Agency Costs, Net Worth, and Business Fluctuations

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This paper develops a simple neoclassical model of the business cycle in which the condition of borrowers' balance sheets is a source of output dynamics. The mechanism is that higher borrower net worth reduces the agency costs of financing real capital investments. Business upturns improve net worth, lower agency costs, and increase investment, which amplifies the upturn; vice versa, for downturns. Shocks that affect net worth (as in a debt-deflation) can initiate fluctuations.

Many students of the business cycle have suggested that the condition of firm and household balance sheets (equivalently, the state of borrower "solvency" or "credit-worthiness") is an important determinant of macroeconomic activity. For example, Frederic Mishkin (1978) and Ben Bernanke (1983) argued that the weakness of borrowers' balance sheets contributed to the severity of the Great Depression, while Otto Eckstein and Allen Sinai (1986) put firm balance sheet variables at the center of their analysis of cyclical dynamics. Numerous studies have connected balance sheet conditions with household and firm spending decisions.

In this paper we present a formal analysis of the role of borrowers' balance sheets in the business cycle. Our vehicle is a modified "real business cycle" model, in which a characteristic of the investment technology is an asymmetry of information between the entrepreneurs who organize and manage physical investment and the savers from whom they borrow. Specifically, we assume a "costly state verification" problem, as in Robert Townsend (1979, 1988). This informational asymmetry makes the Modigliani-Miller theorem inapplicable, opening up the possibility of an interesting interaction between real and "financial" (i.e., balance sheet) factors.

Several aspects of balance sheets are potentially of interest to macroeconomists: The particular balance sheet variable upon which we focus is borrower net worth.[1] Net worth is important, we believe, for the following reason: Whenever there is an asymmetry of information between borrowers and lenders, optimal financial arrangements will typically entail deadweight losses (agency costs), relative to the first-best perfect-information equilibrium; these costs manifest themselves as a higher cost of "external," as compared to "internal," funds. For the particular model used here, and for most standard principal-agent models, it is true that the greater the level of net worth of the potential borrower, the less will be the expected agency costs implied by the optimal financial contract.[2] Thus periods of financial "distress" (when borrower net worth is low) are also times of relatively high agency costs in investment.

At the macroeconomic level, the proposition that borrower net worth and the agency costs of investment are inversely correlated has at least two significant implications.

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[1]More specifically, the focus is on "collateralizable" net worth, as opposed to, for example, human capital. For simplicity of modeling, we do not distinguish in this paper among assets that are more or less easy to sell or borrow against. The issues raised by varying balance sheet liquidity are deserving of further research.

[2]This proposition is quite general. For example, in his analysis of the perhaps more familiar Bengt Holmstrom, 1979, principal-agent setup, in which agents' unobserved actions affect project returns, David Sappington, 1983, demonstrated a similar inverse relationship between the agent's wealth and the agency costs of the principal-agent relationship. See Bernanke and Mark Gertler, 1987, for another example and for references. For a model in which this result need not hold, see Joseph Stiglitz and Andrew Weiss, 1987.

First, since borrower net worth is likely to be procyclical (borrowers are more solvent during good times), there will be a decline in agency costs in booms and a rise in recessions. We will show that this is sufficient to introduce investment fluctuations and cyclical persistence into an environment which is rigged to exhibit neither of these features when agency costs are not present; a kind of accelerator effect emerges. Second, shocks to borrower net worth which occur independently of aggregate output will be an initiating source of real fluctuations. A possible example of this is the "debt-deflation," first analyzed by Irving Fisher (1933): During a debt-deflation, because of an unanticipated fall in the price level (or, alternatively, a fall in the relative price of borrowers' collateral, for example, farmland), there is a decline in borrower net worth. This has the effect of making those individuals in the economy with the most direct access to investment projects suddenly un-creditworthy (i.e., the agency costs associated with lending to them are high). The resulting fall in investment has negative effects on both aggregate demand and aggregate supply. We perform a preliminary analysis of the macro effects of a shock to borrower net worth using the model developed below.

We have tried to conduct our analysis solely from first principles. In particular, we derive the form of all financial arrangements endogenously, and we do not rule out randomizing strategies and lotteries. The model is thus necessarily simple, and our analysis should be viewed as an attempt to obtain qualitative insights, rather than to provide an empirically realistic description of real-financial interactions. Other papers in this area which proceed in a general manner similar to ours include those of Roger Farmer (1984), Bruce Greenwald and Joseph Stiglitz (1986), and Stephen Williamson (1987).

The plan of this paper is as follows: Section I lays out the assumptions of the model. Section II analyzes the benchmark-perfect information case. The equilibrium in this case is rigged to involve no business cycle dynamics (investment is constant and output fluctuations are serially independent). Section III introduces asymmetric information and agency costs. Section III, Parts A, B consider optimal lending contracts and the entrepreneurial investment decision for this case. Implications for macroeconomic equilibrium dynamics are investigated in Section III, Parts C, D; we show that, in contrast to the perfect-information case, the economy with agency costs exhibits persistent fluctuations in investment and output, and that redistributions between borrowers and lenders (as in a debt-deflation) have real aggregate effects. Section IV concludes. Additional results on the nature of the optimal contract under "costly state verification" are presented in the Appendix.

I. The Model

Our starting point is a generic "real business cycle model," that is, a stochastic neoclassical growth model. This framework allows us to illustrate starkly the role of financial factors, since in the standard version of the real business cycle model (for example, Edward Prescott, 1986), the assumption of perfect markets implies that financial structure is irrelevant. Specifically, we study an overlapping generations (OG) model, in the general form used by Peter Diamond (1965). The OG approach has the advantage of providing a tractable framework for dynamic general equilibrium analysis, into which heterogeneity among borrowers and lenders is easily incorporated. The OG setup also allows us to abstract (for the present paper) from long-term financial relationships.[3] The "generations" in our model should be thought of as representing the entry and exit of firms from credit markets, rather than as literal generations; a "period" in our model should therefore be interpreted as the length of a typical financial contract (for example, a bank loan).

As in Diamond (1965) we will assume that each generation of individuals lives for two periods; and that individuals are able to earn labor income only in the first period of life,

[3] For equilibrium analyses of the implications of long-term relationships in agency settings, see Edward Green, 1987, and Gertler 1988.

so that they must save to finance second-period consumption. In Diamond's paper it is assumed that saving can be done either by investing in physical capital or by purchasing government bonds: For an expositional reason that will be explained, we make consumption-good inventories, rather than government bonds, the alternative mode of savings to capital investment. Our model also differs from Diamond's original (which was non-stochastic) in that, in the spirit of the real business cycle literature, we allow for shocks to the aggregate production function.

The modifications to Diamond's model just described are minor and have no particularly surprising implications. The significant distinction between our model and Diamond's is that we replace his simple capital production technology (in which output is transformed into capital one-for-one) with a technology that involves asymmetric information. Specifically, we assume that only the entrepreneurs who direct physical investment can costlessly observe the returns to their individual projects; outside lenders must jointly incur a fixed cost to observe those returns. This "costly state verification" model was first analyzed by Townsend (1979, 1988);[4] he showed that the optimal financial arrangements in this setting will involve (most likely randomized) auditing strategies by lenders, which introduce dissipative agency costs into the process. A main goal of this paper is to draw a connection between the condition of borrower balance sheets and these agency costs, and to demonstrate how this connection may play a role in the business cycle.

The detailed assumptions of the model are now stated.

Time. Time is infinite in the forward direction and is divided into discrete periods indexed by t.

Agents. There are overlapping generations of two-period lived agents (and an initial "old" generation in period zero). It will be convenient to assume that there are a countable infinity of agents in each generation. (An implication of this assumption is that we will generally have to deal in per capita, rather than aggregate, quantities.)

There are two classes of agents. An exogenous fraction η of individuals in each generation are called "entrepreneurs." The rest of the population will be called "lenders." Entrepreneurs and lenders differ in endowments and preferences; much more importantly, they differ in that only entrepreneurs have direct access to the investment technology (see below).

The class of entrepreneurs is itself not homogeneous: We will assume that individual entrepreneurs are indexed by a parameter ω, which in the population of entrepreneurs is uniformly distributed on $[0,1]$. Low-ω entrepreneurs will have a lower cost of investment, and thus may be thought of as more "efficient." (Again, see below.)

Goods. There are two goods, a capital good and an output good. Output produced in a given period t may be consumed by agents during t, or it may be invested in the production of the capital good (which becomes available for use in $t+1$). We also allow output to be stored directly as an inventory. The gross rate of return on storage is r, $r \geq 1$; that is, a unit of output stored in t yields r units in $t+1$.

Capital cannot be consumed but can be used in the production of output. Capital is assumed to depreciate fully in one period (this is expositional reasons only).

Production Technologies. There are separate production technologies for output and for capital. The output good is produced by a constant returns technology using capital and labor. We will assume below that labor supplies are fixed;[5] we may therefore write the production function in per capita[6] terms. For any period t, the production function for per capita output y_t is assumed to be

$$(1) \qquad y_t = \tilde{\theta}_t f(k_t),$$

[4]See also Douglas Gale and Martin Hellwig, 1985.

[5]We focus here on explaining investment fluctuations rather than employment fluctuations. Extensions of the results to the case with variable employment is straightforward in principle.

[6]Throughout "per capita" means "per member of a given generation."

where k_t is the amount of capital per head, and $\tilde{\theta}_t$ is a random aggregate productivity shock. We assume that some production can take place without capital, that is, $f(0) > 0$. We take the random variable $\tilde{\theta}$ to be i.i.d. over time, to be distributed continuously over a finite positive support, and to have a mean equal to θ.

Output in period t can be transformed into period-$(t+1)$ capital (without the use of labor) by means of an investment technology. This investment technology comes in discrete, nondivisible units, called "projects." Each entrepreneur is endowed with one of these projects (and we assume that it is too costly to trade or transfer a project away from the original owner). A project belonging to an entrepreneur of type ω takes as input exactly $x(\omega)$ units of the output good y, where $x(.)$ is increasing in ω. With less than $x(\omega)$ units of y, nothing is produced, and the marginal product of increments of y to a project that already has its requisite quantity of input is zero.

Any project that is undertaken in t produces a quantity of capital, which is available for use in $t+1$. The amount of capital produced by a given project is a discrete random variable with possible outcomes κ_i, $i=1,2,\dots,n$, with $\kappa_j \geq \kappa_k$ for $j>k$. (In the main text we will focus on the case $n=2$.) The probability of outcome κ_i is π_i, and the expected outcome is κ. Note that project outcomes do not depend on the entrepreneur's type ω, although the quantity of inputs does (high-ω entrepreneurs require higher inputs); this is a simple way of motivating an upward-sloping supply curve of capital goods. The distribution of outcomes is identical *ex ante* across projects and is not affected by any action or effort of the individual entrepreneur.

To introduce issues of asymmetric information into the model, we assume that the realized outcome of any particular investment project is costlessly observable only by the entrepreneur who operates (was endowed with) that project. Other agents in the economy can learn the realized returns of a given project only by employing an auditing technology. This technology absorbs γ units of the capital good when operated, but reveals the outcome of the audited project to everyone in the economy and without error.[7] An entrepreneur who underreports the return to his project and is not audited can enjoy extra consumption equal to the marginal product of his extra capital. We assume that it is not possible, without auditing, to infer the outcome of a particular entrepreneur's project, for example, it is not possible for others to observe the entrepreneur's second-period holdings of capital or his realized consumption. We will assume that random auditing is feasible; that is, lenders can pre-commit to auditing with some probability (which may depend on the announced outcome). Finally, it makes things a bit simpler to assume that project outcomes are realized, announcements are made, and auditing takes place before the current value of $\tilde{\theta}$ is known; thus, incentive constraints relevant to decisions in t need depend only on expected values of functions of $\tilde{\theta}_{t+1}$.

Investment projects undertaken in a given period have mutually independent outcomes, so that there is no aggregate (per capita) uncertainty about the quantity of capital produced, that is, expected and actual capital per head are the same. Let i_t be the number of investment projects undertaken in t per capita, and let h_t be the fraction of projects initiated in t that are audited. (Both i_t and h_t will be endogenous in general equilibrium.) For any period t, then, next-period capital stock per head, k_{t+1}, is given by

$$k_{t+1} = (\kappa - h_t\gamma)i_t. \tag{2}$$

We also assume

$$\theta f'(0)\kappa > rx(0) + \gamma, \tag{3}$$

$$\theta f'(\kappa\eta) < rx(1). \tag{4}$$

[7]Alternatively, we could have assumed that auditing results are private information to the auditor. Then a role would arise for zero-profit intermediaries between lenders and entrepreneurs. These intermediaries would internalize all auditing costs and, by holding perfectly diversified portfolios, could eliminate the need to be monitored by depositors (see Douglas Diamond, 1984, and Stephen Williamson, 1987).

(3) and (4) will be sufficient to guarantee that it is always profitable for some but not all entrepreneurs to operate.

Endowments. Every individual has a fixed-labor endowment, which must be used during the first period of life. The labor endowment of an entrepreneur is L^e, the endowment of a lender is L. As a normalization, we assume that the economywide per capita labor endowment, $\eta L^e + (1-\eta)L$, is equal to one; this way we avoid carrying around the distinction between per capita and per labor-input variables.

Preferences. Individual preferences are defined over lifetime consumption (there is no disutility of labor). We assume that entrepreneurs care about only expected consumption when old, that is, they are risk-neutral and do not consume when young. Lenders consume in both periods; lenders born in t have identical utility functions of the form

$$U(z_t^y) + \beta E_t(z_{t+1}^o), \tag{5}$$

where z_t^y and z_{t+1}^o are the consumption of the representative period-t lender when young and old, respectively, $U(.)$ is of the usual concave form, and β is a discount factor.

The key restriction imposed by our specification of preferences is that both borrowers and lenders in t are risk-neutral with respect to period-$(t+1)$ consumption; as in Sappington (1983), the assumption of risk-neutrality permits us to concentrate on the role of the agent's wealth in mitigating agency costs, rather than on issues of risk-sharing. The assumptions that entrepreneurs and lenders have different utility functions and, in particular, that entrepreneurs do not consume when young are inessential.

We will focus on the behavior of this model economy in a competitive market environment. In such an environment, our agents' labor supply and consumption/saving behavior are easy to describe. Labor is supplied inelastically, so that, if the market wage per unit of labor endowment is w_t, entrepreneurs have per capita incomes of $w_t L^e$ and lenders have per capita incomes of $w_t L$. (By our normalization assumption, overall per capita income of the young generation is w_t.) Entrepreneurs do not consume when young, so average entrepreneurial saving, S_t^e, is given simply by

$$S_t^e = w_t L^e. \tag{6}$$

Entrepreneurial saving will be an important variable in the subsequent analysis.

Lenders do consume in the first period, so that their saving depends on the interest rate as well as the wage. We will make assumptions to guarantee that saving always exceeds capital formation ((see (9) below), so there is always storage of inventories in equilibrium. Thus the marginal rate of return is fixed at r, the rate of return to storage. Maximization of (5) implies that there is an optimal consumption for lenders when young, denoted $z_y^*(r)$. Average savings by lenders, S_t, is thus

$$S_t = w_t L - z_y^*(r). \tag{7}$$

The main import of (6) and (7) is the establishment of a direct link between wages (marginal productivities) and saving. This link, not empirically unreasonable in itself, is supposed to proxy for the more general idea that savings (and wealth) are greater when the economy is doing well.

We turn now to characterizing the rest of the competitive equilibrium for our model economy.

II. Equilibrium with Perfect Information

As a benchmark, we first consider the competitive equilibrium of our model when auditing is free ($\gamma = 0$), so that information is perfect. We begin by solving for equilibrium in period t, given the inherited capital stock per head, k_t; we then turn to the (trivial) dynamics.

Let $\hat{q}_{t+1}$ be the expected (as of t) relative price of capital in $t+1$; then $\hat{q}_{t+1}\kappa$ is the expected gross return from each investment project. The opportunity cost of investing for a type-ω entrepreneur is $rx(\omega)$. Assuming that entrepreneurs invest when they can earn nonnegative profits, the efficiency level $\bar{\omega}$ of the entrepreneur who is just indifferent between investing and storing satisfies

$$\hat{q}_{t+1}\kappa - rx(\bar{\omega}) = 0. \tag{8}$$

The projects of entrepreneurs with efficiency levels ω of $\bar{\omega}$ or better (i.e., $\omega \leq \bar{\omega}$) produce an expected surplus, relative to storage. (Note that $\bar{\omega}$ is a function of $\hat{q}_{t+1}$.)

We assume, as noted in the previous section, that economywide savings always exceed the amount required by profitable projects

$$\eta S^e + (1-\eta)S > \int_0^{\bar{\omega}} x(\omega)\,d\omega, \tag{9}$$

for any $\bar{\omega}$, for any realization of θ, and for any inherited level of k_t. (For this to be plausible, the entrepreneurial sector needs to be a relatively small part of the economy.) Thus some saving always funds inventory accumulation in equilibrium and the marginal rate of return is always r.

The interesting issue is the joint determination, in period t, of $\hat{q}_{t+1}$ and the next-period capital stock per head, k_{t+1}. Let i_t be the number of projects undertaken (per capita) in t. Then we have

$$i_t = \bar{\omega}\eta, \tag{10}$$

$$k_{t+1} = \kappa i_t. \tag{11}$$

(10) follows from the observation that any entrepreneur of efficiency level $\bar{\omega}$ or better (which, since ω is uniform, is a fraction $\bar{\omega}$ of all entrepreneurs) will find it profitable to invest when the cost of funds is r. Thus (10) states that investment per capita equals the fraction of entrepreneurs who invest times the fraction of the population who are entrepreneurs. (11) says that the per capita future capital stock will be the average productivity of an investment project (which, by the law of large numbers, is non-stochastic) times the per capita number of projects.

Combining (8), (10), and (11) yields a "capital supply curve" for the perfect information case (call it the SS curve):

$$\hat{q}_{t+1} = rx(k_{t+1}/\kappa\eta)/\kappa \quad [\mathrm{SS}] \tag{12}$$

The SS curve is upward-sloping (see Figure 1). A higher expected value of $\hat{q}_{t+1}$ raises the number of entrepreneurs who can profitably invest, so that a larger share of savings is devoted to capital formation instead of to consumption good inventories.

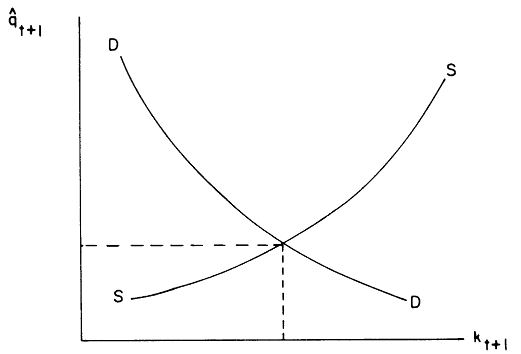

FIGURE 1

The "capital demand curve" for the perfect information case, DD, is just the condition that the expected price of capital equals its expected marginal product

$$\hat{q}_{t+1} = \theta f'(k_{t+1}) \quad [\mathrm{DD}], \tag{13}$$

where, recall, θ is the mean of $\tilde{\theta}_{t+1}$ and f' denotes the derivative. The DD curve is downward-sloping (see Figure 1); the marginal product of capital is higher when the capital stock (per head) is smaller. In each period t, $\hat{q}_{t+1}$ and k_{t+1} are determined as the solution of (12) and (13), that is, as the intersection of the capital supply and demand curves in Figure 1.[8]

The dynamics in the perfect information case are extremely simple: Since (12) and (13) are independent of period-t state variables, $\hat{q}$ and k are constant over time. Investment is fixed and the quantity of production of the output good fluctuates in proportion to the (serially uncorrelated) productivity shock. The amount of consumption is positively serially correlated, since in high-productivity periods there is both more consumption and more inventory accumulation.

[8]The solution exists and is unique. Existence is guaranteed by (3) and (4), uniqueness by the fact that DD always slopes downward and SS always slopes upward.

We have thus developed a benchmark case in which investment is constant. This was the motivation for introducing inventories which pay a fixed-gross yield: The presence of this fixed-return mode of saving has the effect of making the supply of investment funds perfectly elastic with respect to the interest rate, while investment demand (which, in the absence of information problems, depends only on the expected marginal productivity of capital and the marginal cost of producing new capital) is fixed over time. In contrast, when information asymmetries are present, investment demand will vary and be history-dependent.

III. Equilibrium with Asymmetric Information

A. *The Optimal Financial Contract*

We now re-introduce imperfect information ($\gamma > 0$) and begin the process of deriving the dynamic macroeconomic equilibrium for this case. This is done in stages: We begin by considering the situation of an entrepreneur who with certainty intends to undertake his project,[9] but for whom the required project input exceeds his personal savings ($x(\omega) > S^e$). In this case, the entrepreneur must borrow in order to invest. Our task is to determine the optimal arrangements under which this borrowing can take place.

The entrepreneur is assumed to be borrowing from a lender (or consortium of lenders) who have an opportunity cost of funds r. At this point our analysis is partial equilibrium, in that we assume that the entrepreneur's own savings (S^e), the expected relative price in the next period of the produced capital good ($\hat{q}$), and the safe rate of return (r) are taken as exogenous.

The optimal contract is found by application of the revelation principle. Formally, the entrepreneur's problem is to maximize his expected next-period consumption, subject to the constraints that (i) the lender(s) receive an expected rate of return of no less than r, (ii) the entrepreneur has no incentive to lie about realized project outcomes, and (iii) the state-contingent consumptions and auditing probabilities[10] are feasible. The control variables are outcome-dependent auditing probabilities and the entrepreneur's realized consumption levels, which may be contingent both on the project outcome and on whether an audit has occurred.

The Appendix formally states the problem and gives a number of results for the n-state case. For the main text, we choose to specialize to the case $n = 2$. This is for the sake of concreteness; for $n = 2$, it is possible to write out the optimal contract explicitly, while for a larger number of states we have only been able to obtain indirect characterizations. It is worth stressing, however, that the n-state optimal contract does have the "net worth property," that expected agency costs are decreasing in the amount of entrepreneurial savings contributed to the project. Therefore, we can safely claim that allowing for an arbitrary number of possible project outcomes in our macroeconomic analysis would not affect the qualitative nature of our results.

With $n = 2$, there are two possible project outcomes: In state 1 (which occurs with probability π_1), the project produces κ_1 units of the capital good; in state 2 (probability π_2) it produces κ_2 units. State 1 is the "bad" state ($\kappa_1 < \kappa_2$). For an entrepreneur of type ω, the amount borrowed is $x(\omega) - S^e$, and the lenders' required expected return is $r(x(\omega) - S^e)$.

The Appendix shows that, under the optimal contract no auditing occurs when the best possible state (here, state 2) is an-

[9] We ignore for the moment his option of putting his savings into consumption-good inventories.

[10] We are allowing general random auditing strategies, which (as Townsend, 1979, first pointed out) may be significantly more efficient than nonrandom strategies. An implication of permitting random auditing is that the optimal contract will not be in the form of a debt contract, as it is when auditing is nonrandom (Dilip Mookherjee and Ivan Png, 1987; Townsend, 1988). Importantly, our macro results are essentially the same whether stochastic auditing is permitted or not. Thus, we do not have to rely on financial contracts taking a particular debt or equity form. For our purposes, the important distinction is between internal and external finance, not between debt and equity per se.

nounced. Thus, for $n=2$, lenders audit only when the entrepreneur declares the bad state (state 1). Let p be the probability of an audit in the bad state, let c_i be the entrepreneur's consumption payoff when he announces state i $(i=1,2)$ and is not audited, and let c^a be his consumption payoff when he announces the bad state and is audited.[11] Then the optimal contract is found by choosing the vector $\{p, c_1, c_2, c^a\}$ to solve[12]

$$(14) \quad \max \pi_1(pc^a+(1-p)c_1)+\pi_2 c_2$$

subject to

$$(15) \quad \pi_1[\hat{q}\kappa_1 - p(c^a+\hat{q}\gamma)-(1-p)c_1] + \pi_2[\hat{q}\kappa_2 - c_2] \geq r(x-S^e),$$

$$(16) \quad c_2 \geq (1-p)(\hat{q}(\kappa_2-\kappa_1)+c_1),$$

$$(17) \quad c_1 \geq 0,$$

$$(18) \quad c^a \geq 0,$$

$$(19) \quad 0 \leq p \leq 1,$$

where $\hat{q}$ is the expected (next-period) relative price of capital.

Constraint (15) (which specializes the appendix inequality (A2)) requires that lenders receive an expected return of r; this constraint can be shown always to bind. Constraint (16) (which corresponds to the appendix inequality (A3)) is the truth-telling constraint on the entrepreneur; it requires that the contract be structured so that the entrepreneur has no incentive to misreport the good state as the bad state. (16) binds if $p>0$. Constraints (17) and (18) require that the entrepreneur's consumption in the bad state be nonnegative.[13] These "limited liability" constraints restrict the entrepreneur's ability to pay lenders if the project's outcome is bad; as we shall see, the presence of these constraints is the basic reason that the entrepreneur's net worth is important. (19) is a feasibility constraint on p.

The optimal contract for $n=2$ (the solution to (14)–(19)) is relatively simple. There are two regimes: In the first regime, the entrepreneur's net worth is sufficiently large that he is able to pay lenders their required return even in the worst state.[14] That is,

$$(20) \quad \hat{q}\kappa_1 \geq r(x(\omega)-S^e).$$

There is no agency problem in this case, since the entrepreneur can always pay off. Optimal auditing probabilities are always zero, and the lender's payoff is independent of the project's outcome. This might be called the "full-collateralization" case, since the entrepreneur's contribution is so large relative to the input requirement that the lenders face no idiosyncratic risk.[15] The entrepreneur's expected consumption in the full-collateralization case, $\hat{c}_{fc}$, is the expected project output less the required return to lenders:

$$(21) \quad \hat{c}_{fc} = \hat{q}\kappa - r(x(\omega)-S^e),$$

where, recall, $\kappa = \pi_1\kappa_1+\pi_2\kappa_2$ is the mean project output.

If entrepreneurial savings S^e are insufficient, so that (20) fails, we are in the "incomplete collateralization" case, and there will be positive agency costs. In this case the incentive constraint (16) and the "limited liability" constraints (17) and (18) are binding,[16] as well as the outside return constraint

[11] More precisely, c^a is the payoff if the entrepreneur is audited and found to be telling the truth. The optimal payoff if the entrepreneur is audited and found to be lying is easily shown to be zero.

[12] The dependence of the control variables and of x on ω is suppressed in (14)–(19).

[13] A separate restriction for c_2 is unnecessary, since (16) and (19) imply $c_2 \geq 0$.

[14] Recall that we are assuming that project outcomes are realized and announcements made before θ_{t+1} (and thus q_{t+1}) is known. Thus, this ability to repay needs to hold only for the expected value of q, not for the realized value. The alternative assumption complicates the analysis slightly, because incentive constraints would depend on the realized value of q_{t+1}; but qualitative results are unchanged.

[15] If $\kappa_1 = 0$, then "full collateralization" requires $S^e \geq x(\omega)$.

[16] (17) and (18) bind because it is optimal to concentrate the entrepreneur's payoff in the good state, thereby minimizing his incentive to misreport.

(15) (which always binds). The optimal auditing probability p, conditional on the entrepreneur's announcement of state 1, is now given by

$$p=\frac{r(x(\omega)-S^e)-\hat{q}\kappa_1}{\pi_2\hat{q}(\kappa_2-\kappa_1)-\pi_1\hat{q}\gamma}. \tag{22}$$

The equation (22) is obtained from (15) through (18), which all hold with equality in this case.

The optimal auditing probability p is just sufficient to guarantee that the entrepreneur will report honestly when the good state occurs. Under the assumption that $\pi_2(\kappa_2-\kappa_1)-\pi_1\gamma>0$, which we will maintain, p is always positive when there is incomplete collateralization ((20) fails). (It can also be shown that, whenever expected entrepreneurial consumption is positive, $p<1$.) The optimal auditing probability, and thus expected agency costs (which we identify with expected auditing costs, equal to $\pi_1 p\hat{q}\gamma$), is decreasing in the entrepreneur's contribution to the project, S^e. The intuition for the inverse relation of S^e and expected auditing costs is as follows: When S^e is low, lenders require a large total return, which reduces the entrepreneur's consumption in the good state. (The entrepreneur's consumption in the bad state is always optimally zero.) With a low c_2, the entrepreneur has less at risk if he falsely claims the bad outcome when the good state has occurred; thus he must be audited more frequently.

Expected entrepreneurial consumption when there is incomplete collateralization, $\hat{c}_{ic}$, is given by

$$\hat{c}_{ic}=\alpha\{\hat{q}\kappa-r(x(\omega)-S^e)-\pi_1\hat{q}\gamma\}, \tag{23}$$

where $\alpha\equiv[\pi_2\hat{q}(\kappa_2-\kappa_1)]/[\pi_2\hat{q}(\kappa_2-\kappa_1)-\pi_1\hat{q}\gamma]>1$. Note that $\partial\hat{c}_{ic}/\partial S^e=\alpha r>r$; when collateralization is incomplete, the return to "inside" funds exceeds the return to "outside" funds. This is because additional inside funds not only replace outside funds but also reduce expected agency costs. Hence the average "cost of capital" in this model depends upon the mixture of internal and external finance.

B. *The Entrepreneurial Investment Decision*

The derivation of the optimal financial contract assumed that the entrepreneur is committed both to undertaking his investment and to contributing all of his personal savings to the project. As the next step toward constructing a market equilibrium, we now consider the effects of relaxing these provisional assumptions.

In the perfect information case, we distinguished two types of entrepreneurs, those that could profitably invest and those that could not. In the imperfect information case, it turns out, we must allow for three types of entrepreneurs. For any given period t, let $\underline{\omega}$ and $\bar{\omega}$ be the levels of entrepreneurial ability that satisfy

$$\hat{q}\kappa-rx(\underline{\omega})-\hat{q}\pi_1\gamma=0, \tag{24}$$

$$\hat{q}\kappa-rx(\bar{\omega})=0. \tag{25}$$

Entrepreneurs with efficiency levels less than $\underline{\omega}$ have projects whose expected net return[17] is positive, *even if announcements that the bad state has occurred precipitate auditing with probability one* ($p=1$). Call entrepreneurs with $\omega\le\underline{\omega}$ "good" entrepreneurs. Entrepreneurs with efficiency levels $\omega\le\bar{\omega}$, on the other hand, are guaranteed to have positive expected net returns only if there is no auditing ($p=0$), that is, when there are no dissipative agency costs; designate entrepreneurs in this range but who are not "good" (i.e., $\underline{\omega}<\omega\le\bar{\omega}$) as "fair" entrepreneurs.[18] Finally, "poor" entrepreneurs ($\omega>\bar{\omega}$) have projects that have negative expected net returns even if agency costs are zero.

Note again that, as in Section II, both $\underline{\omega}$ and $\bar{\omega}$ are (increasing) functions of the expected relative price of capital, $\hat{q}$. Thus, our

[17]Defined as the expected value of output, less the opportunity cost of inputs and expected auditing costs.

[18]$\bar{\omega}$ is defined exactly as in the perfect information case; compare (8). Thus, for a given $\hat{q}$, both "good" and "fair" entrepreneurs would be "profitable" under perfect information. (Note, though, that the value of $\hat{q}$ in equilibrium is likely to differ in the two cases.)

classification of entrepreneurs is conditional on the value of $\hat{q}$.

Also, for any given ω, let us define the "full-collateralization" level of entrepreneurial saving, $S^*(\omega)$, to be the quantity that exactly satisfies (20). That is,

$$(26) \qquad S^*(\omega) = x(\omega) - (\hat{q}/r)\kappa_1.$$

An entrepreneur of type ω who contributes savings in amount greater than or equal to $S^*(\omega)$ to his project will be able to borrow and invest with zero probability of auditing (and thus with no expected agency costs). $S^*(\omega)$ is a (decreasing) function of $\hat{q}$.

We are now in a position to represent the opportunity sets of different types of entrepreneurs graphically (see Figure 2). For each class of entrepreneurs (good, fair, or poor), the solid line graphs expected entrepreneurial consumption (conditional on undertaking the project) as a function of the amount of savings contributed by the entrepreneur.[19] The dotted line, which in each graph is a ray from the origin with slope r, is the opportunity cost of saving, as determined by the alternative storage technology.

The optimal choices of each class of entrepreneur are easy to discover using Figure 2. Consider first the poor, or inefficient, entrepreneurs. For this group, the total return to storage exceeds the return to investment for any level of savings. Thus, poor entrepreneurs will put their savings into inventory (equivalently, become lenders) and will not undertake their projects.

Good entrepreneurs are in the opposite situation. As long as the quantity of savings that the entrepreneur contributes to his project is less than the full-collateralization level $S^*(\omega)$, the marginal (and average) return to investing in the project is greater than the return to holding inventories. Thus the good entrepreneur will put all of his savings into

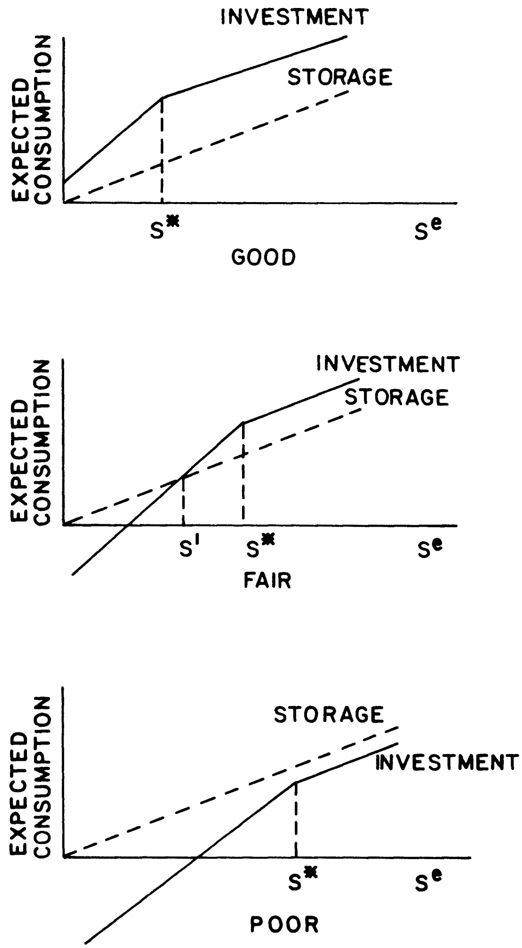

FIGURE 2

his own project,[20] up to the point where his contribution equals $S^*(\omega)$; beyond this point, he is indifferent between investing in his own project and either storing inventories or lending to others. If the good entreprenur's total savings are less than $S^*(\omega)$, his project will be audited with positive probability, so that agency costs are present. If $S^e \geq S^*(\omega)$, the project can be undertaken with zero-agency costs.

The fair entrepreneur's case is a bit more complicated. First, note that his opportunity set has three regions: If $S^e < S'(\omega)$ (where

[19]This line is defined by (23) for $S^e \leq S^*(\omega)$ and by (21) for $S^e > S^*(\omega)$, for a representative ω in each range. Figure 2 ignores the nonnegativity constraint on entrepreneurial consumption. This is harmless, since, as we shall see, entrepreneurs will not want to invest in the range where nonnegativity binds.

[20]Recall that we are assuming risk-neutrality, so that diversification is not an issue.

S' is defined, as in the diagram, as the level of savings at which the total returns to storage and investment are equal), the entrepreneur will store or lend rather than invest. If $S'(\omega) \leq S^e < S^*(\omega)$, then the entrepreneur will invest (contributing all his funds to the project), and will face a positive auditing probability. Finally, if $S^e \geq S^*(\omega)$, the entrepreneur will invest and will contribute enough to the project to ensure full collateralization. (He will be indifferent about the disposition of his savings in excess of $S^*(\omega)$.) Thus the fair entrepreneur's decision about whether to invest or store, as well as the auditing probability if he does invest, may depend on his level of savings.

We say "*may* depend" because of an interesting wrinkle that arises in this case. The upper envelope of the dashed and solid lines, which defines the fair entrepreneur's opportunity set, is *convex* between zero and $S^*(\omega)$. This means that the (risk-neutral) intermediate-quality entrepreneur in principle would be happy to enter a fair lottery. In particular, he would like to risk his savings in a lottery that pays $S^*(\omega)$ with probability $S^e/S^*(\omega)$, zero otherwise. An entrepreneur who wins this gamble would become fully collateralized and would be able to invest without agency costs; a loser gets zero consumption. *Ex ante*, this gamble improves the fair entrepreneur's expected utility.[21]

This incentive for extra risk-taking seems to arise generically in models in which agency costs are decreasing in the wealth of the agent (so that there may be increasing returns to wealth over a range).[22] It is a legitimate objection to our approach that lotteries of this sort are not seen in reality.[23] Presumably risk-aversion, which we exclude, is the major explanation. Any other factor which introduces concavity into the relationship between returns and wealth (for example, if agency cost savings diminish as wealth rises; see Bernanke-Mark Gertler, 1987) would also reduce the incentive for this sort of gambling.

For present purposes, in the spirit of maintaining internal consistency, we will assume that this "savings lottery" among the fair entrepreneurs (or equivalently, between the fair entrepreneurs and, say, lenders) does take place. (Our basic macro results are essentially the same whether we allow the lottery or rule it out arbitrarily.) Under this lottery, a fraction $g(\omega) = S^e/S^*(\omega)$ of entrepreneurs of type ω win their gamble and become fully collateralized investors; the rest get zero consumption and do not invest.

The outcomes of the good and fair entrepreneurs show two contrasting ways in which the quantity of borrower wealth affects investment efficiency. All investors with $\omega \leq \bar{\omega}$ would invest in a world without information problems,[24] since the net returns to their projects when there are no agency costs are positive. With asymmetric information, all "good" entrepreneurs still invest, but they do so with positive expected agency costs. These agency costs decrease in the level of entrepreneurial savings, S^e. Only a fraction of "fair" entrepreneurs invest;[25] those that do experience no agency costs. This occurs because, as a class, the fair entrepreneurs become essentially self-financing. (On net, the fair entrepreneurs are able to borrow from lenders only the difference between full collateralization and the input cost of their projects.) Thus, investment by the intermediate class of entrepreneurs is restricted essentially to the amount of "internal equity" they can generate. The result that entrepreneurs known to be more efficient can borrow externally (albeit with a higher cost

[21] This can be shown formally by modifying the problem (14)–(19) to allow the entrepreneur to enter any fair savings lottery. Only intermediate-quality entrepreneurs will actively desire to enter such a lottery, because of the shape of their payoff functions; good and poor entrepreneurs will be indifferent.

[22] See Bernanke–Gertler, 1987, for another example. Although he does not consider them, lotteries would also seem to ameliorate the principal-agent problem studied by Sappington.

[23] It does seem, though, that people who need a "stake," say to open a business, may exhibit risk-loving behavior.

[24] Poor entrepreneurs, with $\omega > \bar{\omega}$, do not invest in either case.

[25] If lotteries were ruled out, it would still be the case that only a fraction of fair entrepreneurs invest; agency costs would preclude the relatively less efficient ones from undertaking projects.

of funds externally than internally), but that more marginal projects must be largely self-financing, is at least suggestive of real-world arrangements.

C. *Within-Period Equilibrium*

We show now how the expected price and the quantity of new capital are determined within a period, given the inherited capital stock, and assuming $\gamma > 0$.

In any period t, the inherited per capita capital stock k_t is predetermined. With labor supplied inelastically, output is determined by the production function and the random productivity shock θ (compare (1)). The wage and therefore lender and entrepreneurial saving in t are determined.

We would like to know the supply and demand curves for capital. Consider the determination of capital supply, for a given expected relative price of capital, $\hat{q}$.[26] For $\omega \le \underline{\omega}$, define $p(\omega)$ to be the probability that an entrepreneur of type ω is audited (in the bad state). The function $p(\omega)$ is defined by

$$(27)\quad p(\omega) = \max\left(\frac{rx(\omega) - \hat{q}\kappa_1 - rS^e}{\hat{q}(\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma}, 0\right)$$

for $\omega \le \underline{\omega}$ (compare (22)). $p(\omega)$ is decreasing in $\hat{q}$ and in S^e; $p(\omega) = 0$ for $S^e \ge S^*(\omega)$.

Fair entrepreneurs (with types between $\underline{\omega}$ and $\bar{\omega}$), because of the "collateralization lottery," do not face the agency cost of auditing when they invest; but only the fraction of fair entrepreneurs who win the lottery are able to invest. Let $g(\omega)$, defined for $\underline{\omega} < \omega \le \bar{\omega}$, be the fraction of fair entrepreneurs of type ω who can invest (and $1 - g(\omega)$ be the fraction who are excluded). Using the fact that $g(\omega) = S^e/S^*(\omega)$, and substituting from (26), we have

$$(28)\quad g(\omega) = \min\left(\frac{rS^e}{rx(\omega) - \hat{q}\kappa_1}, 1\right)$$

for $\underline{\omega} < \omega \le \bar{\omega}$. The quantity $g(\omega)$ increases in $\hat{q}$ and S^e, and for $S^e \ge S^*(\omega)$, we have $g(\omega) = 1$.

Again, entrepreneurs of type $\omega > \bar{\omega}$ do not invest.

Total capital formation (per head) in this case is given by

$$(29)\quad k_{t+1} = \left[\kappa\underline{\omega} - \pi_1\gamma\int_0^{\underline{\omega}} p(\omega)\, d\omega\right]\eta + \left[\kappa\int_{\underline{\omega}}^{\bar{\omega}} g(\omega)\, d\omega\right]\eta,$$

where the expression in the first set of brackets reflects capital formation (net of auditing costs) by good entrepreneurs, and the second expression in brackets is capital formation by fair entrepreneurs. (29) can be rewritten as

$$(30)\quad k_{t+1} = \left\{\kappa\bar{\omega} - \left[\int_0^{\underline{\omega}} \pi_1\gamma p(\omega)\, d\omega + \int_{\underline{\omega}}^{\bar{\omega}} \kappa(1 - g(\omega)\, d\omega\right]\right\}\eta \quad [\text{SS}].$$

(30) is the capital supply curve for the $\gamma > 0$ case. It is depicted in Figure 3 as the $S'S'$ curve, along with the perfect information capital supply curve (SS) (derived in Section II) for reference. Several points can be made about the $S'S'$ curve.

First, $S'S'$ lies to the left of SS, that is, capital supply is always less in the imperfect information case. ((From (9) and (10), $k_{t+1} = \kappa\bar{\omega}\eta$ when $\gamma = 0$; from (30), $k_{t+1} \le \kappa\bar{\omega}\eta$ when $\gamma > 0$.) This is because imperfect collateralization when $\gamma > 0$ increases the agency costs for those projects undertaken and (perhaps more significantly) leads to a decline in the number of projects that can be profitably initiated.

Second, the $S'S'$ curve is upward-sloping in $(\hat{q}_{t+1}, k_{t+1})$ space. This can be verified by differentiating the expression for k_{t+1} in (30) with respect to $\hat{q}_{t+1}$, using (27), (28), and the definitions of $\underline{\omega}$ and $\bar{\omega}$ ((24) and (25)). (Note that the dependence of the cutoff efficiency levels $\underline{\omega}$ and $\bar{\omega}$ on $\hat{q}_{t+1}$ must be explicitly taken into account.) Since as $\hat{q}$ gets large enough the system approaches "full collateralization" ($p(\omega)$ and $1 - g(\omega)$ approach zero), the $S'S'$ and SS curves coincide at high values of $\hat{q}$.

[26] $\hat{q}$ means $\hat{q}_{t+1}$. We continue to drop the time subscript where there is no ambiguity.

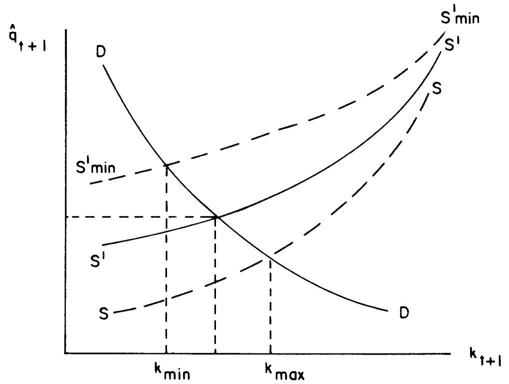

FIGURE 3

Third, unlike that of the SS curve, the position of the $S'S'$ curve depends on a period-t state variable, namely, entrepreneurial savings S^e (which enters into the expressions for $p(\omega)$ and $1-g(\omega)$). High values of S^e (which move the system closer to full collateralization) push the $S'S'$ curve down toward the SS curve; lower values of S^e move the $S'S'$ curve up and away from the SS curve. $S'S'$ reaches its farthest point from SS when S^e is at its minimum value.[27] The dashed line marked $S'S'_{(\min)}$ in Figure 3 describes this boundary.

The determination of the demand for capital is much simpler: Capital demand in the $\gamma > 0$ case is given by the identical DD curve as in the $\gamma = 0$ case (equation (13)). The intersection of $S'S'$ and DD (see Figure 3) determines capital formation in period t. Output which is saved in t but is not invested is stored, to be consumed in the subsequent period. This fully determines the within-period equilibrium.[28]

Two useful comparative statics results follow directly. First, consider the effect of a rise in current income, emanating from an increase in either the inherited capital stock k_t or the value of the productivity shock θ_t. In either case young entrepreneurs (as well as young lenders) will accumulate more savings. Higher entrepreneurial saving (S^e) lowers agency costs and therefore shifts the $S'S'$ curve down to the right, raising k_{t+1} and lowering $\hat{q}_{t+1}$. This effect is not present in the perfect information case. We see, therefore that the presence of agency costs induces a channel of dependence of investment on income as long as the incentive constraint binds for some entrepreneurs.

Second, imagine a redistribution of (labor) endowment from entrepreneurs to lenders, that is, raise L and lower L^e so that $\eta L^e + (1-\eta)L$ is still equal to one. The motivation for this exercise is to model an aspect of "debt-deflation," a situation in which a combination of unindexed debt contracts and unexpected deflation redistributes wealth from the debtor class to the creditor class.[29] A fall in L^e lowers S^e, shifting the $S'S'$ up to the left; $\hat{q}_{t+1}$ rises and k_{t+1} falls. Thus a redistribution from borrowers to lenders depresses capital spending. The intuition is that lower entrepreneurial wealth raises the agency costs associated with capital finance, reducing the net return to investment.[30]

D. *Dynamics*

We are now equipped to consider aggregate dynamics for the $\gamma > 0$ case.

As we have already seen, the (benchmark) perfect information ($\gamma = 0$) case has no interesting dynamics; the capital stock is fixed

[27] Given L^e, the minimum possible value of S^e occurs when wages are minimum, which in turn occurs when capital per head is zero and $\tilde{\theta}$ is at its minimum possible value. Assumptions made above suffice to guarantee that this minimal wage is positive.

[28] Condition (3) guarantees that the $S'S'_{(\min)}$ curve intersects the vertical axis below the DD curve, so that investment is positive no matter how severe the agency problem. For an analysis of investment collapse induced by agency problems, see Bernanke-Gertler, 1987.

[29] The original discussion of debt-deflation is Fisher, 1933. See Bernanke, 1987, and James Hamilton, 1987, for some evidence that debt-deflation was an important feature of the Great Depression. Why debt contracts are in practice typically unindexed is a deep puzzle which we will not discuss here.

[30] If we had assumed diminishing rather than constant returns to the storage technology, the debt-deflation (by driving a larger share of savings into storage, the alternative asset) would also cause the safe rate of return to fall; this is the "flight to quality" phenomenon. Note though that, since $\hat{q}$ rises, debt-deflation cannot explain a stock market crash without introducing additional factors (such as aggregate demand externalities).

and production varies only with the productivity shock $\tilde{\theta}$. The $\gamma > 0$ case is different because of the dependence of the capital supply curve on entrepreneurial savings S^e. The $S'S'$ curve is shifted by variations in either the current capital stock k_t or the productivity shock $\tilde{\theta}_t$, either of which affects the value of the entrepreneurs' labor endowments and thus their savings. Thus future capital depends on both current capital and productivity, leading to a nontrivial dynamics.

Consider how a productivity shock is propagated over time when $\gamma > 0$. In the informationally constrained region, a (temporary) rise in $\tilde{\theta}$ stimulates investment by increasing entrepreneurial net worth (since incomes increase). The $S'S'$ curve shifts rightward. The expansion persists because the rise in the future capital stock makes investment in the subsequent period higher than it would otherwise be. Through the same mechanism, negative productivity shocks may induce a persistent investment downturn. This is our attempt to capture in a formal model the following sort of intuition: In good times, when profits are high and balance sheets are healthy, it is easier for firms to obtain outside funds. This stimulates investment and propagates the good times. Conversely, poor financial health in bad times reduces investment and reinforces the decline in output. Note again that this rationalizes a sort of accelerator effect of income on investment; note also that countercyclical agency costs are crucial to the story.

The dynamic effects of productivity disturbances may be asymmetric in this setup. (Sharp investment downturns are more likely than sharp upturns.) For example, suppose the initial level of capital equals the value the economy attains under perfect information; denote this value as $k_{\max}$. Next, for the case $k_t = k_{\max}$, let θ^* be the minimum value of $\tilde{\theta}$ which generates a level of S^e large enough to make all "good" and "fair" entrepreneurs fully collateralized.[31] Diagrammatically, θ^* is the minimum realization of $\tilde{\theta}$ which makes the capital supply curve ($S'S'$) exactly overlap the perfect information supply curve (SS), given $k_t = k_{\max}$. In this case, a realization of $\tilde{\theta}$ above θ^* has no effect on investment. The $S'S'$ does not move outward since all the efficient entrepreneurs are already fully collateralized. In contrast, a realization of $\tilde{\theta}$ below θ^*, by pushing some entrepreneurs below full collateralization and moving the $S'S'$ curve left, induces an investment downturn.

An explicit characterization of the stochastic steady state of this model cannot be obtained without some additional assumptions (for example, about functional forms). We may note several points, however: First, as long as some part of the support of $\tilde{\theta}$ is below θ^*, then even if the economy begins at $k_{\max}$, there is some probability that it will be in the informationally constrained region in the next period. Second, if the economy begins at the minimum possible equilibrium capital stock $k_{\min}$ (at the intersection of the DD and $S'S'_{\min}$ curves), and assuming that $\tilde{\theta}$ is a nondegenerate and continuously distributed random variable, the capital stock will almost certainly rise over time. Third, independent of initial conditions, the equilibrium capital stock in each period (it is easy to show) will lie in the interval $[k_{\min}, k_{\max}]$. We conclude that for most plausible parameterizations the system will be in the interior of the informationally constrained region with some positive probability in any given period, even asymptotically.

A distributional shock, as described in Section III, Part C, will also initiate interesting dynamics. In particular, a redistribution from borrowers to lenders that does not affect total income will lower investment not only in the current period, but for a number of subsequent periods as well. Thus balance sheet considerations may initiate, as well as propagate, cyclical fluctuations.

IV. Conclusion

We have constructed a simple neoclassical model of intrinsic business cycle dynamics in which borrowers' balance sheet positions play an important role. The critical insight

[31] Given (6), (25), (26), and (13), θ^* is defined by $\theta^* = [x(\bar{\omega}) - (\hat{q}(k_{\max})/r)\kappa_1]/[f(k_{\max}) - f'(k_{\max})k_{\max}]$.

is that the agency costs of undertaking physical investments are inversely related to the entrepreneur's/borrower's net worth. As a result, accelerator effects on investment emerge: Strengthened borrower balance sheets resulting from good times expand investment demand, which in turn tends to amplify the upturn; weakened balance sheets in bad times do just the opposite. The aggregate effects of productivity shocks may be asymmetric (since the agency problem may only bind on the "down" side). Further, redistributions or other shocks that affect borrowers' balance sheets (as may occur in a debt-deflation) will have aggregate real effects.

We have investigated extensions of this approach in related work. Our 1987 paper studies the macroeconomic implications of agency costs in a richer model of the investment process. In that model, projects differ *ex ante* (not just *ex post*, as in the costly state verification model), borrowers are able to obtain private information about project quality by incurring an evaluation cost, and borrowers must decide whether to proceed with projects that they have evaluated. The analysis of that model shows that the concept of "agency costs" relevant to macroeconomic fluctuations is much broader than the monitoring costs of the present paper: "Agency costs" should include any deviation from first-best outcomes associated with the necessity of external finance (whether it be through debt or other instruments). This result is important for interpreting the model empirically. Our companion paper also verifies the robustness of this basic approach to variations in assumptions about endowments and the information structure, and to permitting coalitions among entrepreneurs.

We have not discussed policy implications in the present paper. While, as in most OG models, the competitive solution of our model economy is not guaranteed to be Pareto optimal, it is efficient in a limited, intra-generational sense.[32] Issues of efficiency and policy are taken up at greater length in our 1987 paper. In particular, that paper discusses whether a policy of "debtor bailouts" (redistributions from lenders to borrowers) may be desirable when borrower net worth is low. Also addressed there is the issue of whether agency costs typically lead to "under"- or "over" - investment on average.

Finally, it is important to find out whether the qualitative results of this paper go through when borrowers and lenders are able to make contacts that last many periods. This has been done by Gertler (1988). In an n-period setting, he shows that the concept of "borrower net worth" should be augmented to include not just current endowments (as in the present paper), but also the "most secure" portion of expected future profits; thus, agency costs depend not only on current wealth but also on expected future conditions. He demonstrates that this can induce additional interesting cyclical dynamics into the aggregate economy.

APPENDIX: OPTIMAL CONTRACTING WITH STOCHASTIC AUDITING

This appendix studies the optimal financial contract between risk-neutral[33] entrepreneurs and lenders when there is private information about project outcomes but lenders have access to a costly auditing technology, as described in the text. We allow explicitly for a randomized auditing strategy by the lenders. As in the main text, we are assuming that borrowing and investment occurs in a given period t, and that project realization, auditing, and "settling up" by entrepreneurs and lenders occurs in $t+1$. Settling up is done via transfers of the produced capital good, and takes place before the period-$(t+1)$ value of capital, in terms of the consumption good, is known. Time subscripts are omitted below for legibility.

There are n possible outcomes of the investment project. In state i, κ_i units of the capital good are produced. Assume $0 \leq \kappa_1 < \kappa_2 < \cdots < \kappa_n$ and denote the probability of the ith outcome by π_i, $\pi_i > 0$. After

[32] Our dynamic equilibrium replicates the solution to a planning problem in which there are restrictions on intergenerational trades and the planner is not allowed to manipulate the relative price of capital in order to relax incentive constraints.

[33] The assumption of risk-neutrality differentiates our analysis from that of Townsend, 1988, and Mookherjee and Png, 1987, who consider the risk-averse case. Interestingly, the risk-neutral case seems to avoid some apparent anomalies that can arise in the optimal contract with risk aversion.

privately observing the true state j, the entrepreneur announces a state, say k, to the lenders. The lenders can verify the true state only by incurring an auditing cost of γ units of capital. We assume that lies of the form $k<j$ are feasible; in this case, the entrepreneur can "hide" the extra capital $\kappa_j-\kappa_k$. The expected value of this hidden capital is $\hat{q}(\kappa_j-\kappa_k)$ units of consumption, where $\hat{q}$ is the expected relative price of capital. Lies of the form $k>j$ are assumed infeasible, that is, the entrepreneur cannot show the lenders produced capital that does not exist.

We look for the optimal incentive-compatible contract. Let c_i be the entrepreneur's contractual consumption when he announces outcome i and is not audited, and let c_i^a be his consumption when he announces i, is audited, and is found to be telling the truth. (It is straightforward to show that the entrepreneur's optimal consumption when he is audited and found to be lying is zero (see Mookherjee and Png, 1987); we impose this from the beginning.) We allow a general stochastic auditing strategy: The lenders can commit in advance to auditing an announcement of outcome i with probability p_i. The total input cost of the project is x (here we hold the entrepreneur's "efficiency," ω, fixed). The entrepreneur's contribution is his savings S^e, and the interest rate is r, so that the lenders' total required return is $r(x-S^e)$. The entrepreneur's (borrower's) formal problem is

$$\text{(A1)} \qquad \max_{\{c_i^a,c_i,p_i\}} \sum_{i=1}^{n} \pi_i\left(p_i c_i^a+(1-p_i)c_i\right)$$

subject to

$$\text{(A2)} \qquad \sum_{i=1}^{n} \pi_i\left[\hat{q}\kappa_i-\left(p_i c_i^a+(1-p_i)c_i\right)-\hat{q}p_i\gamma\right] \geq r(x-S^e) \qquad (\lambda_1)$$

$$\text{(A3)} \qquad p_i c_i^a+(1-p_i)c_i \geq (1-p_j)\left(c_j+\hat{q}(\kappa_i-\kappa_j)\right) \qquad i=2,\dots,n \quad j<i \qquad (\lambda_{2ij})$$

$$\text{(A4)} \qquad c_i \geq 0 \qquad i=1,2,\dots,n \qquad (\lambda_{3i})$$

$$\text{(A5)} \qquad c_i^a \geq 0 \qquad i=1,2,\dots,n \qquad (\lambda_{4i})$$

$$\text{(A6)} \qquad p_i \geq 0 \qquad i=1,2,\dots,n \qquad (\lambda_{5i})$$

$$\text{(A7)} \qquad 1 \geq p_i \qquad i=1,2,\dots,n \qquad (\lambda_{6i})$$

where the multipliers associated with each set of constraints are in the right margin in parentheses, and $\hat{q}$ is the expected value of q_{t+1}. The entrepreneur's objective, (A1), is to maximize expected consumption, subject to the constraint that lenders receive their required return (A2), the truth-telling constraint (A3), nonnegativity constraints on c_i and c_i^a, (A4) and (A5), and the restriction that auditing probabilities be between zero and one, (A6) and (A7). The first-order conditions for c_1^a, c_i^a $(i=2,\dots,n)$, c_1, c_i $(i=2,\dots,n-1)$, c_n, p_1, p_i $(i=2,\dots,n-1)$, and p_n are, respectively,

$$\text{(A8)} \qquad \pi_1 p_1(1-\lambda_1)+\lambda_{41}=0$$

$$\text{(A9)} \qquad \pi_i p_i(1-\lambda_1)+p_i\sum_{j=1}^{i-1}\lambda_{2ij}+\lambda_{4i}=0 \qquad i=2,\dots,n$$

$$\text{(A10)} \qquad \pi_1(1-p_1)(1-\lambda_1) -(1-p_1)\sum_{k=2}^{n}\lambda_{2k1}+\lambda_{31}=0$$

$$\text{(A11)} \qquad \pi_i(1-p_i)(1-\lambda_1)+(1-p_i)\sum_{j=1}^{i-1}\lambda_{2ij} -(1-p_i)\sum_{k=i+1}^{n}\lambda_{2ki}+\lambda_{3i}=0 \qquad i=2,\dots,n-1$$

$$\text{(A12)} \qquad \pi_n(1-p_n)(1-\lambda_1) +(1-p_n)\sum_{j=1}^{n-1}\lambda_{2nj}+\lambda_{3n}=0$$

$$\text{(A13)} \qquad \pi_1(c_1^a-c_1)(1-\lambda_1)-\lambda_1\pi_1\hat{q}\gamma +\sum_{k=2}^{n}\left(c_1+\hat{q}(\kappa_k-\kappa_1)\right)\lambda_{2k1}+\lambda_{51}-\lambda_{61}=0$$

$$\text{(A14)} \qquad \pi_i(c_i^a-c_i)(1-\lambda_1)-\lambda_1\pi_i\hat{q}\gamma +(c_i^a-c_i)\sum_{j=1}^{i-1}\lambda_{2ij} +\sum_{k=i+1}^{n}\left(c_i+\hat{q}(\kappa_k-\kappa_i)\right)\lambda_{2ki}+\lambda_{5i}-\lambda_{6i}=0 \qquad i=2,\dots,n-1,$$

$$\text{(A15)} \qquad \pi_n(c_n^a-c_n)(1-\lambda_1) -\lambda_1\pi_n\hat{q}\gamma+(c_n^a-c_n)\sum_{j=1}^{n-1}\lambda_{2nj}+\lambda_{5n}-\lambda_{6n}=0.$$

From (A8) or (A9), it is immediate that $\lambda_1 \geq 1$, so that the lenders' return constraint (A2) always binds. Adding the difference between the LHS and RHS of

(A2) to the objective (A1) reveals that the problem is unchanged if we replace (A1) with

$$\text{(A1)}' \qquad \min_{\{c_i^a, c_i, p_i\}} \sum_{i=1}^{n} \pi_i \hat{q} p_i \gamma.$$

Thus we have

Result 1. The optimal contract minimizes expected auditing costs, subject to the constraints (A2)–(A7).

Result 1 and the fact that (A2) binds imply that expected auditing costs under the optimal contract are nondecreasing in the return required by lenders (the RHS of (A2)). For fixed r, this required return is decreasing in S^e, the collateral of the entrepreneurs. Thus we have

Result 2. Expected auditing costs under the optimal contract are nonincreasing in the quantity of the entrepreneur's collateral S^e (and they are strictly decreasing in S^e when expected auditing costs are positive at the initial point).

We have noted that $\lambda_1 \geq 1$. There are two interesting subcases, $\lambda_1 = 1$ and $\lambda_1 > 1$. If $\lambda_1 = 1$, then we are in the case of no auditing; that is, $p_i = 0$, all i. (Proof: If $\lambda_1 = 1$, then from (A9) we have $p_i \lambda_{2ij} = 0$ ($i = 2, \ldots, n$; $j < i$). (A12) and the fact that $p_n \lambda_{2nj} = 0$ implies $\lambda_{2nj} = 0$, $j < n$. Using (A11) and working recursively backward from $i = n-1$, we conclude $\lambda_{2ij} = 0$ ($i = 2, \ldots, n$; $j < i$.) From (A13)–(A15), this implies $\lambda_{5i} > 0$, all i; that is, $p_i = 0$.) On the other hand, if $p_i > 0$ for any i, then $\lambda_1 > 1$. (Proof: If some $p_i > 0$, then $\lambda_{5i} = 0$. Suppose that $\lambda_1 = 1$. From (A13)–(A15), $\lambda_{5i} = 0$ implies that some λ_{2ij} or λ_{2ki} must be positive. But, as shown just above, this implies $\lambda_1 > 1$, a contradiction.)

Consider first the no-auditing case ($\lambda_1 = 1$). With no auditing there is no deadweight loss; the "first best" is attained. The next result characterizes when this is possible.

Result 3. The optimal contract involves no auditing if and only if the lender's required return is less than the value of the worst possible outcome of the project; that is, $p_i = 0$, all i, iff $r(x - S^e) \leq \hat{q}\kappa_1$.

PROOF:

Suppose $p_i = 0$, all i. From (A3), this implies $c_i \geq \hat{q}(\kappa_i - \kappa_1)$, $i = 2, \ldots, n$. Substituting this into (A2) yields $r(x - S^e) \leq \hat{q}\kappa_1$, which proves sufficiency. Now suppose $r(x - S^e) \leq \hat{q}\kappa_1$. Then the contract $\{c_i = \hat{q}\kappa_i - r(x - S^e)$, $p_i = 0$, c_i^a irrelevant$\}$ satisfies the constraints and involves no auditing. Since auditing costs are minimized, this contract is optimal, by Result 1.

When $r(x - S^e) > \hat{q}\kappa_1$, we are in the case $\lambda_1 > 1$, and the optimal contract involves some positive probability of auditing. We give a few results for this case ($\lambda_1 > 1$ is maintained).

Result 4. In any state in which there is a positive probability of auditing, the entrepreneur receives positive consumption only if he is audited; that is, $p_i > 0 \rightarrow c_i = 0$.

PROOF:

Our proof is for $i = 2, \ldots, n-1$; similar arguments apply for $i = 1$ and $i = n$. Assume $1 > p_i > 0$. (If $p_i = 1$, the value of c_i is irrelevant.) Comparing (A11) and (A9), note that the first two terms of (A11) are proportional to $-\lambda_{4i}$. If $\lambda_{4i} > 0$, then (A11) implies $\lambda_{3i} > 0$ and we are done. Suppose that $\lambda_{4i} = 0$. Then, in (A14), the first and third terms, which are proportional to λ_{4i}, disappear. Since $\lambda_{5i} = 0$, for (A14) to hold there must be some $k > i$ such that $\lambda_{2ki} > 0$. (A11) then again implies $\lambda_{3i} > 0$, so that $c_i = 0$.

Result 5. The entrepreneur receives no consumption in the worst state; $c_1^a = c_1 = 0$.

PROOF:

$\lambda_{41} > 0$ (if $p_1 > 0$) and $\lambda_{31} > 0$ (if $p_1 < 1$) follow immediately from (A8) and (A10).

Result 6. Let $\hat{c}_i = p_i c_i^a + (1 - p_i)c_i$ be the entrepreneur's expected consumption in state i. Then $\hat{c}_i$ is nondecreasing in i, that is, the entrepreneur does better in better states.

PROOF:

For some $\hat{c}_i$, we wish to show that $\hat{c}_k \geq \hat{c}_i$, any $k > i$. $\hat{c}_1 = 0$, so let $i > 1$. If $\lambda_{3i} > 0$ and $\lambda_{4i} > 0$, then $\hat{c}_i = 0$ and the result is immediate. Suppose instead then that either $\lambda_{3i} = 0$ or $\lambda_{4i} = 0$. Then from (A11) or (A9), there exists some $j < i$ such that $\lambda_{2ij} > 0$. This implies $\hat{c}_i = (1 - p_j)(c_j + \hat{q}(\kappa_i - \kappa_j))$. For any $k > i$, we know from (A3) that $c_k \geq (1 - p_j)(c_j + \hat{q}(\kappa_k - \kappa_j)) > (1 - p_j)(c_j + \hat{q}(\kappa_i - \kappa_j)) = \hat{c}_i$. Thus expected consumption is actually strictly increasing in the range where it is positive.

Result 7. There is never any auditing in the highest state; $p_n = 0$.

PROOF:

Suppose $p_n > 0$. Then $\lambda_{5n} = 0$ and, from Result 4, $c_n = 0$. Now if $c_n^a = 0$ also, (A15) can hold only if $\lambda_{5n} > 0$, and we have a contradiction. Suppose instead that $c_n^a > 0$. Then $\lambda_{4n} = 0$. Comparing (A15) with (A9), we see that the first and third terms of (A15), which are proportional to λ_{4n}, must be zero. But then once again (A15) can hold only if $\lambda_{5n} > 0$, a contradiction.

Result 8. The probability of auditing is nonincreasing in the announced state (p_i is nonincreasing in i).

PROOF:

For any p_i, $i = 2, \ldots, n-1$, we wish to show that $p_{i-1} \geq p_i$. If $p_i = 0$, this is trivial, so take $p_i > 0$. By Result 4, $c_i = 0$. Now there are two possibilities to consider, $c_i^a = 0$ and $c_i^a > 0$.

Suppose $c_i^a = 0$. Then for (A14) to hold, there must be some $k > i$ such that $\lambda_{2ki} > 0$. Thus $\hat{c}_k = (1 - p_i)(\hat{q}(\kappa_k - \kappa_i))$, where $\hat{c}_k$ is defined as in Result 6. We know that $\hat{c}_k \geq (1 - p_{i-1})(c_{i-1} + \hat{q}(\kappa_k - \kappa_{i-1}))$. Since $c_{i-1} + \hat{q}(\kappa_k - \kappa_{i-1}) > \hat{q}(\kappa_k - \kappa_i)$, it must be that $p_{i-1} \geq p_i$.

If $c_i^a > 0$, then $\lambda_{4i} = 0$, and the first and third terms of (A14), which together are proportional to λ_{4i}, equal zero. For (A14) to hold, there must again be some $k > i$ such that $\lambda_{2ki} > 0$, and the argument is the same as before.

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