{L}$</td>
<td>-1,179.160</td>
<td>-531.650</td>
<td>-507.133</td>
</tr>
</tbody>
</table>
Table 6. Observed and Predicted Ln Wage Sequences, Primary-Sector Proportions, and Dismissal Rates for Entire Sample and by Schooling Group [standard deviations in brackets]

<table>
<thead>
<tr>
<th>Dist. type</th>
<th>ln(w1)</th>
<th>ln(w2)</th>
<th>ln(w3)</th>
<th>ln(w4)</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entire sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary</td>
<td>1.830</td>
<td>2.175</td>
<td>2.272</td>
<td>2.296</td>
<td>.938</td>
<td>.896</td>
<td>.866</td>
</tr>
<tr>
<td>Marginal</td>
<td>1.830</td>
<td>2.142</td>
<td>2.206</td>
<td>2.207</td>
<td>.131</td>
<td>.121</td>
<td>.115</td>
</tr>
<tr>
<td>Marginal</td>
<td>1.854</td>
<td>1.917</td>
<td>1.990</td>
<td>2.050</td>
<td>.150</td>
<td>.141</td>
<td>.094</td>
</tr>
<tr>
<td>(observed)</td>
<td>[.427]</td>
<td>[.394]</td>
<td>[.435]</td>
<td>[.427]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>HS sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary</td>
<td>1.691</td>
<td>1.795</td>
<td>1.882</td>
<td>1.952</td>
<td>.923</td>
<td>.882</td>
<td>.851</td>
</tr>
<tr>
<td>Marginal</td>
<td>1.691</td>
<td>1.774</td>
<td>1.840</td>
<td>1.889</td>
<td>.171</td>
<td>.145</td>
<td>.140</td>
</tr>
<tr>
<td>Marginal</td>
<td>1.703</td>
<td>1.774</td>
<td>1.827</td>
<td>1.902</td>
<td>.151</td>
<td>.161</td>
<td>.116</td>
</tr>
<tr>
<td>(observed)</td>
<td>[.329]</td>
<td>[.319]</td>
<td>[.375]</td>
<td>[.388]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>COL sample</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Primary</td>
<td>2.016</td>
<td>2.149</td>
<td>2.244</td>
<td>2.311</td>
<td>.940</td>
<td>.901</td>
<td>.875</td>
</tr>
<tr>
<td>Marginal</td>
<td>2.016</td>
<td>2.125</td>
<td>2.195</td>
<td>2.241</td>
<td>.114</td>
<td>.105</td>
<td>.098</td>
</tr>
<tr>
<td>(predicted)</td>
<td>[.400]</td>
<td>[.400]</td>
<td>[.400]</td>
<td>[.400]</td>
<td>[.400]</td>
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<td></td>
</tr>
<tr>
<td>Marginal</td>
<td>2.040</td>
<td>2.092</td>
<td>2.190</td>
<td>2.231</td>
<td>.148</td>
<td>.117</td>
<td>.068</td>
</tr>
<tr>
<td>(observed)</td>
<td>[.460]</td>
<td>[.407]</td>
<td>[.420]</td>
<td>[.402]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** Entries in dismissal columns in rows labeled "primary" refer to the proportion of the cohort in the primary sector in period t + 1 under the model.

imply a slower selection process and hence lower rates of wage growth than do the estimates reported in Table 3. No formal tests between the specifications reported in Tables 3 and 4 are attempted because the models are not nested. Comparisons between the two specifications are only meant to illustrate the sensitivity of structural parameter estimates to the introduction of measurement error. This is a disturbing situation because the measurement-error process is not a product of the model structure.

Table 5 contains estimates of the model in which exogenous separations are allowed to occur in the primary sector with probability $\lambda_p$ (conditional on no dismissal for cause taking place in the period) and $\lambda_s$ in the secondary sector (14c). Because I need to assume that exogenous dismissals (in either the primary or secondary sector) have no effect on welfare, these dismissals are really no different from pure measurement error from the standpoint of the behavioral model being estimated. Nonetheless, the interpretation of these parameters as corresponding to real measurable events probably enhances substantive interest in them.

I find that estimated monitoring rates are quite low, as is dispersion in the heterogeneity distribution, in comparison with the pure measurement-error model results contained in Table 3. Other behavioral parameter estimates are similar across the two specifications. The differences in the estimates of $\pi$ and $\alpha$ across the two specifications imply quite different wage growth and dismissal outcomes. In terms of the exogenous dismissal parameters, dismissals in the secondary sector are much more probable than they are in the primary sector. The estimated probability of a secondary-sector dismissal is .229 using the entire sample, whereas the comparable figure for the primary sector is .069.

To get some sense of the ability of the various specifications of the model to fit the data, I present the implied means and standard deviations of ln wages and dismissals over the sample period and compare them with the actual sample values in Table 6. In constructing predicted values for means and standard deviations, I used estimates from the model that allows for exogenous sector-specific dismissal rates (Table 5).

For the homogeneous model (top panel), I see that there is a very large mean ln wage increase in the primary sector between periods 1 and 2 of 19% coincident with the relegation of 6.2% of employees to the secondary sector in the second period. Average ln wage gains after the second period are modest; for example, ln wage growth in the primary sector between periods 2 and 3 is only 4.5%. Under the model structure, the only variation in primary-sector wages comes from measurement error, which has a constant standard deviation of .397. Because all individuals are employed in the primary sector in the first period by construction, the marginal distribution of wages in this period is the same as that in the primary sector. In the second period, only 93.8% of wage observations are from the primary sector with the remainder coming from the secondary sector. Thus, the mean of the unconditional distribution of ln wages is the weighted average of the mean ln wages in the two sectors. The variance in ln wages consists of within variance, attributable solely to measurement error, and variance in mean ln wages across the two sectors. The mean and standard deviations of ln wages are computed in a similar fashion for periods 3 and 4. Comparing the predicted marginal means and standard deviations with the observed values, I see that the model underestimates mean ln wages in the first period and overestimates them in the subsequent
periods. The model underestimates the standard deviation of In wages in the first period and tends to overestimate them in later periods.

The three columns on the right side of the table indicate the performance of the model in terms of predicting dismissal rates. The figures associated with the row labeled “Primary” are predictions from the model of the proportion of the cohort in the primary sector after each of the first three periods. Using estimates of exogenous and “true” dismissal probabilities for the cohort, I constructed the predicted proportion of reported dismissals in each period. Thus, the model predicts that 13.1% of the sample would report a dismissal during period 1, whereas the observed proportion is 15%. The dismissal predictions are of a similar degree of accuracy for periods 2 and 3.

The bottom two panels of Table 6 contain the same exercise repeated for the high school and college groups. For both schooling groups, mean In wages tend to be more accurately predicted than was the case under the homogeneity assumption. The increasing standard deviations over the first four periods predicted by the model are not very consistent with the data for either of the groups, which exhibit little systematic change in this sample moment over the period. Dismissals tend to be slightly less well predicted in the disaggregated models than in the homogeneous model with the exception of period 2.

I conclude my empirical analysis by conducting some comparative statics exercises, the results of which are summarized in Figure 2. In conducting these exercises, I have used the point estimates of the labor-market parameters for the total sample obtained under the exogenous dismissal model (Table 5). In the figure, I explore the effects of alternatively changing the monitoring rate, the parameter characterizing the distribution of $H(\alpha)$, and the product price ($\pi$) on the equilibrium primary-sector wages and dismissal rates in the first 20 periods of labor-market participation. In Figure 2(a), we see that increasing the product price by 1% results in a change of the primary-sector wage of more than 1% in the first few periods as this change decreases the rate of shirking in the population. An increase in the rate of monitoring has a small positive effect on the wages in the second and third period as shirkers are more quickly detected and more individuals supply effort. Increasing the dispersion in the distribution of $H$ decreases the equilibrium wage in the early periods as a higher proportion of agents shirk.

Figure 2(b) contains the elasticities of primary-sector dismissals with respect to the same three parameters. Note that, although increasing the monitoring rate increases dismissal rates in the first few periods, it reduces dismissal rates in all subsequent periods by producing a more heavily selected distribution and increasing the critical values $\xi_i$. Increases in the product price have a strong negative effect on dismissal rates, whereas increases in $\alpha$ increase dismissals at every point in the life cycle.

5. CONCLUSION

The model presented in this article incorporates moral hazard in employment relationships to provide a central role for “involuntary” separations in the wage-growth process of young labor-market participants. As opposed to most models of moral hazard applied to the labor market, increasing wages are not motivated by long-term contracting considerations but instead result from the operation of a dynamic selection process in which individuals inefficient at production in the primary sector are systematically discovered and dismissed. By primary-sector firms competing for an improving quality distribution of employees, a positively sloped age–earnings profile is produced.

By limiting attention to increasing wage sequences, I have been able to specify conditions sufficient to ensure the existence of a unique equilibrium; the most critical of the assumptions employed would appear to involve restrictions on the information set of firms and concavity of the distribution of productive inefficiency. Although the concavity assumption is restrictive, I believe that, in regard to empirical implementation of the model, it is not particularly troublesome. Experimentation with other concave distributions leads me to believe that in fact it is empirically difficult to distinguish between members of even this relatively small class.

Although a large proportion of employee–firm separations are reported to be “involuntary,” the vast majority of moves are not so characterized by employees. A major deficiency of the model presented here is the exclusion of “quits.” If voluntary moves are made for exogenous rea-
sons, little modification of the preceding model is required if prospective employers are able to correctly distinguish between applicants on the basis of the reason for their separation from their previous employer. If this is not the case, the situation would be much more like the one described by Greenwald (1986). All movers would be treated equivalently, and dismissed individuals would receive a subsidy by being mixed in with those not dismissed. This would in turn reduce the incentive to supply effort on the job. Empirical evidence suggesting a significant difference in postseparation wages by reason for separation supports the contention that potential employers do not treat all applicants in an identical manner.

I believe that my empirical results have suggested that dismissals do have relatively long-term effects on subsequent wage realizations and that the behavioral model put forth in the article is capable of parsimoniously summarizing wage and dismissal processes during the initial stages of the labor-market career. My future work along this line will attempt to incorporate voluntary quits into this general framework and to provide other mechanisms for wage growth in both the primary and secondary sectors of the economy so as to provide a more complete story of turnover and wage growth over the life cycle.

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APPENDIX A: PROOF OF EXISTENCE AND UNIQUENESS OF THE EQUILIBRIUM

By Proposition 2, all equilibrium wage sequences in the class of increasing sequences taking values in the interval $(0, \rho]$ have limit point $\rho$. In the class of increasing sequences, the mapping between wage sequences and critical value sequences is one-to-one, so equilibrium can be equivalently characterized in terms of either.

Using Proposition 1, we can establish the recursion

$$
\xi_t = \beta \xi_{t+1} + MW_{t+1}, \quad t = 1, 2, \ldots
$$

(A.1)

By the definition of competitive equilibrium in period $t+1$,

$$
w_{t+1} = \rho H_{t+1}(\xi_{t+1}; \xi_1, \ldots, \xi_t),
$$

(A.2)

where I explicitly represent the dependence of the distribution function at time $t+1$ on previous critical values $\xi_1, \ldots, \xi_t$ for clarity. Then, in equilibrium,

$$
\xi_t = \beta \xi_{t+1} + \rho MH_{t+1}(\xi_{t+1}; \xi_1, \ldots, \xi_t),
$$

(A.3)

and we see that in equilibrium the critical effort levels are (implicitly) recursively determined.

Consider the determination of $\xi_1$ given an arbitrary value of $\xi_2 = a > 0$:

$$
\xi_1 = \beta a + \rho MH_2(a; \xi_1),
$$

(A.4)

where any solution $\xi_1$ is constrained to lie in the interval $(0, a]$. At $\xi_1 = 0$, the right side of (A.4) is equal to $\beta a + \rho MH(a) > 0$, and when $\xi_1 = a$, the right side of (A.4) is equal to $\beta a + \rho MH(a)/(1 - \pi H(a))$. The right side of (A.4) is concave in $\xi_1$ on $(0, a]$. Thus, if $a \geq \beta a + \rho MH(a)/(1 - \pi H(a))$, there exists a unique solution $\xi_1(a)$, which satisfies the requirements of the equilibrium. Let the set of all $a$ for which solutions exist be denoted $A_2$. Because $\rho MH(a)/(1 - \pi H(a))$ is concave in $a$, this set is connected and is of the form $A_2 = [a_2, \infty)$, where $a_2$ is defined by $0 = (1 - \beta) a_2 - \rho MH(a_2)/(1 - \pi H(a_2))$. For $a \in A_2$, consider the partial derivative

$$
\frac{\partial \xi_1}{\partial a} = \frac{\beta + \rho MH_2(a; \xi_1)}{1 - \rho M \{\partial H_2(a; \xi_1)/\partial \xi_1\}} > 0
$$

(A.5a)

at any solution to (A.4). Moreover,

$$
\frac{\partial^2 \xi_1}{\partial a^2} = \frac{\rho M \{\partial h_2(a; \xi_1)/\partial a\}}{D} + \left\{\frac{\rho M [\beta + \rho MH_2(a; \xi_1)]}{D^2}\right\}
$$

\times \frac{\partial^2 H_2(a; \xi_1)}{\partial \xi_1 \partial a} < 0,
$$

(A.5b)

where $D$ refers to the denominator in the right side of (A.5a). Thus, on $A_2$, $\xi_1(a)$ is a concave function.

Now consider the determination of $\xi_1$ and $\xi_2$ given $\xi_3 = a > 0$:

$$
\xi_2 = \beta a + \rho MH(a; \xi_1, \xi_2),
$$

(A.6)

where I define the function $\tilde{\xi}_1(\xi_2)$ as equal to the nonzero solution to (A.4) if $\xi_2 \in A_2$ and as equal to 0 if $\xi_2 \notin A_2$. I look for solutions of (A.6) for which $\xi_2 \in [0, a]$. At $\xi_2 = 0[= \xi_1 = 0]$, the right side of (A.6) is equal to $\beta a + \rho MH(a)$. At $\xi_2 = a$, the right side of (A.6) is equal to $\beta a + \rho MH(a; \tilde{\xi}_1(a), a)$, which is a concave function of $a$. On the interval $(0, a]$,

$$
\frac{\partial \text{RHS}[A.6]}{\partial \xi_2} = \rho M \left\{\frac{\partial H_3(a; \xi_1, \xi_2)}{\partial \xi_2} + \frac{\partial H_3(a; \xi_1, \xi_2)}{\partial \xi_1} \frac{\partial \tilde{\xi}_1}{\partial \xi_2}\right\} > 0
$$

(A.7a)

and

$$
\frac{\partial^2 \text{RHS}[A.6]}{\partial \xi_2^2} = \rho M \left\{\frac{\partial^2 H_3(a; \xi_1, \xi_2)}{\partial \xi_2^2} + \frac{\partial^2 H_3(a; \xi_1, \xi_2)}{\partial \xi_1^2} \left[\frac{\partial \xi_1}{\partial \xi_2}\right]^2\right\}
$$

(A.7b)
\[
+ \frac{2}{2} \frac{\partial^2 H_3(a; \xi_1, \xi_2)}{\partial \xi_1 \partial \xi_2} \frac{\partial^2 \xi_1}{\partial \xi_2^2} + \frac{\partial H_3(a; \xi_1, \xi_2)}{\partial \xi_1} \frac{\partial^2 \xi_1}{\partial \xi_2^2} < 0
\]

so that RHS(A.6) is a concave function of \( \xi_2 \).

Then there exist unique solutions \( \hat{\xi}_1(\hat{\xi}_2(a)) \) and \( \hat{\xi}_2(a) \) with \( \hat{\xi}_2(a) > 0 \) for all \( a \) such that \( a \geq \beta a + \rho M H(a; \hat{\xi}_1(a), a) \). Denote this connected set by \( A_3 = [a_3, \infty) \). To see that \( a \in A_3 \Rightarrow \hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(a)))))))))) > 0 \), consider a value \( \hat{a} \in A_3 \) such that \( \hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(\hat{\xi}_1(\hat{\xi}_2(a)))))))))))))) = 0 \). Then \( \hat{a} \) would have to be such that the following equation yielded a solution for \( \hat{\xi}_2(a) \):

\[
\hat{\xi}_2 = \beta a + \rho M H_3(a; 0, \hat{\xi}_2(a)).
\]

At \( \hat{a} \), the solution to \( \hat{A}_8(a) \) would be \( \hat{\xi}_2(\hat{a}) = \hat{a} \). From (7), observe that \( H_3(a; 0, q) = H_2(a; q) \). Then

\[
\hat{\xi}_1 = \beta a + \rho M H_2(a; \hat{\xi}_1)
\]

yields a nonzero solution for \( \hat{\xi}_1 \) for \( a \geq \hat{a} \) in this case), which is a contradiction. Then all \( a \in A_3 \) produce nonzero unique solutions for \( \hat{\xi}_1 \) and \( \hat{\xi}_2 \) with \( \hat{\xi}_1(\hat{\xi}_2(a)) \leq \hat{\xi}_2(a) \) and \( a_3 \geq a_2 \).

In period \( t \), I write

\[
\hat{\xi}_t = \beta a + \rho M H_{t+1}(a; \xi_1, \xi_2, \ldots, \xi_{t-1}, \xi_t, \xi_t)
\]

(9)

to determine \( \hat{\xi}_t \) and by implication the entire path of critical values from period 1 through \( t \). By extension of the argument given previously, the right side of (9) is a concave function of \( \hat{\xi}_t \). Then if \( a \in A_{t+1} \), there exists unique nonzero solutions for all \( \hat{\xi}_1, \ldots, \hat{\xi}_t \).

Let

\[
\hat{A} \equiv \lim_{t \to \infty} A_t = [\lim_{t \to \infty} a_t, \infty).
\]

Now

\[
a_\infty \equiv \lim_{t \to \infty} a_t = \rho M/(1 - \beta)
\]

so that for any \( a \geq a_\infty \) there exists a unique increasing wage sequence with all elements positive. By Proposition 2, any nonzero equilibrium wage sequence must converge to \( \rho \) in the limit, which implies that the associated critical value sequence must converge to the limit point \( \rho M/(1 - \beta) \), which is equal to \( a_\infty \). Thus there exists a unique equilibrium critical value sequence (which implies a unique wage sequence) in the class of increasing and bounded sequences for all \( \rho > 0, \beta \in (0, 1), \pi \in (0, 1], \) and concave \( H \).

APPENDIX B: COMPUTATION OF THE EQUILIBRIUM

In this appendix I describe the computation of the Nash equilibrium termination contract described in the text. The algorithm is used in the computation of the ML estimates of the behavioral and measurement-error distribution parameters.

Computation of the equilibrium wage sequence will produce two types of approximation error. The first source of approximation error is generated by replacing the operator \( T \) with the operator \( T_S \) in which all rows after row \( S \) are replaced with the value \( \rho \). Because agents are assumed to be infinitely lived and because an equilibrium wage of \( \rho \) is only attained asymptotically, it is clear that some approximation error is induced by implicitly setting the probability of shirking equal to 0 at times \( S + 1, S + 2, \ldots \). The second source of approximation error is more standard and arises because I only iterate on the operator \( T_S \) a finite number of times rather than the infinite number strictly required to define an exact fixed point for this monotone operator.

To compute the finite \( S \) approximation to the infinite-horizon equilibrium, my strategy will be as follows. First, define an equilibrium wage sequence for the problem in which the free-entry (zero-profit) assumption is satisfied for periods 1 through \( S \) and after which wages are set to \( \rho \). I establish that for each \( S \geq 1 \) there exists a unique fixed point for this operator. I show that the fixed point of \( T_S \) converges to the fixed point of \( T \) uniformly as \( S \) gets indefinitely large.

The following result follows directly from Proposition 3:

Result B.1. Define the map

\[
T_S(\{w_S\}) = \left[ \begin{array}{c}
\rho H(\xi_1(\{w_S\}_{S+2}^\infty)) \\
\vdots \\
\rho H_S(\xi_S(\{w_S\}_{S=S+1}^\infty)) \\
\rho \\
\vdots 
\end{array} \right] (B.1)
\]

on the space of increasing wage sequences on \( (0, \rho] \). \( T_S \) has a unique fixed point.

Of course, the fixed point of \( T_S \) will have the property that all sequence elements beginning with \( S + 1 \) are equal to \( \rho \).

For any given value of \( \rho, \beta, \pi, \) and \( H \), let \( \{w_*^S\} \) denote the fixed point associated with \( T \) and let \( \{w_*^S(S)\} \) denote the fixed point associated with \( T_S \).

Proposition B.1.

\[
\lim_{S \to \infty} d_\infty(\{w_*^S(S)\}, \{w_*^S\}) = 0.
\]

Proof. For any \( \{w_S\} \) in the class of increasing sequences on \( (0, \rho] \),

\[
[T_S - T](\{w_S\}) = \left[ \begin{array}{c}
\rho[H(\xi_1(\{w_S\}_{S=2}^\infty)) - H(\xi_1(\{w_S\}_{S=2}^\infty))] \\
\vdots \\
\rho[H_S(\xi_S(\{w_S\}_{S=S+1}^\infty)) - H_S(\xi_S(\{w_S\}_{S=S+1})] \\
\rho[1 - H_{S+1}(\{w_S\}_{S=S+1}^\infty)) \\
\vdots 
\end{array} \right] .
\]


Because the operators \( T_S \) and \( T \) coincide for the first \( S \) elements of the wage sequence and because

\[
\sup_{q > S} \rho |1 - H_q(\xi_q(\bar{w}_s)_{s=q+1})| = \rho |1 - H_{S+1}(\xi_{S+1}(\bar{w}_s)_{s=S+2})|,
\]

then

\[
d_\infty(T_S(\{w_s\}), T(\{w_s\})) = \rho |1 - H_{S+1}(\xi_{S+1}(\bar{w}_s)_{s=S+2})|.
\]

From (7),

\[
\lim_{S \to \infty} H_{S+1}(\xi_{S+1}(\{w_1\})) = 1
\]

for all bounded increasing sequences, so

\[
\lim_{S \to \infty} d_\infty(T_S(\{w_s^*\}), T(\{w_s^*\})) = 0
\]

\[
\Rightarrow \lim_{S \to \infty} d_\infty(\{w_s^*\} - T_S(\{w_s^*\}), \{w_s^*\} - T(\{w_s^*\})) = 0
\]

\[
\Rightarrow \lim_{S \to \infty} d_\infty(\{w_s^*\} - T_S(\{w_s^*\}), 0) = 0
\]

\[
\Rightarrow \lim_{S \to \infty} d_\infty(\{w_s^*\}(S), \{w_s^*\}) = 0.
\]

For computational purposes, I make use of the triangle inequality to determine whether or not a given value of \( S \) produces an equilibrium \( \{w_s^*(S)\} \) "sufficiently close" to the equilibrium of the infinite-horizon problem \( \{w_s^*\} \). Now \( \{w_s^*(S)\} \) converges uniformly to \( \{w_s^*\} \), so for any \( \epsilon/2 > 0 \) there exists an \( S(\epsilon/2) \) such that

\[
d_\infty(\{w_s^*(S(S/2))\}, \{w_s^*\}) < \epsilon/2.
\]

By the triangle inequality,

\[
d_\infty(\{w_s^*(S(S/2) + 1)\}, \{w_s^*(S(S/2))\}) \leq d_\infty(\{w_s^*(S(S/2) + 1)\}, \{w_s^*\}) + d_\infty(\{w_s^*(S(S/2))\}, \{w_s^*\}) \leq \epsilon.
\]

Then \( S \) is "sufficiently large" if \( \sup S \mid w_s^*(S) - w_s^*(S + 1) \mid \leq \epsilon. \)

The second approximation issue concerns the computation of the equilibrium wage sequence for any given value of \( S \). Using the monotone operator \( T_S \), I solve for the equilibrium wage sequence by iterating on the initial wage vector \( \rho \). By the same argument as that given previously regarding the selection of \( S \), I have that

\[
d_\infty(\{w_s^{K+1}(S)\}, \{w_s^K(S)\}) \leq d_\infty(\{w_s^{K+1}(S)\}, \{w_s^K(S)\}) + d_\infty(\{w_s^K(S)\}, \{w_s^K(S)\}),
\]

where \( \{w_s^r(S)\} = T_{S-r}^{-1}(\rho), r = 2, 3, \ldots, \) and \( \rho \) denotes an infinite dimensional sequence with all elements equal to \( \rho \). Because the sequence \( \{w_s^r(S)\} \) is uniformly convergent with limit point \( \{w_s^*(S)\} \), for any \( \nu/2 > 0 \) there exists some value \( \nu/2 \) such that

\[
\lim_{S \to \infty} H_{S+1}(\xi_{S+1}(\{w_s^r(S)\})) = 1
\]

Operationally, I stop iteration on \( T_S \) after iteration \( K \) when

\[
d_\infty(\{w_s^{K+1}(S)\}, \{w_s^K(S)\}) \leq \nu, \text{ where } \nu \text{ is some positive constant.}
\]

There are two sources of approximation error; the bound on the total approximation error is simply the sum of the upper bounds on the individual sources of error. Thus, for a given \((\epsilon/2, \nu/2)\) pair, using an \( S(\epsilon/2) \) approximation to the infinite-horizon problem and iterating on the \( T_S \) operator \( K(\nu/2) \) times yields a sequence of primary-sector wages having the property that the absolute value of the difference between each element and its corresponding element in the infinite-horizon equilibrium sequence is no greater than \((\epsilon + \nu)/2\).

To recapitulate, I compute the (approximate) equilibrium wage and dismissal profile using the following procedure:

1. Choose positive constants \( \epsilon/2 \) and \( \nu/2 \).
2. Set \( S \), beginning with \( S = 1. \) Using the operator \( T_S \), iterate until (B.2) is satisfied, beginning with \( K = 2 \). Denote the value of the wage sequence at the final iteration by \( \{w_s^*(S)\} \).
3. Repeat step 2 for \( S + 1 \).
4. Compute \( M_S \equiv d_\infty(\{w_s^*(S + 1)\}, \{w_s^*(S)\}) \). If \( M_S \leq \epsilon + \nu \), the (approximate) equilibrium wage sequence for the infinite-horizon problem is \( \{w_s^*(S + 1)\} \). If \( M_S > \epsilon + \nu \), repeat the operation beginning with step 2 using \( S + 1 \).

References


