Appendix III: Models of Ambiguity

Subjective Expected Utility (SEU)

The general form of the SEU model is

$$U(f) = \int_S u \cdot f \, d\pi$$

where $\pi \in \Delta S$ is a subjective probability over states of the world and $u : \mathbf{R}_+ \to \mathbf{R}$ is a cardinal utility index over tokens. If $\pi = (\frac{1}{3}.\frac{1}{3}, \frac{1}{3})$ and $u(t) = -\exp\{-\rho t\}$, this becomes:

$$U_{\text{SEU}}(\mathbf{x}; \rho) = \tfrac{1}{3} \cdot - \exp\{-\rho x_1\} + \tfrac{1}{3} \cdot - \exp\{-\rho x_2\} + \tfrac{1}{3} \cdot - \exp\{-\rho x_3\}.$$

Maxmin Expected Utility (MEU)

The general form of the MEU model is

$$U(f) = \min_{\pi \in \Pi} \int_S u \circ f d\pi,$$

where $\Pi \subseteq \Delta S$ is a set of distributions over states. Here, we assumed Π corresponds to the set of possible distributions specified in the experiment, $\Pi = \{\pi : \pi_2 = \frac{1}{3}\}$. Maintaining the functional form for u, this generates:

$$U_{\text{MEU}}(\mathbf{x}; \rho) = \begin{cases} -\frac{2}{3} \exp\{-\rho x_1\} - \frac{1}{3} \exp\{-\rho x_2\} & \text{if} \quad x_1 \leq x_3 \\ -\frac{2}{3} \exp\{-\rho x_3\} - \frac{1}{3} \exp\{-\rho x_2\} & \text{if} \quad x_1 > x_3 \end{cases}.$$

Recursive Expected Utility (REU)

The general form of the REU model is

$$U(f) = \int_{\Delta S} \varphi \left(\int_S u \circ f \, d\pi \right) d\mu,$$

where $\mu \in \Delta(\Delta S)$ is a probability over probabilities on S and $\phi : u(\mathbf{R}_+) \to \mathbf{R}$ is a possibly nonlinear transformation over expected utility levels. Assuming $\phi(z) = -\exp\{-\alpha z\}$ and μ is uniformly distributed over $\{\pi : \pi_2 = \frac{1}{3}\}$, this becomes:

$U_{\text{REU}}(\mathbf{x};\alpha_0,\rho) =$

$$\frac{1}{\alpha_0}\int_0^{\frac{2}{3}} -\exp\left\{-\alpha_0\left(\begin{array}{c} -\pi_1\exp\{-\rho x_1\} \\ -\frac{1}{3}\exp\{-\rho x_2\} - \left(\frac{1}{3}-\pi_1\right)\exp\{-\rho x_3\}\end{array}\right)\right\} d\pi_1,$$

with α_o defined implicitly as a function of α as follows:

$$\alpha = \left(\frac{30}{1-\exp\{30\alpha_0\}}\right)\alpha_0.$$

The normalization of α in terms of α_0 is to control for differences in risk aversion across subjects. Generally, comparison of ambiguity aversion across individuals is a delicate matter. In the REU model, if the concavity of the ambiguity aggregator is naively measured by α_0, its values can only be sensibly related when risk attitudes are similar. This is because the range of utilities, $\left[-1,-e^{-100\rho}\right]$, is a nonlinear contraction of $[0,100]$, the range of tokens.[1]

To aid intuition, recall that the CARA coefficient mechanically decreases if wealth is denominated in cents rather than dollars. Analogously, the utility for wealth converts wealth levels to utility values and the rate of this nonlinear conversion depends on ρ. Therefore, the estimated ρ mechanically influences the estimated raw ambiguity coefficient α_0. A smaller ρ denominates utility for wealth in wider units (dollars), while a larger ρ denominates utility for wealth in narrower units (cents). While raw ambiguity coefficients α_0 are directly comparable across subjects which share ρ, this comparison is more delicate across subjects with different risk attitudes.

To facilitate more intuitive comparisons, we chose to normalize the scale of utility units for wealth so the thirty tokens is 30 units and zero tokens is 0 units.[2] If one subject's normalized ambiguity coefficient α is higher than another's, then she will be more averse to ambiguous assets returning 30 tokens in good states and 0 tokens in bad ones. The implicit equation can

[1] For example, if $\rho = 0.01$, then the difference in cardinal utility between 100 and 0 tokens is 0.632, while if $\rho = 0.001$, this difference is 0.095.

[2] More concretely, the ambiguity coefficient is normalization can be viewed in the following manner:

$$\alpha = \left(\frac{30}{1-\exp\{30\alpha_0\}}\right)\alpha_0 = \left(\frac{\overbrace{30-0}^{\Delta\text{tokens}}}{\underbrace{-\exp\{-30\alpha_0\}-(-\exp\{0\alpha_0\})}_{\Delta\text{utility}}}\right)\alpha_0.$$

be obviously altered to normalize the comparison for different token levels. where the parameter α reflects the curvature of the second-order expected utility index, thus measuring absolute ambiguity aversion.

α-Maxmin Expected Utility (α-MEU)

The general form of the REU model is

$$U(f) = \alpha \cdot \min_{\pi \in C} \int u \circ f \, d\pi + (1-\alpha) \cdot \max_{\pi \in C} \int u \circ f \, d\pi,$$

where $C \subseteq \Delta S$ is a set of distributions over states and $\alpha \in [0,1]$ reflects the relative weight of the worst versus the best possible belief given f. If we maintain the assumptions from the MEU model that $C = \{\pi : \pi_2 = \frac{1}{3}\}$ and u has CARA form, this specializes to:

$$U_{\alpha\text{-MEU}}(\mathbf{x}; \alpha, \rho) = \begin{cases} \begin{array}{c} -\frac{2}{3}\alpha \exp\{-\rho x_1\} - \frac{1}{3}\exp\{-\rho x_2\} \\ -\frac{2}{3}(1-\alpha)\exp\{-\rho x_3\} \end{array} & \text{if} \quad x_1 \leq x_3 \\ \begin{array}{c} -\frac{2}{3}\alpha \exp\{-\rho x_3\} - \frac{1}{3}\exp\{-\rho x_2\} \\ -\frac{2}{3}(1-\alpha)\exp\{-\rho x_1\} \end{array} & \text{if} \quad x_1 > x_3 \end{cases}.$$

Contraction Representation

The general form of this utility is:

$$U(f) = \min\left\{ \int_S u \circ f \, d\pi : \pi \in (1-\epsilon)\{s(P)\} + \epsilon P \right\},$$

where $s(P) \in \Delta S$ is the Steiner point of the set P.[3] In the experiment, the objective set of priors is $P = \{\pi \in \Delta\{x, y, z\} : \pi(y) = \frac{1}{3}\}$, whose Steiner point is $s(P) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Maintaining the CARA function over tokens, the utility function over portfolios reduces to:

$$U_{\mathrm{C}}(\mathbf{x}; \epsilon, \rho) = \begin{cases} \begin{array}{c} \frac{1-\epsilon}{3}\pi \cdot -\exp\{-\rho x\} + \\ \frac{1}{3} \cdot -\exp\{-\rho y\} + \frac{(1+\epsilon)}{3} \cdot -\exp\{-\rho z\} \end{array} & \text{if } x \leq z \\ \begin{array}{c} \frac{1+\epsilon}{3}\pi \cdot -\exp\{-\rho x\} + \\ \frac{1}{3} \cdot -\exp\{-\rho y\} + \frac{(1-\epsilon)}{3} \cdot -\exp\{-\rho z\} \end{array} & \text{if } z < x \end{cases}.$$

This is exactly α-MEU parameterized so $\alpha = \frac{1-\epsilon}{2}$. Since the contraction model is not separately identified, we provide estimates only for the α-MEU, which can then be interpreted for the contraction model through the explained change of variables.

[3] The convex combination of two sets A and B is defined as the union of their pointwise convex combinations: $\lambda A + (1-\lambda)B = \{\lambda a + (1-\lambda)b : a \in A, b \in B\}$.