Econometrics II

Midterm Examination

Spring 1999

Please answer all the questions and show all of your work. If you think a question is ambiguous, clearly state how you interpret it before providing an answer.

1. (24 points) The dependent variable y_i is related to the exogenous variable x_i (both scalars) as follows:

$$y_i = \beta_0 + \alpha^{-1} x_i + \varepsilon_i,$$

where ε_i is independently distributed (over i) with mean 0 and variance τx_i^2. The covariate $x_i > 0$ for all i, and the parameters β, α, and τ are unknown.

   1. Define $\beta_1 \equiv \alpha^{-1}$, and let $\beta = (\beta_0\ \beta_1)'$. Is the ordinary least squares (OLS) estimator of β unbiased? Is it consistent? Is it efficient?
   2. Can you define a consistent and asymptotically more efficient estimator of β? If so, define this estimator in some detail.
   3. Define a consistent estimator of α. Is this estimator also unbiased? Explain why or why not.
   4. Describe how you would compute the asymptotic sampling distribution of your estimator of α. Provide as much detail as possible.

2. (24 points) Consider the following two equation system:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_i + \varepsilon_i \\ z_i &= \alpha_0 + \alpha_1 m_i + u_i,\ i = 1, ..., I, \end{aligned}$$

where the disturbance vector $\xi_i = (\varepsilon_i\ u_i)$ is independently distributed (over i) with $E(\xi_i) = (0\ 0)'$ and

$$E(\xi_i \xi_i') = \begin{bmatrix} \tau m_i^2 & 0 \\ 0 & \delta x_i^2 \end{bmatrix}.$$

(That isn't a typo - the conditional variance of the error term in each equation is a function of the regressor in the other equation.). The regressors x_i and m_i always assume positive values in the population.

1. Is OLS unbiased and consistent when estimation is performed on an equation-by-equation basis?
2. Describe a consistent and asymptotically efficient estimator of β and α, if one exists. Is your estimator a single equation estimator or a "systems" estimator (as in Seemingly Unrelated Regression)?
3. Say that economic theory tells us that $\beta_1 = \alpha_1$. Describe a consistent and asymptotically efficient estimator that imposes this restriction. Provide as much detail as possible.

3. (30 points) Consider the following panel data model. You have access to two observations for each of I individuals, with

$$\begin{aligned} y_{i1} &= \beta_0 + \beta_1 x_{i1} + \varepsilon_{i1} \\ y_{i2} &= \beta_0 + \beta_1 x_{i2} + \varepsilon_{i2}, \end{aligned}$$

where

$$\begin{aligned} \varepsilon_{i1} &= \eta_i + \xi_{i1} \\ \varepsilon_{i2} &= \lambda \eta_i + \xi_{i2}, \end{aligned}$$

and where ξ_{it} is independently and identically distributed over all (i, t) with mean 0 and variance σ_ξ^2. Assume that $x_{i2} \neq x_{i1}$ for all i.

1. Assume $E(\eta_i | x_{i1}, x_{i2}) = 0$ and $E(\eta_i^2 | x_{i1}, x_{i2}) = \sigma_\eta^2$ all i. Assume that λ, σ_η^2, and σ_ξ^2 are unknown. Define a feasible GLS estimator of β, if one exists. Provide details.
2. Assume that $E(\eta_i | x_{i1}, x_{i2}) \neq 0$. Assuming that λ and σ_ξ^2 are unknown, is it possible to define a consistent estimator of any element of the vector β? Explain why or why not.
3. Assume that $E(\eta_i | x_{i1}, x_{i2}) \neq 0$ but λ is known. Define a consistent estimator of any or all elements of β in this case, if such an estimator exists.

4. (22 points) You want to estimate the following linear regression model:

$$y_i = \beta x_i + \varepsilon_i,$$

where ε_i is independently and identically distributed with mean 0 and variance σ_ε^2. Unfortunately, you are unable to observe y_i directly. Instead, you have a "noisy" measure of y_i,

$$y_i^* = y_i + u_i,$$

where u_i is independently and identically distributed with mean 0 and variance σ_u^2.

1. Is the estimator obtained by regressing y_i^* on x_i,

$$\hat{\beta} = \frac{\sum_i y_i^* x_i}{\sum_i x_i^2},$$

a consistent estimator of β? Show why or why not.

2. If $E(u_i) = \gamma$ for all i, is the OLS estimator $\hat{\beta}$ consistent? If not, find the asymptotic bias associated with the estimator.