

Final (Take-Home) Exam

*The Formulation and Estimation of Equilibrium Models
of Household and Labor Market Behavior*

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*Please answer five (5) of the following six (6) questions as fully as possible, and show your work. All questions have equal weight. You are welcome to consult with each other at the early stages of responding to the questions, but each student should formulate their own final response to each question. You will have one week to complete the exam; please give your completed examination to Ela no later than 12 (noon) on Wednesday, **November 25**.*

If you have some (clarifying) questions about the exam I will be happy to respond by e-mail. If I think any issue raised is important enough, I will cc all students working on the exam when responding.

Good luck!

1. In a number of individual-level data sets, sample members are often observed to enter jobs directly from the out of the labor force (*OLF*) state, that is, they do not indicate that they were in the unemployment state (*U*) prior to finding the job. Although some analysts, like Clark and Summers, argue that the states *U* and *OLF* are largely one in the same, Flinn and Heckman (1983) found that exit rates into employment from *U* were substantially higher than those from *OLF*. Thus, the states do not appear to be the same in a behavioral sense.

Within a stationary environment, assume that individuals face an exogenous wage offer distribution $F(w)$, and that there is no on-the-job search. When in the unemployment state, the individual pays an instantaneous cost of c [i.e., $c < 0$] and receives offers at the rate λ_u . If the individual is in the *OLF* state, the individual receives offers at a rate λ_o [$0 < \lambda_o < \lambda_u$] and receives an instantaneous utility flow of b . The utility flow parameter b is a random variable as well. Given a current realization of b , the duration of time until a new b is realized is exponentially distributed with parameter δ . Each new value of b is an independently and identically distributed draw from the distribution G . All accepted jobs end at rate η , and this is the only way jobs end.

- (a) Is it possible for the rate of exit from *OLF* into *E* to be the same as the rate of exit from *U* into *E*? Why or why not?

- (b) Compare the accepted wage distributions for people entering a job from the state OLF with those accepting a job from the state U . Are they the same or not? What accounts for any differences or the fact that they are identical?
- (c) Write down the steady state employment rate for your model.
- (d) Is it possible for agents to go from U to OLF in your model? Is it possible to observe transitions from OLF to U ? Characterize the rates of transition between these states if they are positive.
2. Individuals meet firms at an exogenous rate λ . Once they have met the firm, the value of the match is revealed to the worker and the firm, which is denoted by θ . The matching distribution is given by $G(\theta)$. If a match θ generates an employment contract, the worker is paid a wage $w(\theta)$ and the job is dissolved at an exogenous rate η . The “threat point” of the potential employee is the value of continuing search, denoted V_n , and the threat point of the employer is 0. The value of the wage is determined using a Nash bargaining framework, so that

$$w(\theta, V_n) = \arg \max_w (V_e(w) - V_n) \left(\frac{\theta - w}{\rho + \eta} \right),$$

where ρ is the instantaneous discount rate, and $\rho + \eta$ is the effective discount rate of the firm.

- (a) Write down the labor market dynamics generated by this model in as much detail as possible. In particular, find $V_e(w)$, $w(\theta, V_n)$, and V_n . What is the probability density function of accepted wages? What is the steady state unemployment rate?
- (b) Workers and firms can increase the value of the match by purchasing health insurance. The instantaneous price of health insurance is $\phi > 0$. If health insurance is purchased the match the total value of the match improves and becomes $a\theta$, where $a > 1$ [due to the increased healthiness of the worker]. Then the Nash bargaining problem becomes

$$(w, d)(\theta, V_n) = \arg \max_{w, d} (V_e(w) - V_n) \left(\frac{a^d \theta - w - d\phi}{\rho + \eta} \right),$$

where d equals 1 if health insurance is purchased and equals 0 if not, and we have assumed that the firm actually pays the health insurance premium directly to the insurance company. Describe the equilibrium outcomes associated with this model as you did in part (a). In equilibrium, will some jobs be covered by health insurance and others not? If so, what proportion of jobs will be covered by health insurance, and what is the relationship between the presence of health insurance and wages? Provide some intuition for your results.

3. You have access to the following information. Individuals are asked to report information on their first 16 months spent in the labor market. All individuals in the sample began their labor market career in the unemployment state. The length of time individual i spends in unemployment is recorded as t_i^u , where $t_i^u \leq 16$. If $t_i^u = 16$, the individual was still unemployed at the end of the 16 month period, so that this spell was “right-censored.” If the unemployment spell is right-censored, then $c_i^u = 1$, otherwise $c_i^u = 0$. If the unemployment spell ended before the observation period, the accepted wage, w_i , is recorded, as is the length of the employment spell during the sample observation period. The length of the employment spell is t_i^e . If $t_i^u < 16$ and $t_i^u + t_i^e < 16$, then the completed spell of employment is observed, and $c_i^e = 0$. If $t_i^u < 16$ and $t_i^u + t_i^e \geq 16$, then $c_i^e = 1$. You have access to the information:

$$\{t_i^u, c_i^u, t_i^e, c_i^e, w_i\}_{i=1}^N.$$

Consider estimation of a stationary search model in which there is no on-the-job search. The rate of arrival of offers while unemployed is $\lambda > 0$, the separation rate from jobs is $\eta > 0$, the utility flow in the unemployment state is b , and the discount rate is $\rho > 0$. Individuals are assumed to be expected wealth-maximizers, and as such, employ a reservation wage strategy in deciding whether to accept a job offer. The distribution of job offers is given by G .

You *know* the true value of the discount rate to be $\rho = .01$. The wage offer distribution is lognormal with cumulative distribution function

$$G(w; \mu_w, \sigma_w) = \Phi\left(\frac{\ln w - \mu_w}{\sigma_w}\right)$$

and probability density function

$$g(w; \mu_w, \sigma_w) = (w\sigma_w)^{-1} \phi\left(\frac{\ln w - \mu_w}{\sigma_w}\right),$$

where Φ is the c.d.f. of a standard normal random variable and ϕ is its corresponding density function.

- (a) Determine whether the model is identified given the data available to you. You should assume that the number of individuals with employment spell information is ‘large.’ Develop estimators, in as much detail as possible, of all estimable functions of the parameters.
- (b) If you only had access to wage information from this sample, could you consistently estimate the population wage offer distribution parameters μ_w and σ_w ? Why or why not?

4. A population of individuals have (quasi-linear) utility functions given by

$$u_i(c, l) = \alpha_i \ln l + (1 - \alpha_i)c,$$

where l is leisure, c is consumption of a market good with price 1, and α_i is a individual-specific preference parameter, where $\alpha_i \in (0, 1)$. Their total time endowment is T , and $c_i = w_i(T - l_i) + Y_i$, where Y_i is individual i 's nonlabor income and w_i is their wage rate. The distribution of α in the population is given by

$$G(\alpha) = \alpha^\delta, \alpha \in (0, 1), \delta > 0.$$

The wage offer w is independently distributed with respect to α , and has the cumulative distribution function

$$F(w) = 1 - \exp(-\delta w)$$

in the population. Thus the model is completely characterized in terms of the two primitive parameters α and δ .

- (a) You have access to a very limited amount of information from a random sample of size N from the population. You know the average hours worked for the employed subsample, and you know their average wages. Are the primitive parameters identified on the basis of this information? If so, derive a consistent estimator of these parameters.
 - (b) If you also know the proportion of the sample that is employed, can you derive a “better” set of estimators (still consistent) of the parameters than those in part (a)? If so, describe your new estimator and how you would implement it.
5. Husbands and wives have individual utility functions defined over private goods and a public good. Spouse j has a utility function given by

$$u_j(x_j, z) = \lambda_j \ln x_j + (1 - \lambda_j) \ln z, \quad j = 1, 2,$$

where x_j is the private good consumed by spouse j and z is a public good consumed by both household members. Thus, the household consumes three goods in all, x_1 , x_2 , and z , and you are to assume that the price of each is unity (i.e., $p_{x_1} = p_{x_2} = p_z = 1$).

You have access to data from the Consumer Expenditure Survey (CEX), and based on prior information regarding which good is public and which are private, for a random sample of households you are able to assemble information on $\{x_{1i}, x_{2i}, z_i, Y_{1i}, Y_{2i}\}_{i=1}^N$, where Y_{ji} is the income of spouse j in household i , and $Y_{1i} > 0$ and $Y_{2i} > 0$ for all households in the population.

All households determine consumption allocations within a (static) Nash equilibrium, in which each spouse contributes z_j to the purchase of the public good, and

z_j is a “best response” to the contribution of the other spouse, $z_{j'}$. A household is characterized by the state variables

$$S = (\lambda_1 \lambda_2 Y_1 Y_2),$$

and households are i.i.d. draws from the population distribution function F_S .

Based on the sample information available, is F_S nonparametrically identified? If not, why not? If so, propose a consistent estimator of it.

6. Demand for a good x is given by

$$x = \delta \frac{I}{p_x},$$

where I is income and p_x is the price of the good, which is assumed to be equal to 1. You have access to a random sample of N observations on x^* and I^* , where x^* and I^* are possibly noisy measures of x and I , respectively, with the sample information summarized by $\{x_i^*, I_i^*\}_{i=1}^N$. If they exist, define consistent estimators of model parameters under the following four alternative sets of assumptions. All parameters included in a model specification, including those characterizing the measurement error process, are to be considered unknown and to be estimated.

- (a) $I_i^* = I_i$; $x_i^* = x_i + \varepsilon_i$, ε_i is independently and identically distributed (i.i.d.) with mean and variance $(0, \sigma_\varepsilon^2)$; and δ is common to all population members.
- (b) $I_i^* = I_i + u_i$, u_i i.i.d. $(0, \sigma_u^2)$; $x_i^* = x_i$; δ common to all population members.
- (c) $I_i^* = I_i$; $x_i^* = x_i$; δ i.i.d. $(\bar{\delta}, \sigma_\delta^2)$.
- (d) $I_i^* = I_i$; $x_i^* = x_i$; δ i.i.d with distribution $G(\delta) = \delta^\eta$.