Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires∗

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Abstract
Recent papers interpret micro-level findings of greater cyclicality in the wages of new hires as evidence for flexible wages of new hires, thus concluding that wage rigidity is not an empirically plausible mechanism for resolving the unemployment volatility puzzle. We analyze data from the Survey of Income and Program Participation (SIPP) to argue that greater cyclicality of wages for new hires reflects cyclicality in the composition of match quality across new hires from employment. After controlling for the cyclicality of wages for new hires from other jobs, we find no evidence for greater wage flexibility for new hires from unemployment. In light of our empirical findings, we study an equilibrium model of unemployment with staggered Nash bargaining, heterogeneous match quality, and on-the-job search. Workers in bad matches vary their search intensity according to the probability of finding a better match, generating cyclicality in the contribution of bad-to-good transitions to total job-to-job flows. Using simulated data from our model, we compute measures of new hire wage cyclicality analogous to those found in the literature and show that cyclical match composition in our model generates spurious evidence of new hire wage flexibility of comparable in magnitude to what we estimate from the SIPP. The model is also successful in accounting for the cyclicality of aggregate wages and the dynamics of unemployment.

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1 Introduction

Aggregate wage data suggests relatively little variation in real wages as compared to output and unemployment. This consideration has motivated incorporating some form of wage rigidity in quantitative macroeconomic models to help account for business cycle fluctuations. The approach traces to the early large scale macroeconometric models and remains prevalent in the recent small scale DSGE models. It has also been true for searching and matching models of the labor market in the tradition of Diamond, Mortensen and Pissarides. For example, Shimer (2005) and Hall (2005) show that incorporating wage rigidity greatly improves the ability of these models to account for unemployment fluctuations.

An influential paper by Pissarides (2009), however, argues that the aggregate data may not provide the relevant measure of wage stickiness: what matters for employment adjustment is the wages of new hires, which need to be disentangled from aggregate measures of wages. In this regard, there is a volume of panel data evidence beginning with Bils (1985) that finds that wages of new hires are substantially more cyclical than those of existing workers. Pissarides then interprets this evidence as suggesting that the relevant measures suggest a high degree of wage flexibility, calling into question efforts to incorporate wage rigidity into macroeconomic models.

In this paper we revisit the issue of the flexibility of new hire wages and the associated implications for aggregate unemployment fluctuations. We argue that this interpretation of the new hire wage cyclical confounds wage flexibility and the phenomenon of cyclical upgrading, whereby workers move into higher paying jobs during expansions. We adopt a novel empirical strategy to separate contractual wage flexibility from cyclical match quality. Guided by an existing empirical and theoretical literature, we argue that cyclical changes in match quality should be more prevalent among employed workers. We first present fresh evidence on new hire wage cyclical using a detailed new panel data set and argue that the typical findings from the literature are driven by cyclical in match quality of new hires from employment. We then develop a quantitative macroeconomic model that is able to account for both the aggregate and panel data evidence.

Key to our approach is the fact that the existing empirical literature identifies new hire wage cyclical using an econometric specification that does not distinguish between new hires from unemployment and those from coming other jobs. First, following Haefke, Sonntag, and van

\[^1\] See for example, Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), Gertler, Sala and Trigari (2008), Galí, Smets and Wouters (2012), and Christiano, Eichenbaum and Trabandt (2014).

\[^2\] Gertler and Trigari (2009), Hall and Milgrom (2008), and Blanchard and Galí (2010) build on this approach and model the wage setting mechanism in greater detail.
Rens (2013), we note that the key wage for understanding unemployment fluctuations is that of new hires from unemployment. Second, we argue that in pooling these two types of new hires, the prototypical regression conflates possible wage flexibility of new hires from unemployment with procyclical improvements of match quality for new hires from employment. Drawing upon existing literatures on cyclical upgrading and job-to-job changes, we argue that excess wage cyclicality for workers from employment should be interpreted as evidence of cyclical match quality.

To address these concerns, we construct a unique dataset from the Survey of Income and Program Participation (SIPP) that allows us to separately estimate the wage cyclicality of new hires from unemployment from those making job-to-job transitions. We first show that by pooling the two types of new hires with our data, we can replicate the typical result of the existing literature: new hire wages appear to be more flexible than the wages of continuing workers. When we estimate separate terms for both types of new hires, however, we find no evidence of excess wage cyclicality for new hires coming from unemployment, but substantial evidence of procyclical match improvement for workers making job-to-job transitions.

To drive our point home, we develop a search and matching model with wage rigidity in the form of staggered wage contracting, variable job match quality, and on-the-job search. We show that the model is consistent with both the aggregate data and the panel data evidence. All the ingredients we add are critical for achieving this consistency. One interesting implication of the model is that it produces job reallocation, leading to a kind of sullying effect of recessions, as originally conceived by Barlevy (1992).

Our results are aligned with a rich literature on earnings growth and job-to-job transitions. Beginning with Topel and Ward (1992), an extensive empirical literature has documented that a large fraction of the wage increases experienced by a given worker occur through job-to-job transitions. Such job movements can be understood as employed workers actively searching for higher paying jobs, along the lines of Burdett and Mortensen (1998). A related theoretical literature has concluded that such match improvements are more easily realized during expansions than during recessions (Barlevy, 1992). In contrast, “job-ladder” models offer no systematic prediction for wage changes of workers searching from unemployment, as they are assumed to adopt a reservation wage strategy that is not contingent on their most recent wage. Moreover, a recent paper of Lise and Robin (2013) has found evidence of a non-negligible “cleansing effect” where, conditional on finding a job, workers from unemployment are predicted to find better jobs during recessions, implying the possibility of countercyclical match quality for workers from unemployment. We conclude that, while other mechanisms could be in play to generate cyclical
match quality for new hires from unemployment, the existing literature suggests that it should be most apparent for workers making job-to-job changes.

While the interpretation of flexible wages for new hires is still prevalent in the literature, other papers have documented that the addition of finer controls weakens the case for flexible wages. Gertler and Trigari (2009) argue that evidence of new hire wage cyclicality is driven by match quality; after introducing fixed-effects at the person-job level, they find no evidence of new hire wage cyclicality. Martins, Solon, and Thomas (2010) use data from Portugal to isolate new hire wage cyclicality within jobs; their estimates suggest that new hire wage cyclicality is roughly the same as that for continuing workers. Using administrative data from the LEHD on quarterly job movements and associated changes in earnings, Hyatt and McEntarfer (2012) provide evidence that wages for workers with 3 to 8 months of nonemployment between jobs are far less cyclical than for workers who make job changes within a quarter (whom they identify as job-to-job changers). Although the analysis we present is different than that of Hyatt and McEntarfer, we view the overall consistency of our results as important cross-validation.

Haefke, Sonntag and van Rens (2013) use cross-sectional data from the CPS and recover point estimates suggestive of greater wage cyclicality of new hires from unemployment, although not statistically significant. This contrasts with our results, where we recover point estimates that are suggestive of no excess wage cyclicality among new hires from unemployment. We suspect that the panel aspect of our data permits sharper controls for unobserved heterogeneity and compositional effects. Hagedorn and Manovskii (2013) develop indirect measures of match quality to argue that empirical claims of new hire wage volatility and implicit wage contracts à la Beaudry and Dinardo (1991) are unfounded. We view our paper as a complement to these two approaches: we demonstrate that it is important to separately estimate wage cyclicality for new hires from unemployment precisely due to cyclical changes in match quality for workers switching jobs. We then go a step further in analyzing our results in the context of an equilibrium model of business cycles and unemployment volatility.

Section 2 provides the new panel data evidence. Section 3 describes the model and Section 4 presents the numerical results. Concluding remarks are in Section 5.

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3 Real wages in their data, however, are largely driven by large exchange rate devaluations, quite different from the type of real wage variation measured in the U.S.

4 Aside from certain methodological differences, our data span a longer time period, allow for far more precise measures of duration of non-employment between jobs, and contain measures of hourly wages (as opposed to quarterly earnings).
2 Data and Empirics

This section presents new evidence on the wage cyclicality of new hires. We do so using a rich new data set. We first show that we are able to replicate the existing evidence showing greater cyclicality of the wages of new hires relative to existing workers. We then proceed to show that (i) there is no evidence of excess wage cyclicality for workers hired from unemployment, (ii) but there is substantial evidence for procyclicality in match quality for job changers. Put simply, we show that the evidence is consistent with new hires having the same degree of wage flexibility as existing workers. We first describe the data and then move to the estimation.

2.1 Data

We use data from the Survey of Income and Program Participation (SIPP) from 1990 to 2012. The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. Over our sample period the Census Bureau introduced eight panels. The starting years were 1990-1993, 1996, 2001, 2004, and 2008. The average length of time an individual stays in a sample ranges from 32 months in the early samples to 48 in the more recent ones.

Most key features of the SIPP are consistent across panels. Each household within a panel is interviewed every four months, a period referred to as a wave. During the first wave that a household is in the sample, the household provides retrospective information about employment history and other background information for working age individuals in the household. At the end of every wave, the household provides detailed information about activities over the time elapsed since the previous interviews, including job transitions that have occurred within the wave. Although each wave contains data for four months, we restrict our sample to the final wave for observations to mitigate the SIPP “seam effect”, whereby survey respondents “project current circumstances back onto each of the four prior months…” (Census Bureau, 2001, pg.1-6).

The SIPP has several features that make it uniquely suited for our analysis. Relative to other commonly used panel data sets (e.g., the PSID or the NLSY), the SIPP is larger, contains multiple representative cohorts, and is assembled from information collected at a high frequency (e.g. surveys are every four months as opposed to annually). The high frequency structure of the data is crucial for constructing precise measurements of employment status and wages. As the SIPP contains multiple cohorts, at each point in time the sample is always representative of
the U.S. population, unique among widely used panel datasets.

Crucial to our approach is that the SIPP maintains consistent job IDs. Fujita and Moscarini (2013) document that, starting with the 1996 SIPP wave, a single job may be assigned multiple IDs for an identifiable subset of survey respondents. In the appendix, we develop a procedure that exploits a feature of the SIPP employment interview module that allows us to identify jobs that may have been assigned multiple IDs. We find a substantial number of recalls, corroborating Fujita and Moscarini’s finding that recalls compose a significant fraction of transitions to employment from non-employment. We do not include these observations as new hires in our analysis; if these workers receive wages that are only as cyclical as “stayers”, they would bias the estimation of wage cyclicity of new hires from unemployment downwards.

The appendix provides greater detail on the data. It also describes the construction of the variables we use in the estimation.

2.2 Baseline Empirical Framework

We begin with a simple statistical framework to study the response of individual level wages to changes in aggregate conditions that has been popular in the literature, beginning with Bils (1985). Let $w_{ijt}$ be the wage of individual $i$ in job $j$ at time $t$, $x_{ijt}$ individual level characteristics such as education and experience as well as a time trend, $u_t$ the unemployment rate, $\text{I}(\text{new}_{ijt})$ an indicator variable that equals unity if the worker is a new hire and zero if not, and $\alpha_i$ an individual fixed effect. The measurement equation for wages is then given by

$$\log w_{ijt} = x_{ijt}'\pi_x + \pi_u \cdot u_t + \pi_n \cdot \text{I}(\text{new}_{ijt}) + \pi_{nu} \cdot \text{I}(\text{new}_{ijt}) \cdot u_t + \alpha_i + \epsilon_{ijt}$$  \hspace{1cm} (1)

where $\epsilon_{ijt}$ is random error term.

The inclusion of the unemployment rate in the regression is meant to capture the influence of cyclical factors on wages, while the interaction of the new hire dummy with the unemployment rate is meant to measure the extra cyclicity of new hires wages. In particular, the coefficient $\pi_u$ can be interpreted as the semi-elasticity of wages with respect to unemployment, while $\pi_u + \pi_{nu}$ gives the corresponding semi-elasticity for new hires. The key finding in the literature is that $\pi_{nu}$ is negative (along with $\pi_u$), suggesting greater cyclical sensitivity of new hires’ wages.

At this point we make two observations: First, with exception of Haefke, Sonntag and van Rens (2013), the literature typically does not distinguish between new hires coming from unem-

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5Included among the many studies regressing individual level wages on some measure of unemployment as a cyclical indicator are Barlevy (2001); Beaudry and Dinardo (1991); Deveraux (2001); Hagedorn and Manovskii (2013); Martins, Solon, and Thomas (2012); Shin (1991); and Solon, Barsky and Parker (1994).
ployment and those coming from other jobs. Second, since changes in wages of workers making job-to-job transitions include variations of quality across jobs, cyclical movements in job match quality will bias the new hire effect for workers coming from employment. We turn to these issues shortly.

We first show that with our data we can obtain the results in the literature. To obtain consistent coefficient estimates of equation (1), it is necessary to account for the presence of unobserved heterogeneity implied by $\alpha_i$ that may be correlated with observables. The convention in the literature, accordingly, is to use either a first difference or a fixed effects estimator, depending on the properties of the error term. The low serial correlation of the error terms in the exercises we perform suggests that the fixed effects estimator is preferred. However, since Bils (1985) and others used a first difference estimator, we show the results are robust to either approach.

The regressions are based on monthly data. For comparability to Bils (1985), we only use observations for men between the ages of 20 and 60. Accordingly, unemployment is the prime age unemployment rate. As our measure of hourly wages, we use job-specific earnings. In cases in which an hourly wage is directly available, we use that as our measure. In cases in which an hourly wage is not directly available, we use job-specific earnings divided by the product of job-specific hours per week and job-specific weeks per month. Wages are deflated by a three months average of the monthly PCE. Finally, we define “new hires” as individuals who are in the first four months of their tenure on a job. The appendix provides additional information on variable construction, including the individual level characteristics we use.

Table 1 presents the results. The first column presents the estimates of equation (1) using fixed effects and the second presents estimates using first differences. The results are robust across specifications. Similar to Bils (1985), we find that new hires’ wages are significantly more cyclical than those for existing workers. When estimating the equation in first differences, the semi-elasticity of new hire wages is $-1.445$, compared to $-0.448$ for continuing workers. With fixed effects, the new hire semi-elasticity is estimated to be $-1.409$, compared to $-0.162$ for continuing workers. In both specifications, the semi-elasticity is significant at the 1% level for continuing workers; and the new hire differential is significant at the 1% level. We find no evidence of serial correlation in the predicted errors, implying that fixed effects are more efficient than first differences. Hence, our preferred estimates come from the fixed effects regression.

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6While we have monthly information, as we noted earlier we only use wage information from the final month of each four month wave to avoid the “seam” effect.

7Note that given this definition we will only have one wage observation for a new hire since we only use the final month of a four month wave to obtain wage data.
While we recover precise coefficient estimates that imply both continuing worker wage cyclical-
ity and a new hire effect, our estimates reveal less cyclicality than most of the existing
literature. Using annual NLSY data from 1966-1980, Bils (1985) finds a continuing worker semi-
elasticity of 0.6, versus 3.0 for changers. Barlevy (2001) uses both PSID and NLSY through
1993 and recovers a semi-elasticity of 3.0 for job changers. We speculate that our lower esti-
mates are due mainly the high-frequency of our data (every four months as opposed to every
year). If workers are on staggered multi-period contracts (as will be the case in the quantita-
tive model we present), then a smaller fraction of wages are likely to be adjusted over a four month
interval than would be the case annually. In any case, our quantitative model will generate data
consistent with the degree of wage cyclicality suggested by the evidence in Table 1.

2.3 Robustness of the New Hire Effect

We now present evidence that the estimated new hire effect in the literature reflects misspecifi-
cation and not greater wage cyclicality of new hires.

We begin by distinguishing new hires that come from unemployment from those that come
from other jobs. As Haefke, Sonntag, and van Rens (2013) emphasize, the hiring margin that is
key for generating unemployment volatility in search and matching models with sticky wages is
that of workers coming from unemployment, not that of workers making job-to-job transitions.
Yet most empirical studies do not distinguish between new hires that are job changers and
workers hired from unemployment, as in Bils (1985).\footnote{Indeed, Bils (1985) uses the term “job changers” as opposed to “new hires” and offers a more general interpre-
tation of his results than has been adopted by others since.}

Accordingly, it is important to isolate the wage behavior for new hires coming from unem-
ployment. To do so, we estimate a variant of (1) that allows for a separate new hire effect for
workers coming from non-employment and workers making direct job-to-job transitions:

$$
\log w_{ijt} = x_{ijt} \pi_x + \pi_u \cdot u_t + \pi_{nE} \cdot I(new_{ijt} \& EN_{Eijt}) \cdot u_t + \pi_{nE} \cdot I(new_{ijt} \& EE_{Eijt}) \cdot u_t \\
+ \pi_{nE} \cdot I(new_{ijt} \& EN_{Eijt}) + \alpha_i + e_{ijt},
$$

where we use the notation $EN$ to signify workers with an intervening spell of non-employment
and $EE$ to signify workers who made direct job-to-job transitions.

Table 2 presents the results. For robustness, we consider three different measures of what
constitutes a new hire from non-employment. In the baseline case presented in the first column
we use the broadest measure: all new hires who did not receive a wage in the previous month,
independent of how long the unemployment spell. The second column addresses the concern that new hires from nonemployment that have only missed one months pay might in fact be job-changers taking a short break between jobs. Accordingly, for this case we lump those new hires with only one month of non-employment with job-changers. In the third column for ENE transitions we consider only new hires coming from short-term unemployment, which we consider to be a spell nine months or less. Here we address the concern that there may be something unusual about the wage behavior of those coming from long-term unemployment, leading us to estimate a separate coefficient for these workers. Finally, the fourth column addresses at the same time both concerns of long-term unemployment and short break in-between jobs.

As the table shows, for all three specifications, the new hire effect disappears for workers coming from unemployment. The coefficient $\pi^E_{nu}$ is not statistically significant in each case. Thus, for new hires coming from unemployment, wages are no more cyclical than those for existing workers. Moreover, while the new hire effect disappears for workers coming from unemployment, we find substantial evidence of procyclical changes in match quality for job changers, and indeed, the coefficient on the job-changer interaction term is higher than the coefficient on the interaction term for the baseline regressions in table 1 where both types of new hires are pooled together.

Although we expect to obtain more efficient estimates from the fixed effects regressions, it will be convenient to use estimates of the various interaction terms when we examine our results with a quantitative model. We proceed identically in table 3 as in table 2, except we estimate the regression in first differences. Our results do not change: we find no evidence of wage cyclicality for new hires from unemployment, but recover a negative and statistically significant on the interaction term for new hires from employment.

We regard the negative coefficient on $\pi^E_{nu}$ as indicative of procyclical match quality for employed workers. Pissarides (2009) interprets a negative coefficient on the new hire term pooled across all new hires as indicative of greater flexibility in new hire wages. According to this interpretation, the results from table 2 might suggest that firms have substantial leeway to adjust the wages of new hires from employment but not of new hires from unemployment. First, we note that such an interpretation would still not detract from one of our main points: the primal wage for studying the volatility of unemployment is that of new hires from unemployment. Second, we find such an interpretation implausible: it is hard to rationalize a bargaining mechanism whereby new hires from unemployment receive a less flexible wage than new hires from employment.

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\[9\] We can reject the null hypothesis that the wage cyclicity for new hires from non-employment equals the wage cyclicity for new hires from employment at the 5% level.
Instead, we interpret our results in line with (i) an empirical literature finding that job changers realize substantial wage gains from switching jobs (Topel and Ward, 1992), and (ii) a theoretical literature arguing that is easier for workers in employment to locate better matches during expansions than recessions (Barlevy, 2002).

How does our interpretation map into the regression equation (2)? Note that while the regression specification allows for time-invariant person fixed effects, it does not control for match specific effects that differ across jobs. Suppose that the error term $e_{ijt}$ in the regression equation (1) takes the form

$$e_{ijt} = q_{ij} + \varepsilon_{ijt}$$

where $q_{ij}$ represents unobserved match quality. If workers find better matches when the unemployment rate is low – or similarly, if the share of workers moving from bad to good matches of total job flows procyclical – then changes in $e_{ijt}$ across jobs should be correlated with the change in the unemployment rate due to cyclical changes in $q_{ij}$:

$$\text{Cov}(\Delta q_{ij}, \Delta U_t) < 0.$$  

It follows that, among new hires from employment, the error term $e_{ijt}$ will be correlated with the unemployment rate $U_t$ in differences (if the regression is estimated in first-differences) and mean deviations (if the regression is estimated in fixed effects). As a consequence, the estimated coefficient intended to identify the excess cyclicality of new hires wages, $\pi_{nu}^{EE}$, will be biased downward. If so, estimates of a negative value of $\pi_{nu}^{EE}$ would reflect composition bias rather than greater cyclicality of new hire wages.

We emphasize further that the compositional bias stemming from procyclical match quality is likely greater for job changers than for new hires coming from unemployment. As Barlevy (2002) emphasizes, job upgrading is an important reason for employment-to-employment transitions and this upgrading is highly procyclical. By contrast, a worker coming from non-employment may be more inclined to accept a bad match in a boom, which may dominate the alternative of being unemployed. And as is emphasized in the search literature, accepting a bad match does not preclude the worker from trying to find a better one.

Figure 1 illustrates how cyclical match quality may bias the estimated new hire effect for workers transitioning between jobs. The figure portrays wages on two possible jobs for the worker: a high wage job that is a good match and a low wage job that is a bad one. For each job the dotted line is the wage absent cyclical effects. The solid line is the wage including cyclical effects. Suppose the worker starts out in a bad job match and then when the first boom comes,
the worker moves to a good match. The worker’s wage jumps partly due to cyclical factors but mostly due to improved match quality. Absent a control for match quality, it appears that the worker’s wage as new hire was far more cyclical than that of an existing worker. What is needed is a way to control for the component of the wage change that is due to match quality, thus isolating the true cyclical variation.

One possible way to control for match quality would be to introduce fixed effects at the person-job level and identify wage flexibility for new hires from wage variation within a job. In previous iterations of this paper, we estimated such a regression and found no evidence for new hire wage flexibility. To the extent that initial wages and unemployment are highly persistent, however, we believe that a longer panel would be required to recover estimates of greater wage flexibility for new hires, should wage flexibility be the driving force behind a negative estimate of $\pi_{un}^{EE}$.

We view our regression estimates as conditional moments from the data. In the next section, we develop a model of equilibrium unemployment with on-the-job search, variable match quality, and wage stickiness for new hires. We follow the typical strategy of targeting steady state quantities (such as the average wage growth of new hires from employment and unemployment) and leave the cyclical moments as model outcomes. We find that the model is successfully able to simultaneously match the untargeted micro and macro moments.

Crucially, in our model, new hires wages are no more flexible than those for existing workers; yet data generated from the model will give rise to the appearance of new hire wage flexibility when evaluated by the typical regression from the literature.

3 Model

In keeping with a vast literature, we model employment fluctuations using a variant of the Diamond, Mortensen, and Pissarides search and matching model. Our starting point is a simple real business cycle model with search and matching in the labor market, similar to Merz (1995) and Andolfatto (1996). As in these papers, we minimize complexity by imposing complete consumption insurance. Our use of the real business cycle model is also meant for simplicity. It will become clear that our central point of how a model with wage rigidity can account for the micro wage evidence will hold in a richer macroeconomic framework.11

10 Preliminary versions of this work can be found in Gertler and Trigari (2009), section C.1., pg. 71.

11 Similarly, Gertler and Trigari (2009) have investigated the role of staggered Nash bargaining within a real business cycle model with technology shocks as the only driving force. Gertler, Sala and Trigari (2008) have then
We make two main changes to the Merz/Andolfatto framework. First we allow for staggered wage contracting with wage contracts determined by Nash bargaining, as in Gertler and Trigari (GT, 2009). Second, we allow for both variable match quality and on-the-job search with variable search intensity. These features will generate procyclical job ladder effects, in the spirit of Barlevy (2002) and Menzio and Shi (2011). As we will show, both these variants will be critical for accounting for both the macro and micro evidence on unemployment and wage dynamics.

3.1 Search, Vacancies, and Matching

There is a continuum of firms and a continuum of workers, each of measure unity. Workers within a firm are either good matches or bad matches. A bad match has a productivity level that is only a fraction $\phi$ of that of a good match, where $\phi \in (0, 1)$. Let $n_t$ be the number of good matches within a firm that are working during period $t$ and $b_t$ the number of bad matches. Then the firm’s effective labor force $l_t$ is the following composite of good and bad matches:

$$l_t = n_t + \phi b_t$$  \hspace{1cm} (5)

Firms post vacancies to hire workers. Firms with vacancies and workers looking for jobs meet randomly (i.e., there is no directed search). The quality of a match is only revealed once a worker and a firm meet. Match quality is idiosyncratic. A match is good with probability $\xi$ and bad with complementary probability $1 - \xi$. Hence, the outcome of a match depends neither on ex-ante characteristics of the firm or the worker. Whether or not a meeting becomes a match depends on the realization of match quality and the employment status of the searching worker.

Workers search for jobs both when they are unemployed and when they are employed. Before search occurs, matches are subject to an exogenous separation shock. With probability $\nu$, workers will search on-the-job; absent successful search that generates a new match at a different firm, these workers will remain at the firm for another period. With probability $1 - \nu$, the match is terminated. Workers who are subject to the separation shock and do not successfully find a job by the end of the period will be unemployed at the start of the next period.

There are three general types of searchers: the unemployed, the employed, and the recently separated. We first consider the unemployed. Let $\bar{n}_t = \int_n n_t \, di$ and $\bar{b}_t = \int_n b_t \, di$ be the total number of workers who are good matches and who are bad matches, respectively, where firms verified that the insights on the role of the contracting structure in generating plausible movements in the labor share and the relevant labor market variables carry over to a more general setup that features multiple sources of cyclical fluctuations and additional propagation mechanisms.
are indexed by $i$. The total number of unemployed workers $\bar{u}_t$ is then given by

$$\bar{u}_t = 1 - \bar{n}_t - \bar{b}_t. \tag{6}$$

We assume that each unemployed worker searches with a fixed intensity, normalized at unity. Under our parameterization, it will be optimal for a worker from unemployment to accept both good and matches.

The second type of searchers we consider are those who search on the job. Absent other considerations, the only reason for an employed worker to search is to find a job with improved match quality. In our setting, the only workers who can improve match quality are those currently in bad matches. We allow such workers to search with variable intensity $\varsigma_{bt}$. As has been noted in the literature, however, not all job transitions involve positive wage changes (see Flinn, 2002). Accordingly, we suppose that workers in good matches may occasionally leave for idiosyncratic reasons, e.g. locational constraints. We assume that these workers search with fixed intensity $\varsigma_n$ and only accept other good matches.

Finally, we assume that the fraction $1 - \nu$ of workers separated during period $t$ search with fixed intensity $\varsigma_u$. Such workers are either hired by another firm to work in the subsequent period or remain unemployed. As is the case with workers searching from unemployment, workers separated within the period will find it optimal to both accept good or bad matches. We include such flows to be consistent with the observation that workers observed making job-to-job transitions sometimes are observed to take pay cuts; and, as pointed out by Christiano, Eichenbaum, and Trabandt (2013), flows in the data that appear to be job-to-job transitions may in fact be separations immediately followed by successful job search. We assume $\varsigma_u < 1$, consistent with the notion in the search literature that, for various reasons, employed workers search with less efficiency than unemployed workers (e.g., employed workers have less time to devote to search than unemployed workers).

We derive the total efficiency units of search effort $\bar{s}_t$ as a weighted sum of search intensity

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12Strictly speaking, with staggered wage contracting, workers in good matches may want to search if their wages are (i) sufficiently below the norm and are (ii) not likely to be renegotiated for some time. However, because the fraction of workers likely to be in this situation in our model is of trivial quantitative importance, due to the transitory nature on average of wage differentials due to staggered contracting, we abstract from this consideration.

13For similar reasons, structural econometric models formulated to assess the contribution of on-the-job search to wage dispersion in a stationary setting often include a channel for exogenous, non-economic job-to-job transitions with wage drops. Examples include Jolivet et al. (2006) and Lentz and Mortensen (2012).

14Note that as the expected gains from search for such workers is zero up to a first order, there is no loss of generality in assuming fixed search intensity.
across the three types:

\[ \bar{s}_t = \bar{u}_t + \nu(\zeta_b \bar{b}_t + \zeta_n \bar{n}_t) + (1 - \nu)\zeta_u(\bar{n}_t + \bar{b}_t) \]  

(7)

The first term reflects search intensity of the unemployed; the second term, the search intensity of the employed; the third, the search intensity of workers separated within the period. As we will show, the search intensity of bad matches on the job will be procyclical. Furthermore, the cyclical sensitivity of the efforts of workers in bad matches to find better jobs will ultimately be the source of procyclical movements in match quality and new hire wages.

The aggregate number of matches \( \bar{m}_t \) is a function of the efficiency weighted number of searchers \( \bar{s}_t \) and the number of vacancies \( \bar{v}_t \), as follows:

\[ \bar{m}_t = \sigma_m \bar{s}_t \bar{v}_t^{1-\sigma}, \]  

(8)

where \( \sigma \) is the elasticity of matches to units of search effort and \( \sigma_m \) reflects the efficiency of the matching process.

The probability \( p_t \) a unit of search activity leads to a match is:

\[ p_t = \frac{\bar{m}_t}{\bar{s}_t} \]  

(9)

The probability the match is good \( p_t^a \) and the probability it is bad \( p_t^b \) are given by:

\[ p_t^a = \xi p_t \]  

(10)

\[ p_t^b = (1 - \xi) p_t \]  

(11)

The probability for a firm that posting a vacancy leads to a match \( q_t^m \) is given by

\[ q_t^m = \frac{\bar{m}_t}{\bar{v}_t} \]  

(12)

Not all matches lead to hires, however, and hires vary by quality. The probability \( q_t^a \) a vacancy leads to a good quality hire and the probability \( q_t^b \) it leads to a bad quality one are given by

\[ q_t^a = \xi q_t^m \]  

(13)

\[ q_t^b = (1 - \xi) \left( 1 - \frac{\nu(\zeta_b \bar{b}_t + \zeta_n \bar{n}_t)}{\bar{s}_t} \right) q_t^m \]  

(14)
Since all workers accept good matches, $q_t^n$ is simply the product of the probability of a match being good conditional on a match, $\xi$, and the probability of a match, $q_t^m$. By contrast, since on the job searchers do not accept bad matches, to compute $q_t^b$ we must net out the fraction of searchers who are doing so on the job, $\nu(\varsigma_b b_t + \varsigma_n n_t)/\bar{s}_t$.

Finally, we can express the expected number of workers in efficiency units of labor that a firm can expected to hire from posting a vacancy, $q_t$, as

$$q_t = q_t^n + \phi q_t^b$$

It follows that the total number of new hires in efficiency units is simply $q_t v_t$.

### 3.2 Firms

Firms produce output $y_t$ using capital and labor according to a Cobb-Douglas production technology:

$$y_t = z_t k_t^\alpha l_t^{1-\alpha},$$

where $k_t$ is capital and $l_t$ labor in efficiency units. Capital is perfectly mobile. Firms rent capital on a period by period basis. They add labor through a search and matching process that we describe shortly. The current value of $l_t$ is a predetermined state.

Labor in efficiency units is the quality adjusted sum of good and bad matches in the firm (see equation (5)). It is convenient to define $\gamma_t \equiv b_t/n_t$ as the ratio of bad to good matches in the firm. We can then express $l_t$ as the follow multiple of $n_t$:

$$l_t = n_t + \phi b_t = (1 + \phi \gamma_t) n_t,$$

where as before, $\phi \in (0,1)$ is the productivity of a bad match relative to a good one. The labor quality mix $\gamma_t$ is also a predetermined state for the firm.

The evolution of $l_t$ depends on the dynamics of both $n_t$ and $b_t$. Let $\rho_t^i$ be the probability of retaining a worker in a match of type $i = n, b$. Letting $q_t^i$ denote the probability of filling a vacancy with a worker leading to a match of type $i$, we can express the evolution of $n_t$ and $b_t$ as follows:

$$n_{t+1} = \rho_t^n n_t + q_t^n v_t$$

$$b_{t+1} = \rho_t^b b_t + q_t^b v_t$$

where $\nu(\varsigma_b b_t + \varsigma_n n_t)/\bar{s}_t$. 

15
where $q_i^t\nu_t$ is the quantity of type $i$ matches and where equations (13) and (14) define $q_i^n$ and $q_i^b$. The probability of retaining a worker is the product of the job survival probability $\nu$ and the probability the worker does not leave for a job elsewhere $(1 - \varsigma_i^n\rho_i^n)$:

$$\rho_i^n(t) = \nu(1 - \varsigma_i^n\rho_i^n), \quad i = n, b$$  \hspace{1cm} (20)

It follows from equations (17) and (20) that we can express the survival probability of a unit of labor in efficiency units, $\rho_t$, as the following convex combination of $\rho_i^n$ and $\rho_i^b$ :

$$\rho_t = \rho_i^n + \phi \gamma_t \rho_i^b / (1 + \phi \gamma_t)$$  \hspace{1cm} (21)

The hiring rate in efficiency units of labor, $x_t$, is ratio of new hires in efficiency units $q_t\nu_t$ to the existing stock, $l_t$

$$x_t = \frac{q_t\nu_t}{l_t}$$  \hspace{1cm} (22)

where the expected number of efficiency weighted new hires per vacancy $q_t$ is given by equation (15). The evolution of $l_t$ is then given by:

$$l_{t+1} = (\rho_t + x_t) l_t$$  \hspace{1cm} (23)

It is useful to define $\bar{\gamma}_t^m \equiv (q_b^t\nu_t) / (q_n^t\nu_t) = q_b^t / q_n^t$ as the ratio of newly-formed bad to good matches. Then, making use of equations (15), (17), (18), (19) and (22) to characterize how the quality mix of workers $\gamma_t = b_t / n_t$ evolves over time:

$$\gamma_{t+1} = \frac{\rho_i^b \gamma_t + q_b^t \nu_t / n_t}{\rho_i^n + q_b^t \nu_t / n_t} = \frac{\rho_i^b \gamma_t / (1 + \phi \gamma_t) + x_t \bar{\gamma}_t^m / (1 + \phi \bar{\gamma}_t^m)}{\rho_i^n / (1 + \phi \gamma_t) + x_t / (1 + \phi \bar{\gamma}_t^m)}$$  \hspace{1cm} (24)

We now turn to the firm’s decision problem. Assume that labor recruiting costs are quadratic in the hiring rate for labor in efficiency units, $x_t$, and homogeneous in the existing stock $l_t$.\footnote{We assume quadratic recruiting costs because we have temporary wage dispersion due to staggered contracts and perfectly mobile capital. With proportional costs, all capital would flow to the low wage firms.} Then let $\Lambda_{t,t+1}$ be the firm’s stochastic discount factor, i.e. the household’s intertemporal marginal rate of substitution, $r_t$ the rental rate, and the wage per efficiency unit of labor, $w_t$. Then the firm’s decision problem is to choose capital $k_t$ and the hiring rate $x_t$ to maximize the
discounted stream of profits net recruiting costs subject to the equations that govern the laws
of motion for labor in efficiency units \( l_t \) and the quality mix of labor \( \gamma_t \), and given the expected
paths of rents and wages. In particular, we may express the value of each firm \( F_t(l_t, \gamma_t, w_t) \equiv F_t \) as

\[
F_t = \max_{k_t, x_t} \{ z_t \dot{k}^{\alpha}_{t} l_t^{1-\alpha} - \frac{\kappa}{2} x_t^2 l_t - w_t l_t - r_t k_t + \mathbb{E}_t \{ \Lambda_{t,t+1} F_{t+1} \} \}
\]

subject to equations (23) and (24), and given the values of the firm level states \((l_t, \gamma_t, w_t)\) and
the aggregate state vector. For the time being, we take the firm’s expected wage path as given.
In Section 3.4 we describe how wages are determined for both good and bad workers.

Given constant returns and perfectly mobile capital, the firm’s value \( F_t \) is homogeneous in \( l_t \). The net effect is that each firm’s choice of the capital/labor ratio and the hiring rate is
independent of its size. Let \( J_t \) be firm value per efficiency unit of labor and let \( \dot{k}_t \equiv k_t/l_t \) be its
capital labor ratio. Then

\[
F_t = J_t \cdot l_t \tag{25}
\]

with \( J_t \equiv J_t(\gamma_t, w_t) \) given by

\[
J_t = \max_{k_t, x_t} \{ z_t \dot{k}^{\alpha}_{t} - \frac{\kappa}{2} x_t^2 - w_t - r_t \dot{k}_t + (\rho_t + x_t) \mathbb{E}_t \{ \Lambda_{t,t+1} J_{t+1} \} \}. \tag{26}
\]

subject to (23) and (24).

The first order condition for capital rental is

\[
r_t = \alpha z_t \dot{k}^{\alpha-1}_t. \tag{27}
\]

Given Cobb-Douglas production technology and perfect mobility of capital, \( \dot{k}_t \) does not vary
across firms.

The first order condition for hiring is

\[
\kappa x_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ J_{t+1} + (\rho_t + x_t) \left[ \frac{\partial J_{t+1}}{\partial \gamma_{t+1}} + \frac{\partial J_{t+1}}{\partial w_{t+1}} \frac{\partial w_{t+1}}{\partial \gamma_{t+1}} \right] \frac{\partial \gamma_{t+1}}{\partial x_t} \right] \right\} \tag{28}
\]

The expression on the left is the marginal cost of adding worker, while the one on the right is
discounted marginal benefit. The first term on the right-hand side of (28) is standard: It reflects
the marginal benefit of adding a unit of efficiency labor. The second term reflects a “composition
effect” of hiring. While the firm pays the same recruitment costs for bad and good workers (in
quality adjusted units), bad workers have separate survival rates within the firm due to their
particular incentive to search on-the-job. The composition term reflects the effect of hiring on period-ahead composition, and the implied effect in the value of a unit of labor quality to the firm.\footnote{Under our calibration, the effect will be zero, up to a first order. See appendix for details.}

### 3.3 Workers

We next construct value functions for unemployed workers, workers in bad matches, and workers in good matches. These value functions will be relevant for wage determination, as we discuss in the next section. Importantly, they will also be relevant for the choice of search intensity by workers in bad matches who are looking to upgrade.

We begin with an unemployed worker: Let $U_t$ be the value of unemployment, $V_n^t$ the value of a good match, $V_b^t$ the value of a bad match, and $u_B$ the flow benefit of unemployment. Then, the value of a worker in unemployment satisfies

$$U_t = u_B + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ p_n^t \bar{V}_n^{t+1} + p_b^t \bar{V}_b^{t+1} + (1 - p_t) U_{t+1} \right] \right\}.$$  \hspace{1cm} (29)

where the unconditional job finding probability $p_t$, and the probabilities of finding good and bad matches, $p_n^t$ and $p_b^t$, are given by equations (9) and (10), and where $\bar{V}_n^{t+1}$ and $\bar{V}_b^{t+1}$ are the average values of good and bad matches at time $t + 1$.\footnote{Technically, the average value of employment in the continuation value of $U_t$ should be that of a new hire rather than the unconditional one. However, Gertler and Trigari (2009) show that the two are identical up to a first order. Hence, we use the simpler formulation for clarity. In particular, the unconditional average value for a type $i$ match is $\bar{V}_i^{t+1} = \int V_i^{t+1} dG_{t+1}$, where $G$ denotes the joint distribution of wages and composition, while the average value conditional on being a new hire is given by $\bar{V}_{i,x}^{t+1} = \int V_{i,x}^{t+1} (x_t/\bar{x}) dG_t$, where $\bar{x} = \int x_t dG_t$. Since $w$, $\gamma$ and $x$ in the steady state are identical across firms, $\bar{V}_{i,x}^{t+1} = \bar{V}_i^{t+1}$ up to a first order.}

For workers that begin the period employed, we suppose that the cost of searching as a function of search intensity is given by

$$c(\varsigma_{it}) = \frac{s_0}{1 + \eta} \varsigma_{it}^{1+\eta}$$

where $i = b, n, u$. Let $w_{it}$ be the wage of a type $i$ worker, $i = n, b$. The value of a worker in a bad match is given by

$$V_b^t = \max_{\varsigma_{bt}} \{ w_{bt} + \tau_t - [\nu c(\varsigma_{bt}) + (1 - \nu)c(\varsigma_{ut})] \}
+ \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \nu [(1 - s_u p_{t}^n) \bar{V}_n^{t+1} + s_b p_{t}^n \bar{V}_n^{t+1}] \right.ight.
+ (1 - \nu) \left[ s_u p_{t}^n \bar{V}_n^{t+1} + s_b p_{t}^n \bar{V}_b^{t+1} + (1 - s_u p_t) U_{t+1} \right] \} \}$$  \hspace{1cm} (30)
The flow value is the wage $w_{bt}$ net the expected costs of search plus a term $\tau_t$ we describe below. If the worker “survives” within the firm, which occurs with probability $\nu$, he searches with variable intensity $\varsigma_{bt}$. If he is separated, which occurs with probability $1 - \nu$, he searches with fixed intensity $\varsigma_u$. The first term in the continuation value is the value of continuing in the match, which occurs with probability $\nu(1 - \varsigma_{nt}p_{nt}^n)$. The second term reflects the value of switching to a good match, which occurs with probability $\nu\varsigma_{bt}p_{bt}^n$. The third term and fourth term reflect the value of being separated but immediately finding a good or bad job. The final term reflects the value of being separated into unemployment.

A worker in the bad match chooses the optimal search intensity $\varsigma_{bt}$ according to (30), satisfying
\[
\varsigma_{n_{bt}} = \mathbb{E}_t \left\{ \Lambda_{t,t+1}p_{nt}^n \left( \bar{V}_{nt}^{b} - V_{nt}^{b} \right) \right\}.
\] (31)

Search intensity varies positively with the product of the likelihood of finding a good match, $p_{nt}^n$, and the net gain of doing so, i.e. the difference between the value of good and bad matches. One can see from equation (31) how the model can generate procyclical search intensity by workers in bad matches. The probability of finding a good match will be highly procyclical and the net gain roughly acyclical. Thus, the expected marginal gain from search will be highly procyclical, leading to procyclical search intensity.

If there is dispersion of wages among bad matches due staggered contracting, then search intensities can differ across these workers. To simplify matters, we assume that the family provides an insurance scheme that smooths out search intensities across its family members, much in the same way it offers consumption insurance. In particular, we assume that there is a transfer scheme that insures that the sum of the wage and the transfer equals the average wage across matches, $\bar{w}_{bt}$. In particular, $\tau_t = (\bar{w}_{bt} - w_{bt})$, which implies $w_{bt} + \tau_t = \bar{w}_{bt}$. With the transfer, the discounted marginal benefit to search (the right side of equation (31)) does not depend on worker-specific characteristics, so that $V_{bt}^b = \bar{V}_{bt}^b$. Search intensity is thus the same across all workers in bad matches.

The value of a worker in a good match is analogous to the value function for a bad match.
\[
V_t^n = w_{nt} - \left[ \nu c(s_n) + (1 - \nu)c(s_n) \right]
+ \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \nu(1 - \varsigma_{nt}p_{nt}^n) + s_n p_{nt}^n \bar{V}_{nt}^n \right] 
+ (1 - \nu)[s_n p_{nt}^n \bar{V}_{nt}^n + s_n p_{nt}^n \bar{V}_{nt}^b + (1 - \varsigma_{nt+1}u_{nt+1})] \right\}.
\] (32)

One key difference is that on-the-job search intensity is fixed for good matches. Note that up to a first order, however, there are zero expected gains from search given that workers in good
matches only move to other good matches. Hence, we rule out variable search by workers in good matches without loss of generality.

For technical simplicity, we suppose that the search intensities of good and bad workers are identical in steady state (i.e., $s_{bt} = s_{tn}$ in steady state). This will imply that in the steady state, good and bad matches will have the same level of expected longevity a firm, which will in turn help simplify the impact of the distribution of workers between good and bad matches on the equilibrium (in the first order approximation).

### 3.4 Nash Wage

As in GT, workers and firms divide the joint match surplus via staggered Nash bargaining. For simplicity, we assume that the firm bargains with good workers for a wage. Bad workers then receive the fraction $\phi$ of the wage for good workers, corresponding to their relative productivity. Thus if $w_t$ is the wage for a good match within the firm, then $\phi w_t$ is the wage for a bad match. It follows that $w_t$ corresponds to the wage per unit of labor quality. We note that this simple rule for determining wages for workers in bad matches approximates the optimum that would come from direct bargaining. It differs slightly due mainly to differences in duration of good and bad matches with firms. The gain from imposing this simple rule is that we need only characterize the evolution of a single type of wage. Importantly, in bargaining with good workers, firms also take account of the implied costs of hiring bad workers.

Our assumptions are equivalent to having the good workers and firms bargain over the wage per unit of labor quality $w_t$. For the firm, the relevant surplus per worker is $J_t$, derived in section 3.2 (equation (26)). For good workers, the relevant surplus is the difference between the value of a good match and unemployment:

$$H_t = V^n_t - U_t$$

As in GT, the expected duration of a wage contract is set exogenously. At each period, a firm faces a fixed probability $1 - \lambda$ of renegotiating the wage. With complementary probability, the wage from the previous period is retained. The expected duration of a wage contract is then $1/(1 - \lambda)$. Workers hired in between contracting periods receive the prevailing firm wage per unit of labor quality $w_t$. Thus in the model there is no new hire effect: Adjusting for relative productivity the wages of new hires are the same as for existing workers.

Let $w^*_t$ denote the wage per unit of labor quality of a firm renegotiating its wage contract in
the current period. The wage $w^*_t$ is chosen to maximize the Nash product of a unit of labor quality to a firm and a worker in a good match, given by

$$H_t^\eta J_t^{1-\eta}$$

subject to

$$w_{t+1} = \begin{cases} w_t & \text{with probability } \lambda \\ w^*_{t+1} & \text{with probability } 1 - \lambda \end{cases}$$

where $w^*_{t+1}$ is the wage chosen in the next period if the parties are able to re-bargain and where $\eta$ is the households relative bargaining power.

Let $H^*_t \equiv H_t(\gamma_t, w^*_t)$ and $J^*_t \equiv J_t(\gamma_t, w^*_t)$ (where $H_t \equiv H_t(\gamma_t, w_t)$ and $J_t \equiv J_t(\gamma_t, w_t)$). Then the first order condition for $w^*_t$ is given by

$$\eta \frac{\partial H^*_t}{\partial w^*_t} J^*_t = (1 - \eta) \left( -\frac{\partial J^*_t}{\partial w^*_t} \right) H^*_t$$

where

$$\frac{\partial H^*_t}{\partial w^*_t} = 1 + \nu (1 - \varsigma_n p^\alpha_t) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1}}{\partial w^*_t} \right\}$$

and

$$\frac{\partial J^*_t}{\partial w^*_t} = -1 + (\rho_t + x_t) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}}{\partial w^*_t} \right\}.$$ 

Under multi-period bargaining, the outcome depends on how the new wage settlement affects the relative surpluses in subsequent periods where the contract is expected to remain in effect. The net effect, as shown in GT, is that up a first order approximation the contract wage will be an expected distributed lead of the target wages that would arise under period-by-period Nash bargaining, where the weights on the target for period $t + i$ depend on the likelihood the contract remains operative, $\lambda^i$.

In general, the new contract wage will be a function of the firm level state $\gamma_t$ (the ratio of bad to good matches), as well as the aggregate state vector. However, given our assumptions that steady state search intensities are the same for good and bad matches and that wages are proportional to productivity, $w^*_t$ is independent of $\gamma_t$ in the first order approximation. Accordingly,

---

18 We suppress the dependence of $w^*$ and similar objects on the firm’s composition in the notation.

19 For simplicity, we omit additional terms in the expression for $\partial H^*/\partial w^*$ that will be zero up to a first order. See the appendix for details.
to a first order, we can express the evolution of average wages $\bar{w}_t$ as

$$
\bar{w}_t = (1 - \lambda)w^*_t + \lambda \bar{w}_{t-1}
$$

where $1 - \lambda$ is the fraction of firms that are renegotiating and $\lambda$ is the fraction that are not and where the average wage per unit of labor quality is defined by

$$
\bar{w}_t = \int w \, dG_t(\gamma, w)
$$

with $G_t(\gamma, w)$ denoting the time $t$ fraction of units of labor quality employed at firms with wage less than or equal to $w$ and composition less than or equal to $\gamma$. (See the appendix for details.)

### 3.5 Households: Consumption and Saving

We adopt the representative family construct, following Merz and Andolfatto, which essentially generates perfect consumption insurance. There is a measure of families on the unit interval, each with a measure one of workers. The distribution of unemployed and employed workers (along with their wages) within the family is that of the aggregate distribution. Before making allocating resources to per-capita consumption and savings, the family pools all wage and unemployment income. Additionally, the family owns diversified stakes in firms that pay out profits. The household can then assign consumption $\bar{c}_t$ to members and save in the form of capital $\bar{k}_t$, which is rented to firms at rate $r_t$ and depreciates at the rate $\delta$.

Let $\Omega_t \equiv \Omega(s_t)$ be the value of the representative household. Then,

$$
\Omega_t = \max_{\bar{c}_t, \bar{k}_{t+1}} \{ \log(\bar{c}_t) + \beta \mathbb{E}_t \Omega_{t+1} \}
$$

subject to

$$
\bar{c}_t + \bar{k}_{t+1} + \frac{\bar{\varsigma}_0}{1 + \eta_0} \left\{ \left[ \nu \varsigma_n^{1+\eta} + (1 - \nu) \varsigma_u^{1+\eta} \right] \bar{n}_t + \left[ \nu \varsigma_b^{1+\eta} + (1 - \nu) \varsigma_u^{1+\eta} \right] \bar{b}_t \right\} = \bar{w}_t \bar{n}_t + \phi \bar{w}_t \bar{b}_t + (1 - \bar{n}_t - \bar{b}_t) w_0 + (1 - \delta + r_t) \bar{k}_t + T_t + \Pi_t,
$$

and

$$
\bar{n}_{t+1} = \rho^n_t \bar{n}_t + \xi p_t \bar{s}_t
$$

$$
\bar{b}_{t+1} = \rho^b_t \bar{b}_t + \xi \gamma_t \rho_t \bar{s}_t
$$
where Π_t are the profits from the household’s ownership holdings in firms and T_t are lump sum transfers from the government.

The first-order condition from the household’s savings problem gives

\[ 1 = (1 - \delta + r) \mathbb{E}_t \{ \Lambda_{t,t+1} \} \]  

(43)

where \( \Lambda_{t,t+1} \equiv \beta \tilde{c}_t / \bar{c}_{t+1} \).

### 3.6 Resource Constraint, Government Policy, and Equilibrium

The resource constraint states that the total resource allocation towards consumption, investment, vacancy posting costs, and search costs is equal to aggregate output:

\[
\bar{y}_t = \bar{c}_t + \bar{k}_{t+1} - (1 - \delta) \bar{k}_t + \frac{\kappa}{2} \int x_i^2 t_i d_i + \frac{s_0}{1 + \eta_c} \left( [\nu \bar{c}_n^{1+\eta_c} + (1 - \nu) \bar{c}_n^{1+\eta_c}] \bar{n}_t + [\nu \bar{c}_b^{1+\eta_c} + (1 - \nu) \bar{c}_b^{1+\eta_c}] \bar{b}_t \right). 
\]

(44)

The government funds unemployment benefits through lump-sum transfers:

\[
T_t + (1 - \bar{n}_t - \bar{b}_t) u_b = 0.
\]

(45)

A recursive equilibrium is a solution for (i) a set of functions \( \{ J_t, V^n_t, V^b_t, U_t \} \); (ii) the contract wage \( w^*_t \); (iii) the hiring rate \( x_t \); (iv) the subsequent’s period wage rate \( w_{t+1} \); (v) the search intensity of a worker in a bad match \( \varsigma_{bt} \); (vi) the search intensity of a worker in a bad match \( \varsigma_{bt} \); (vii) the average wage and hiring rates, \( \bar{w}_t \) and \( \bar{x}_t \); (viii) the capital labor ratio \( \bar{k}_t \); (ix) the average consumption and capital, \( \bar{c}_t \) and \( \bar{k}_{t+1} \); (x) the average employment in good and bad matches, \( \bar{n}_t \) and \( \bar{b}_t \); (xi) the density function of composition and wages across workers \( dG_t(\gamma, w) \); and (xii) a transition function \( Q_{t,t+1} \). The solution is such that (i) \( w^*_t \) satisfies the Nash bargaining condition (36); (ii) \( x_t \) satisfies the hiring condition (28); (iii) \( w_{t+1} \) is given by the Calvo process for wages (35); (iv) \( \bar{w}_t \) satisfies the first-order condition for search intensity of workers in bad matches (31); (v) \( r_t \) satisfies (27); (vi) \( \bar{w}_t = \int_{w,\gamma} w dG_t(\gamma, w) \) and \( \bar{x}_t = \int_{w,\gamma} x dG_t(\gamma, w) \); (vii) the rental market for capital clears, \( \bar{k}_t = \bar{k}_{t+1} / (\bar{n}_t + \phi \bar{b}_t) \); (viii) \( \bar{c}_t \) and \( \bar{k}_{t+1} \) solve the household problem (39); (ix) \( \bar{n}_t \) and \( \bar{b}_t \) evolve according to (41) and (42); (x) the evolution of \( G_t \) is consistent with \( Q_{t,t+1} \); (xi) \( Q_{t,t+1} \) is defined in the appendix.
3.7 New Hire Wages and Job-to-Job Flows

Here we describe how our model is able to capture the panel data evidence on new hire wage cyclicality, despite new hires’ wages being every bit as sticky as those for existing workers (conditional on match quality). To do that, we derive an expression for the average wage growth of job changers that permits to interpret the semi-elasticity of job changers’ wage to changes in unemployment that is implied by the model.

The model includes two types of job-to-job transitions: those due to on-the-job search and those due to separations followed by search and finding of a new job within the same period, with no spell of unemployment between the jobs. Since workers searching on the job only accept good matches, the first type of transitions leads to bad-to-good and good-to-good job flows. The second type of transitions instead leads to the full range of job-to-job flows. For example, the bad-to-good job flow has two components: the first results from on-the-job search, \( \nu \tilde{\omega}_b \xi_t b_t \); the second from match separation and finding of a new job within the period, \( (1 - \nu) \gamma_t \xi_t b_t \).

Let \( \bar{g}^w_t \) denote the average wage growth of continuing workers, \( g^{JC}_t \) the average wage growth of new hires who are job changers, and \( c^w_t \) the component of \( g^{JC}_t \) due compositional effects (i.e. changes in match quality across jobs). Further, let \( \delta_{BG,t} \) be the share of flows moving from bad to good matches out of total job flows at time \( t \) and let \( \delta_{GB,t} \) be the share moving from good to bad matches. Then to a first order (see the appendix for details) we can express average wage growth for changers:

\[
\bar{g}^{JC}_t = (1 - \omega) \bar{g}^w_t + \omega \tilde{c}^w_t
\]

with

\[
\bar{g}^w_t = \tilde{w}_t - \tilde{w}_{t-1}
\]

\[
\tilde{c}^w_t = \pi_{BG} \tilde{\delta}_{BG,t-1} - \pi_{GB} \tilde{\delta}_{GB,t-1}
\]

where \( \tilde{z} \) denotes log deviations of variable \( z \) from steady state and \( \omega \in [0, 1) \) is the steady state share of average job changer wage growth that is due to changes in match quality. As shown in the appendix, the parameters \( \omega, \pi_{BG}, \) and \( \pi_{GB} \) are all positive and are functions of model primitives.

Equation \( 46 \) indicates that average wage growth for job changers is a convex combination of average wage growth for existing workers and a composition component. Absent the composition effect (i.e. if \( \omega = 0 \)), average wage growth for job changers would look no different than for
continuing workers. With the composition effect present, however, cyclical variation of the composition of new match quality enhances the relative volatility of job changers’ wages.

In particular, the cyclical composition effect \( c^w_t \) varies positively with share in total job flows of workers moving from bad to good matches, \( \delta_{BG,t-1} \), and negatively with the share movement from good to bad, \( \delta_{GB,t-1} \). As we have discussed, the search intensity by workers in bad matches, \( \bar{\varsigma}_t \), is highly procyclical, leading to \( \delta_{BG,t-1} \) being procyclical and \( \delta_{GB,t-1} \) countercyclical. The dynamics of the shares also depends on the average firm composition, \( \bar{\gamma}_t \), determining the relative stocks of bad and good matches available to make a job-to-job transition. During expansions composition slowly improves (\( \bar{\gamma}_t \) decreases) so that over time less workers in bad matches remain available to make a bad-to-good transition and more workers in good matches can make a good-to-bad transition. Specifically, after substituting the expressions for the flow shares (see the appendix for details), the compositional component can be rewritten as

\[
\hat{c}^w_t = \pi_\gamma \hat{\bar{\gamma}}_{t-1} + \pi_\varsigma \hat{\bar{\varsigma}}_{t-1}
\]  

where the parameters \( \pi_\gamma \) and \( \pi_\varsigma \) are positive and functions of model primitives. In the next section, we show that the net effect of procyclical search intensity and countercyclical composition is that \( c^w_t \) is procyclical, i.e. the composition effect on job changers’ enhance wage growth in good times and weakens it in bad times. In this way the model can produce the kind of cyclical movements in match quality that can lead to estimates of new hire wage cyclicity that suffer from the kind of composition bias we discussed in Section 2. We demonstrate this concretely in the next section by showing that data generated from the model will generate estimates of a new hire effect on wages for job changers, even though new hires’ wages have the exact same cyclical as for existing workers.

Note also that the model features no match quality effect for workers searching from unemployment, as workers from unemployment accept good and bad matches alike. This is consistent with the estimates from the empirical section, which show that new hires coming from unemployment have the same wage cyclicity as continuing workers.

4 Results

In this section we present some simulations to show how the model can capture both the aggregate evidence on unemployment fluctuations and wage rigidity and the panel data evidence on the relative cyclicity of new hires’ versus continuing workers’ wages. We first describe the
calibration before turning to the results.

4.1 Calibration

We adopt a monthly calibration. There are 16 parameters in the model for which we must select values. We calibrate 10 of the parameters using external sources. Five of the externally calibrated parameters are common to the macroeconomics literature: the discount factor, $\beta$; the capital depreciation rate, $\delta$; the "share" of labor in the production technology, $\alpha$; and the autoregressive parameter and standard deviation for the productivity process, $\rho_z$ and $\sigma_z$. Our parameter choices are standard: $\beta = 0.99^{1/3}$, $\delta = 0.025^3$, $\alpha = 1/3$, $\rho_z = 0.95^{1/3}$, and $\sigma_z = 0.007$.\textsuperscript{20,21}

Five more parameters are specific to the search literature. Our choice of the matching function elasticity with respect to searchers, $\sigma$, is 0.4, guided by the estimates from Blanchard and Diamond (1989).\textsuperscript{22} We set the worker’s bargaining power $\eta$ to 0.5, as in GT. We normalize the matching function constant, $\sigma_m$, to 1.0. We set the elasticity of search costs, $\eta\sigma$, to 1.0. This is close to the value estimated from Danish data by Christensen et al. (2005), 1.19. We choose $\lambda$ to target the average frequency of wage changes. Taylor (1999) argues that medium to large-size firms adjust wages roughly once every year; this is validated by findings from microdata by Gottschalk (2005), who concludes that wages are adjusted roughly every year. To be conservative, we set $\lambda = 8/9$, implying that wages are renegotiated on average every 3 quarters, which is consistent with the estimates in Gertler, Sala and Trigari (2009). We consider an alternative calibration with $\lambda = 11/12$, implying an average duration between negotiations of one year. The parameter values are given in Table 4.

The remaining six parameters are jointly calibrated to match model-relevant moments measuring aggregate labor flows, individual-level wage dynamics, and the value of leisure. We calibrate the inverse productivity premium, $\phi$; the probability that a new match is good, $\xi$; the hiring cost parameter, $\kappa$; the scale parameter of the search cost, $\varsigma_0$; the flow value of unemployment, $u_b$; and the separation probability, $(1 - \nu)$ to match six moments: the average wage

\textsuperscript{20}Note that, in contrast to the frictionless labor market model, the term $\alpha$ does not necessarily correspond to the labor share, since the labor share will in general depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below $\alpha$, here we simply follow convention by setting $\alpha = 2/3$.

\textsuperscript{21}The parameter $\sigma_z$ is chosen to target the standard deviation of output.

\textsuperscript{22}This value lays slightly outside the range of values identified by Pissarides and Petrongolo (2001) and well below the value estimated by Shimer (2005). Note that in these papers, only the unemployed search and enter the matching function, while searchers in our model comprise both unemployed and employed workers. When we simulate data from our model and estimate the matching function elasticity under the assumption that only the unemployed search, we recover an elasticity in excess of 0.6.
change of workers making E-E transitions in our data; average wage loss of workers making an E-N-E transition; the U-E probability; the E-E probability; the relative value of non-work; and the E-U probability. Although there is not a one-to-one mapping of parameters to moments, there is a sense in which the identification of particular parameters are more informed by certain moments than others. We use this informal mapping to provide a heuristic argument of how the various parameters are identified.

We calibrate $\phi$ to target the average wage change of workers making direct job-to-job transitions in our data, 4.80%; holding everything constant, a higher $\phi$ implies a smaller (positive) average percentage wage increase for job changers. We recover $\phi = 0.70$. We calibrate $\xi$ to match the average wage loss of workers making an E-N-E transition, 6.20%. Holding fixed the inverse productivity premium $\phi$ and the steady state value of $\gamma$, a lower $\xi$ corresponds to a lower probability of finding a good match from unemployment; and hence, a lower $\xi$ generates a larger wage loss for workers making E-N-E transitions. We recover $\xi = 0.02$.

We calibrate the separation probability $(1 - \nu)$ to match the empirical E-U probability of 0.026. Note that separated workers have the opportunity to find a new job and avoid unemployment. Hence, the E-U in the model equals $(1 - \nu)(1 - s_u \tilde{p})$, implying $(1 - \nu) = 0.06$ (where $\tilde{z}$ denotes steady state of a variable $z$). The hiring cost parameter, $\kappa$, determines the resources that firms place into recruiting, and hence, influences the probability that a worker finds a job. We set the steady state job finding probability $\tilde{p}$ to match the monthly U-E transition probability, 0.45; and then calibrate $\kappa$ to be consistent with $\tilde{p}$. We restrict $s_u = s_m = \tilde{s}_b$ and note that a higher search cost implies a lower E-E flow. We calibrate $\varsigma_i$ to match an E-E flow of 0.029; we obtain $\varsigma_0 = 0.06$.

We calibrate the flow value of unemployment $u_B$ to be the sum of the relative utility of leisure and unemployment insurance. We adopt the utility component estimated by Hall and Milgrom (2008) of 0.46 and the take-up weighted UI component estimated by Chodorow-Reich Karabarbounis (2014) of 0.041. Hence, we set $u_B$ equal to 0.501. In our setting, the relative value of nonwork activities satisfies

$$\bar{u}_T = \frac{u_B + \frac{s_0}{1 + \kappa} \left[ \nu \varsigma_i^{1+\eta} + (1 - \nu) \varsigma_u^{1+\eta_u} \right]}{\bar{a} + (\kappa/2)\bar{x}^2},$$

where $\bar{a} = (1 - \alpha) \tilde{y}/\tilde{l}$. Note that the value of nonwork includes saved search costs from on-the-job search and the value of work includes saved vacancy posting costs. Finally, when taking the model to the data, we assume that workers employed in bad matches suffer a lower disutility of leisure, scaled by the inverse productivity premium, $\phi$. This also makes the period surplus from
unemployment in bad match proportional to period surplus in a good match: \( \phi w + (1 - \phi)u_B - u_B = \phi (w - u_B) \).

The full list of parameter values and targeted moments are given in Table 5. Having fully calibrated the model, we now evaluate whether it provides an accurate description of aggregate and individual-level dynamics. We first test the ability of the model to match the cyclical properties of aggregate unemployment and wages. Second, we assess the ability of the model to generate the correct relative cyclicality in wage growth for job changers versus continuing workers.

### 4.2 Model Simulations of Aggregate and Panel Data Evidence

We first explore whether the model provides a reasonable description of labor market volatility. In particular, we compare the model implications to quarterly U.S. data from 1964:1 to 2013:2. We take quarterly averages for monthly series in the data. Given that the model is calibrated to a monthly frequency, we take quarterly averages of the model simulated data series.

We measure output \( y \) as real output in the nonfarm business sector. The wage \( w \) is average per worker earnings of production and non-supervisory employees in the private sector, deflated with the PCE. Total employment \( n + b \) is measured as all employees in the nonfarm business sector. Unemployment \( u \) is civilian unemployment 16 years and older. Vacancies \( v \) are a composite help-wanted index computed by Barnichon (2010) combining print and online help-wanted advertising. The data and model output are detrended with an HP filter with the conventional smoothing parameter.

To explore the how the model works to capture the aggregate data, we first compute impulse responses to a one percent shock to productivity. The solid line is the response of the baseline model with staggered wage contracting and the dashed line is the model with period-by-period Nash bargaining. The model with wage rigidity produced an enhanced response of output and the various labor market variables, relative to the flexible wage case. This result is standard in the literature dating back to Shimer (2005) and Hall (2005) and in close keeping with Gertler and Trigari (2009), who use a similar model of staggered wage contracting, but without job-to-job transitions. We see that the addition of job-to-job transitions does not alter the main implications of wage rigidity for aggregate dynamics.

We begin by computing a variety of business cycle moments obtained from stochastic simulation obtained from feeding in a random sequence of productivity shocks. We do not mean to suggest that productivity shocks are the main business cycle driving forces. Rather, the simple real business cycle model offers a convenient way of studying the model implications for
unemployment and wage dynamics.

We first consider the model implications of an impulse response to a one percent increase in productivity. The plots are given in Figure 2. To highlight the role of staggered contracting, we plot the model generated output for the benchmark case ($\lambda = 8/9$) and the flexible wage case ($\lambda = 0$). Under period-by-period contracting, the model implications are reminiscent of those of the standard Nash bargaining model discussed by Hall (2005) and Shimer (2005). Wages immediately increase following a technology shock, whereas employment, unemployment, and vacancy posting respond only gradually and by very little. In the case with staggered contracting, the pattern is reversed: wages adjust gradually and only modestly, whereas there are large and immediate changes in employment, unemployment, and vacancies. We also find a greater increase in the job-finding probability under staggered bargaining. Additionally, we see that for both period-by-period and staggered bargaining, the stock of workers in good matches increases while the stock of workers in bad matches decreases; however, the quantitative magnitude of the change is far greater for the economy with staggered bargaining.

Table 6 compares the various business cycle statistics and measures of labor market volatility generated by the model with the data. The top panel gives the empirical standard deviations, autocorrelations, and correlations with output of output, wages, employment, unemployment, and vacancies. All standard deviations are normalized relative to output. The bottom panel computes the same statistics using the model. Here we use our baseline assumption that wage contracts have an expected duration of three quarters.

Overall the model does a reasonable job of accounting for the relative volatility of unemployment (5.41 in the model versus 5.74 in the data) and for wages (0.45 versus 0.48). As is common in the literature, the model understates the volatility of employment and overstates the volatility of vacancies. In the former case, the absence of a labor force participation margin is relevant and in the latter, error in measuring vacancies. Consistent with Shimer (2005) and Hall (2005), the wage inertia induced by staggered contracting is critical for the ability of the model to account for the volatility of unemployment. This result is robust to allowing for on-the-job search and procyclical match quality.

We next turn to the model’s ability to account for the panel data evidence. We use a stochastic simulation of the model to generate a time series on the unemployment rates and on the wages of new hires versus continuing workers. We then estimate equation (1) using the simulated data. Table 7 compares the results from the panel data (the first column) with those obtained from data from our model with wage contracts fixed for three quarters on average (the second column) and four quarters on average (the third column). Note that the estimates of
cyclical wage elasticities for continuing versus new workers are very similar in both cases. The model is thus able to produce estimates suggesting relatively greater cyclicality of new hires’ wages in a magnitude consistent with the evidence. The estimated excess cyclicality, however, is clearly an artifact of composition bias: After controlling for match quality, new hires’ wages in the model are nearly exactly as cyclical as they are for continuing workers.

Our baseline model has wage contracts fixed for three quarters on average. In the last column we explore the implications of having period-by-period Nash bargaining for wage determination. While the new hire effect remains, the estimated wage elasticities are too large (by a factor of roughly 6.5 for continuing workers). Thus, to account for the panel data estimates it is necessary to have not only procyclical movements in new hires’ match quality but also some degree of wage inertia as, for example, produced by staggered multi-period contracting.

Figures 3 and 4 illustrates how compositional effects influence wage dynamics. We repeat the experiment of a one percent increase in TFP. Figure 3 then reports impulse responses for labor in efficiency units, good matches, bad matches and job flows between good and bad matches. In the wake of the boom, labor quality increases. Underlying this increase is a rise in good matches and a net fall in bad matches. The rise in good matches is in part due to good matches being hired out of unemployment: But it is mostly due to an increase in the job flow share of workers moving from bad to good matches and a decline in the reverse flow share, as the two bottom left panels indicates. This pattern in the net flows also leads to a net decline in bad matches. 

Figure 4 the decomposes the response of new hires’ wage growth into the part due to the growth of contracts wages and the part due to compositional effects, using equations (46), (47), and (?). The sold line in the top panel is total new hires’ wage growth, the dashed line is the part due to composition, and the dashed line is average contract wage growth. As the figure illustrates, most of the new hires’ wage response is due to compositional effects. The bottom panel then relates the compositional effect mainly to the increase in the share of job flows moving from bad to good matches.

Finally, while our motivation for introducing procyclical job reallocation is to account for the panel data evidence, we note that it also generates interesting implications for the cyclical behavior of productivity. In particular, total factor productivity in the model depends on the allocation of workers between good and bad matches. To see this, we take the production function (16) and the definition of labor quality (17) to obtain an expression for how productivity

\footnote{In gross term there are bad matches due to workers being hired from unemployment. The behavior of the job to job flows swamps this effect however.}
depends on the quality composition, measured by $\gamma_t = b_t/n_t$:

\[
y_t = z_t k_t^\alpha (n_t + \phi b_t)^{1-\alpha} = z_t \left( \frac{1 + \phi \gamma_t}{1 + \gamma_t} \right)^{1-\alpha} k_t^\alpha (n_t + b_t)^{1-\alpha}
\]

where the term $z_t \left( \frac{1 + \phi \gamma_t}{1 + \gamma_t} \right)^{1-\alpha}$ is the effective level of TFP. Loglinearizing this term yields the effect of cyclical reallocation on cyclical productivity:

\[
\hat{z}_t - (1 - \alpha) \frac{1}{1 + \gamma} \frac{1 - \phi}{1 + \phi \gamma} \hat{\gamma}_t
\]

Since $\hat{\gamma}_t$ is countercyclical, the effect of labor reallocation on productivity is procyclical.

In Figure 6 we report the response of the endogenous component of productivity $e_t$ to a one percent increase in the exogenous component $z_t$, where $\hat{e}_t$ can be expressed as

\[
\hat{e}_t = -(1 - \alpha) \frac{1}{1 + \gamma} \frac{1 - \phi}{1 + \phi \gamma} \hat{\gamma}_t.
\]

The endogenous component adds a small – roughly 0.23 percent at the peak – but highly persistent effect on productivity, as the top panel suggests. The bottom panel shows the effect on output: the improvement in aggregate match quality due to the reallocation of labor leads to a similarly modest but persistent increase in output. Hence, for this particular experiment, the impact of the endogenous component of TFP on output is relatively modest. Consider a different experiment, however, where output is reduced by the amount it fell during the Great Recession (roughly ten percent relative to trend). A back-of-the-envelope calculation based on Figure 6 would then suggest that the fall in output would be accompanied by a persistent drop in productivity of more than a percentage point due to the endogenous reallocation of labor.

5 Concluding Remarks

We present panel data evidence suggesting that the excess cyclicality of new hires’ wages relative to existing workers may be an artifact of compositional effects in the labor force that have not been sufficiently accounted for in the existing literature. We reinforce this point by developing a model of aggregate unemployment that generates quantitative implications consistent with both macro and micro data. In the model, new hires’ wages are the same as continuing workers of
the same match productivity; but, as we find in our estimates from panel data, new hire wages appear to be more cyclical due to the procyclicality of job quality in new matches. Our bottom line: it is reasonable for macroeconomists to continue to make use of wage rigidity to account for economic fluctuations. The focus should be on how best to model wage rigidity rather than whether it is appropriate to model at all.

Finally, our model of unemployment fluctuations with staggered wage contracting differs from much of the literature in allowing a channel for procyclical job-to-job transitions. For many purposes, it may be fine to abstract from this additional channel. However in major recessions like the recent one, a slowdown in job reallocation is potentially an important factor for explaining the overall slowdown of the recovery. Recent studies by Haltiwanger, Hyatt and McEntarfer (2013) and Moscarini and Postel-Vinay (2014) provide evidence that the rate of job-to-job transitions has not recovered relative to the overall job-finding rate in the current recovery. Our model provides a hint about how the slowdown in job reallocation might feedback into other economic activity. It might be interesting to explore these issues and consider other factors, such as financial market frictions, that have likely hindered the reallocation process in the recent recession.

\footnote{Notable exceptions include Menzio and Shi (2011) and Moscarini and Postel-Vinay (2013).}
References


### Table 1: “Bils regressions” and the new hire effect

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<td>I(new)</td>
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| Estimator                      | Fixed Effects    | First Differences |
|                                | 379,104          | 321,397          |

Robust standard errors in parenthesis

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 2: Job changers (EE) vs. new hires from unemployment (ENE), fixed effects

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<td>−</td>
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<td>−0.067***</td>
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<tr>
<td></td>
<td>−</td>
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<td>(0.0087)</td>
<td>(0.0087)</td>
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|                          | 0.019           | 0.004    | 0.046    | 0.011    |
| $P(\pi_{U,n}^{EE} = \pi_{U,n}^{ENE})$ | 0+               | 1+       | (0, 9]   | (1, 9]   |
| Unemp spell for ENE      | 375,649         | 375,649  | 375,649  | 375,649  |
| No. of fixed effects     | 56,878          | 56,878   | 56,878   | 56,878   |

Robust standard errors in parenthesis

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 3: Job changers (EE) vs. new hires from unemployment (ENE), first differences

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\( P(π_{EE}^{U,n} = π_{ENE}^{U,n}) \)

|                      | 0.163           | 0.171    | 0.424    | 0.463    |

Unemp spell for ENE

|                      | 0+              | 1+       | (0, 9]   | (1, 9]   |

No. observations

|                      | 318,771         | 318,771  | 318,771  | 318,771  |

Robust standard errors in parenthesis

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Table 4: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ = $0.997 = 0.99^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$ = 0.008 = 0.025/3</td>
<td></td>
</tr>
<tr>
<td>Production function parameter</td>
<td>$\alpha$ = 0.33</td>
<td></td>
</tr>
<tr>
<td>Technology autoregressive parameter</td>
<td>$\rho_z$ = $0.983 = 0.95^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>Technology standard deviation</td>
<td>$\sigma_z$ = 0.0075</td>
<td></td>
</tr>
<tr>
<td>Elasticity of matches to searchers</td>
<td>$\sigma$ = 0.4</td>
<td></td>
</tr>
<tr>
<td>Bargaining power parameter</td>
<td>$\eta$ = 0.5</td>
<td></td>
</tr>
<tr>
<td>Matching function constant</td>
<td>$\sigma_m$ = 1.0</td>
<td></td>
</tr>
<tr>
<td>Search cost elasticity</td>
<td>$\eta_k$ = 1.0</td>
<td></td>
</tr>
<tr>
<td>Baseline renegotiation frequency</td>
<td>$\lambda$ = 0.889 (3 quarters)</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Jointly calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Inverse productivity premium</td>
<td>0.70</td>
<td>Average E-E wage increase (4.80%)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Prob. of good match</td>
<td>0.02</td>
<td>Average E-N-E wage decrease (6.20%)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Hiring cost parameter</td>
<td>165.73</td>
<td>U-E probability (0.45)</td>
</tr>
<tr>
<td>$\varsigma_0$</td>
<td>Scale parameter or search cost</td>
<td>0.06</td>
<td>E-E probability (0.029)</td>
</tr>
<tr>
<td>$u_b$</td>
<td>Flow value of unemployment</td>
<td>1.91</td>
<td>Relative value, non-work (0.501)</td>
</tr>
<tr>
<td>$1 - \nu$</td>
<td>Separation probability</td>
<td>0.05</td>
<td>E-U probability (0.026)</td>
</tr>
</tbody>
</table>
Table 6: Aggregate statistics

<table>
<thead>
<tr>
<th></th>
<th>$y$</th>
<th>$w$</th>
<th>$n + b$</th>
<th>$u$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S. Economy, 1964:1-2013:02</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.48</td>
<td>0.64</td>
<td>5.74</td>
<td>6.38</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.88</td>
<td>0.88</td>
<td>0.94</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.79</td>
<td>−0.87</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model Economy, $\lambda = 8/9$ (3 quarters)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.50</td>
<td>0.26</td>
<td>4.59</td>
<td>7.14</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.84</td>
<td>0.95</td>
<td>0.88</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>0.72</td>
<td>0.92</td>
<td>−0.92</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model Economy, $\lambda = 11/12$ (4 quarters)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.45</td>
<td>0.31</td>
<td>5.41</td>
<td>8.15</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.85</td>
<td>0.96</td>
<td>0.89</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>0.64</td>
<td>0.92</td>
<td>−0.92</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model Economy, $\lambda = \infty$ (Flex wages)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.74</td>
<td>0.16</td>
<td>2.76</td>
<td>4.73</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.82</td>
<td>0.80</td>
<td>0.91</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.83</td>
<td>−0.83</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Table 7: Wage semi-elasticities: All new hires

<table>
<thead>
<tr>
<th></th>
<th>SIPP</th>
<th>Model, 3Q</th>
<th>Model, 4Q</th>
<th>Model, flex</th>
</tr>
</thead>
<tbody>
<tr>
<td>UR</td>
<td>−0.45</td>
<td>−0.88</td>
<td>−0.53</td>
<td>−2.41</td>
</tr>
<tr>
<td>UR \cdot I(new)</td>
<td>−1.00</td>
<td>−1.17</td>
<td>−1.08</td>
<td>−2.64</td>
</tr>
</tbody>
</table>
Figure 1: Composition bias and new hire wage cyclicality
Figure 2: Impulse responses of employment to productivity shock
Figure 3: Labor market composition and job flows
Figure 4: Wage growth and components

\[ g_{JC}, \delta_{GB, t-1} \]
Figure 5: TFP, productivity, and output
6 Appendix: data

We use data from the Survey of Income and Program Participation (SIPP) from 1990 to 2012. The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. From 1990 to 1993, the Census Bureau would introduce a new panel on an annual basis, where each panel is administered for a period of 32 to 40 months. Hence for certain years in the early 1990s, data is available from multiple panels, each consisting from around 15,000 to 24,000 households. Starting in 1996, the Census changed the structure of the survey to follow contiguous panels. Since the redesign, new panels have been introduced in 1996, 2001, 2004, and 2008; for each of these panels, the Census has followed a larger sample of households (e.g. 40,188 in 1996) over a longer period.

Some information is only updated once per wave; other information is updated at monthly or weekly frequencies. Employment information, for example, is collected at various frequencies. For instance, employment status is reported at weekly but earnings are only reported monthly. Job identifiers are recorded only at a four-month frequency; that is, job identifiers do not change within a wave. We combine monthly earnings records specific to each job to discern the pattern of job flows and sources of earnings over the wave.

The SIPP has several advantages relative to other commonly used panel data sources such as the PSID or the NLSY. Relative to the PSID, the SIPP follows a larger number of households, is nationally representative, and has more frequent observations. For the purposes of this paper, the PSID also suffers the disadvantage that it is difficult to identify wage earnings with a particular job in years where multiple jobs are held. Relative to the NLSY, the SIPP follows a larger number of households, but more importantly, multiple cohorts. Relative to both surveys, the SIPP suffers the disadvantage that it follows any particular individual for a shorter overall duration. But as mentioned before, the SIPP collects rich retrospective information that get around problems of left-censoring; in particular, we observe start dates for jobs held during the first wave but started prior to the first interview. We then use precise measures of job transitions to determine the following sequence of jobs spells for the entirety of the sample.\footnote{For each wave, the survey contains fields for up to two jobs. The survey maintains longitudinally consistent job IDs for each individual and tracks certain job-specific characteristics at a monthly frequency, including earnings. We follow the procedure detailed in cite Stinson (2003) to correct inconsistent job identification variables for the 1990 to 1993 panels. We use monthly earnings data within waves to determine at which job the individual is working and for what months the individual is working at each potential job. From these data, we determine within a wave whether an individual made a job transition; and whether the job transition was characterized by an intervening period of non-employment.}
6.1 Identifying distinct jobs

The SIPP maintains job-specific longitudinally consistent employment information over waves for which an individual is reports non-zero employment. For such case, the SIPP maintains distinct job identifiers for identical jobs, allowing users to distinguish new jobs from “recalls” (to adopt the terminology of Fujita and Moscarini, 2014). Table 8 gives an example employment history of an individual who works at a job, spends four months in non-employment, but returns to the same job. The SIPP correctly records that the individual returned to the job that she left.

The SIPP resets employment records for individuals who are without employment for an entire wave. If individuals return to a previously held job after spending an entire wave in non-employment, the SIPP will incorrectly record the individual as starting a new job. Hence, a single job can be given multiple job identifiers. Table 9 gives a sample employment history of an individual who works at a job, spends an entire wave out of work, and then returns to the same job. As in the previous example, the individual spends four months not working; but because those four months happen to fall over the entirety of a wave, the job is given a new identifier when the individual returns to work. For such individuals, we could mistakenly label a recall to be a job-to-job transition across separate jobs.

We exploit an additional source of information recorded by the SIPP to identify potential recalls. Every time that a distinct job identifier is associated with an individual, the survey also adds a start date. This is indicated by the box around “start date” in the third row of table 9. When we observe a start date that falls before the date that the SIPP purges the mapping of start dates to unique jobs, we have a good indication that the “new job” is in fact a recall.

To what extent do respondents report the date that they began the job, inclusive of employment gaps, versus the date that they last began a contiguous employment spell? We note that the survey question recording start dates is explicitly designed to identify the start date to be the former of the two, as it is designed to distinguish jobs that began within the wave from jobs that began before the wave.

For example, in the 1996 panel, respondents are asked “Did [FIRST AND LAST NAME] begin [HIS HER] employment with [NAME OF EMPLOYER] at some time between [MONTH1] 1st and today?” (variable STRTJB). If individuals respond in the affirmative, they are asked about the month and day within the wave that the job began (STRTREFP). Otherwise, they are asked to give their “BEST estimate” of the year, month, and date that the job began (variables
To identify potential recalls, we apply the following criterion: for individuals with an incomplete employment record – e.g. respondents who have spent a complete wave in non-employment – we consider any job with a start date prior to the period of non-employment (the date at which the SIPP purges internal employment records) as a potential recall, and we do not count the individual as a new hire.

We illustrate our criterion in tables 10 and 11. In table 10, we observe an individual work a wave at Job A, spend an entire wave in non-employment, and then start work at Job B in wave 3. The start date of Job B is before the “gap date”, and hence, it is more likely that Job B is the same as Job A. Hence, we do not consider the individual as a new hire at Job B. In table 11, we similarly observe an individual work at Job A, spend a wave in non-employment, and then work at Job B; however, the start date for job B in this instance is after the gap date, and hence, we consider the worker to be a new hire in wave 3.

We apply the gap date criterion with two small additions: first, for a subset of job dissolutions, workers report the cause of the dissolution. If the worker reports that he left the pre-gap job to take another job, we do preclude the possibility that the post-gap job is a recall to the first job. Second, if the start date at a post gap job is missing or statistically imputed, we identify the job as a potential recall and do not count the worker as a new hire.

We find a substantial number of potential recalls across the post-1996 panels, corroborating Fujita and Moscarini’s findings of recalls as a pervasive feature of the U.S. labor market. Table 12 gives a tabulation of potential recalls among all SIPP respondents who report a full wave without employment.

---

26 See the 1996 Panel Wave 02 Questionnaire at http://www.census.gov/content/dam/Census/programs-surveys/sipp/questionnaires/1996/SIPP%201996%20Panel%20Wave%202-02-20Core%20Questionnaire.pdf
Table 8: Two separate employment spells, one job, correct IDs. Job ID preserved across contiguous employment spells because individual reports employment for each wave.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>A</td>
<td>09/95</td>
<td>M1</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>A</td>
<td>09/95</td>
<td>M2-M4</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 9: Two separate employment spells, one job, incorrect IDs. Job ID information is lost when individual spends an entire wave without employment. At wave 3, the job is incorrectly coded as being a new job and the start date is asked again.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 10: Two separate employment spells, “gap date” falls after reported job start date for job “B”. Rule out wave 3 job as “new hire”.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
<th>Gap Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
<td>05/96</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>11/95</td>
<td>M1-M4</td>
<td>A</td>
<td>05/96</td>
</tr>
</tbody>
</table>
Table 11: Two separate employment spells, “gap date” is prior to reported job start date for job “B”. Count wave 3 job as “new hire”.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
<th>Gap date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
<td>05/96</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>08/96</td>
<td>M1-M4</td>
<td>A</td>
<td>05/96</td>
</tr>
</tbody>
</table>

Table 12: Potential recall in the SIPP

<table>
<thead>
<tr>
<th>Panel</th>
<th>No. of jobs post 4 month empl. gap</th>
<th>% Potential recalls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>13,360</td>
<td>33.20</td>
</tr>
<tr>
<td>2001</td>
<td>8,605</td>
<td>38.81</td>
</tr>
<tr>
<td>2004</td>
<td>16,675</td>
<td>49.22</td>
</tr>
<tr>
<td>2008</td>
<td>23,776</td>
<td>52.49</td>
</tr>
</tbody>
</table>
7 Appendix: model

We now derive the log-linear equations that describe the first-order dynamics of the model. Most of the derivations are standard or are similar to those in Gertler and Trigari (2009). Relative to GT, where the only persistent firm-specific state variable was wages, here we must also keep track of composition. As might be expected, there is a non-trivial interplay between composition and wages at the firm level. Composition is inherited from the previous period and influences the wage through the Nash wage bargain; the wage influences next-period composition through hiring. We introduce a simplifying steady state restriction that lends analytic and computational tractability to the analysis.

We first state five steady-state results that will simplify the derivation of the log-linear equations. We establish how these properties will be used to linearize the “retention motive” in hiring, wherein firms vary the hiring rate to vary next-period composition. We then go over the relevant equations for determining the Nash wage: the worker and firm surpluses, certain derivatives of the surpluses, and the Nash first order condition. We then derive recursive log-linear expressions for the firm and worker surpluses. We derive similar expressions for the derivative of surpluses, which act as discount factors that differ across firms and worker in the determination of the Nash wage under staggered contracting. We derive log-linear expressions for the wage growth of job changers and the flow shares of job-to-job changers. Then, we prove the steady-state results that we repeatedly invoke in deriving recursive log-linear expressions for the worker and firm surplus and discount factors and to linearize the retention term in hiring. In the last section, we tie up a final loose end and define the operator mapping the distribution function from period \( t \) to period \( t + 1 \).

7.1 Some useful steady-state results

Labor force composition affects firms through the average retention rate of an efficiency unit of labor. Let \( \tilde{z} \) denotes the steady state of variable \( z \). Assume that \( \zeta_n = \zeta_b \), so that \( \hat{\rho}_n = \hat{\rho}_b = \hat{\rho} \) (i.e., retention rates of good and bad workers are the same in steady state). Then we obtain the following steady state results:

1. \( \text{Var}_t(\gamma_t) = 0 \)
2. \( \partial \gamma_{t+1}/\partial x_t = 0 \)
3. \( \partial w^*_t(\gamma_t)/\partial \gamma_t = 0 \)
4. \( \partial J_t(\gamma_t, w_t)/\partial \gamma_t = 0 \)

5. \( \partial H_t(\gamma_t, w_t)/\partial \gamma_t = 0 \)

Intuitively, these results guarantee that composition evolves as though it were an aggregate state variable. We make extensive use of all the results to derive recursive log-linear equations for the worker and firm surpluses, log-linear equations for the worker and firm discount factors, and to prove that within-firm composition has no first order effects on hiring rates.

We will invoke these results in the following subsections and then prove them at the end of the appendix.

7.2 Hiring equation

In the main text, we derive the first order condition for hiring. Given that next period wage equals this period wage \( w_t \) with probability \( \lambda \) and next period contract wage \( w^*_{t+1}(\gamma_{t+1}) \) with probability \( 1 - \lambda \), we can write the hiring condition at a firm with composition \( \gamma_t \) and wage \( w_t \) as

\[
\kappa_x t (\gamma_t, w_t) = \mathbb{E}_t \{ \Lambda_{t, t+1} \left[ \lambda J_{t+1} (\gamma_{t+1}, w_t) + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})) \right] \} + \omega_t (\gamma_t, w_t),
\]

where the second term represents a non-standard retention motive in hiring:

\[
\omega_t (\gamma_t, w_t) = [\rho_t (\gamma_t) + x_t (\gamma_t, w_t)] \times
\mathbb{E}_t \left\{ \Lambda_{t, t+1} \left[ \lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1}))}{\partial \gamma_{t+1}} \right] \right\}.
\]

The firm cares about composition for the implied retention rate of a unit of labor quality (represented by the first two terms in square brackets) and through possible effects of firm composition on future renegotiated wages (the third term).

Since we will prove that \( \partial J/\partial \gamma, \partial w^*/\partial \gamma, \partial \gamma'/\partial x \) are all equal to 0 in the steady state, it follows that up to a first order \( \omega_t (\gamma_t, w^*_t(\gamma_t)) = 0 \).
### 7.3 Staggered Nash bargaining

Consider the problem of a renegotiating firm and workers in good matches. We can write the surplus of workers in good matches $H_t(\gamma_t, w^*_t(\gamma_t))$ as

$$H_t(\gamma_t, w^*_t(\gamma_t)) = w^*_t(\gamma_t) - u_b - [\nu c(s_n) + (1 - \nu) c(s_u)]$$

$$+ E_t \{ \Lambda_{t,t+1} [\nu s_n p_t^a H_{t+1} - (1 - \nu) s_n p_t \bar{H}_t^a] \}$$

$$+ \nu (1 - s_n p_t^a) E_t \{ \Lambda_{t,t+1} [\lambda H_{t+1} (\gamma_{t+1}, w^*_t(\gamma_t))]$$

$$+ (1 - \lambda) H_{t+1} (\gamma_{t+1}, w^*_t(\gamma_{t+1})) \}$$

with

$$\bar{H}_t^a \equiv \xi (\bar{V}_t^a - U_t) + (1 - \xi) (\bar{V}_t^b - U_t).$$

Similarly, we can write firm surplus $J_t(\gamma_t, w^*_t(\gamma_t))$ as

$$J_t(\gamma_t, w^*_t(\gamma_t)) = a_t - w^*_t(\gamma_t) - \frac{\kappa}{2} x_t (\gamma_t, w^*_t(\gamma_t))^2$$

$$+ [\rho_t(\gamma_t) + x_t (\gamma_t, w^*_t(\gamma_t))] \times$$

$$E_t \{ \Lambda_{t,t+1} [\lambda J_{t+1} (\gamma_{t+1}, w^*_t(\gamma_t))]$$

$$+ (1 - \lambda) J_{t+1} (\gamma_{t+1}, w^*_t(\gamma_{t+1})) \}$$

with

$$\kappa x_t (\gamma_t, w^*_t(\gamma_t)) = E_t \{ \Lambda_{t,t+1} [\lambda J_{t+1} (\gamma_{t+1}, w^*_t(\gamma_t))]$$

$$+ (1 - \lambda) J_{t+1} (\gamma_{t+1}, w^*_t(\gamma_{t+1})) \}$$

$$+ \omega_t (\gamma_t, w^*_t(\gamma_t))$$

and

$$a_t \equiv (1 - \alpha) z_t I_t^\alpha .$$

The first order condition for Nash bargaining is

$$\eta \frac{\partial H_t(\gamma_t, w^*_t(\gamma_t))}{\partial w^*_t(\gamma_t)} J_t(\gamma_t, w^*_t(\gamma_t)) = (1 - \eta) \left( - \frac{\partial J_t(\gamma_t, w^*_t(\gamma_t))}{\partial w^*_t(\gamma_t)} \right) H_t(\gamma_t, w^*_t(\gamma_t))$$

Note that the outcome of the bargaining problem will generally depend on labor composition within the firm. Since $\text{Var}_t(\gamma_t) = 0$ up to a first order, it is of no matter for studying the first-order model dynamics.
where the $\partial H^*/\partial w^*$ and $-\partial J^*/\partial w^*$ act as cumulative discount factors applied by the worker and the firm to value the contract wage stream. The worker discount factor is given by

$$
\frac{\partial H_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} = 1 + \nu (1 - \varsigma_n p^n_t) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \right\}
$$

with

$$
\psi_t(\gamma_t, w_t^*(\gamma_t)) = \nu (1 - \varsigma_n p^n_t) \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial \gamma_{t+1}} \right.ight.
$$

$$
+ (1 - \lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial \gamma_{t+1}} \bigg] \times
$$

$$
\left\{ \frac{\partial \gamma_{t+1}}{\partial x_t} \frac{\partial x_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right\} \bigg\}
$$

Since we will prove that $\partial H/\partial \gamma$, $\partial w^*/\partial \gamma$, $\partial \gamma/\partial x$ are all equal to 0 in the steady state, it follows that up to a first order $\psi_t(\gamma_t, w_t^*(\gamma_t)) = 0$.

The firm discount factor is given by

$$
\frac{\partial J_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} = -1 + \left[ \rho_t(\gamma_t) + x_t(\gamma_t, w_t^*(\gamma_t)) \right] \lambda \times
$$

$$
\mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right\}
$$

where we have used the fact that $J_t(\gamma_t, w_t^*(\gamma_t))$ is maximized with respect to $x_t(\gamma_t, w_t^*(\gamma_t))$, so that in taking the derivative with respect to $w_t^*(\gamma_t)$, we can hold $x_t(\gamma_t, w_t^*(\gamma_t))$ fixed at its optimal value.

### 7.3.1 Surplus of workers in good matches

We now develop a first order approximation of the Nash condition. We start by loglinearizing $H_t(\gamma_t, w^*(\gamma_t))$. We will establish that composition is first-order equivalent across firms. This permits us to drop composition as a separate argument of the value function and the contract
wage and allow it to be captured by the aggregate state. We can then write

\[ H_t(w_t^*) = w_t^* - u_b - [\nu c(\varsigma_n) + (1 - \nu) c(\varsigma_u)] \]

\[ + \nu \mathbb{E}_t \{ \Lambda_{t,t+1} [\nu \varsigma_n \rho_t H_{t+1} - (1 - (1 - \nu) \varsigma_u) p_t H_{t+1}^a] \} \]

\[ + \nu (1 - \varsigma_n \rho_t) \mathbb{E}_t \{ \Lambda_{t,t+1} (w_{t+1}^*) \} \]

\[ + \nu (1 - \varsigma_n \rho_t^n) \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} [H_{t+1} (w_{t+1}^*) - H_{t+1} (w_{t+1}^*)] \} + \rho \lambda \beta \mathbb{E}_t \{ \Lambda_{t,t+1} H_{t+1} (w_{t+1}^*) \} \]

Loglinearizing, we obtain:

\[ \hat{H}_t(w_t^*) = \left( \frac{\bar{w}}{\bar{H}} \right) \bar{w}_t^* \]

\[ + (\nu - \hat{\rho}) \beta \mathbb{E}_t \left\{ \bar{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \]

\[ - (1 - (1 - \nu) \varsigma_u) \hat{\rho} \beta \left( \frac{\bar{H}_a}{\bar{H}} \right) \mathbb{E}_t \left\{ \bar{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \]

\[ + \hat{\rho} \beta \mathbb{E}_t \left\{ - \frac{\nu - \hat{\rho}}{\hat{\rho}} \bar{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \]

\[ + \hat{\rho} \lambda \beta \left( \bar{w} / \bar{H} \right) \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_t^* \} \]

where

\[ \hat{\epsilon} = \frac{\partial H_t(w_t^*)}{\partial w_t^*} \bigg|_{ss} = \frac{1}{1 - \hat{\rho} \lambda \beta} \]

and

\[ \hat{H}_{t+1}^a = \left( \xi \frac{\hat{H}}{\hat{H}^a} \right) \left( \hat{V}_t^a - \hat{U}_t \right) + \left( 1 - \xi \frac{\hat{H}}{\hat{H}^a} \right) \left( \hat{V}_t^b - \hat{U}_t \right) \]

Further simplifying gives

\[ \hat{H}_t(w_t^*) = \left( \frac{\bar{w}}{\bar{H}} \right) \left[ w_t^* + \hat{\rho} \lambda \beta \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \} \right] \]

\[ - (1 - (1 - \nu) \varsigma_u) \hat{\rho} \beta \left( \frac{\hat{H}_a}{\hat{H}} \right) \mathbb{E}_t \left\{ \bar{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \]

\[ + (\nu - \hat{\rho}) \beta \mathbb{E}_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1} \right\} \]

\[ + \hat{\rho} \beta \mathbb{E}_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1} (w_{t+1}^*) \right\} . \]
7.3.2 Surplus of firms

Combining the expression of the firm surplus with the hiring condition and using the same notation as for the worker surplus in good matches, we can write the firm surplus as

\[ J_t(w_t^*) = a_t - w_t^* + \frac{\kappa}{2} x_t(w_t^*)^2 
+ \rho_t \mathbb{E}_t \{ \Lambda_{t,t+1} J_{t+1}(w_{t+1}^*) \} 
+ \lambda \rho_t \mathbb{E}_t \{ \Lambda_{t,t+1} [J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)] \} \]

where we have dropped the term \( \omega_t(\gamma_t, w_t^*(\gamma_t)) \) that is 0 up to a first order, as previously established.

Loglinearizing yields

\[ \tilde{J}_t(w_t^*) = \left( \frac{\tilde{a}}{\tilde{J}} \right) \tilde{a}_t - \left( \frac{\tilde{w}}{\tilde{J}} \right) \tilde{w}_t^* + \left( \frac{\kappa \tilde{x}^2}{\tilde{J}} \right) \tilde{x}_t(w_t^*) 
+ \tilde{\rho} \beta \mathbb{E}_t \{ \tilde{\rho}_t + \tilde{\Lambda}_{t,t+1} + \tilde{J}_{t+1}(w_{t+1}^*) \} 
- \lambda \tilde{\rho} \beta \left( \frac{\tilde{w}}{\tilde{J}} \right) \tilde{\mu} \mathbb{E}_t \{ \tilde{w}_t^* - \tilde{w}_{t+1}^* \}, \]

where

\[ \tilde{\mu} = \frac{\partial J_t(w_t^*)}{\partial w_t^*} \bigg|_{ss} = \frac{1}{1 - \lambda \beta}. \]

Rearranging further,

\[ \tilde{J}_t(w_t^*) = \left( \frac{\tilde{a}}{\tilde{J}} \right) \tilde{a}_t - \left( \frac{\tilde{w}}{\tilde{J}} \right) \left[ \tilde{w}_t^* + \lambda \tilde{\rho} \beta \tilde{\mu} \mathbb{E}_t \{ \tilde{w}_t^* - \tilde{w}_{t+1}^* \} \right] 
+ \left( \frac{\kappa \tilde{x}^2}{\tilde{J}} \right) \tilde{x}_t(w_t^*) 
+ \tilde{\rho} \beta \mathbb{E}_t \{ \tilde{\rho}_t + \tilde{\Lambda}_{t,t+1} + \tilde{J}_{t+1}(w_{t+1}^*) \} 
- \lambda \tilde{\rho} \beta \left( \frac{\tilde{w}}{\tilde{J}} \right) \tilde{\mu} \mathbb{E}_t \{ \tilde{w}_t^* - \tilde{w}_{t+1}^* \}. \]

7.3.3 Worker and firm discount factors

Following the same simplifications as for the worker and firm surpluses, we can write the firm and worker discount factors as:

\[ \frac{\partial H_t(w_t^*)}{\partial w_t^*} = 1 + \nu (1 - \varsigma_0 p_t) \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} \frac{\partial H_{t+1}(w_t^*)}{\partial w_t^*} \} \]

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\[
\frac{\partial J_t(w_t^*)}{\partial w_t^*} = -1 + [\rho_t + x_t(w_t^*)] \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(w_{t+1}^*)}{\partial w_{t+1}^*} \right\}
\]

where we have dropped from the worker discount factor the term \( \psi_t(\gamma_t, w_t^*(\gamma_t)) \) that is 0 up to a first order. Now define the variables \( \epsilon_t(w) \) and \( \mu_t(w) \) as follows:

\[
\epsilon_t(w) \equiv \frac{\partial H_t(w)}{\partial w}, \quad \mu_t(w) \equiv -\frac{\partial J_t(w)}{\partial w}
\]

We can then write the worker and firm discount factors in bargaining as

\[
\epsilon_t = 1 + \nu (1 - \varsigma_n) \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} \epsilon_{t+1} \}
\]

\[
\mu_t(w_t^*) = -1 + [\rho_t + x_t(w_t^*)] \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} \mu_{t+1}(w_{t+1}^*) \}
\]

and their corresponding loglinear expressions as

\[
\hat{\epsilon}_t = \tilde{\rho} \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\epsilon}_{t+1} \} - (\nu - \tilde{\rho}) \beta \lambda \hat{\rho}
\]

\[
\hat{\mu}_t(w_t^*) = \beta \lambda [\tilde{\rho} \hat{\rho} + (1 - \tilde{\rho}) \tilde{x}_t(w_t^*)] + \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\mu}_{t+1}(w_{t+1}^*) \}
\]

To derive a recursive expression for \( \hat{\mu}_t(w_t^*) \), first note that we have

\[
\mathbb{E}_t \{ \hat{\mu}_{t+1}(w_{t+1}^*) - \hat{\mu}_{t+1}(w_{t+1}^*) \} = -(1 - \tilde{\rho}) \lambda \hat{\mu} \left( \tilde{w}/\tilde{J} \right) \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}
\]

Combining, we obtain

\[
\hat{\mu}_t(w_t^*) = \beta \lambda [\tilde{\rho} \hat{\rho} + (1 - \tilde{\rho}) \tilde{x}_t(w_t^*)] + \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\mu}_{t+1}(w_{t+1}^*) \}
\]

\[
- \beta \lambda (1 - \tilde{\rho}) \lambda \hat{\mu} \left( \tilde{w}/\tilde{J} \right) \beta \lambda \hat{\mu} \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}.
\]

### 7.4 Wage growth of job changers

In this section, we derive expressions for the flow shares of the various types of job-to-job flows; and we derive an expression for the average wage growth of job changers.

#### 7.4.1 Job-to-job flows

We have two types of job-to-job transitions: those due to on-the-job search and those due to separations followed by search and finding of a new job within the same period. The latter are initiated by a match separation shock, but there is no spell of unemployment between the jobs.
We have the following job-to-job flows:

Bad to good : 
\[ (1 - \nu) \varsigma_u + \nu \varsigma_b \bar{\gamma}_t \]

Good to bad : 
\[ (1 - \nu) \varsigma_a (1 - \bar{\xi}) p_t \bar{n}_t \]

Bad to bad : 
\[ (1 - \nu) \varsigma_u (1 - \bar{\xi}) p_t \bar{b}_t \]

Good to good : 
\[ [(1 - \nu) \varsigma_u + \nu \varsigma_n] \bar{\xi} p_t \bar{n}_t \]

Summing over the flows we obtain total job flows as:

\[ [(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)] p_t \bar{n}_t \]

The shares of flows over total flows then are defined as:

\[
\delta_{BB,t} = \frac{(1 - \nu) \varsigma_u (1 - \bar{\xi}) \bar{\gamma}_t}{(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)}
\]

\[
\delta_{BG,t} = \frac{[(1 - \nu) \varsigma_u + \nu \varsigma_b] \bar{\xi}}{(1 - \nu) \varsigma_u (1 - \bar{\xi}) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)}
\]

\[
\delta_{GB,t} = \frac{[(1 - \nu) \varsigma_u + \nu \varsigma_n] \bar{\xi}}{(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)}
\]

\[
\delta_{GG,t} = \frac{[(1 - \nu) \varsigma_u + \nu \varsigma_n] \bar{\xi}}{(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)}
\]

We also define the component of the share of bad-to-good flows due to on-the-job search, \( \delta_{BGS,t} \), given by:

\[
\delta_{BGS,t} = \frac{\nu \varsigma_b \bar{\xi} \bar{\gamma}_t}{(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \bar{\xi} (\varsigma_n + \varsigma_b \bar{\gamma}_t)}
\]

### 7.4.2 Average wage growth of job changers

Let \( \bar{g}_t^{w_c} \) denote the average wage growth of continuing workers and \( \bar{g}_t^{JC} \) the average wage growth of job changers.

Up to a first order, \( \bar{g}_t^{JC} \) can be written as:

\[
\bar{g}_t^{JC} = \delta_{BB,t-1} \log \left( \frac{\phi \bar{w}_t}{\phi \bar{w}_{t-1}} \right) + \delta_{GG,t-1} \log \left( \frac{\bar{w}_t}{\bar{w}_{t-1}} \right) + \\
\delta_{BG,t-1} \log \left( \frac{\bar{w}_t}{\phi \bar{w}_{t-1}} \right) + \delta_{GB,t-1} \log \left( \frac{\phi \bar{w}_t}{\bar{w}_{t-1}} \right)
\]
Simplifying, we obtain:

\[ \tilde{g}^{JC}_t = \tilde{g}^w_t + c^w_t \]

with

\[ \tilde{g}^w_t = \log \left( \frac{\tilde{w}_t}{w_{t-1}} \right) \]

and

\[ c^w_t = (- \log \phi) (\delta_{BG,t-1} - \delta_{GB,t-1}) \]

Thus, average wage growth of new hires that are job changers equals average wage growth of continuing workers plus a composition component measuring the change in match quality among job changers. The composition component equals 0 if match quality is homogeneous \((\phi = 1)\).

Loglinearizing the average gross wage growth of job changers we obtain:

\[ \tilde{g}^{JC}_t = \frac{1}{1 + \tilde{c}^w_t \tilde{g}^w_t} + \frac{\tilde{c}^w_t}{1 + \tilde{c}^w_t \tilde{c}^w_t} \]

Loglinearizing the compositional effect we obtain:

\[ \tilde{c}^w_t = \frac{1}{\delta_{BG} - \delta_{GB}} \left( \tilde{\delta}_{BG,1} \tilde{\delta}_{BG,t-1} - \tilde{\delta}_{GB} \tilde{\delta}_{GB,t-1} \right) \]

with

\[ \tilde{\delta}_{BG,t} = \frac{1}{1 + \tilde{\gamma}} \tilde{\gamma}_t + \frac{1 - \tilde{\delta}_{BG}}{\tilde{\delta}_{BG}} \tilde{\delta}_{BGS} \tilde{s}_{bt} \]

\[ \tilde{\delta}_{GB,t} = -\frac{\tilde{\gamma}}{1 + \tilde{\gamma}} \tilde{\gamma}_t - \tilde{\delta}_{BGS} \tilde{s}_{bt} \]

Rearranging, we find the expression relating the composition effect to variable search intensity of workers in bad matches and firm average composition:

\[ \tilde{c}^w_t = \frac{\tilde{\delta}_{BG} + \tilde{\gamma} \tilde{\delta}_{GB}}{(1 + \tilde{\gamma}) \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right) \tilde{\gamma}_{t-1} + \frac{1 - \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right)}{\left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right) \tilde{\delta}_{BGS} \tilde{s}_{b,t-1}} \]

### 7.5 Derivation of steady-state results

We now derive the steady-state results invoked at the beginning of the appendix.
7.5.1 Evolution of composition

Recall the expression for the retention rates of good and bad workers:

\[ \rho_i^t = \nu (1 - \varsigma_i \rho_i^n), \quad i = n, b. \]

When the steady state search intensities, \( \varsigma_n \) and \( \tilde{\varsigma}_b \), are equal, so too are the steady state retention rates.

Recall the expression of the survival probability of a unit of labor quality:

\[ \rho_t = \frac{\rho^n_t + \phi \gamma_b \rho^n_t}{1 + \phi \gamma_t}. \]

It is easy to establish that \( \rho_t^b = \rho_t^n \) in the steady state implies that the partial derivative of \( \rho_t \) with respect to composition is equal to 0 in the steady state:

\[ \left. \frac{\partial \rho_t}{\partial \gamma_t} \right|_{ss} = \frac{\phi (\rho_t^b - \rho_t^n)}{(1 + \phi \gamma_t)^2} = 0. \]

Now recall the expression for the dynamics of composition:

\[ \gamma_{t+1} = \frac{\rho_t^b \gamma_t / (1 + \phi \gamma_t) + x_t \tilde{\gamma}_m / (1 + \phi \tilde{\gamma}_m)}{\rho_t^n / (1 + \phi \gamma_t) + x_t / (1 + \phi \tilde{\gamma}_m)}. \]

Evaluating at the steady state, gives

\[ \tilde{\gamma} = \tilde{\gamma}_m. \]

Then, it is straightforward to establish that, under our steady state assumption, the partial derivative of \( \gamma_{t+1} \) with respect to \( x_t \) equals 0:

\[ \left. \frac{\partial \gamma_{t+1}}{\partial x_t} \right|_{ss} = \frac{\phi (\rho_t^b - \rho_t^n) \gamma_t}{(1 + \phi \gamma_t)^2 \left( \tilde{\gamma}_m \rho_t^n - \gamma_t \rho_t^b \right)} = 0. \]

Now derive the log-linear equation for the evolution of composition:

\[ \tilde{\gamma}_{t+1} = \tilde{\rho} \tilde{\gamma}_t + (1 - \tilde{\rho}) \tilde{\gamma}_m - (\nu - \tilde{\rho}) \tilde{\varsigma}_b. \]

This expression makes clear that, up to a first order, the dynamics of composition are not driven by any firm-specific variable. This implies that composition evolves equally at all firms and
independently of the individual firm’s history of wages and composition; in particular, starting from steady state, the time path of composition across firms is first-order equivalent, so that

\[ \gamma_t = \bar{\gamma}_t, \]

and hence, \( \text{Var}_t(\gamma_t) = 0 \) up to a first order.

### 7.5.2 The effect of composition on wages and firm and worker values

We show that \( \partial w_t / \partial \gamma_t = 0 \) in steady state. In doing so, we also show that in the steady state \( \partial J_t / \partial \gamma_t = 0 \) and \( \partial H_t / \partial \gamma_t = 0 \).

Define

\[
F_t(\gamma_t, w^*_t(\gamma_t)) = \eta \epsilon_t(\gamma_t, w^*_t(\gamma_t)) J_t(\gamma_t, w^*_t(\gamma_t)) - (1 - \eta) \mu_t(\gamma_t, w^*_t(\gamma_t)) H_t(\gamma_t, w^*_t(\gamma_t))
\]

Since \( F_t(\gamma_t, w^*_t(\gamma_t)) = 0 \) by the surplus sharing condition, we have

\[
\frac{\partial w^*_t(\gamma_t)}{\partial \gamma_t} = -\frac{\partial F_t(\gamma_t, w^*_t(\gamma_t)) / \partial \gamma_t}{\partial F_t(\gamma_t, w^*_t(\gamma_t)) / \partial w^*_t(\gamma_t)}
\]

from the implicit function theorem.

The term \( \partial F_t(\gamma_t, w^*_t(\gamma_t)) / \partial \gamma_t \) satisfies

\[
\frac{\partial F_t(\gamma_t, w^*_t(\gamma_t))}{\partial \gamma_t} = \eta \frac{\partial \epsilon_t(\gamma_t, w^*_t(\gamma_t))}{\partial \gamma_t} J_t(\gamma_t, w^*_t(\gamma_t)) + \eta \epsilon_t(\gamma_t, w^*_t(\gamma_t)) \frac{\partial J_t(\gamma_t, w^*_t(\gamma_t))}{\partial \gamma_t}
\]

\[
- (1 - \eta) \frac{\partial \mu_t(\gamma_t, w^*_t(\gamma_t))}{\partial \gamma_t} H_t(\gamma_t, w^*_t(\gamma_t))
\]

\[
- (1 - \eta) \mu_t(\gamma_t, w^*_t(\gamma_t)) \frac{\partial H_t(\gamma_t, w^*_t(\gamma_t))}{\partial \gamma_t}.
\]

We will show that at steady state \( \partial H / \partial \gamma, \partial J / \partial \gamma, \partial \epsilon / \partial \gamma \) and \( \partial \mu / \partial \gamma \) are all proportional to \( \partial w^*(\gamma) / \partial \gamma \), so that \( \partial w^*(\gamma) / \partial \gamma = 0 \) at steady state.
7.5.2.1 Effect of composition on worker surplus

For any composition \(\gamma_t\) and wage \(w_t\), we can write \(\frac{\partial H_t (\gamma_t, w_t)}{\partial \gamma_t}\) as follows:

\[
\frac{\partial H_t (\gamma_t, w_t)}{\partial \gamma_t} = \nu (1 - \zeta n p_t^N) \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial H_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial w_{t+1}^* (\gamma_{t+1})} \right] \right\}
\]

where we have used the fact that \(J_t (\gamma_t, w_t)\) is maximized with respect to \(x_t(\gamma_t, w_t)\), so that in taking the derivative with respect to \(\gamma_t\), we can hold \(x_t(\gamma_t, w_t)\) fixed at its optimal value.

Evaluating at steady state, using \(\frac{\partial \gamma'}{\partial x} = 0\) and rearranging:

\[
\frac{\partial H}{\partial \gamma} \left( 1 - \rho \beta \frac{\partial \gamma'}{\partial \gamma} \right) = \rho \beta (1 - \lambda) \frac{\partial H}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{\partial \gamma'}{\partial \gamma}
\]

The steady state value of \(\partial H/\partial \gamma\) is proportional to the steady state value of \(\partial w^*(\gamma)/\partial \gamma\); hence, if \(\partial w^*(\gamma)/\partial \gamma\) is equal to zero at steady state, so is \(\partial H/\partial \gamma\).

7.5.2.2 Effect of composition on firm surplus

For any composition \(\gamma_t\) and wage \(w_t\), we can write \(\frac{\partial J_t (\gamma_t, w_t)}{\partial \gamma_t}\) as

\[
\frac{\partial J_t (\gamma_t, w_t)}{\partial \gamma_t} = \frac{\partial \rho_t (\gamma_t)}{\partial \gamma_t} \times \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial J_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1})) \right] \right\}
\]

where we have used the fact that \(J_t (\gamma_t, w_t)\) is maximized with respect to \(x_t(\gamma_t, w_t)\), so that in taking the derivative with respect to \(\gamma_t\), we can hold \(x_t(\gamma_t, w_t)\) fixed at its optimal value.
Evaluate at steady state using \( \partial \rho / \partial \gamma = 0 \) gives
\[
\frac{\partial J}{\partial \gamma} \left( 1 - \beta \frac{\partial \gamma'}{\partial \gamma} \right) = \beta \left( 1 - \lambda \right) \frac{\partial J \partial w^*(\gamma)}{\partial w} \frac{\partial \gamma'}{\partial \gamma}
\]

The steady state value of \( \partial J / \partial \gamma \) is proportional to the steady state value of \( \partial w^*(\gamma) / \partial \gamma \).

### 7.5.2.3 Effect of composition on worker discount factor

For any composition \( \gamma_t \) and wage \( w_t \), we can write \( \partial \epsilon_t (\gamma_t, w_t) / \partial \gamma_t \) as
\[
\frac{\partial \epsilon_t (\gamma_t, w_t)}{\partial \gamma_t} = \nu \left( 1 - s_n p_t \right) \lambda E_t \left\{ \Lambda_{t,t+1} \frac{\partial \epsilon_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right\} + \nu \left( 1 - s_n p_t \right) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right] + (1 - \lambda) \frac{\partial H_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \frac{\partial w_{t+1}^* (\gamma_{t+1})}{\partial \gamma_{t+1}} \right\} \times \frac{\partial x_t (\gamma_t, w_t)}{\partial \gamma_t} \partial \left( \partial \gamma_{t+1} / \partial \gamma_t \right)
\]
where, for simplicity, we have used already that \( \partial \gamma'/\partial x = 0 \) at steady state. Evaluate at steady state:
\[
\frac{\partial \epsilon}{\partial \gamma} \left( 1 - \rho \lambda \frac{\partial \gamma'}{\partial \gamma} \right) = \rho \left[ \frac{\partial H}{\partial \gamma} + (1 - \lambda) \frac{\partial H \partial w^*(\gamma)}{\partial w} \right] \frac{\partial x \partial (\partial \gamma'/\partial x)}{\partial \gamma}
\]
which is proportional to \( \partial w^*(\gamma) / \partial \gamma \) since \( \partial H / \partial \gamma \) is proportional to \( \partial w^*(\gamma) / \partial \gamma \).

### 7.5.2.4 Effect of composition on firm discount factor

For any composition \( \gamma_t \) and wage \( w_t \), we can write \( \partial \mu_t (\gamma_t, w_t) / \partial \gamma_t \) as
\[
\frac{\partial \mu_t (\gamma_t, w_t)}{\partial \gamma_t} = \left[ \frac{\partial \rho_t (\gamma_t)}{\partial \gamma_t} + \frac{\partial x_t (\gamma_t, w_t)}{\partial \gamma_t} \right] \lambda E_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right\}
+ \left[ \rho_t (\gamma_t) + x_t (\gamma_t, w_t) \right] \lambda E_t \left\{ \Lambda_{t,t+1} \frac{\partial \mu_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right\}
\]
Evaluating at steady state,
\[
\frac{\partial \mu}{\partial \gamma} \left( 1 - \lambda \beta \frac{\partial \gamma'}{\partial \gamma} \right) = \frac{\partial x}{\partial \gamma} \lambda \beta \frac{\partial J}{\partial w}
\]
Now consider $x_t(\gamma_t, w_t)$ from the hiring condition and calculate $\partial x_t(\gamma_t, w_t)/\partial \gamma_t$.

We have

$$\kappa \frac{\partial x_t(\gamma_t, w_t)}{\partial \gamma} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1})}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right] \frac{\partial \gamma_{t+1}}{\partial \gamma} \right\}$$

$$+ \frac{\partial \omega_t(\gamma_t, w_t)}{\partial \gamma_t}$$

with

$$\frac{\partial \omega_t(\gamma_t, w_t)}{\partial \gamma_t} = [\beta_t(\gamma_t) + x_t(\gamma_t, w_t)] \times$$

$$\mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1})}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right] \frac{\partial (\partial \gamma_{t+1}/\partial x_t)}{\partial \gamma_t} \right\}$$

where, for simplicity, to write $\partial \omega_t(\gamma_t, w_t)/\partial \gamma_t$ we have used already that $\partial \gamma'/\partial x = 0$ at steady state.

Evaluating at steady state,

$$\kappa \frac{\partial x}{\partial \gamma} = \beta \left[ \frac{\partial J}{\partial \gamma} + (1 - \lambda) \frac{\partial J \partial w^*(\gamma)}{\partial \gamma} \right] \frac{\partial \gamma'}{\partial \gamma} + \frac{\partial \omega}{\partial \gamma}$$

with

$$\frac{\partial \omega}{\partial \gamma} = \beta \left[ \frac{\partial J}{\partial \gamma} + (1 - \lambda) \frac{\partial J \partial w^*(\gamma)}{\partial \gamma} \right] \frac{\partial (\partial \gamma'/\partial x)}{\partial \gamma}$$

Thus, the steady state value of $\partial x/\partial \gamma$ is proportional to the steady state value of $\partial w^*(\gamma)/\partial \gamma$. This implies that the steady state value of $\partial \mu/\partial \gamma$ is also proportional to $\partial w^*(\gamma)/\partial \gamma$.

Since all the terms comprising $F$ are proportional to $\partial w^*(\gamma)/\partial \gamma$ and the factors of proportionality are not all equal to zero, we have that $\partial w^*(\gamma)/\partial \gamma$ is equal to zero.
7.6 Transition function

We now define the law of motion for the distribution function, $G_t$. Let $C$ and $W$ be the sets of possible compositions and wages. Define the Cartesian product of the worker/firm state space to be $S \equiv C \times W$ with $\sigma$-algebra $\Sigma$ with typical subset $S = (C \times W)$. Define the transition function $Q_{s,s'}((\gamma, w), C \times W)$ as the probability that an individual retained or hired by a firm characterized by $(\gamma, w)$ transits to the set $C \times W$ next period when the aggregate state transits from $s$ to $s'$. Then $Q_{s,s'}$ satisfies

$$Q_{t,t+1}((\gamma_t, w_t), C \times W) = I(\gamma_{t+1}(\gamma_t, w_t) \in C) \times \left[ (1 - \lambda)I(w_t^*(\gamma_{t+1}(\gamma_t, w_t)) \in W) \frac{x_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{x}_t + \bar{\rho}_t} \right. \\
\left. + \lambda I(w_t \in W) \frac{x_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{x}_t + \bar{\rho}_t} \right]$$

where $I(\cdot)$ is the indicator function. Then,

$$G_{t+1}(C \times W) = \int_{(\gamma, w) \in C \times W} Q_{t,t+1}((\gamma_t, w_t), (C \times W)) dG_t(\gamma_t, w_t)$$

Note that the distribution of wages and composition is not necessarily degenerate up to a first order, as is typically the case in models with Poisson adjustment. Here, the optimal wage could be history dependent within a firm through the firm’s composition. Given our steady state results on the dynamics of firm composition, however, both wages and composition are degenerate up to a first order.