Problem 1: Efficient Risk Sharing in an Edgeworth Box Economy

Brad and Angelina have formed a private company to market their services as actors. Brad owns 40% of the shares and Angelina owns 60%. They expect an income of $100 million in a normal year, but if Angelina gets pregnant their income could fall to $60 million. People Magazine puts the probability of pregnancy at 1/4. Neither Brad nor Angelina has any assets besides shares in the company.

Brad and Angelina have different attitudes toward risk. If Angelina spends $c \geq 0$ dollars on consumption, her utility from consumption is $u(c) = c$. If Brad spends $c \geq 0$ dollars on consumption, his utility is $u(c) = \ln c$, where $\ln c$ denotes the natural logarithm of the number $c \geq 0$. When faced with a choice under uncertainty, both Brad and Angelina are assumed to maximize the expected value of utility.

1. What are the states of nature in this problem? What are their probabilities?
2. What are the contingent commodities? How would you describe a contingent commodity bundle?
3. Describe the individual's preferences using a utility function defined on contingent commodity bundles.
4. Describe an allocation for this economy? When is the allocation attainable?
5. What conditions must be satisfied in order for an allocation to be Pareto-efficient?
6. What does the set of Pareto-efficient allocations look like?
7. What do you notice about the co-movement of consumption between the two individuals?
8. Now suppose that there is a large number of individuals like Brad and Angelina who can trade contingent commodities at market-clearing prices. Describe the maximization problems that represent Brad's and Angelina's individual decisions.

9. Derive Brad's demand for contingent commodities when the equilibrium prices are $p = (1, 3)$. What are the optimal bundles for Angelina at these prices? Hint: Draw a diagram illustrating her indifference curve and budget constraint.

10. What are the quantities traded in a competitive equilibrium of this economy?