Household Behavior and the Marriage Market

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Motivation

- Modeling household behavior requires some assumptions regarding the objectives of the agents, their mode of interaction, their information sets, home production technologies, etc.

- Most analyses have taken a strong position on the mode of interaction of HH members, ranging from a dictatorial agent (or social planner) in the unitary case, through Nash equilibrium, to cooperation between the spouses (the “collective” model or Nash bargaining being prominent examples).

- In this paper we attempt to determine the manner in which households behave through predictions derived concerning marriage patterns.
Earlier Theoretical Considerations

- They recognize that achieving efficient outcomes is often costly, and is typically not a best response.
- An alternative to efficient outcomes is Nash equilibrium in best responses.
- The “separate spheres” terminology refers to the case in which the household produces at least 2 public goods, with each spouse at a corner with respect to their contributions to one of the goods.
- Thus, the “noncooperative” equilibrium looks decentralized with each spouse deciding on their resource allocations to the subset of public goods to which they alone contribute.
- Other authors who have examined noncooperative household behavior include Chen and Woolley (2001) and Lechene and Preston (2011).
Welfare Implications

- The sustainability of the marriage itself may be influenced by whether efficient payoffs can be attained (though we don’t consider this in what is essentially a static model).
- If the welfare of children is a public good in the household, efficient outcomes result in higher child welfare (see Flinn (2000)).
- If leisure is a private good, moving to efficient outcomes will, in general, result in increases in the labor supply of both spouses. Thus a planner interested in aggregate labor supply would do well to implement policies that promote efficient intrahousehold time allocations.
The assumptions of the models (regarding preferences and production possibilities) are identical.

That paper considered four alternative models of household behavior:

1. Nash equilibrium (in reaction functions)
2. Pareto weighted utility (with a constant weight $\alpha_0$)
3. Constrained Pareto weighted utility (each individual had to attain at least their welfare level in the Nash equilibrium)
4. Endogenous household interaction (HDI)

The 4th model was the more general, and households endogenously sorted between behavioral rule (1) and behavioral rule (3)
Using data from the PSID, we found that 75 percent of households were “cooperative” and 25 percent were “noncooperative.”

We showed that “tests” comparing the performance of behavioral rules were impossible without a restrictive parameterization. In short, when working with saturated models, the fits of models (1)-(3) were identical and $\alpha_0$ was not identified.

Under this parameterization, the EHI model performed significantly better than the others.

The current paper conducts tests without restricting the parameterization of the model, and thus is more general than the test performed in the other paper.

However, we are forced to assume that all households behave either according to (1) or to (3).
The Household’s Decision Problem

- Two agent households. Preferences of agent $i$ are given by

$$u_i(l_i, K) = \lambda_i \ln l_i + (1 - \lambda_i) \ln K,$$

where $l_i$ is the leisure of spouse $i$ and $K$ is a public good produced in the household.

- The household public good is produced according to

$$K = \tau_1^{\delta_1} \tau_2^{\delta_2} M^{1-\delta_1-\delta_2}, \quad \delta_i \in (0, 1), \quad \delta_1 + \delta_2 \leq 1,$$

where $\delta_i$ is a technology parameter associated with spouse $i$ and total household income is given by

$$M = w_1 h_1 + w_2 h_2 + Y,$$

where $w_i$ is the wage offer of spouse $i$. 
The time constraints of the spouses are given by

\[ h_i + \tau_i + l_i = T, \ i = 1, 2, \]

where \( h_i \) is time in the market, \( \tau_i \) time in home production, and \( l_i \) leisure (the private good).

The household’s problem is to set time allocation decisions,

\[ (h_1, \tau_1, h_2, \tau_2) \]

given a choice set described by the parameters

\[ (w_1, Y_1, w_2, Y_2, T), \]

where the price of purchased inputs for the household production technology is set to 1.
Two equation system for each spouse’s reaction function. Let $a_i \equiv (h_i, \tau_i)$. For spouse 1,

$$a_1^*(a_2) = \arg \max_{a_1} u_1(a_1, a_2),$$

where the maximization is given nonlabor income $(Y + w_2 h_2)$ and given the time input of agent 2 in household production, $\tau_2$.

The unique Nash equilibrium is defined by

$$a_1^N = a_1^*(a_2^N)$$

$$a_2^N = a_2^*(a_1^N),$$

with the Nash equilibrium payoff to spouse $i$ given by

$$V_i^N \equiv u_i(a_1^N, a_2^N).$$
Efficient Outcomes (No Side Constraints)

- We define an efficient outcome using the Benthamite Social Welfare function
  \[
  (a_1^{PO}, a_2^{PO})(\alpha) = \arg \max_{a_1, a_2} \alpha u_1(a_1, a_2) + (1 - \alpha) u_2(a_1, a_2),
  \]
  with \( \alpha \in (0, 1) \).

- By varying \( \alpha \) we trace out the Pareto frontier,
  \[
  (u_1(a_1^{PO}(\alpha), a_2^{PO}(\alpha)), u_2(a_1^{PO}(\alpha), a_2^{PO}(\alpha))), 0 < \alpha < 1.
  \]

- Then why would any household choose *not* to implement the cooperative solution?
Efficiency is a nice property for a household time allocation to have, but efficient outcomes may be “unreasonable” on other dimensions.

Efficient outcomes require particular types of coordination, trust, etc. to implement, as was emphasized by Lundberg and Pollak (1993).

Nash equilibrium actions are unique in our example and don’t require coordination (being best responses). They seem like a natural side constraint on the efficient outcomes set.

Let $\alpha_0$ denote a “notional” Pareto weight, which might be culturally determined, for example.
Define critical values

\[ \alpha(V_1^N) : u_1(a_1^{PO}(\alpha(V_1^N)), a_2^{PO}(\alpha(V_1^N))) = V_1^N \]

\[ \bar{\alpha}(V_2^N) : u_2(a_1^{PO}(\bar{\alpha}(V_2^N)), a_2^{PO}(\bar{\alpha}(V_2^N))) = V_2^N \]

Given the properties of the utility and home production functions, these values \((\alpha, \bar{\alpha})(V_1^N, V_2^N)\) are unique, and \(\alpha(V_1^N) < \bar{\alpha}(V_2^N)\).

The \(CPO(\alpha_0)\) actions are:

\[
\{ a_1^{CPO}(\alpha_0), a_2^{CPO}(\alpha_0) \} = \\
\begin{cases} 
\{ a_1^{PO}(\alpha_0), a_2^{PO}(\alpha_0) \} & \text{if } \alpha(V_1^N) \leq \alpha_0 \leq \bar{\alpha}(V_2^N) \\
\{ a_1^{PO}(\alpha(V_1^N)), a_2^{PO}(\alpha(V_1^N)) \} & \text{if } \alpha_0 < \alpha(V_1^N) \\
\{ a_1^{PO}(\bar{\alpha}(V_2^N)), a_2^{PO}(\bar{\alpha}(V_2^N)) \} & \text{if } \bar{\alpha}(V_2^N) < \alpha_0
\end{cases}
\]

The restriction is manifested in a restricted connected set of Pareto weights and actions associated with them.

In DBF (2012) we show that the solutions are unique.
Figure 2
Pareto Frontier and Admissible Solutions
To simplify the econometric specification and make the identification discussion more transparent, we condition on the fact that both spouses work. We lost 12 percent of the sample, but are able to condition on wage draws. Missing information on any household decision or potentially observable state variable would greatly complicate the identification strategy.

We observe $X = (h_1, \tau_1, h_2, \tau_2; w_1, w_2, Y)$. Our objective is to estimate parameters characterizing the population distributions of $S = (\lambda_1, \lambda_2, \delta_1, \delta_2)$ and $\alpha_0$, if possible.

We spend some time considering identification issues.
Households are described by the complete state variable vector
\[ S = (\lambda_1 \lambda_2 \delta_1 \delta_2 w_1 w_2 Y), \]
with the first four elements unobserved \((S_U)\) and the last three observed \((S_O)\).

The actions \(A = (h_1 \tau_1 h_2 \tau_2)\) are observed.

We assume no measurement error in \(S_O\) and \(A\).

We will say that the distribution of the state variables, \(F_S\), is nonparametrically identified under behavioral rule \(B\) if
\[ A = A_B(S_U S_O) \]
is invertible, i.e.,
\[ S_U = S_U(A, S_O; B) \]
is a function.

If so, \(\hat{F}_S(B)\) is the empirical distribution function of \((S_U(A, S_O; B) S_O)\).
We have the following proposition:

**Theorem**

Let $\mathcal{R}$ be the set of rules that determine time allocations in the household and that are invertible in the sense defined above. Then all $R \in \mathcal{R}$ are equivalent descriptions of the sample information.

- Under our functional form assumptions, $NE \in \mathcal{R}$. Then a consistent estimator of the distribution of $S$ is given by $\hat{F}_S(\cdot; NE)$.
- We show that PO and CPO (models 2 and 3) both are elements of $\mathcal{R}$ for a given value of $\alpha_0$.
- Since the model is saturated excluding the parameter $\alpha_0$, it is not identified.
- For certain values of $\alpha_0$ and $(S_O, A)$, the inverse does not map into $(0, 1)$ for $\lambda_1$ and/or $\lambda_2$ for the PO and CPO models.
- We set $\alpha = \alpha_0 = 0.5$, and found that the mapping was into the right space for all parameters.
Marital sorting patterns provide some information on intrahousehold behavior.

We can determine whether marriage patterns are more consistent with noncooperative or efficient behavior.

We can think of this as testing based on out-of-sample predictions, which in this case consists of unmodeled behavior in the same sample of individuals.

We find fairly strong support for all households behaving cooperatively.

This should be taken with a grain of salt, since Del Boca and Flinn (2012) found that in their most general model in which households endogenously chose to be noncooperative or cooperative, 25 percent behaved noncooperatively. Thus the restriction that all behave in the same manner is strong.
There exist $N$ males and $N$ females.

An individual of gender $i$ is characterized by $(\lambda_i, \delta_i, w_i, y_i)$, and the distribution of these characteristics in the gender $i$ population inhabiting the marriage market is

$$G_i(\lambda, \delta, w, y).$$

A household is defined by the vector of state variables

$$S = (\lambda_1, \delta_1, w_1, y_1) \cup (\lambda_2, \delta_2, w_2, y_2).$$

Given a value of $S$, time allocations are made given the rule $R$. 
Household time allocation choices are denoted $C = (h_1, h_2, \tau_1, \tau_2)$, and we have

$$C = R(S).$$

We consider the two rules discussed previously:

- Noncooperative Nash equilibrium (in reaction functions) ($NE$)
- Constrained Pareto optimal ($CPO$)

We didn’t consider the $PO$ rule since it performed poorly in DBF (2012) when running the horse race between the four specifications.
Let male $i$ and female $j$ be characterized by

\[
m_i = (\lambda_{1i}, \delta_{1i}, w_{1i}, y_{1i})
\]
\[
f_j = (\lambda_{2j}, \delta_{2j}, w_{2j}, y_{2j})
\]

Preference ordering of male $i$ over possible mates is $P(m_i)$, and $P(f_j)$ is similarly defined.

A marriage market is defined by $(M, F; P)$, where

\[
P = \{P(m_1), ..., P(m_N); P(f_1), ... P(f_N)\}
\]

is the collection of preferences in the population, $M = \{m_1, ..., m_N\}$, and $F = \{f_1, ..., f_N\}$.

A matching $\mu$ is a one-to-one correspondence from the set $M \cup F$ onto itself of order 2 (that is $\mu^2(x) = x$) such that $\mu(m) \in F$ and $\mu(f) \in M$. We refer to $\mu(x)$ as the mate of $x$. 
The matching $\mu$ is individually rational if each agent is acceptable to his or her mate. That is, a matching is individually rational if it is not blocked by any (individual) agent.

A matching $\mu$ is stable if it is not blocked by any individual or any pair of agents.

In our application

$$ S_{ij} = m_i \cup f_j. $$

Under rule $R$

$$ C_{ij}(R) = R(S_{ij}). $$

Plug these into the utility functions of $i$ and $j$ to get $V_i(j; R)$. This is the value to individual $i$ of being matched with individual $j$ under behavioral rule $R$.

Note that our situation is one of non-transferable utility.
The given $R$, the preference ordering of $i$ is given by

$$P(m_i | R) = f_{(1)}^i(R), f_{(2)}^i(R), ..., f_{(N)}^i(R),$$

Preference orderings of females are determined in an analogous manner.

A marriage market is defined by $(M, F; R)$, since $(M, F, R) \rightarrow P$, so that

$$(M, F, P(M, F, R))$$
The set of stable matchings associated with marriage market \((M, F, R)\) is given by \(\Theta(M, F, R)\) which is never empty. There in general exist a number of stable matchings.

Male-preferred equilibrium proceeds as:

1. Each male not tentatively matched with a female makes an offer to the preferred one in the set of those who have not previously rejected him.
2. Each woman (tentatively) accepts the proposal that yields the maximum payoff to her among the set of those made this round and the best one from all previous rounds. No man can make another offer to a woman who has previously rejected him.
3. Process continues until no man makes an offer to any woman.

Female preferred stable matching equilibrium is found in the same way after reversing the roles of the men and women in terms of match proposal.
Adding Match Heterogeneity

- For both "realism" and for econometric purposes, it is useful to add a pairwise-valuation that is not strictly a function of the $S_{ij}$.
- Assume that utility of male $i$ matched with female $j$ is given by

$$\tilde{u}_i(a_i, a_j, \varepsilon_{ij}; S_{ij}) = u_i(a_i, a_j; S_{ij}) + \varepsilon_{ij}$$

and for female $j$ matched to male $i$

$$\tilde{u}_j(a_i, a_j, \varepsilon_{ij}; S_{ij}) = u_j(a_i, a_j; S_{ij}) + \varepsilon_{ij}.$$ 

- Then we show that

$$\tilde{V}_i^R(\varepsilon_{ij}) = V_i^R + \varepsilon_{ij}$$
$$\tilde{V}_j^R(\varepsilon_{ij}) = V_j^R + \varepsilon_{ij}$$

for both the $NE$ and $CPO$ cases.
Characteristics

<table>
<thead>
<tr>
<th>Individual</th>
<th>$\lambda$</th>
<th>$\delta$</th>
<th>$w$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>0.20</td>
<td>0.30</td>
<td>10.00</td>
<td>140.00</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.05</td>
<td>0.25</td>
<td>9.00</td>
<td>100.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>0.10</td>
<td>0.60</td>
<td>10.00</td>
<td>50.00</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.30</td>
<td>0.15</td>
<td>18.00</td>
<td>20.00</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.15</td>
<td>0.10</td>
<td>12.00</td>
<td>150.00</td>
</tr>
<tr>
<td>$m_3$</td>
<td>0.40</td>
<td>0.30</td>
<td>20.00</td>
<td>150.00</td>
</tr>
</tbody>
</table>
## Female Payoffs (without match shocks)

### Female Payoffs

<table>
<thead>
<tr>
<th>Male</th>
<th>NE</th>
<th>CPO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$</td>
<td>$f_2$</td>
</tr>
<tr>
<td>$m_1$</td>
<td>5.166</td>
<td>5.579</td>
</tr>
<tr>
<td>$m_2$</td>
<td>5.358</td>
<td>5.823</td>
</tr>
<tr>
<td>$m_3$</td>
<td>4.672</td>
<td>4.972</td>
</tr>
</tbody>
</table>
Male Payoffs (without match shocks)

<table>
<thead>
<tr>
<th>Male</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>4.992</td>
<td>5.195</td>
<td>4.586</td>
<td>4.994</td>
<td>5.195</td>
<td>4.586</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>5.371</td>
<td>5.618</td>
<td>4.840</td>
<td>5.434</td>
<td>5.618</td>
<td>4.856</td>
</tr>
<tr>
<td>( m_3 )</td>
<td>4.584</td>
<td>4.748</td>
<td>4.333</td>
<td>4.585</td>
<td>4.749</td>
<td>4.334</td>
</tr>
</tbody>
</table>
We assume throughout that $\varepsilon_{ij} \sim i.i.d. N(0, \sigma_\varepsilon^2)$.

In this example marriage market of size 3, we report the marriage patterns for 100,000 draws of $\{\varepsilon_{ij}\}_{i=1, j \geq i}^{3, 3}$.

We vary the size of $\sigma_\varepsilon$, with $\sigma_\varepsilon \in \{0, 0.1, 0.2\}$.

As we allow $\sigma_\varepsilon \rightarrow \infty$, all marriage sorting patterns become equally likely.
Table 1
Marriage Patterns from Example

<table>
<thead>
<tr>
<th>Marriages (m – f)</th>
<th>( \sigma_z = 0 )</th>
<th>( \sigma_z = 0.1 )</th>
<th>( \sigma_z = 0.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( NE )</td>
<td>( CPO )</td>
<td>( NE )</td>
</tr>
<tr>
<td>1 – 1</td>
<td>1.000  1.000</td>
<td>0.927  0.862</td>
<td>0.640  0.587</td>
</tr>
<tr>
<td>2 – 2</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.001  0.001</td>
</tr>
<tr>
<td>3 – 3</td>
<td>0.000  0.000</td>
<td>0.071  0.136</td>
<td>0.275  0.328</td>
</tr>
<tr>
<td>1 – 2</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.007  0.008</td>
</tr>
<tr>
<td>2 – 1</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.005  0.007</td>
</tr>
<tr>
<td>3 – 2</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.072  0.068</td>
</tr>
<tr>
<td>1 – 3</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.002  0.002</td>
</tr>
<tr>
<td>2 – 2</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.072  0.068</td>
</tr>
<tr>
<td>3 – 1</td>
<td>0.000  0.000</td>
<td>0.000  0.000</td>
<td>0.072  0.068</td>
</tr>
</tbody>
</table>
- $N$ household sample from PSID

- Males $m_1$ through $m_N$, females $f_1$ through $f_N$, where indexes are arbitrary

- $\Gamma_D$ associates the index of a wife in the data with the index of her husband as $f_{\Gamma_D(i)}$. [66 = $\Gamma_D(1)$ indicates that female 66 in the sample is married to male number 1].

- In the data, $s_k = \{m_k, f_{\Gamma_D(k)}\}$, $k = 1, ..., N$. These are the male and female characteristics in sample household $k$. 
Problematic to think of all individuals in the sample as belonging to the same marriage market, although this is what is commonly assumed in analyzing marriage patterns using a TU setup.

A marriage market of size $n$ is given by

$$\sigma = \{m(1_m), m(2_m), \ldots, m(n_m); f(1_f), f(2_f), \ldots, f(n_f)\},$$

$(j_g)$ indicating the index of the male or female with the $j^{th}$ smallest index among their gender $g$.

Denote the marital partner of male $(i_m)$ in marriage market $\sigma$ by $(j_f) = \Gamma_\sigma((i_m)), i_m = 1, \ldots, n$.

A marriage market $\sigma$ is closed if all $m \in \sigma$ have the same potential marital partners $f \in \sigma$ and no others and if all $f \in \sigma$ have the same potential marital partners $m \in \sigma$ and no others.

A size $n$ closed marriage market $\sigma_n$ is constructed by taking $n$ draws without replacement from the populations $G_1$ and $G_2$. 
Choosing Between R

- Now in any given marriage market of size $n$ that includes $i$ and $\Gamma_D(i)$, that is, a husband and his wife, we will either have $\Gamma_\sigma(i) = \Gamma_D(i)$ or not.

- Conditional probability function

\[
P(\Gamma_\sigma(i) = \Gamma_D(i) | (m_i, f_{\Gamma_D(i)} \tilde{\sigma}_{-i}(n)), R) = \begin{cases} 1 & \text{if } \Gamma_\sigma(i) = \Gamma_D(i) \\ 0 & \text{if } \Gamma_\sigma(i) \neq \Gamma_D(i) \end{cases}
\]

- Objective to compare the consistency of marriage patterns observed under $R = \text{NE}$ and $\text{CPO}$.

- Consider marriage markets to be randomly formed.

- The marginal probability of observing male $i$ married to female $\Gamma_D(i)$ under behavioral rule $R$ is

\[
P(s_i^D | R, n) = \int P(\Gamma_\sigma(i) = \Gamma_D(i) | (m_i, f_{\Gamma_D(i)} \tilde{\sigma}_{-i}(n)), R) dG_{-i}(\tilde{\sigma}_{-i}(n)).
\]

where $\tilde{\sigma}_{-i}(n)$ is all of the other male and female types in this (sub-) marriage market.
Sample Likelihood Construction

- Sequence of terms $P(s_i^D | R, n), i = 1, ..., N$.
- $P(s_i^D | R, n)$ is independent of any other spousal pairing $s_j^D$, $j = 1, ..., n; j \neq i$. We think of the likelihood that two married couples in the data actually belonging to the same (sub-) marriage market as being negligible.
- Then
  \[ P(s_1^D, s_2^D, \ldots, s_N^D | R, n) = \prod_{i=1}^{N} P(s_i^D | R, n). \]
- It follows that the log likelihood is given by
  \[ \ln L(R, n) = \sum_{i=1}^{N} \ln P(s_i^D | R, n). \]
- No parameters appear in this, it being solely a function of $G_1$, $G_2$, and $R$ (except for the parameter $\alpha_0$ in the CPO specification, which we hold fixed at 0.5).
So far $G_1$ and $G_2$ assumed known. Must modify the probability of an observed marriage match to

$$
\hat{P}(s_i^D | R, n) = \int P(\Gamma_{\sigma}(i) = \Gamma_D(i) | (m_i(R), f_{\Gamma_D(i)}(R), \tilde{\sigma}_{-i}(n)), R)
\times d\hat{G}_{-i}(\tilde{\sigma}_{-i}(n) | R),
$$

where $\hat{G}_{-i}$ signifies that the distribution of characteristics of potential spouses is estimated.

Note that as $N \to \infty$, we have

$$
\lim_{N \to \infty} \hat{G}_{-i} = G_{-i} = G
$$

These are used to form

$$
\ln \tilde{L}(R, n) = \sum_{i=1}^{N} \ln \hat{P}(s_i^D | R, n).
$$

One could think of the estimation problem as choice of $R \in \mathbb{R}$,

$$
\hat{R}(n) = \arg \sup L(R, n).
$$
Empirical Analysis

- 282 married couples from the 2006-2007 PSID.
- Aged 25-49, married within last 5 years.
- No children less than 6
- Difficult to get information on non-labor income by spouse. We simply imputed each as

\[ y_{ik} = \frac{w_{ik}}{w_{ik} + w'_{ik}} y_k, \quad i = 1, 2 \]

in household \( k \).
In the first stage, we back out the primitive parameters under each of the two $R$ considered.

See that the time allocation decisions in household production are always efficient, thus these estimates are invariant across $R$.

Individuals look like they value the private good, leisure, more when using the CPO.

Remaining rows are simply data.
Table 2
Means and (Standard Deviations) of Individual Characteristics

\( N = 282 \)

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Husband</th>
<th></th>
<th>Wife</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NE</td>
<td>CPO</td>
<td>NE</td>
<td>CPO</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.401</td>
<td>0.372</td>
<td>0.603</td>
<td>0.579</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.121)</td>
<td>(0.103)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.070</td>
<td>0.095</td>
<td>0.070</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.062)</td>
<td>(0.064)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>( h )</td>
<td>42.414</td>
<td>36.800</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(10.979)</td>
<td>(11.620)</td>
<td></td>
<td></td>
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<tr>
<td>( \tau )</td>
<td>6.688</td>
<td>10.979</td>
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<tr>
<td></td>
<td>(6.215)</td>
<td>(8.183)</td>
<td></td>
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</tr>
<tr>
<td>( w )</td>
<td>21.063</td>
<td>18.097</td>
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</tr>
<tr>
<td></td>
<td>(13.185)</td>
<td>(12.096)</td>
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<tr>
<td>( Y )</td>
<td>37.893</td>
<td>32.895</td>
<td></td>
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<tr>
<td></td>
<td>(89.609)</td>
<td>(71.031)</td>
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</tbody>
</table>
Testing Results

- Criterion utilized is

\[
\ln \tilde{L}(CPO, n) - \ln \tilde{L}(NE, n) = \\
\sum_{i=1}^{N} \ln \int P(\Gamma_\sigma(i) = \Gamma_D(i) | (m_i(CPO) f_{\Gamma_D(i)}(CPO) \tilde{\sigma}_{-i}(n)), CPO) d\hat{G}_{-i}(\sigma)
\]

\[\sum_{i=1}^{N} \ln \int P(\Gamma_\sigma(i) = \Gamma_D(i) | (m_i(NE) f_{\Gamma_D(i)}(NE) \tilde{\sigma}_{-i}(n)), NE) d\hat{G}_{-i}(\tilde{\sigma})
\]

- A positive value is evidence for cooperative behavior (given by \textit{CPO}).
Bootstrapping

- We repeat the computation of the loglikelihood ratio under replication samples drawn from our data set.
- The unit of observation is the married couple, so that in a replication sample we will have multiple draws of a subset of the original households.
- This could cause problems in the GS assignment procedure, since if a sub-marriage market contained two or more identical males or females, ties in the preference orderings would be produced.
- This would create nonunique stable matches, even given the male-proposer assumption.
- Fortunately, the addition of idiosyncratic match values means that even observationally-equivalent individuals will be differentially preferred by individuals on the other side of the market.
We formed (sub-) marriage markets of size $n = 2, \ldots, 7$.

We set $\sigma_\varepsilon = 0.1$. It would be interesting to estimate this value as a free parameter, but given the computational time requirements for the GS implementation, numerical integration, and bootstrapping, this was beyond our means at present.

We performed the analysis for the entire sample and for each of four census regions.

Over the actual sample and all bootstrap replications, the loglikelihood ratio was positive except for 5 cases.

There was 1 bootstrap replication in the East, and 4 in the West that produced negative values.
Table 3
Log Likelihood Values
U.S.
(N = 282)

<table>
<thead>
<tr>
<th>Marriage Market Size (n)</th>
<th>NE</th>
<th>CPO</th>
<th>Difference (CPO – NE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-175.448</td>
<td>-164.691</td>
<td>10.757</td>
</tr>
<tr>
<td>3</td>
<td>-279.686</td>
<td>-260.192</td>
<td>19.494</td>
</tr>
<tr>
<td>4</td>
<td>-369.825</td>
<td>-343.993</td>
<td>25.832</td>
</tr>
<tr>
<td>5</td>
<td>-432.488</td>
<td>-399.996</td>
<td>34.492</td>
</tr>
<tr>
<td>6</td>
<td>-485.183</td>
<td>-445.058</td>
<td>40.125</td>
</tr>
<tr>
<td>7</td>
<td>-529.887</td>
<td>-492.423</td>
<td>37.646</td>
</tr>
</tbody>
</table>
Figure 1
Ln Likelihood Difference
Entire U.S.
Testing between competing models of household behavior is typically performed using over-identified models created by imposing arbitrary parametric restrictions.

We utilized saturated models, under which the behavioral models are identical in a statistical sense, to form “out of sample” predictions on marriage market behavior.

We found that the marital patterns were more consistent with the hypothesis that households behave in a constrained-efficient manner.

Of course, the model is highly stylized and relies on rigid functional form assumptions.

Even the class of saturated models is large and the researcher will always have to choose a parameterization. Thus any empirical analysis will always rely on untestable functional form assumptions.