Unemployment Fluctuations with Staggered Nash Wage Bargaining*

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Abstract

A number of authors have recently emphasized that the conventional model of unemployment dynamics due to Mortensen and Pissarides has difficulty accounting for the relatively volatile behavior of labor market activity over the business cycle. We address this issue by modifying the MP framework to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects of average wages on the bargaining process. We then show that a reasonable calibration of the model can account well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this respect.

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1 Introduction

A long standing challenge in macroeconomics is accounting for the relatively smooth behavior of real wages over the business cycle along with the relatively volatile behavior of employment. A recent body of research, beginning with Shimer (2005a), Hall (2005a) and Costain and Reiter (2008), has re-ignited interest in addressing this challenge. These authors show that the conventional model of unemployment dynamics due to Mortensen and Pissarides (hereafter "MP") cannot account for the key cyclical movements in labor market activity, at least for standard calibrations of parameters. The basic problem is that the mechanism for wage determination within this framework, period-by-period Nash bargaining between firms and workers, induces too much volatility in wages. This exaggerated procyclical movement in wages, in turn, dampens the cyclical movement in firms' incentives to hire. Shimer (2004) and Hall (2005a) proceed to show that with the introduction of ad hoc wage stickiness, the framework can account for employment volatility. Of course, this begs the question of what are the primitive forces that might underlie this wage rigidity.

A rapidly growing literature has emerged to take on this puzzle. Much of this work attempts to provide an axiomatic foundation for wage rigidity, explicitly building up from assumptions about the information structure, and so on.[1] To date, due to complexity, this work has focused mainly on qualitative findings and has addressed quantitative issues only in a limited way.[2]

In this paper we take a pragmatic approach to modelling wage rigidity, with the aim of developing a framework that is tractable for quantitative analysis. In particular, we retain the intuitively appealing feature of Nash bargaining, but modify the conventional MP model to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and its existing workforce. Assuming that all employees of the same productivity receive the same wage, workers hired in between contract settlements receive the existing contract wage. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. The gain over a simple ad hoc wage adjustment mechanism is that the key primitive parameter of the model is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in a wage equation. In this way, the staggered contracting structure provides more discipline in evaluating the model than do simple ad hoc adjustment mechanisms.

[1] Examples include Kennan (2006), Menzio (2005), and Shimer and Wright (2004), and Hall and Milgrom (2008). Others have explored flexible wage alternatives: e.g., Hagedorn and Manovskii (2006), Krause and Lubik (2004), Mortensen and Nagypal (2007), and Rotemberg (2006). See Hall (2005c) for a survey.

[2] An exception is Menzio (2005) who presents a calibrated model with endogenous wage rigidity. His model does well except for wages, which are too smooth. We instead focus on explaining the joint dynamics of labor market activity and wages.

The use of time dependent staggered price and wage setting, of course, is widespread in macroeconomic modelling, beginning with Taylor (1980) and Calvo (1983). More recently, Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) have found that staggered wage contracting is critical to the empirical performance of the recent vintage of dynamic general equilibrium macroeconomic frameworks (i.e., sticky prices alone are not sufficient). There are, however, some important distinguishing features of our approach. First, macroeconomic models with staggered wage setting typically have employment adjusting along the intensive margin. That is, wage stickiness enhances fluctuations in hours worked as opposed to total employment. As a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firm's and workers have an on-going relationship. If wages are not allocational, of course, then wage rigidity does not influence model dynamics. By contrast, in the model we present, wages affect employment at the extensive margin: They influence the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005a), in this kind of setting the Barro's critique does not apply.

A second key difference involves the nature of the wage contracting process. In the conventional macroeconomic models, monopolistically competitive workers set wages. Here, firms and workers bargain over wages in a setting with search and matching frictions. As a consequence, some interesting "spillover" effects emerge of the average market wage on the contract wage. These spillover effects are a product of the staggered contract/bargaining environment. They introduce additional stickiness in the movement of real wages, much the same way that real rigidities enhance nominal price stickiness in models of staggered price setting (e.g., Kimball, 1995, and Woodford, 2003).

In section 2 we present the basic ingredients of the model, as well as the definition of the equilibrium. In section 3, to develop some intuition about how the model works, we characterize the steady state and then derive a set of simple dynamic equations for wages and hiring, obtained by considering a local approximation of the model about the steady state. We also exposit the spillover effects that influence the wage bargaining process, contributing to overall wage stickiness. One additional distinguishing feature of the setup is that a "horizon effect" emerges that influences the bargaining process, since firms care about the implications of the contract wage for future hires, while workers do not. While the horizon effect is interesting from a theoretical perspective, it turns out to not be quantitatively important in our baseline calibration. In section 4 we examine the empirical performance of the model and show that the framework does a good job of accounting for the basic features of the U.S. data, including wage dynamics.

In section 5, we address three key issues concerning the robustness of our analysis. First, we show that under our baseline calibration, wages almost surely lie within the bargaining set over the life of the contract, which justifies our solution to both the firm's decision problem and the overall bargaining problem. Second, we show that the losses to a firm and its workers from opting for multi-period as opposed to period-by-period contracts are quite small, which provides some support for our contract structure. Finally, Pissarides (2007) has recently argued that the evidence on the relative cyclicality of wages of new hires is inconsistent with our assumption that, in between contracts, the wages of new hires are tied to those of similarly productive existing workers. We argue that this is not the case and present some new evidence in support of our contention.

Concluding remarks are in section 6. Finally, the appendix provides an explicit derivation of all the key results, including the steady state of the model. It also presents the complete loglinearized model.

2 The Model

The framework is a variation of the Mortensen and Pissarides search and matching model (Mortensen and Pissarides, 1994, Pissarides, 2000). The main difference is that we allow for staggered multi-period wage contracting. Within the standard framework, workers and firms negotiate wages based on period-by-period Nash bargaining. We keep the Nash bargaining framework, but in the spirit of Taylor (1980) and Calvo (1983), only a fraction of firms and workers re-set wages in any given period. As well, they strike a bargain that lasts for multiple periods. We assume that workers hired in between contracting periods receive the contract wage for existing workers of similar productivity. Here the idea is that due to scale economies in bargaining, employment terms are negotiated only periodically: Firms do not negotiate separate terms for the relatively small percentage of workers who enter in-between contracting periods. In the spirit of Hall (2005a), the contract wage provides the wage norm for newly hired workers. Though we defer to section 5 a discussion of the formal evidence on this issue, we note in the meantime that survey evidence from both the U.S. (Bewley, 1999) and the Euro area (Galuscak et al., 2008) supports this proposition.

For technical reasons, there are two other differences from MP. First, because it will turn out to be important for us to distinguish between existing and newly hired workers at a firm, we drop the assumption of one worker per firm and instead allow firms to hire a continuum of workers. We assume constant returns to scale, however, which greatly simplifies the bargaining problem. Second, we drop the conventional assumption of a fixed cost per vacancy opened and instead assume that firms face quadratic adjustment costs of adjusting employment size.[3] The reason is as follows: With staggered wage setting, there will arise a dispersion of wages across firms in equilibrium. Quadratic costs of adjusting employment ensures a determinate equilibrium in the presence of wage dispersion.

We embed our search and matching framework within a simple intertemporal general equilibrium framework in order to study the dynamics of unemployment and wages. Following Merz (1995) and others, we adopt the representative family construct, which effectively involves introducing complete consumption insurance.

Finally, since the environment is stationary, we omit the use of time subscripts and denote the current value of any variable y with y and the next period value with y'.

[3] We allow for quadratic costs of adjusting employment as opposed to vacancies to keep the algebra as simple as possible. The main cyclical properties of the model are preserved under either approach.

2.1 Unemployment, Vacancies and Matching

Let us now be more precise about the details: There is a continuum of infinitely lived workers and a continuum of infinitely lived firms, each of measure one. Each firm employs n workers in the current period. It also posts v vacancies in order to attract new workers for the next period of operation. The total number of vacancies and employed workers are $\bar{v} = \int_i v di$ and $\bar{n} = \int_i n di$ where firms are indexed by i. The total number of unemployed workers searching for a job is

$$\bar{u} = 1 - \bar{n}.$$

Following convention, we assume that the aggregate number of new hires or "matches", $\bar{m}$, is a function of unemployed workers and vacancies, as follows:

$$\bar{m} = \sigma_m \bar{u}^{\sigma} \bar{v}^{1-\sigma}. \tag{1}$$

where the parameter σ_m reflects the efficiency of the matching process. The current probability a firm fills a vacancy, q, is given by

$$q = \frac{\bar{m}}{\bar{v}}. \tag{2}$$

Similarly, the probability an unemployed worker finds a job, p, is given by

$$p = \frac{\bar{m}}{\bar{u}}. \tag{3}$$

Both firms and workers take q and p as given.

Finally, each firm exogenously separates from a fraction $1 - \rho$ of its workers each period, where ρ is the probability a worker "survives" with the firm until the next period. Accordingly, within our framework fluctuations in unemployment will be due to cyclical variation in hiring as opposed to separations. Both Hall (2005b,c) and Shimer (2005a,b) argue that this characterization is consistent with recent U.S. evidence.

2.2 Firms

Every period, each firm produces output, y, using capital, k, and labor, n, according to the following Cobb-Douglas technology:

$$y = zk^{\alpha} n^{1-\alpha}, \tag{4}$$

where z is a common productivity factor which is stochastic and follows a first order Markov process. As we noted earlier, because we will have wage dispersion across firms, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs. For simplicity, we assume capital is perfectly mobile across firms and that there is a competitive rental market in capital.

It is convenient to define the hiring rate, x, as the ratio of new hires, qv, to the existing workforce, n:

$$x = \frac{qv}{n}. \tag{5}$$

Note that the firm knows the current hiring rate with certainty, since it knows the likelihood q that each vacancy it posts will be filled. Next period, in turn, total workforce is the sum of the number of surviving workers, ρn, and new hires, xn:

$$n' = (\rho + x)\, n. \tag{6}$$

Let $\mathbf{s}$ be the set of variables that define the aggregate state; let $\beta\Lambda(\mathbf{s}, \mathbf{s}')$ be the firm's discount rate, where the parameter β is the household's subjective discount factor and where $\Lambda(\mathbf{s}, \mathbf{s}')$ is defined below. Let r be the rental rate of capital; and let w be the firm's wage rate. Assume that the costs of adjusting the workforce are quadratic and proportional to the existing workforce size. Then, conditional on the current wage and employment, the value F of each firm may be expressed as:

$$F(n, w, \mathbf{s}) = \max_{k,x} \left\{ zk^{\alpha} n^{1-\alpha} - wn - \frac{\kappa}{2} x^2 n - rk + \beta E\left\{ \Lambda\left(\mathbf{s}, \mathbf{s}'\right) F\left(n', w', \mathbf{s}'\right) | n, w, \mathbf{s} \right\} \right\}, \tag{7}$$

subject to equation (6). For the time being we take the firm's expected wage path as given but then in section 2.4 describe the wage determination process.

With constant returns to scale with perfectly mobile capital, the marginal product of a worker equals the average product. The latter in turn is independent of the employment size of the firm. Given this homogeneity in the firms' technology, we argue that it is reasonable to restrict attention to a "fundamentals-based" solution for the path of wages, where the outcome of the wage bargain is similarly independent of the firm's scale.[4] It then follows that the value of the firm conditional on the current wage is linear in its current employment level, as follows:

$$F(n, w, \mathbf{s}) = J(w, \mathbf{s})\, n, \tag{8}$$

where, letting $\breve{k} = k/n$ denote the firm's capital/employment ratio:

$$J(w, \mathbf{s}) = \max_{\breve{k},x} \left\{ z\breve{k}^{\alpha} - w - \frac{\kappa}{2} x^2 - r\breve{k} + (\rho + x)\, \beta E\left\{ \Lambda\left(\mathbf{s}, \mathbf{s}'\right) J\left(w', \mathbf{s}'\right) | w, \mathbf{s} \right\} \right\}. \tag{9}$$

[4] In section 2.4 we describe the wage bargain in detail. Given our assumptions on technology, the outcome of the current wage bargain is indeed independent of the firm's scale, so long as the relevant parties believe that the same is true of the outcome of future wage bargains. We have not explored whether there may be self-fulfilling solutions where firm scale matters to current wages simply because the parties believe it will matter in the future. We believe our "fundamentals-based" solution is the most natural starting point, given the environment, and defer to future research whether or not there may be interesting sunspot equilibria to consider.

At any time, the firm maximizes its value by choosing the hiring rate (by posting vacancies) and its capital/employment ratio, given the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future. We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain.

The first order condition for the capital/employment ratio is simply:

$$r = \alpha z \breve{k}^{\alpha-1}, \tag{10}$$

where, due to Cobb-Douglas production and perfectly mobile capital, $\breve{k}$ is the same across firms.

The first order condition for hiring equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa x = \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) J \left(w', \mathbf{s}' \right) |w, \mathbf{s} \right\}. \tag{11}$$

Combining equations yields the following forward looking difference equation for the hiring rate:

$$\kappa x = \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \left(a' - w' + \frac{\kappa}{2} x'^2 + \rho \kappa x' \right) |w, \mathbf{s} \right\}, \tag{12}$$

where a denote the current marginal product of labor (i.e., $a = (1-\alpha)y/n$), which is independent of the firm. The hiring rate thus depends on a discounted stream of the firm's expected future surpluses from the marginal worker: the sum of net earnings at the margin, $a' - w'$, and saving on adjustment costs, $\frac{\kappa}{2} x'^2$.

Observe that the only firm-specific variable the hiring rate x depends on is the wage w. Thus, all firms with the same wage w choose the same hiring rate x, regardless of their respective employment size.

2.3 Workers

In this sub-section we develop an expression for a worker's surplus from employment, which becomes a critical determinant of the outcome of the wage bargain. Given constant returns to scale, a workers welfare is independent of the employment size of the firm at which he is working. Given this consideration and given that we effectively have complete consumption insurance, we may index workers simply by their respective current wage, w. That is, all workers with wage w will have the same welfare, regardless of which firm they are employed.

Accordingly, we can express the current value of employment (in consumption good equivalents) as $V(w, \mathbf{s})$. Let $U(\mathbf{s})$ be the current value of unemployment, then $V(w, \mathbf{s})$ is given by

$$V(w, \mathbf{s}) = w + \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \left[\rho V \left(w', \mathbf{s}' \right) + (1-\rho) U \left(\mathbf{s}' \right) \right] |w, \mathbf{s} \right\}. \tag{13}$$

Note that this value depends on the worker's wage, w, as well as the likelihood the worker will remain employed in the subsequent period.

To construct the value of unemployment, we first define $\bar{V}_x(\mathbf{s})$ as the average value of employment conditional on being a new worker in the current period. The subscript x is meant to denote that we are averaging $V(w, \mathbf{s})$ across new workers, i.e., workers who were hired in the previous period. Let $G(w, \mathbf{s})$ be the cumulative distribution function of wages in state $\mathbf{s}$, that is, the fraction of workers who have a wage less than or equal to w, given the macro state $\mathbf{s}$. Then $\bar{V}_x(\mathbf{s}')$ is given by

$$\bar{V}_x(\mathbf{s}') = \int_w V(w', \mathbf{s}') \frac{x(w, \mathbf{s})}{\bar{x}} dG(w, \mathbf{s}), \tag{14}$$

where $x(w, \mathbf{s})$ is the hiring rate at firms with wage w in state $\mathbf{s}$ and $\bar{x}$ is the average hiring rate, given by:[5]

$$\bar{x} = \int_w x(w, \mathbf{s})\, dG(w, \mathbf{s}). \tag{15}$$

Next, let b be the flow value from unemployment, taken to be unemployment benefits. Then, $U(\mathbf{s})$ may be expressed as

$$U(\mathbf{s}) = b + \beta E\left\{\Lambda(\mathbf{s}, \mathbf{s}')\left[p\bar{V}_x(\mathbf{s}') + (1-p)\, U(\mathbf{s}')\right] | \mathbf{s}\right\}, \tag{16}$$

where, as before, p is the probability of finding a job for the subsequent period. The value of unemployment thus depends on the current flow value b and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that is currently unemployed is $\bar{V}_x(\mathbf{s}')$, the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.[6]

The worker surplus at a firm paying a wage w, $H(w, \mathbf{s})$, and the average worker surplus conditional on being a new hire, $\bar{H}_x(\mathbf{s})$, are given by:

$$H(w, \mathbf{s}) = V(w, \mathbf{s}) - U(\mathbf{s}), \tag{17}$$

and

$$\bar{H}_x(\mathbf{s}) = \bar{V}_x(\mathbf{s}) - U(\mathbf{s}). \tag{18}$$

It follows that:

[5] $\bar{V}_x(\mathbf{s}')$ is thus distinct from the unconditional average value of employment $\bar{V}(\mathbf{s}') = \int_w V(w', \mathbf{s}')\, dG(w', \mathbf{s}')$. However, since in the steady state wages and hiring rates are identical across firms, $\bar{V}_x(\mathbf{s}')$ and $\bar{V}(\mathbf{s}')$ are identical in the steady state and have similar dynamics outside the steady state, up to a first order approximation. Key to the latter result is that in the steady state the distribution of wages is degenerate.

[6] There is accordingly no directed search. Note, however, that wage differentials across firms are only due to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

$$H(w,\mathbf{s}) = w - b + \beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\left[\rho H\left(w',\mathbf{s}'\right) - p\bar{H}_x\left(\mathbf{s}'\right)\right]|w,\mathbf{s}\right\}. \tag{19}$$

2.4 Nash Bargaining and Wage Dynamics

As we noted in the introduction, we restrict the form of the wage contract to call for a fixed wage per period over an exogenously given length of time. Given these restrictions on the form of the contract, workers and firms use a Nash bargaining approach to determine the contract wage.

We introduce staggered multi-period wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability $1-\lambda$ that it may renegotiate the wage. This adjustment probability is independent of its history. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at $1/(1-\lambda)$. The coefficient λ is thus a measure of the degree of wage stickiness that can be calibrated to match the data.[7] This simple Poisson adjustment process, further, implies that it is not necessary to keep track of individual firms' wage histories, which makes aggregation simple. In the end, the model will deliver a simple loglinear relation for the evolution of wages that is the product of Nash bargaining in conjunction with staggered wage setting.

Firms that enter a new wage agreement negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage.[8] When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the wage paid the previous period. As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for workers who arrive in between contracting periods and instead, in Hall's terminology, have the existing contract wage provide the wage norm for these new employees. Of course, the newly hired workers recognize that they will be able to renegotiate wage at the next round of contracting.

Before proceeding we note that the multi-period contracting introduces a non-standard element to our Nash bargaining problem. In contrast to the conventional setup, the wage negotiated for the current period affects the hiring rate since there is a probability λ that the contract will continue the next period. As we show in appendix B, this makes the bargaining set non-convex, unlike the standard Nash problem. However, the departure from convexity is not quantitatively large. Thus, as we show in the appendix, under our calibration, the first order conditions define a global optimum. It is true that the non-convex bargaining set opens up the possibility that a wage lottery may be preferred. However, as we show in the appendix, under reasonable conditions the gains

[7] This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

[8] To be clear, with constant returns, one could either think of the firm bargaining with each marginal worker individually or bargaining with a union that wishes to maximixe average worker surplus.

from the lottery are small and easily offset by small transactions costs of running and enforcing the lottery. Thus in what follows we restrict attention to deterministic wage contracts.

Let w^* denote the wage of a firm that renegotiates in the current period. As just noted, the firm negotiates with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the contract wage w^* is chosen to solve

$$\max_{w} H(w, \mathbf{s})^{\eta} J(w, \mathbf{s})^{1-\eta}, \tag{20}$$

s.t.

$$w' = \begin{cases} w \text{ with probability } \lambda, \\ w^{*\prime} \text{ with probability } 1-\lambda, \end{cases} \tag{21}$$

where $w^{*\prime}$ is the wage in the subsequent period if the firm is able to renegotiate and where $J(w, \mathbf{s})$ and $H(w, \mathbf{s})$ are defined as in equations (9) and (19), respectively. We emphasize that $w^{*\prime}$ is chosen optimally at $t+1$ and is thus independent of w.

The first order necessary condition for the Nash bargaining solution is given by

$$\eta \epsilon(\mathbf{s}) J(w^*, \mathbf{s}) = (1-\eta) \mu(w^*, \mathbf{s}) H(w^*, \mathbf{s}), \tag{22}$$

where $\epsilon(\mathbf{s}) \equiv \partial H(w, \mathbf{s}) / \partial w$ is the effect of a rise in the contract wage on the worker's surplus, while $\mu(w, \mathbf{s}) \equiv -\partial J(w, \mathbf{s}) / \partial w$ is minus the effect of a rise in the contract wage on the firm's surplus, and where

$$\epsilon(\mathbf{s}) = 1 + (\rho\lambda\beta) E\left\{\Lambda(\mathbf{s}, \mathbf{s}') \epsilon(\mathbf{s}') | \mathbf{s}\right\}, \tag{23}$$

and[9]

$$\mu(w, \mathbf{s}) = 1 + [\rho + x(w, \mathbf{s})] (\lambda\beta) E\left\{\Lambda(\mathbf{s}, \mathbf{s}') \mu(w, \mathbf{s}') | \mathbf{s}\right\}. \tag{24}$$

Observe that $\epsilon(\mathbf{s})$ is effectively the cumulative discount factor the worker uses to value the contract wage stream, while $\mu(w, \mathbf{s})$ is that for the firm. Since the hiring rate $x(w, \mathbf{s})$ must be non-negative, $\mu(w, \mathbf{s}) \geq \epsilon(\mathbf{s})$, implying that the firm places a greater weight on the future than does the worker. Intuitively, the firm has a longer horizon than the worker because it cares about the effect of the current wage contract on payments not only to the existing workforce, but also to new workers who enter under the terms of the existing contract. A worker, on the other hand, only cares about wages during his or her tenure at the firm.

It is possible to rewrite equation (22) as

$$\chi(w^*, \mathbf{s}) J(w^*, \mathbf{s}) = [1 - \chi(w^*, \mathbf{s})] H(w^*, \mathbf{s}), \tag{25}$$

[9] As the appendix shows, the dependency of $\mu(w, \mathbf{s})$ on w (via the effect of w on $x(w, \mathbf{s})$) is the source of the non-convexity in the bargaining set. However, because this effect is not quantitatively large, the Nash first order conditions still define a global optimum over the space of deterministic wage contracts.

with

$$\chi(w,\mathbf{s}) = \frac{\eta}{\eta + (1-\eta)\,\mu(w,\mathbf{s})\,/\epsilon(\mathbf{s})}. \tag{26}$$

Equation (26) is a variation on the conventional sharing rule, where the relative weight χ depends not only on the worker's bargaining power η, but also on the differential firm/worker horizon, reflected by the term μ/ϵ. Note that in the limiting case of $\lambda = 0$, $\mu/\epsilon = 1$ and $\chi = \eta$, as in the conventional case of period-by-period wage bargaining. With $\lambda > 0$, however, χ is less than η (since μ/ϵ exceeds unity). In analogy to Binmore, Rubinstein and Wolinsky (1986), because the horizon effect effectively makes the worker more impatient than the firm, it effectively reduces the worker's bargaining power from η to χ.

It is also straightforward to show that in the limiting case of $\lambda = 0$, the wage that solves the bargaining problem is given by

$$w^*_{\lambda=0} = \eta\left[a + (\kappa/2)\,x\,(w^*,\mathbf{s})^2\right] + (1-\eta)\left[b + p\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\bar{H}_x\left(\mathbf{s}'\right)|\mathbf{s}\right\}\right]. \tag{27}$$

As in the conventional formulation, with period-by-period Nash bargaining, the wage is a convex combination of what a worker contributes to the match (the first term on the right) and what the worker loses by accepting a job (the second term), where the weights depend on the relative bargaining power.[10] With multi-period contracting, however, the contract wage will also depend on expected future conditions. As we show in section 3.2, with $\lambda > 0$, up to a first order, w^* equals an expected discounted sum of current and expected future values of a target wage w^o. The target wage w^o, further, is the wage that would arise under period-by-period Nash bargaining, modified to allow for the horizon effect.

We next characterize the relation between the contract wage w^* and the evolution of the average wage $\bar{w}$ across workers, given by

$$\bar{w} = \int_w w dG(w,\mathbf{s}). \tag{28}$$

Since the fraction of firms and workers that renegotiate contracts is a random draw across the population and since all firms and workers that renegotiate choose the same contract wage, by the law of large numbers we can express the wage index recursively as

$$\bar{w}' = (1-\lambda)w^{*\prime} + \lambda\int_w w\frac{\rho + x\,(w,\mathbf{s})}{\rho+\bar{x}}dG\,(w,\mathbf{s})\,, \tag{29}$$

where $1-\lambda$ is the fraction of workers who are renegotiating and λ is the fraction who are not. The next period average wage, $\bar{w}'$, is thus a convex combination of the next period contract wage, $w^{*\prime}$, and the average wage across the population of workers that do not renegotiate, given by the integral

[10] One small difference from the standard formulation is that due to our quadratic adjustment cost formulation, the saving on adjustment costs that is a component of the worker's value to the match is given by $(\kappa/2)\,x^2$.

in the second term. As we show below, up to a first order, this integral can be approximated by the current period average wage, $\bar{w}$.[11]

Finally, given the process for the evolution wages, the fraction of workers with wage w evolves according to

$$dG\left(w, \mathbf{s}'\right) = \begin{cases} \lambda \dfrac{\rho + x\left(w, \mathbf{s}\right)}{\rho + \bar{x}} dG\left(w, \mathbf{s}\right) & \forall \;\; w \neq w^{*\prime} \\ \lambda \dfrac{\rho + x\left(w, \mathbf{s}\right)}{\rho + \bar{x}} dG\left(w, \mathbf{s}\right) + \left(1 - \lambda\right) & if \;\; w = w^{*\prime} \end{cases} \tag{30}$$

where $dG\left(w, \mathbf{s}\right)$ is the fraction of workers with wage w in state $\mathbf{s}$ and $dG\left(w, \mathbf{s}'\right)$ is the corresponding fraction in state $\mathbf{s}'$. Given the staggered wage contracting, the aggregate productivity shock creates dispersion of wages across workers. It is straightforward to show, however, that in the deterministic steady state all workers receive the target wage and that in the absence of new shocks the dispersion of wages converges to this degenerate distribution.

2.5 Consumption and Saving

Following Merz (1995) and others, we use the representative family construct, which gives rise to perfect consumption insurance. In particular, the family has of continuum of employed workers at all firms and unemployed workers, representative of the population at large. The family pools their incomes before choosing per capita consumption and asset holdings. In addition to wage income and unemployment income, the family has a diversified ownership stake in firms, which pays out profits. Finally, households may either consume $\bar{c}$, or save in the form of capital $\bar{k}$, which they rent to firms at the rate r. Let $\Omega\left(\mathbf{s}\right)$ be the value function for the representative household. Then the maximization problem may be expressed as

$$\Omega\left(\mathbf{s}\right) = \max_{\bar{c}, \bar{k}'} \left\{ \log\left(\bar{c}\right) + \beta E \left\{ \Omega\left(\mathbf{s}'\right) | \mathbf{s} \right\} \right\}, \tag{31}$$

subject to

$$\bar{c} + \bar{k}' = \bar{w}\bar{n} + \left(1 - \bar{n}\right) b + \left(1 - \delta + r\right) \bar{k} + T + \Pi, \tag{32}$$

where T is lump sum transfers from the government and Π is profits received from firms, taken as given by the households.

Given the law of large numbers, total household employment $\bar{n}$ evolves according to

$$\bar{n}' = \left(\rho + \bar{x}\right) \bar{n}, \tag{33}$$

[11] The integral $\int_w w \frac{\rho + x(w,S)}{\rho + \bar{x}} dG\left(w, S\right)$ is not quite identical to $\bar{w} = \int_w w dG\left(w, S\right)$. However, given that the hiring rates are identical across firms in the steady state, the dynamics of this integral are equivalent to the dynamics of $\bar{w}$, up to a first order. Key to the result is that the steady state wage distribution is degenerate as is the hiring rate.

where the family takes the average hiring rate $\bar{x}$ as given.

Then the first order necessary condition for consumption/saving yields:

$$1 = (1 - \delta + r)\,\beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)|\mathbf{s}\right\}, \tag{34}$$

where $\Lambda(\mathbf{s}, \mathbf{s}') = \bar{c}/\bar{c}'$.

2.6 Resource Constraint, Government Policy and Equilibrium

We complete the model with the following resource constraint, which divides output between consumption, investment and adjustment costs:

$$\bar{y} = \bar{c} + \bar{k}' - (1 - \delta)\,\bar{k} + \frac{\kappa}{2}\int_i x^2 n di. \tag{35}$$

where $\bar{y} = \int_i y di$ and $\bar{k} = \int_i k di$. Finally, lump sum transfers adjust to balance the government budget constraint:

$$T + (1 - \bar{n})\,b = 0 \tag{36}$$

where unemployment insurance payments, b, are given exogenously. This completes the set of model equations.

We next define the equilibrium:

A recursive competitive equilibrium is a solution for: (i) a set of functions $\{J(w, \mathbf{s})$, $V(w, \mathbf{s})$, $H(w, \mathbf{s})$, $U(\mathbf{s})$, $\bar{V}_x(\mathbf{s})$, $\bar{H}_x(\mathbf{s})\}$; (ii) the contract wage $w^*(\mathbf{s})$; (iii) the worker and firm discount factors, $\epsilon(\mathbf{s})$ and $\mu(w, \mathbf{s})$; (iv) the hiring rate $x(w, \mathbf{s})$; (v) the subsequent period's wage rate $w(\mathbf{s}')$; (vi) the rental rate of capital $r(\mathbf{s})$; (vii) the average wage and hiring rate, $\bar{w}(\mathbf{s})$ and $\bar{x}(\mathbf{s})$; (viii) the density function of wages across workers $dG(w, \mathbf{s})$; (ix) average consumption and capital, $\bar{c}(\mathbf{s})$ and $\bar{k}(\mathbf{s}')$; (x) and average employment $\bar{n}(\mathbf{s})$. The solution is such that: (i) $w^*(\mathbf{s})$ solves the bargaining condition (22); (ii) $\epsilon(\mathbf{s})$ and $\mu(w, \mathbf{s})$ satisfy (23) and (24); (iii) $x(w, \mathbf{s})$ satisfies the hiring condition (12); (iv) $w(\mathbf{s}')$ is given by the Calvo process for wages (21); (v) $r(\mathbf{s})$ satisfies (10); (vi) $\bar{w}(\mathbf{s})$ and $\bar{x}(\mathbf{s})$ are defined as in (28) and (15); (vii) $dG(w, \mathbf{s})$ evolves according to (30); (viii) $\bar{n}(\mathbf{s})$ evolves according to (33); (ix) $\bar{c}(\mathbf{s})$ and $\bar{k}(\mathbf{s}')$ satisfy (34) and (35); (x) and the rental market for capital clears, that is, $\breve{k}(\mathbf{s}) = \bar{k}(\mathbf{s})/\bar{n}(\mathbf{s})$.

3 Model Characteristics

To gain some intuition, we first present some of the key steady state conditions and then describe several of the key relationships that help characterize the dynamics outside the steady state.

3.1 Steady State

Here we describe the steady state for the labor market, conditional on the steady state marginal product of labor $\tilde{a}$, where the "tilde" denotes the steady state value of a variable. We defer to appendix A the complete steady state model. As we show in the appendix, $\tilde{a}$ is determined as in the conventional neoclassical growth model and is independent of the labor market equilibrium.

The key labor market variables are the hiring rate, $\tilde{x}$, the wage $\tilde{w}$, the job finding probability $\tilde{p}$, unemployment $\tilde{u}$, and vacancies $\tilde{v}$. The key behavioral relations are the hiring condition and the solution for the wage bargain:

$$\kappa\tilde{x} = \beta\left(\tilde{a} - \tilde{w} + \frac{\kappa}{2}\tilde{x}^2 + \rho\kappa\tilde{x}\right), \tag{37}$$

$$\tilde{w} = \tilde{\chi}\left(\tilde{a} + \frac{\kappa}{2}\tilde{x}^2 + \kappa\tilde{p}\tilde{x}\right) + (1-\tilde{\chi})\, b, \tag{38}$$

with

$$\tilde{\chi} = \frac{\eta}{\eta + (1-\eta)\,\tilde{\mu}/\tilde{\epsilon}}, \quad \tilde{\mu} = \frac{1}{1-\lambda\beta}, \quad \tilde{\epsilon} = \frac{1}{1-\rho\lambda\beta},$$

where $\tilde{\chi}$ is the steady state effective worker bargaining power (after taking into account horizon effects on bargaining), $\tilde{\mu}$ is the steady state firm discount factor and $\tilde{\epsilon}$ is the steady state worker discount factor. Within a local region of the steady state, the hiring condition relates $\tilde{x}$ inversely to $\tilde{w}$, since the net marginal benefit to the firm of expanding employment is decreasing in both of these variables. Conversely, the wage equation relates $\tilde{w}$ positively to $\tilde{x}$, since firm surplus is increasing in this variable while the worker's surplus from the match is decreasing, everything else equal. In addition, $\tilde{w}$ varies positively with $\tilde{p}$ since the worker's outside option is increasing in the job finding probability. Note that the staggered contracting structure affects the steady state only via the impact on the effective bargaining power parameter $\tilde{\chi}$. As we discussed earlier, the different horizon of the firm and worker has the effect of reducing the worker's relative bargaining power, which, everything else equal leads to a lower wage and higher hiring rate.

Next we observe that in the steady state the hiring rate equals the job separation rate:

$$\tilde{x} = 1 - \rho. \tag{39}$$

Equations (37), (38), and (39) thus determine $\tilde{x}$, $\tilde{w}$, and $\tilde{p}$. Given $\tilde{x}$ and $\tilde{p}$, $\tilde{u}$ is pinned down by the condition that new hires by firms must equal the number of unemployed workers who find jobs:

$$\tilde{x}(1-\tilde{u}) = \tilde{p}\tilde{u}. \tag{40}$$

Finally, the condition the new matches are given by the matching functions pins down vacancies:

$$\tilde{p}\tilde{u} = \sigma_m \tilde{u}^{\sigma}\tilde{v}^{1-\sigma}. \tag{41}$$

3.2 Wage/Hiring Dynamics and Spillover Effects

To gain some intuition, we next derive loglinear versions of the two central equations of the model that govern labor market dynamics outside the steady state: the relations for wages and hiring. Let $\widehat{z}$ denote the percent deviation of variable z from its steady state value $\tilde{z}$. Then, as appendix C shows, loglinearizing the first order condition for Nash bargaining, equation (25), yields a first order forward looking equation for the contract wage,

$$\widehat{w}^{*} = (1-\tau)\,\widehat{w}^{o}\left(w^{*},\mathbf{s}\right) + \tau E\widehat{w}^{*\prime}, \tag{42}$$

with

$$\tau = \frac{(\rho\beta\lambda)\,\psi}{1+(\rho\beta\lambda)\,\psi},$$

where $\psi = \tilde{\chi}\tilde{\mu} + (1-\tilde{\chi})\,\tilde{\epsilon}$, and where the forcing variable $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$ is given by

$$\widehat{w}^{o}\left(w^{*},\mathbf{s}\right) = \tilde{\chi}\left[\varphi_{a}\widehat{a} + \varphi_{x}\widehat{x}\left(w^{*},\mathbf{s}\right)\right] + (1-\tilde{\chi})\,\varphi_{p}E\left[\widehat{p} + \widehat{\bar{H}}_{x}\left(\mathbf{s}'\right) + \widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right] + \widehat{\Phi}\left(w^{*},w^{*\prime},\mathbf{s},\mathbf{s}'\right), \tag{43}$$

with

$$\widehat{\Phi}\left(w^{*},w^{*\prime},\mathbf{s},\mathbf{s}'\right) = \varphi_{\chi}E\left[\widehat{\chi}\left(w^{*},\mathbf{s}\right) - \rho\beta\widehat{\chi}\left(w^{*\prime},\mathbf{s}'\right)\right], \tag{44}$$

where the coefficients φ_{a}, φ_{x}, φ_{p}, and φ_{χ} depend on the primitive model parameters and steady state values, as shown in the appendix.

As in the conventional literature on time-dependent wage and price contracting (Taylor, 1980 and Calvo, 1983), the contract wage depends on an expected discounted sum of the target under perfectly flexible adjustment, in this case $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$, as iterating forward equation (42) suggests. The target $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$ is the first order approximation of the period-by-period Nash bargaining solution for wages, adjusted for the horizon effect. The first term in brackets on the right side of equation (43) is the the loglinear approximation of the worker's contribution to the match and the second is the loglinear approximation of the worker's opportunity cost. The horizon effect enters in two ways: First, the effective bargaining weight $\tilde{\chi}$ replaces the the primitive bargaining weight η in determining the relative input on the target wage of the worker's contribution versus the worker's opportunity cost. Second, fluctuations in the effective bargaining weight can influence $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$, as captured by the term $\widehat{\Phi}\left(w^{*},w^{*\prime},\mathbf{s},\mathbf{s}'\right)$. As we show shortly, though, under our calibration, the impact of the horizon effect on the path of the target wage is small: $\tilde{\chi}$ is close to η and $\widehat{\Phi}\left(w^{*},w^{*\prime},\mathbf{s},\mathbf{s}'\right)$ does not fluctuate much.

Note that the target wage $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$ is computed taking as given that all other firms and workers in the economy are operating on multi-period wage contracts. In moving from partial equilibrium to general equilibrium, it is convenient to distinguish between $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$ and $\widehat{\bar{w}}^{o}$, the target wage that would arise if *all* firms and workers were negotiating period-by-period wage contract (again, adjusting for the horizon effect). By loglinearizing equation (27), we obtain the following relation between $\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)$ and $\widehat{\bar{w}}^{o}$:

$$\widehat{w}^o\left(w^*,\mathbf{s}\right)=\widehat{\bar{w}}^o+\frac{\tau_1}{1-\tau}E\left(\widehat{\bar{w}}'-\widehat{w}^{*\prime}\right)+\frac{\tau_2}{1-\tau}\left(\widehat{\bar{w}}-\widehat{w}^*\right), \tag{45}$$

with

$$\widehat{\bar{w}}^o=\tilde{\chi}\varphi_a\widehat{a}+\left[\tilde{\chi}\varphi_x+(1-\tilde{\chi})\varphi_p\right]\widehat{\bar{x}}+(1-\tilde{\chi})\varphi_p\widehat{p}+\widehat{\bar{\Phi}}, \tag{46}$$

and

$$\widehat{\bar{\Phi}}=\varphi_\chi E\left[\widehat{\chi}\left(\bar{w},\mathbf{s}\right)-(\rho-\tilde{p})\beta\widehat{\chi}\left(\bar{w}',\mathbf{s}'\right)\right], \tag{47}$$

where τ_1 and τ_2 depend on model primitives, as appendix C shows.

The difference between $\widehat{w}^o\left(w^*,\mathbf{s}\right)$ and $\widehat{\bar{w}}^o$ reflects the influence of spillovers of economy-wide average wages on the individual firm and worker bargain. These spillovers work through their effect on the target wage $\widehat{w}^o\left(w^*,\mathbf{s}\right)$ and are measured by the gap between the average wage $\widehat{\bar{w}}$ and the contract wage $\widehat{w}^*$. As we show in the next section, these spillovers magnify the wage stickiness that comes from multi-period contract. The spillover that is most important quantitatively reflects the impact of market wages on the worker's outside option and is captured by the second term in equation (45). If, everything else equal, $E\widehat{\bar{w}}'$ exceeds $E\widehat{w}^{*\prime}$, opportunities are unusually good for workers expecting to move into employment next period. This raises the target wage $\widehat{w}^o\left(w^*,\mathbf{s}\right)$ and in turn the contract wage $\widehat{w}^*$. The reverse happens if $E\widehat{\bar{w}}'$ is below $E\widehat{w}^{*\prime}$. In this way stickiness in market wages introduces inertia into the individual wage bargain. The other types of spillovers, captured by the third term, work through the impact of the wage differential on the firm's relative hiring rate and are not important quantitatively.

We can now derive a set of simple difference equations for wages and hiring. The loglinearized wage index is given by

$$\widehat{\bar{w}}=(1-\lambda)\widehat{w}^*+\lambda\widehat{\bar{w}}_{-1}. \tag{48}$$

Combining this expression with the equations that define the evolution of the contract wage, then yields the following second order difference equation for the aggregate wage:

$$\widehat{\bar{w}}=\gamma_b\widehat{\bar{w}}_{-1}+\gamma_o\widehat{\bar{w}}^o+\gamma_f E\widehat{\bar{w}}', \tag{49}$$

where

$$\begin{aligned}
\gamma_b &= (1+\tau_2)\phi^{-1}\\
\gamma_o &= \varsigma\phi^{-1}\\
\gamma_f &= \left(\tau\lambda^{-1}-\tau_1\right)\phi^{-1}\\
\phi &= (1+\tau_2)+\varsigma+\left(\tau\lambda^{-1}-\tau_1\right)\\
\varsigma &= (1-\lambda)(1-\tau)\lambda^{-1}
\end{aligned}$$

with $\gamma_b+\gamma_o+\gamma_f=1$. Note the forcing variable in the difference equation is the spillover-free target wage $\widehat{\overline{w}}^o$, i.e., the target wage if all firms are bargaining period-by-period, adjusted for the horizon effect (see equation (46)).

Due to staggered contracting, $\widehat{\overline{w}}$ depends on the lagged wage $\widehat{\overline{w}}_{-1}$ as well as the expected future wage $E\widehat{\overline{w}}'$. Solving out for the reduced form of equation (49) yields an expression that relates the wage to the lagged wage and a discounted stream of expected future values of $\widehat{\overline{w}}^o$. Note that the spillover effects, measured by τ_1 and τ_2 work to reduce the relative importance of the expected future wage relative to the lagged wage (by reducing γ_f relative to γ_b). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way as to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, for example, Woodford, 2003).

Note the formulation nests the conventional period-by-period wage bargaining setup. As λ converges to zero (the case of period-by-period wage bargaining), both γ_b and γ_f go to zero and γ_o goes to unity, implying $\widehat{\overline{w}}$ simply tracks $\widehat{\overline{w}}^o$ in this instance. Further, as we showed in section 2.4, the horizon effect on bargaining disappears in that case, meaning that $\widehat{\overline{w}}^o$ becomes identical to the wage that would arise economy-wide under period-by-period bargaining.

Finally, loglinearizing the difference equation for the hiring rate (12) and aggregating economy-wide yields:

$$\widehat{\overline{x}} = E\left[\varkappa_a \widehat{a}' - \varkappa_w \widehat{\overline{w}}' + \widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right) + \beta \widehat{\overline{x}}'\right], \tag{50}$$

where the expressions for the coefficients $\varkappa_a$ and $\varkappa_w$ are reported in the appendix. The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

We defer to the appendix a complete presentation of the loglinear equations of the model.

4 Model Evaluation

4.1 Calibration

We choose a monthly calibration in order to properly capture the high rate of job finding in U.S. data. There are twelve parameters to which we need to assign values. Five are conventional in the business cycle literature: the discount factor, β, the depreciation rate, δ, the "share" parameter on capital in the Cobb-Douglas production function, α, the autoregressive parameter of the technology shock, ρ_z, and standard deviation of the technology shock, σ_z. We use conventional values for all these parameters: $\beta = 0.99^{\frac{1}{3}}$, $\delta = 0.025/3$, $\alpha = 0.33$, $\rho_z = 0.95^{\frac{1}{3}}$, and $\sigma_z = 0.0075$[12]. Note in contrast to the frictionless labor market model, the term $1 - \alpha$ does not necessarily correspond to

[12] σ_z is chosen to target the standard deviation of output.

the labor share, since the latter will in general depend on the outcome of the bargaining process. However, because a wide rage of values of the bargaining power imply a labor share just below $1 - \alpha$, here we simply follow convention by setting $1 - \alpha = 2/3$.[13]

There are an additional six parameters that are specific to the conventional search and matching framework: the job survival rate, ρ, the elasticity of matches to unemployment, σ, the matching function constant, σ_m, the bargaining power parameter, η, the adjustment cost parameter, κ, and the unemployment flow value, b.

We choose the average monthly separation rate $1 - \rho$ based on the observation that jobs last about two years and a half. Therefore, we set $\rho = 1 - 0.035$. We choose the elasticity of matches to unemployment, σ, to be equal to 0.5, the midpoint of values typically used in the literature.[14] This choice is within the range of plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then note that the parameter σ_m can be normalized. Larger values of this parameter only result in smaller values of average vacancies without affecting the steady state properties or the dynamics of the model (see equation (41) in the description of the steady state). We normalize σ_m to 1. To maintain comparability with much of the existing literature, we set the bargaining power parameter η to be equal to 0.5.[15] One of the few studies that provides direct estimates is Flinn (2006), who finds a point estimates of 0.4, close to the value we use. We then use the adjustment cost parameter, κ, and the flow unemployment value, b, to target the average job finding probability, $\tilde{p}$, and the value of $\bar{b}$, defined as the ratio of the unemployment flow value, b, to the steady state flow contribution of the worker to the match, $\tilde{a} + \frac{\kappa}{2}\tilde{x}^2$. We choose $\tilde{p} = 0.45$ to match recent estimates of the U.S. average monthly job finding rate (Shimer, 2005a). Perhaps most controversial is the choice of $\bar{b}$. We follow much of the literature by assuming that the value of non work activities is far below what workers produce on the job (see Hall, NBER Macroannual, 2005, p. 121, for a brief discussion). In particular, we specifically follow Shimer (2005a) and Hall (2005c) and choose $\bar{b} = 0.4$. This requires setting $b = 1.46$ and $\kappa = 148.2$. This parameterization implies a ratio of adjustment costs to output equal to 1 percent. In addition, under the interpretation of b as unemployment benefits, it implies a steady state replacement ratio of 0.42 (since the steady state ratio of the wage to the worker's contribution to the job is 0.956.)

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[13] In our calculations, $1 - \alpha$ equals 0.667 and the labor share 0.646.

[14] The values for σ used in the literature are: 0.24 in Hall (2005a), 0.4 in Blanchard and Diamond (1989), Andolfatto (1994) and Merz (1995), 0.45 in Mortensen and Nagypal (2007), 0.5 in Hagedorn and Manovskii (2006), 0.5 in Farmer (2004), 0.72 in Shimer (2005a).

[15] Flinn (2006) estimates the bargaining power parameter to be in this range. We find that reasonable perturbations of this value (e.g., from 0.4 to 0.6) have no perceptible effects on our quantitative results. In contrast to much of the the literature, we cannot appeal to the Hosios (1990) condition to constrain the parameter in this region since this condition does not generate efficiency within our framework. As we discuss in the working paper version of our model, because we have costs of adjusting employment as opposed to vacancies, efficiency requires that workers' wages are driven to workers' opportunity cost.

Table 1: Values of parameters		
Discount factor	β	0.997
Capital depreciation rate	δ	0.008
Production function parameter	α	0.33
Technology autoregressive parameter	ρ_z	0.983
Technology standard deviation	σ_z	0.0075
Survival rate	ρ	0.965
Elasticity of matches to unemployment	σ	0.5
Bargaining power parameter	η	0.5
Matching function constant	σ_m	1
Adjustment cost parameter	κ	148.2
Unemployment flow value	b	1.46
Renegotiation frequency	λ	0.889

Finally, there is one parameter that is specific to this model: the probability λ that a firm may not renegotiate the wage. We pick λ to match the average frequency of wage contract negotiations. While there is no systematic direct evidence on the frequency of wage negotiations, Taylor (1999) argues that in most medium to large sized firms wages are typically adjusted once per year. He also argues that this pattern characterizes union workers as well as non-union workers, including in the latter workers who do not have formal employment contracts. In addition, based on microeconomic data on hourly wages, Gottschalk (2005) concludes that wage adjustments are most common a year after the last change. This evidence, of course applies primarily to base pay. There are, however, other components such as bonuses that might be adjusted more frequently over the year, though it is very unclear how important these adjustments might be in practice. Nonetheless, to be conservative, for our baseline case we set $\lambda = 8/9$, implying that wage contracts are renegotiated on average once every 3 quarters. We then consider the case of a 4 quarter average contract length as a robustness exercise.

Note finally that as we discussed in the previous section, the implied steady state horizon-adjusted bargaining parameter $\tilde{\chi}$ does not vary much from the primitive parameter η (0.44 versus 0.50).

Our parametrization is summarized in Table 1

4.2 Results

We judge the model against quarterly U.S. data from 1964:1-2005:1. For series that are available monthly, we take quarterly averages. Since the artificial series that the model generates are based on a monthly calibration, we also take quarterly averages of this data.

Most of the data is from the BLS. All variables are measured in logs. Output y is production in the non-farm business sector. The labor share ls and output per worker y/n are similarly from the non-farm business sector. The wage w is average hourly earnings of production workers in the private sector, deflated by the CPI. Employment n is all employees in the non-farm sector. Unemployed u is civilian unemployment 16 years old and over. Vacancies v are based on the help wanted advertising index from the Conference Board. Consumption c is real personal consumption expenditures of nondurables. Investment i is equal to the sum of real private investment plus real personal consumption of durables. Finally, the data are HP filtered with a conventional smoothing weight.

We examine the behavior of the model taking the technology shock as the exogenous driving force. To illustrate how the wage contracting process affects model dynamics, we first examine the impulse responses of the model economy to a unit increase in total factor productivity. The solid line in each panel of Figure 1 illustrates the response of the respective variable for our model. For comparison, the dotted line reports the response of the conventional flexible wage model with period-by-period Nash bargaining (obtained by setting $\lambda = 0$).

Observe that in the conventional case with period-by-period wage adjustment, the response of employment is relatively modest, confirming the arguments of Hall and Shimer. There is also only a modest response of other indicators of labor market activity, such as vacancies, v, unemployment u, labor market tightness, $\theta = v/u$, and the hiring rate x. Wages, by contrast, adjust quickly. The resulting small adjustment of employment leads to output dynamics that closely mimic the technology shock.

By contrast, in the model with staggered multi-period contracting, there is a smooth drawn out adjustment in wages, directly a product of the staggered multi-period contracting. As a consequence, the hiring rate jumps sharply in the wake of the technology shock along with the measures of labor market activity. A substantial rise in employment follows, certainly as compared to the conventional flexible wage case. The lagged rise in employment leads to a humped shaped response of output, i.e., output continues to rise for several periods before reverting to trend, in contrast to the technology shock which reverts immediately. The quadratic adjustment costs of hiring, along with the wage rigidity, contribute to the persistence of the output response.

We next explore how well the model economy is able to account for the overall volatility in the data. The top panel in Table 3 reports the standard deviation, autocorrelation, and contemporaneous correlation with output for the eleven key variables in the U.S. economy. The standard deviations are normalized relative to output.

To provide a benchmark, the second panel present statistics from the model economy for the standard case of period-by-period wage bargaining. As in Shimer (2005a), the model does not come close to generating the relative volatility of the labor market variables that appears in the data.

Table 3: Aggregate Statistics										
	y	w	ls	n	u	v	θ	a	i	c
	U.S. Economy, 1964:1-2005:01									
Relative St. Dev.	1.00	0.52	0.51	0.60	5.15	6.30	11.28	0.61	2.71	0.41
Autocorrelation	0.87	0.91	0.73	0.94	0.91	0.91	0.91	0.79	0.85	0.87
Correlation with y	1.00	0.56	−0.20	0.78	−0.86	0.91	0.90	0.71	0.94	0.81
	Model Economy, $\lambda = 0$ (Flex wages)									
Relative St. Dev.	1.00	0.87	0.09	0.10	1.24	2.72	2.72	0.93	3.11	0.37
Autocorrelation	0.81	0.81	0.58	0.92	0.92	0.90	0.90	0.78	0.80	0.85
Correlation with y	1.00	1.00	−0.54	0.59	−0.59	0.92	0.92	1.00	0.99	0.93
	Model Economy, $\lambda = 8/9$ (3 quarters)									
Relative St. Dev.	1.00	0.56	0.57	0.35	4.44	5.81	9.84	0.71	3.18	0.35
Autocorrelation	0.84	0.95	0.65	0.90	0.90	0.82	0.88	0.76	0.86	0.86
Correlation with y	1.00	0.66	−0.56	0.77	−0.77	0.91	0.94	0.97	0.99	0.90
	Model Economy, $\lambda = 11/12$ (4 quarters)									
Relative St. Dev.	1.00	0.48	0.58	0.44	5.68	7.28	12.52	0.64	3.18	0.34
Autocorrelation	0.85	0.96	0.68	0.91	0.91	0.86	0.90	0.74	0.88	0.86
Correlation with y	1.00	0.55	−0.59	0.78	−0.78	0.93	0.95	0.95	0.99	0.90

In contrast, the third panel shows that the model economy for the baseline case (3 quarters) appears to capture well most of the basic features of the data. It comes reasonably close to matching the relative volatilities and co-movements of the key indicators of labor market activity, including unemployment u, vacancies v and the tightness measure θ. These were the variables emphasized in the Hall/Shimer analysis. The model only captures about sixty percent of the relative volatility of employment. However, here it is important to keep in mind that the framework abstracts from labor force participation, a non-trivial source of cyclical employment volatility. Some further evidence of its overall plausibility is that the model comes close to account for the relative behaviors of both consumption and investment.

A distinguishing feature of our analysis is that we appear to capture wage dynamics. Note that we come very close to matching the relative volatility of wages (0.56 versus 0.52 in the data), their autocorrelation (0.95 versus 0.91 in the data) and the contemporaneous correlation of wages with output (0.66 versus 0.56 in the data).

As we noted earlier, we assumed three quarter average length wage contracts for our baseline case to error on the side of caution, even though the evidence suggests that the modal period of wage adjustments is one year. In the bottom panel of Table 3 we also report statistics based on four quarter average length wage contracts. Interestingly, the performance of the model improves overall. Not surprising, the enhanced wage rigidity raises the volatilities of the labor market variables. In the end, the model tracks the relative volatilities and co-movements of the key labor market variables, u, v, and θ as well as in the baseline case. The model, however, is also now able to capture nearly three quarters of the relative volatility of employment.

Table 4: The Spillover Effect and the Horizon Effect

	Relative Standard Deviations									
	y	w	ls	n	u	v	θ	a	i	c
Model Economy	1.00	0.56	0.57	0.35	4.44	5.81	9.84	0.71	3.18	0.35
Model Economy - No Spillover	1.00	0.69	0.50	0.19	2.50	3.38	5.58	0.83	3.13	0.36
Model Economy - No Horizon	1.00	0.57	0.53	0.35	4.53	5.97	10.05	0.70	3.20	0.34

We next in Table 4 consider several variations of the model designed to illustrate the importance of the bargaining spillovers and of the horizon effect on wage bargaining. As we discussed earlier, the inertia in wage dynamics is not simply a product of staggered multi-period contracting, but also of the spillover effect of economy-wide wages on the individual wage bargain that arises in this kind of environment. To quantify the importance of these spillovers for model dynamics, we simulate the model eliminating the spillover effects on wage dynamics. In particular, we set equal to zero the parameters τ_1 and τ_2, which govern the magnitude of the spillover effects, in equation (49). To be clear, τ_1 and τ_2 are both functions of model primitives that we are holding fixed. Thus the exercise is not a formal counterfactual experiment, but rather an informal way to judge how the spillover mechanism is sharping model dynamics.

As the table makes clear, eliminating the spillovers significantly enhances wage flexibility and reduces employment volatility. When the spillovers are removed, the relative volatility of wages jumps nearly fifty percent, from 0.56 to 0.69 . Conversely, the relative volatility of employment is reduced roughly in half, from 0.35 to 0.19. The other measures of labor activity u, v and θ similarly fall by about half. Overall, as comparison of Tables 3 and 4 suggest, the spillovers are responsible for about a half of the added rigidity in wages relative to the flexible benchmark model and for about two thirds of the added volatility in the labor market.

The last row of Table 4 presents information on the importance of the horizon effect on the wage bargain. The row presents the model statistics for our baseline case, but with the horizon effect shut off (i.e., χ fixed at η). The relative volatilities remain close to those arising in the baseline case. Thus, as we conjectured, the horizon effect is not that important quantitatively.

Finally, it is interesting to compare our analysis with Hagedorn and Manovskii (HM, 2006), who argue that an alternative calibration of the MP model can account for labor market volatility. In particular, these authors find parameters that allow the model to match the low elasticity of wages with respect to productivity present in the data. From micro data, they estimate this elasticity to be 0.47. To have the model match it, they require a very low value of η, the bargaining power of workers, and a very high value of $\bar{b}$, the relative steady state flow value of unemployment, as compared to what is conventional in the literature. In particular, they require η very close to zero, well below the conventionally used value of 0.5, as well as Flinn's (2006) estimate of 0.4. In addition they require $\bar{b}$ close to unity, well above Shimer (2005a) and Hall's (2005c) preferred value of 0.4. A value of $\bar{b}$ close to unity, of course implies that workers are nearly indifferent between employment and unemployment. The overall calibration effectively makes labor supply highly elastic, enabling the model to have large employment movements with moderate wage adjustments. Nonetheless, the resulting calibration is not without controversy.[16]

Interestingly, we find from our macro data that the wage elasticity with respect to labor productivity is 0.53, which is very close to the estimate that the authors obtained from micro evidence. However, as we suggested earlier, we stick with conventional values of η and $\bar{b}$, and instead introduce wage sluggishness by appealing to staggered multi-period contracts. Further, as opposed to picking parameters to match the wage elasticity, we calibrate the average duration of wages contracts to match the evidence. We then ask how well the model explains the wage elasticity (along with other volatilities.) It turns out the model does very well on this accounting, generating a wage elasticity of 0.50, nearly identical to what our data suggests.

In addition, as we observed in Table 3, our model does well at explaining the overall cyclical volatility of wages, including the co-movement with aggregate activity as well as the relative volatility. On the other hand, the HM model does not do well on this dimension, even though it is calibrated to match the wage elasticity with respect to productivity. How can this be? Note that since this elasticity, $el(w,a)$, is effectively a regression coefficient from the regression of log wages on log productivity it equals the product of the correlation $corr(w,a)$ and the relative standard deviations σ_w/σ_a. Since the HM calibration only fixes the product of these two moments, it needs not do well at matching them individually. This turns out to be the case, as we show next.

Table 5 compares values of $el(w,a)$, $corr(w,a)$ and σ_w/σ_a against U.S. data for three models: the conventional Mortensen and Pissarides model, the framework based on the HM calibration, and our baseline model with staggered wage contracting (GT). While the HM model captures $el(w,a)$

[16] In addition to having values of η and $\bar{b}$ that are at variance with the literature, Hornstein, Krusell and Violante (2005) note that the HM calibration implies suspiciously large employment effects from changes in unemployment insurance.

by construction, it misses badly on the other two moments. The correlation between wages and productivity is too high (unity versus 0.62 in the data) while the relative volatility of wages is too low (0.49 versus 0.85). The former outcome is due to the period-by-period Nash bargaining that ties aggregate wage movements to current period productivity. The latter result arises from the low bargaining power of workers which forces their wages close to their reservation values. Thus while the HM model by construction matches $el(w, a)$, it does so by inducing offsetting errors in $corr(w, a)$ and σ_w/σ_a.

By contrast, our model does well at matching not only the wage elasticity but also the correlation of wages and productivity, as well as the relative volatility. The staggered contract structure works to dampen the correlation between productivity and wages. At the same time, because workers have more bargaining power than in the HM calibration, wages are more sensitive to productivity movements, permitting the model to match the data. Again, we stress that our model is calibrated to match the average duration of contract lengths. It is therefore not by construction that we match the wage elasticity, in contrast to HM.

Table 5: Wages and Labor Share Statistics

	$el(w,a)$	$corr(w,a)$	σ_w/σ_a
U.S. data	0.53	0.62	0.85
MP baseline	0.98	1.00	0.98
HM	0.49	1.00	0.49
GT	0.50	0.63	0.79
	$el(ls,a)$	$corr(ls,a)$	σ_{ls}/σ_a
U.S. data	-0.50	-0.60	0.83
MP baseline	-0.02	-1.00	0.02
HM	-0.51	-1.00	0.51
GT	-0.50	-0.63	0.80

Finally, the bottom part of Table 5 shows that similar conclusions apply for the volatility of the labor share. While the HM model does not explain all the relevant moments well, our framework does.

5 Issues

We next address several issues involving the robustness of the assumptions that underlie our analysis.

5.1 Bargaining Set

A key maintained hypothesis in our analysis is that workers and firms can expect that they will not want to voluntarily dissipate their relationship over the life of their relationship. This assumption simplifies how both parties form expectations when they enter relationships. Here we demonstrate that this condition holds to a reasonable approximation. Put differently, under our parametrization, wages have a negligible probability of falling outside the bargaining set. Intuitively, given our Poisson process for contract adjustment, only a very small fraction of contracts will have a duration sufficiently long for the wage to move out of the bargaining set.

The lower and upper limits of the bargaining set are given by, respectively, the reservation wage of the worker and the reservation wage of the firm. We compute these values by finding the wage for the current contract that makes the expected surplus zero for each party, given that the wage will be renegotiated once the contract expires. If the wage stays in the bargaining set as we have defined it, then both parties are willing to stay in the relationship until the next round of contract negotiations. If it doesn't, then one party is not.

To analyze how likely the wage is to stay in the bargaining set we first generate artificial times series from the model and then compute a time series for the firm and worker reservations wages and the contract wage. We then check whether a contract wage set τ periods earlier lies within the current bargaining set.

In Figure 2, for example, we consider a contract that has been in place for the average time of nine months. As the figure shows, at any time t for the simulated series, the contract wage from three quarters earlier lies safely in the bargaining set.

In Figure 3, we consider wage contracts that have been in place for forty months. Even in this case the contract wage stays in the bargaining set, though it edges closer to the boundaries than in the nine month case. However, under our baseline calibration, the probability a contract will remain intact for forty months is less than one percent ($\lambda^{40} = (1 - 1/9)^{40} = 0.0089$). Thus, at the time of a new contract negotiation there is less than a one percent chance that the contract wage will eventually fall out of the bargaining set.

Note that we could have instead employed a "truncated" Poisson process where the probability of renegotiation is Poisson for T periods and then unity for any contract that has lasted T periods. Assuming T is not too large - say 40 months or less - then we could guarantee firms never go out of the bargaining set with certainty.[17] Given our calibration, however, the truncated Poisson generates results nearly identical to what our simple Poisson process gives. The gain from using the simple process is that the algebra stays simple.

[17] We could also allow for idiosyncratic shocks using this approach. In this instance T would have to be less than 40 months, though it could be significantly longer than 9 months, as inspection of Figures 2 and 3 suggests.

5.2 Welfare Costs of Multi-Period Contracts

We have been appealing partly to realism and partly to the notion that bargaining is costly to justify firms and workers negotiating wage contracts only infrequently. It is reasonable to ask, however, how much would the two parties gain if they could costlessly negotiate wages on a period-by-period basis. The answer is very little.

We consider the following experiment. We first compute the firm surplus and the worker surplus for a typical firm/worker pair operating on multi-period contracts and currently renegotiating. We then ask how the surplus measures differ for a firm/worker pair that deviates from all the other pairs by operating on period-by-period contracts.

Combining the recursive formulation for J in equation (9) with the optimality conditions for capital and hiring, equations (10) and (11), leads to the following simple expression for the match surplus of a firm who is currently renegotiating:

$$\begin{aligned} J\left(w^{*},\mathbf{s}\right) &= a-w^{*}-\frac{\kappa}{2}x\left(w^{*},\mathbf{s}\right)^{2}+\left[\rho+x\left(w^{*},\mathbf{s}\right)\right]\kappa x\left(w^{*},\mathbf{s}\right), \\ &= a-w^{*}+\frac{\kappa}{2}x\left(w^{*},\mathbf{s}\right)^{2}+\rho\kappa x\left(w^{*},\mathbf{s}\right). \end{aligned} \tag{51}$$

Consider now the worker's surplus in equation (19). Let us define $\Theta\equiv b+\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)p\bar{H}_{x}\left(\mathbf{s}'\right)|\mathbf{s}\right\}$ as the component of the surplus that is independent of current and expected future wage bargains. We can then express the surplus of a worker who is currently renegotiating as:

$$H\left(w^{*},\mathbf{s}\right)=w^{*}+\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\rho H\left(w',\mathbf{s}'\right)|w^{*},\mathbf{s}\right\}-\Theta. \tag{52}$$

The choice of contract length, of course, influences the volatility of wages and hiring. Since J is linear in w^{*} and quadratic in x, the firm cares only about the volatility of the latter. In this regard, there are two offsetting effects: First, as the third term in the top line of equation (51) indicates, an increase in the variability of the hiring rate raises adjustment costs, which reduces J. Second, and conversely, an increase in the variability of x raises the expected discounted value of future profits, as the last term suggests, implying in turn an increase in J. When expected discounted labor productivity net wages increases, for example, the firm raises its hiring rate, making the latter a signal of the former. By doing more hiring, further, the firm expands its expected size, further raising overall expected profits. The combined effect is that expected future profits are increasing and convex in x.

As equation (51) shows, the positive effect of the variability of x on expected future profits outweighs the negative effect on current adjustments. Thus, overall, J is increasing and convex in x, which suggests that everything else equal the firm prefers volatility in the hiring rate. That J is convex in the hiring rate with convex adjustment costs is a standard result in the literature. In the context of our model, it suggests that firms might actually prefer multi-period contracts since the smoothed behavior of the wage raises hiring volatility.

The worker does care about the volatility of the wage. As equation (52) indicates, H depends on how its value in the subsequent period covaries with the stochastic discount factor, $\Lambda(\mathbf{s}, \mathbf{s}')$. In general, this covariance depends on the covariance of the expected future path of wages with the whole term structure of stochastic discount factors. To the extent wages move procyclically and thus vary inversely with the marginal utility of consumption, wage volatility reduces the worker's surplus from the job. In general, it appears ambiguous as to whether the worker would prefer multi-period or period-by-period contracts. While period-by-period contracts increase the sensitivity of the wage to contemporaneous shocks, multi-period contracts introduce persistent movements in the wage, enhancing the overall unconditional volatility.

To quantify the impact of multi-period contracting on the firm and the worker, we first take quadratic approximations of (51) and (52) around their respective deterministic steady state values. We then simulate the model and evaluate the loss under multi-period contracts versus period-by period contracts. The first column in Table 6 reports the deterministic steady state values of J and H, as well as the composite surplus $NP = H^{\eta}J^{1-\eta}$, under the benchmark parameterization. The next column reports the percentage change in the unconditional value of each variable induced by macroeconomic volatility for our benchmark case where all firms are operating on multi-period contracts. As we have suggested, the firm prefers the volatility, though the gain is very slight: The unconditional value of J increases by 0.15 percent. For the worker, the change differs in both sign and magnitude: The procyclical wage volatility reduces the worker surplus by 2.41 percent.

Table 6: Firm and worker surplus values

Aggregate		$\lambda = 8/9$		$\lambda = 0$
Firm		$\lambda = 8/9$	$\lambda = 0$	$\lambda = 0$
	Steady state value	Change from steady state		Change from steady state
Firm surplus, J	5.20	0.15%	0.10%	0.01%
Worker surplus, H	4.09	-2.40%	-2.36%	-2.19%
Nash product, NP	4.61	-1.18%	-1.18%	-1.14%

We next consider the case where one firm deviates from the others by negotiating period-by-period contracts. One technical complication emerges because we have used the steady state to pin down the adjustment cost parameter κ. Because the steady state depends on the horizon effect on bargaining (through the effect of multi-period contracting on the effective bargaining power parameter $\tilde{\chi}$) the implied value of κ for a firm can differ depending on contract length. We accordingly proceed to do the calculation two different ways: First, for the firm on period-by-period contracts, we keep κ the same as for the other firms by adjusting the primitive bargaining power parameter η to equal $\tilde{\chi}$. In this case, the steady state of the firm under period-by-period contracting is identical to that of a firm under multi-period contracting. Second, we keep η the same across firms and allow κ to vary. Fortunately, because the horizon effect is not quantitatively important under our benchmark calibration, the two approaches yield nearly identical results. Accordingly, Table 6 just reports (in the third column) the results using the first method.

For a firm under period-by-period contracting, the percentage increase in J stemming from macroeconomic volatility is smaller than in the case of multi-period contracting. This is not surprising, given that firms prefer hiring volatility, everything else equal, and that this volatility declines under period-by-period contracting. The difference is very small, though: a 0.10 percent gain versus a 0.15 percent gain. For a worker, the reverse is true. H is larger under period-by-period contracts. Again the margin of difference is small: 2.36 percent loss relative to the steady state with period-by-period contracting versus a 2.41 percent loss with multi-period contracting. Since the gain to the worker offsets the loss to the firm, the composite surplus NP is effectively the same under period-by-period and multi-period contracting.

Finally, we ask how would the parties' bargaining surpluses be affected if all firms switched to period-by-period contracting. Results are shown in the last column of Table 6. In this case, as we saw earlier, there is an overall decline in aggregate volatility. J decreases relative to the other cases and H increases. Overall, the surplus NP improves. But, again, the magnitudes of the changes are very small: under 0.25 percent in each case.

We conclude that under our benchmark formulation there are no strong incentives for firms and workers acting either alone or in concert with others to switch from multi-period to period-by-period wage contracting.[18] Key to this result, though, is that as we showed in the previous sub-section, the contract wage is almost surely in the bargaining set over the life of the contract. If this were not the case, firms and workers would have a much stronger rationale for wanting to consider period-by-period contracting.

[18] Based on realism and convenience, we have restricted attention to a simple multi-period contract with a constant wage per period. It has been suggested that a contract where the wage is declining over time may be preferred to this simple contract. In our view, however, the constraint that new workers are treated the same as (similarly productive) existing workers should apply to the present value of compensation they receive and not simply to the most recent contract wage. Under this interpretation, new hires receive the same expected wage profile as existing workers did when they started the most recent contract. Thus, since changing the timing of wage payments does not change the present value of payments to either existing workers or new hires, there is no benefit to doing so.

Note, though, that even under the alternative scenario where new hires just receive the latest contract wage, the gains from a downward sloping wage profile are not large: Since the benefits of moving from the simple multi-period contract to fully flexible period-by-period contracts are small, it is possible to show that so too are the benefits of moving to a more exotic multi-period contract that is nonetheless still more restrictive than a period-by-period contract.

5.3 Evidence

As we have been noting, a key restriction in our model is that in between contract periods, new hires receive the same wages as existing workers of similar productivity. Here we first discuss existing panel data evidence on the cyclical behavior of new hire wages and then present some additional evidence in support of our hypothesis. We next turn to another kind of evidence on the employment effects of wage contracting that also provides support for our approach.

5.3.1 Wages of new hires

There is a lengthy literature, beginning with Vroman (1977, 1978) that shows that job movers have more cyclical wages than job stayers. Some other notable contributions include Bils (1985) and Barlevy (2001). Vroman's original interpretation of this evidence was that it reflected cyclical movements in the composition of job quality: The fraction of hiring by "high wage" firms increases during booms and declines during recessions. Other examples of a compositional interpretation of the evidence include Barlevy (2001) and Moscarini and Postel-Vinay (2008).

To the extent that it is compositional effects at work, the new hire wage evidence does not contradict our maintained hypothesis. While workers may transition between high and low wage jobs over the cycle, the wages of new hires may still be tied to those of existing workers within the same firm. In this instance wage rigidity within the firm will still amplify the effects of disturbances on hiring.

Pissarides (2007), however, argues that the evidence instead suggests that the wages of new hires within a given firm are more cyclical than those of the existing workers within the same firm. In our view, the evidence is not sufficiently sharp to support this claim for two related reasons: First, in virtually every case the data includes only evidence on workers and not firms, i.e., it does not match workers with firms. Thus, it is not possible to directly compare new hires with existing workers in the same firm. Second, in every case, there is not an adequate effort to control for cyclical changes in the composition of job quality. Simple industry controls are not adequate since, as Vroman notes, these compositional effects can arise within industries as well as across.

To illustrate how the compositional effects may underlay the evidence on new hire wage cyclicality, we draw on some companion empirical work by Gertler, Huckfeldt and Trigari (2008). The data in this study is on individual workers taken from the Survey of Income and Program Participation (SIPP) over the years 1990 to 1996. It includes four panels lasting roughly three years each from 1990, 1991, 1992 and 1993. There is a new sample of individuals in each panel. Each individual within the samples is interviewed every four months. In addition, the study restricts attention to males.

An important advantage of the SIPP is that it is a large representative sample. Moreover, the three year sampling period for individuals allows for cyclical variation. We focus on the 1990-1996 data for several reasons. First, over this period it is possible to obtain good quality information on both job transitions and job tenure, which is critical to the analysis. Second, the period contains the 1990-91 recession and its aftermath, and thus includes considerable fluctuations in aggregate economic activity.

We consider two types of regression equations to evaluate the cyclical sensitivity of new hires' wages: The first follows the literature. The second is a variation that introduces a control for compositional effects. Let $w_{i,j,t}$ be the real wage of worker i in firm j at time t, $X_{i,t}$ be a vector of individual specific factors (e.g., age, sex, education, experience, tenure, etc.), U_t the civilian male unemployment rate and $I(new_{i,j,t})$ an indicator variable that takes on a value of unity if the worker is a new hire and zero otherwise. Following the literature we define a new hire as someone with job tenure under a year. However, the results are robust to using a shorter period of job tenure to define a new hire (e.g. six months).

The specification we consider that follows the literature is given by:

$$\log(w_{i,j,t}) = \pi_x' \cdot X_{i,t} + \pi_u \cdot U_t + \pi_n \cdot I(new_{i,j,t}) \cdot U_t + \gamma_i + \epsilon_{i,j,t}, \tag{53}$$

where γ_i is an individual fixed effect and $\epsilon_{i,j,t}$ is an i.i.d. error term. A typical finding in the literature is that $\pi_n < 0$, implying that new hires have wages that are more procyclical than those of existing workers (since unemployment is countercyclical.)[19]

From the above specification however, it is not possible to tell whether the greater wage cyclicality is due to compositional effects, as originally stressed in the literature, or greater volatility of new hires wages within a given firm, as argued by Pissarides. While the regression controls for individual heterogeneity via the vector of individual characteristic $X_{i,t}$, it does not control for cyclical movements in the composition of job quality for new hires. Suppose, for example, that a highly skilled machinist takes a job as a low paid cab driver in a recession and then is re-employed as a high paid machinist in a boom. In this case there is a cyclical movement in job match quality for the individual that is not captured in equation (53).

A way to modify this specification to control for compositional shifts in match quality is by allowing for a separate fixed effect γ_{ij} for each job j, as follows:

$$\log(w_{i,j,t}) = \pi_x' \cdot X_{i,t} + \pi_u \cdot U_t + \pi_n \cdot I(new_{i,j,t}) \cdot U_t + \gamma_{ij} + \epsilon_{i,j,t}. \tag{54}$$

The only difference between the two specifications is that in the latter we are allowing the fixed effect to vary as the worker changes jobs. To be clear, the intercept γ_{ij} is a combination of a pure individual effect that is independent of the match and a component that reflects match quality that we cannot separately identify. However, given that the pure individual effect is constant, the difference in γ_{ij} across two jobs will capture differences in match quality.

[19] Beaudry and DiNardo (1991) appeal to implicit contracts theory to propose an alternative specification that relates workers to earlier macroeconomic conditions, in particular the minimum unemployment rate over their job tenure, as to current unemployment. However, Gertler, Huckfeldt and Trigari (2008) show that by more carefully controlling for job tenure effects on wages, the minimum unemployment variable loses its explanatory power to current unemployment.

To separate the fixed effect on wages from changing jobs from the cyclical effect of aggregate labor market conditions on wages, it is necessary that we observe a worker on at least one job spell that lasts more than the contract period. That is at least one job spell in our data must be long enough to observe the impact of cyclical variation on the wage. If we assume that one year is a reasonable benchmark for how long a wage contract lasts, then our data set easily satisfies this criteria[20].

Table 7 reports the results for the conventional specification in column one and for the specification that allows for job-specific fixed effects in column two. The results for the conventional specification are consistent with the literature. The coefficient interacting new hires with unemployment, π_n, is significantly negatives and nearly twice the size in absolute value of the coefficient on unemployment, π_u. The results thus suggest that the percentage response of new hires' wages to a change in unemployment is roughly three times as large as it is for existing workers' wages, which is consistent with findings elsewhere.

Once we allow for firm-specific fixed effects, however, the results change dramatically. The new hire effect on wages is no longer statistically significant. That is, after controlling for compositional effects, new hire wages appear no more cyclically sensitive than existing workers' wages. In Gertler, Huckfeldt and Trigari (2008) we show that the results are robust to alternative ways of controlling for composition.

Table 7: Baseline Fixed-Effects Specification

Source: Survey of Income and Program Participation, 1990-93

	(1) Person fixed-effect	(2) Person-job fixed-effect
Unemployment rate	-0.00564***	-0.00576***
	(0.0010)	(0.0009)
Unemployment rate $\cdot\ I(new)$	-0.01042***	0.00193
	(0.0038)	(0.0037)
No. of Obs.	125941	122026
No. of Groups	17897	18234
Min no. of Obs. per Group	2	2
Max no. of Obs. per Group	9	9
Mean no. of Obs. per Group	7.037	6.692

***significant at 1%, **significant at 5%, *significant at 10%

[20] Of the roughly 18000 individuals in our sample, 3989 were "new hires" at some point in the panel. Of these, we observe 3291 with tenure on at least one job strictly greater than a year.

How do we reconcile the results from the two different specifications? If indeed match quality varies across jobs then the failure to allow for a job-specific fixed effect implies there is an omitted variable in the first specification that enters the error term. Further, if the net movement from low to high quality matches is procyclical, then the error term will be negatively correlated with the variable that interacts new hires with unemployment. Under this interpretation, accordingly, the negative estimate of π_n reflects omitted variable bias due to the failure to control for the composition of job match quality.

We do not claim that these results prove definitely that the relatively high cyclicality of new hire wages is due entirely to compositional effects. But they do suggest that at a minimum the existing literature has failed to adequately account for this possibility. As a result, it is not possible to conclude from this literature in its current form that new hires have more cyclical wages than existing workers within the same firm.

5.3.2 Employment effects of wage contracting

A different kind of evidence relevant to our analysis assesses whether, by shaping the response of wages, the multi-period contract structure influences how a firm's hiring responds to aggregate disturbances. The empirical work of which we are aware appears consistent with this prediction. The sharpest results come from a study of the employment and wage dynamics of a panel of Canadian manufacturing firms by Card (1990). The firms Card examined all were on union wage contracts, but the contracts differed by the provisions for indexing to inflation. The experiment Card considered was to analyze the response of firm employment to the average firm wage, using unexpected inflation as an instrument. Because firms differed in indexing provisions, the surprise inflation generated a dispersion in real wages. Card then found in firm-level regressions that movements in the real wage induced by the unanticipated surprise inflation had a significant effect on employment. In this way, restrictions on wage adjustment induced by the contract structure appears to have influenced the response of hiring to an aggregate shock. These results lend support to our general approach.

Olivei and Tenreyro (2007, 2008) also present evidence that wage contracting affects cyclical employment dynamics using aggregate data. In particular, they show that the response of employment to a monetary policy shock has a strong seasonal component: The effects of shocks early in the year are much weaker than of shocks later on. They then show that seasonal patterns in the adjustment of wage contracts can account for this phenomenon: Wage adjustments from new contracts are more frequent early in the year relative to later on. The frequent wage adjustments early in the year dampen the effects of monetary shocks that occur at this time. While their 2007 paper obtains the results for U.S. data, their follow-up paper shows the results are robust to data from other industrialized economies.

5.4 Multiple Shocks

We used as our baseline macroeconomic framework a real business cycle model with a technology shock as the single driving force, partly to follow the literature and partly to keep the analysis sufficiently simple to focus on the details of the wage contracting structure. This raises the issue (applicable to the entire literature) of whether it makes sense to evaluate the model against unconditional data moments if there are other important shocks besides productivity that drive aggregate economic activity.[21]

It is important to recognize, however, that we are not trying to evaluate how well the model with productivity shocks alone matches the data. Rather, we are instead using this simple single-shock model as laboratory to evaluate the implications of different wage contracting structures for labor market volatility. Thus, the moment matching exercise in section 4 should be viewed as a check as to whether indeed the paradigm is a reasonable laboratory for this purpose. That our baseline model with wage rigidity captures the aggregates statistics reasonably well suggests that it is a credible framework to use to study how much the performance deteriorates with wage flexibility.

To be more specific, we believe that our simple quantitative approach provides insights that carry over to a more general setting, at least in a "ballpark" sense. Note that we can re-write our key loglinear relation between the hiring rate and wages, equation (50), as follows:

$$\widehat{\widetilde{x}} = E\left[\varkappa_a(\widehat{\widetilde{y}}' - \widehat{\widetilde{n}}') - \varkappa_w \widehat{\widetilde{w}}' + \widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right) + \beta\widehat{\widetilde{x}}'\right] \tag{55}$$

where $\varkappa_a/\varkappa_w = (1-\alpha)(\widetilde{y}/\widetilde{n})/\tilde{w}$. Because the ratio $\varkappa_a/\varkappa_w$ is very close to unity, hiring effectively depends on the expected evolution of the labor share. Movements in the hiring rate then precipitate associated movements in unemployment and labor market tightness.

What is key for our purposes is that we have a model that, in conjunction with our contracting structure, produces plausible cyclical movements in the labor share and the relevant labor market quantities. Table 3 shows that our baseline meets this standard. With an empirically reasonable benchmark, it is then credible to study the importance of our contracting structure by replacing it with conventional Nash bargaining and then assessing the degree to which performance deteriorates.

The broad insights are likely to carry over to models that allow for alternative shocks and possibly alternative propagation mechanisms, so long as the baseline model with our contracting structure can generate empirically reasonable movements in the labor market variables.[22] In Gertler, Sala, and Trigari (2008) we verify this conjecture by embedding our framework of unemployment with staggered wage contracting within a contemporary quantitative macroeconomic model, of the

[21] If there was an agreed upon way to identify pure productivity shocks, then one could simply look at the data conditional on these shocks. Unfortunately this is not the case, given the difficulty in cleaning off unmeasured factor utilization from the measure of total factor productivity.

[22] Models that feature shocks other than total productivity typically capture realistic movements in labor productivity in part by allowing for variable intensity of capital utilization (e.g. Christiano, Eichenbaum and Evans, 2005, and Smets and Wouters, 2007).

type popularized by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007).[23] The framework allows for multiple sources of business cycle dynamics as well as a rich set of endogenous model dynamics. In addition, we estimate the key model parameters using Bayesian methods. The estimated model indeed captures well the behavior of the labor market variables including the labor share. Replacing our wage contracting structure with period-by-period Nash bargaining then leads to a deterioration in performance similar to what we observed in the simple framework of this paper.

6 Conclusion

We have modified the Mortensen and Pissarides model of unemployment dynamics to allow for staggered multi-period wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects of aggregate wages that influence the bargaining process. We then show that a reasonable calibration of the model can account reasonably well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this regard.

We also presented some evidence to suggest that the high relative cyclicality of new hires' wages may reflect cyclical composition effects as opposed to differences of wage flexibility within a firm between new and existing workers. That is, the evidence is consistent with procyclical movements in the average quality of jobs for new hires. In work in progress we are attempting to incorporate these compositional effects within our framework. We conjecture that so long as both high and low quality employment is procyclical with flexible wages, then the effects of our contracting framework on employment volatility will carry through to this framework.

[23] Christiano, Motto, and Rostagno (2007) similarly embed our labor market structure within a contemporary quantitative macro model. See Trigari (2008) and Walsh (2005) for early attempts to incorporate unemployment into a macroeconomic framework with nominal rigidities.

References

[1] Andolfatto, David (1996). "Business Cycles and Labor Market Search," *American Economic Review* 86, 112-132.

[2] Barlevy, Gadi (2001). "Why Are the Wages of Job Changers So Procyclical," *Journal of Labor Economics* 19 (4), 837-78.

[3] Barro, Robert (1977). "Long-Term Contracting, Sticky Prices and Monetary Policy," *Journal of Monetary Economics* 3 (3), 305-316.

[4] Beaudry, Paul and John DiNardo (1991). "The Effect of Implicit Contracts on the Movement of Wages over the Business Cycle: Evidence from Micro Data," *Journal of Political Economy* 99 (4), 665-688.

[5] Bewley, Truman (1999). "Why Wages Don't Fall During a Recession," Harvard University Press.

[6] Bils, Mark (1985), "Real Wages Over the Business Cycle: Evidence from Panel Data," *Journal of Political Economy* 93 (4), 666-689.

[7] Binmore, Ken, Ariel Rubinstein and Asher Wolinsky (1986). "The Nash Bargaining Solution in Economic Modelling," *The RAND Journal of Economics* 17 (2), 176-88.

[8] Blanchard, Olivier and Peter Diamond (1989). "The Beveridge Curve," *Brookings Papers on Economic Activity* 1, 1-60.

[9] Card, David (1990). "Unexpected Inflation, Real Wages, and Employment Determination in Union Contracts," *American Economic Review* 80 (4), 669-688.

[10] Calvo, Guillermo (1983). "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12, 383-398.

[11] Christiano, Lawrence, Martin Eichenbaum and Charles Evans (2005). "Nominal Rigidities and the Dynamics Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113 (1), 1-45.

[12] Christiano, Lawrence, Roberto Motto and Massimo Rostagno (2007). "Monetary Policy and Stock-Market Boom Bust Cycles," Northwestern University.

[13] Costain, James and Michael Reiter (2008). "Business Cycles, Unemployment Insurance and the Calibration of Matching Models," *Journal of Economic Dynamics and Control* 32 (4), 1120-55.

[14] Farmer, Roger (2006). "Shooting the Auctioneer," mimeo, UCLA.

[15] Flinn, Christopher (2006). "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining and Endogenous Contact Rates," *Econometrica* 74, 1013-1062.

[16] Galuscak, Kamil, Alan Murphy, Daphne Nicolitsas, Frank Smets, Pawel Strzelecki, Matija Vodopivec (2008). "The Determination of Wages of Newly Hired Employees: Survey Evidence on Internal versus External Factors," mimeo, ECB.

[17] Gertler, Mark, Chris Huckfeldt and Antonella Trigari (2008). "Interpreting Wage Cyclicality of New Hires", mimeo, NYU.

[18] Gertler, Mark, Luca Sala and Antonella Trigari (2007). "A Estimated Monetary DSGE Model with Unemployment and Staggered Nash Wage Bargaining," mimeo, NYU.

[19] Gottschalk, Peter (2005). "Downward Nominal Wage Flexibility: Real or Measurement Error?," *Review of Economics and Statistics* 87(3), 556-568.

[20] Hagedorn, Marcus and Iourii Manovskii (2006). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, forthcoming.

[21] Hall, Robert and Paul Milgrom (2008). "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, forthcoming.

[22] Hall, Robert (2006). "The Labor Market and Macro Volatility: A Nonstationary General Equilibrium Analysis," mimeo, Stanford University.

[23] Hall, Robert (2005a). "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95 (1), 50-65.

[24] Hall, Robert (2005b). "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market," *Review of Economics and Statistics* 87(3), 397-407.

[25] Hall, Robert (2005c). "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years," *NBER Macroeconomics Annual*, M. Gertler and K. Rogoff editors, MIT Press, 101-137.

[26] Hornstein, Andreas, Per Krusell and Gianluca Violante (2005). "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Economic Quarterly* (Federal Reserve Bank of Richmond) 91(3), 19-51.

[27] Hosios, Arthur (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies* 57(2), 279-298.

[28] Kennan, John (2006). "Private Information, Wage Bargaining and Employment Fluctuations," NBER Working Paper No. 11967.

[29] Kimball, Miles (1995). "The Quantitative Analytics of The Basic Neomonetarist Model," *Journal of Money, Credit and Banking* 27(4-2), 1241-1289.

[30] Krause, Michael and Thomas Lubik (2004). "On-the-Job Search and the Cyclical Dynamics of the Labor Market," mimeo, Richmond Fed.

[31] Menzio, Guido (2005). "High Frequency Wage Rigidity," mimeo, University of Pennsylvania.

[32] Menzio, Guido and Espen Moen (2006), "Incomplete Self-Enforcing Labor Contracts," mimeo, University of Pennsylvania.

[33] Merz, Monika (1995). "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics* 36, 269-300.

[34] Mortensen, Dale and Eva Nagypal (2007). "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics* 10 (3), 327-347.

[35] Mortensen, Dale and Christopher Pissarides (1994). "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61, 397-415.

[36] Moscarini, Giuseppe and Fabien Postel-Vinay (2008). "The Timing of Labor Market Expansions: New Facts and a New Hypothesis," *NBER Macroeconomics Annual*, forthcoming.

[37] Olivei, Giovanni and Silvana Tenreyro (2008). "Wage Setting Patterns and Monetary Policy: International Evidence," mimeo, London School of Economics.

[38] Olivei, Giovanni and Silvana Tenreyro (2007). "The Timing of Monetary Policy Shocks," *American Economic Review* 97 (3), 636-663.

[39] Petrongolo, Barbara and Christopher Pissarides (2001). "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature* 39(2), 390-431.

[40] Pissarides, Christopher (1985). "Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages," *American Economic Review* 75 (4), 676-690.

[41] Pissarides, Christopher (2000). Equilibrium Unemployment Theory, The MIT Press.

[42] Pissarides, Christopher (2007). "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," The Walras-Bowley lecture, North American Summer Meetings of the Econometric Society, Duke University.

[43] Rotemberg, Julio (2006). "Cyclical Wages in a Search-and-Bargaining Model with Large Firms," *NBER International Seminar on Macroeconomics 2006*, The MIT Press, forthcoming.

[44] Shimer, Robert (2005a). "The Cyclical Behavior of Equilibrium Unemployment, Vacancies, and Wages: Evidence and Theory," *American Economic Review* 95 (1), 25-49.

[45] Shimer, Robert (2005b). "Reassessing the Ins and Outs of Unemployment," mimeo, University of Chicago.

[46] Shimer, Robert (2004). "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association* (Papers and Proceedings) 2, 469-479.

[47] Shimer, Robert and Randall Wright. (2004), "Competitive Search Equilibrium with Asymmetric Information," mimeo, University of Chicago.

[48] Smets, Frank and Raf Wouters (2007). "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach," *American Economic Review,* March.

[49] Taylor, John B. (1980). "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88, 1-23.

[50] Taylor, John B. (1999). "Staggered Price and Wage Setting in Macroeconomics," *Handbook of Macroeconomics,* Volume 1, J.B. Taylor and M. Woodford editors, Elsevier.

[51] Trigari, Antonella, (2008). "Equilibrium Unemployment, Job Flows and Inflation Dynamics," *Journal of Money, Credit and Banking*, forthcoming.

[52] Vroman, Wayne (1977). "Worker Upgrading and the Business Cycle," *Brookings Papers on Economic Activity* 1, 229-52

[53] Vroman, Wayne (1978). "Cyclical Earnings Changes of Low Wage Workers," *Research in Labor Economics* 2.

[54] Walsh, Carl (2005). "Labor Market Search, Sticky Prices and Interest Rate Rules," *Review of Economic Dynamics* 8, 829-49.

[55] Woodford, Michael (2003). Interest and Prices, Princeton University Press.

Appendix A: Steady State

Consumption and savings

$$1 = \beta \left(1 - \delta + \tilde{r}\right)$$

Capital/employment ratio

$$\tilde{r} = \alpha \tilde{z} \left(\tilde{k}/\tilde{n}\right)^{-(1-\alpha)}$$

Marginal product of labor

$$\tilde{a} = \left(1 - \alpha\right) \tilde{z} \left(\tilde{k}/\tilde{n}\right)^{\alpha}$$

Rates

$$\tilde{x} = 1 - \rho$$

Flows

$$\tilde{x} \left(1 - \tilde{u}\right) = \tilde{p}\tilde{u}$$

Matching

$$\tilde{p}\tilde{u} = \sigma_m \tilde{u}^{\sigma} \tilde{v}^{1-\sigma}$$

Hiring

$$\kappa \tilde{x} = \beta \left(\tilde{a} - \tilde{w} + \frac{\kappa}{2}\tilde{x}^2 + \rho \kappa \tilde{x}\right)$$

Wages

$$\tilde{w} = \tilde{\chi} \left(\tilde{a} + \frac{\kappa}{2}\tilde{x}^2 + \kappa \tilde{p}\tilde{x}\right) + \left(1 - \tilde{\chi}\right) b$$

where

$$\tilde{\chi} = \frac{\eta}{\eta + \left(1 - \eta\right) \tilde{\mu}/\tilde{\epsilon}} \qquad \tilde{\mu} = \frac{1}{1 - \lambda\beta} \qquad \tilde{\epsilon} = \frac{1}{1 - \rho\lambda\beta}$$

Resource constraint

$$1 = \frac{\tilde{c}}{\tilde{y}} + \delta \frac{\tilde{k}}{\tilde{y}} + \frac{\kappa}{2}\tilde{x}^2 \frac{\tilde{n}}{\tilde{y}}$$

where

$$\tilde{y}/\tilde{n} = \tilde{z} \left(\tilde{k}/\tilde{n}\right)^{\alpha} \qquad \tilde{y}/\tilde{k} = \tilde{z} \left(\tilde{k}/\tilde{n}\right)^{-(1-\alpha)}$$

Appendix B: Bargaining Set, Global Optimum and Lotteries

We first establish that the bargaining set is not convex, making the problem non-standard. We then show, however, that among the set of deterministic wage contracts, we have a unique global optimum. Finally, we show that while there may be gains from having a lottery over different wage contracts, these gains are quite tiny and easily offset by small transactions costs.

Bargaining set

Let the function $J(w, \mathbf{s}) = F(H(w, \mathbf{s}))$ describe the frontier of the bargaining set, where $H(w, \mathbf{s})$ is worker surplus and $J(w, \mathbf{s})$ is firm surplus. Given that next period's wage w' equals this period's wage w with probability λ and equals next period's target wage $w^{*\prime}$ with probability $1 - \lambda$, we can write

$$H(w, \mathbf{s}) = w - b + \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \left[\rho \lambda H(w, \mathbf{s}') + \rho (1 - \lambda) H(w^{*\prime}, \mathbf{s}') - p \bar{H}_x \left(\mathbf{s}' \right) \right] |\mathbf{s} \right\} \tag{B1}$$

$$J(w, \mathbf{s}) = a - w - (\kappa/2) x^2 (w, \mathbf{s}) + [\rho + x(w, \mathbf{s})] \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \left[\lambda J \left(w, \mathbf{s}' \right) + (1 - \lambda) J(w^{*\prime}, \mathbf{s}') \right] |\mathbf{s} \right\} \tag{B2}$$

with

$$\kappa x(w, \mathbf{s}) = \beta E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \left[\lambda J \left(w, \mathbf{s}' \right) + (1 - \lambda) J(w^{*\prime}, \mathbf{s}') \right] |\mathbf{s} \right\} \tag{B3}$$

The slope of the bargaining set frontier is then given by

$$\frac{\partial J(w, \mathbf{s})}{\partial w} / \frac{\partial H(w, \mathbf{s})}{\partial w} = -\frac{\mu(w, \mathbf{s})}{\epsilon(\mathbf{s})} < 0 \tag{B4}$$

where

$$\epsilon(\mathbf{s}) = 1 + (\rho \lambda \beta) E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \epsilon \left(\mathbf{s}' \right) |\mathbf{s} \right\} > 0 \tag{B5}$$

$$\mu(w, \mathbf{s}) = 1 + [\rho + x(w, \mathbf{s})] (\lambda \beta) E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \mu \left(w, \mathbf{s}' \right) |\mathbf{s} \right\} > 0 \tag{B6}$$

Let $\mu_w(w, \mathbf{s}) \equiv \partial \mu(w, \mathbf{s}) / \partial w$ and $x_w(w, \mathbf{s}) \equiv \partial x(w, \mathbf{s}) / \partial w$. Then we can express the change in the slope as

$$-\frac{\mu_w(w, \mathbf{s})}{\epsilon^2(\mathbf{s})} > 0 \tag{B7}$$

with

$$\mu_w(w, \mathbf{s}) = x_w(w, \mathbf{s}) (\lambda \beta) E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \mu \left(w, \mathbf{s}' \right) |\mathbf{s} \right\} + [\rho + x(w, \mathbf{s})] (\lambda \beta) E \left\{ \Lambda \left(\mathbf{s}, \mathbf{s}' \right) \mu_w \left(w, \mathbf{s}' \right) |\mathbf{s} \right\} < 0 \tag{B8}$$

where

$$x_w(w,\mathbf{s}) = -(\beta\lambda/\kappa) E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\mu\left(w,\mathbf{s}'\right)|\mathbf{s}\right\} < 0 \tag{B9}$$

Since $-\mu_w(w,\mathbf{s}) > 0$, it follows that the function $J = \digamma(H)$ is strictly decreasing and strictly convex, implying that the bargaining set is non-convex. This contrasts with conventional case of period-by-period Nash bargaining where $\mu_w(w,\mathbf{s}) = 0$. In this instance the bargaining set is convex since the change in the slope of the bargaining set is zero. The negative impact of w on μ arises in our case ultimately due to the "horizon effect" on bargaining in the multi-period contract case that we described in the text. In particular, unlike the worker, the firm cares about the implications of the wage contract for new hires that come in subsequent periods. This means that how the firm discounts the future depends on the hiring rate $x(w,\mathbf{s})$, which in turn depends on w. An increase in w thus not only decreases the firm's flow profits per worker, it also reduces the hiring rate, implying lower expected profits in the future (since new hires coming in on this contract receive the higher wage.) The decline in $x(w,\mathbf{s})$ in turn reduces $\mu(w,\mathbf{s})$. This effect, which is not present in the standard case, is responsible for the non-convexity of the bargaining set in the multi-period case.

As we show below, however, the departure from convexity in quantitatively small. This should not be surprising because as we showed in the text, the horizon effect which underlies the non-convexity is trivial quantitatively under our baseline calibration. Further, as we show below, not only is the departure from convexity small, the bargaining frontier is well-behaved: It is strictly decreasing and strictly convex, there are no kinks, that is, the bargaining set is smoothly non-convex. Thus, as we show next, excluding lotteries, our Nash bargaining solution yields a unique global optimum.

Global optimum

We begin by establishing that the first order condition defines an optimum when the economy is within a local region of the steady state and then show that the optimum is unique.

We may express the first order condition for Nash bargaining as requiring that the slope of the bargaining set frontier equals the slope of the Nash product level set:

$$-\frac{\mu(w,\mathbf{s})}{\epsilon(\mathbf{s})} = -\frac{\eta}{1-\eta}\frac{J(w,\mathbf{s})}{H(w,\mathbf{s})} \tag{B10}$$

Similarly, the second order condition for a maximum requires that the change in the slope of the bargaining set frontier is smaller than the change in the slope of the Nash product level sets:

$$-\frac{\mu_w(w,\mathbf{s})}{\epsilon^2(\mathbf{s})} < \frac{\eta}{1-\eta}\frac{1}{H(w,\mathbf{s})}\left[\frac{\mu(w,\mathbf{s})}{\epsilon(\mathbf{s})} + \frac{J(w,\mathbf{s})}{H(w,\mathbf{s})}\right] \tag{B11}$$

As we noted earlier, in the standard case of period-by-period Nash bargaining, $\mu_w(w,\mathbf{s}) = 0$. This implies that the second order condition holds globally in this instance since the right side of the above expression is always positive. That the second order condition holds globally implies that the first order condition defines a unique optimum.

Because $\mu_w(w, \mathbf{s}) > 0$ in our case, we cannot establish analytically that the second order condition always holds. However, we show numerically that under our baseline calibration the second order condition is easily satisfied globally, making the optimum unique.

We begin by establishing that in the steady state the first order necessary condition for our bargaining problem defines a unique optimum. Combining equations (B10) and (B11) implies that for the order condition to define a local optimum, the following condition must hold:

$$-\eta \frac{\mu_w(w, \mathbf{s}) J(w, \mathbf{s})}{\mu^2(w, \mathbf{s})} < 1 \tag{B12}$$

Under our baseline calibration this condition is easily satisfied in the steady state:

$$-\left.\frac{\eta\mu_w(w, \mathbf{s}) J(w, \mathbf{s})}{\mu^2(w, \mathbf{s})}\right|_{ss} = \frac{\eta(1-\rho)\beta\lambda^2}{1-\beta\lambda} = 0.1208 < 1 \tag{B13}$$

Thus at the steady state the first order condition defines an optimum. We also know from the text that the steady state equilibrium is unique. Thus in the steady state, the first order condition for Nash bargaining defines a unique global optimum.

We next show that we have a unique global maximum for the bargaining problem, even if we do not impose that the individual firm and its workers are in steady state. That is, we ask whether an individual firm and its workers might have the incentive to deviate from the steady state Nash bargaining solution, assuming all other firms and workers remain at the steady state. We show that the answer is no.

In particular, suppose that a firm and its workers are contemplating for one contracting spell a wage that is different from the steady state wage. Is there an alternative wage within the bargaining set that satisfies the first order conditions which they may prefer? By constructing the relevant bargaining set, we can show numerically that no other solution exists.

Let w denote a wage within the bargaining set $[\underline{w}, \overline{w}]$ that two bargaining parties are contemplating and, as in the text, let $\tilde{H}$ and $\tilde{J}$ denote steady state worker and firm surplus, respectively. Then in this case, the worker and firm surplus are given by

$$H(w) = w - b + \beta\left[\rho\lambda H(w) + \rho(1-\lambda)\tilde{H} - \tilde{p}\tilde{H}\right] \tag{B14}$$

$$J(w) = \tilde{a} - w - (\kappa/2)x^2(w) + [\rho + x(w)]\beta\left[\lambda J(w) + (1-\lambda)\tilde{J}\right] \tag{B15}$$

with

$$\kappa x(w) = \beta\left[\lambda J(w) + (1-\lambda)\tilde{J}\right] \tag{B16}$$

In addition, the boundaries of the bargaining set are defined by

$$H(\underline{w}) = 0 \tag{B17}$$

$$J(\overline{w}) = 0 \tag{B18}$$

We can next solve the system given by (B14) to (B18) to first construct the bargaining set and then show that the second order condition given by equation (B11) holds globally for this case. Figure 4 portrays the numerical solution for the bargaining set along with the Nash product level sets. There is a unique optimum where the slopes of the two sets are equal. At any other point, the change in the slope of the bargaining set is smaller than that for the Nash product level set. Hence, the first order condition defines a unique global optimum. It is also instructive to observe that even though the bargaining set is not convex, the departure from convexity is both smooth and quantitatively small. This is what ensures that the optimum is unique.

What we have showed thus is that, given that the rest of the economy is in the unique steady state, any individual firm and its workers have no incentive to deviate for one contracting spell from the steady state solution. For simplicity we have assumed that the firm and its workers revert to the steady state solution once the contract has elapsed. All we need to assume, however, is that the two parties expect to be back at the unique steady state at some point in the future. We can use a recursive argument to establish uniqueness, along the lines we have just presented. In particular, we consider that point in the future which is the last contract before the firm and its workers revert to steady state. We then show no alternative is preferable to the steady state, using the exact same argument we have just employed. We can then work all the way back to the present, showing that the firm and its workers have no incentive to deviate today.

Thus we have proved that there is a unique global optimum for the individual firm when the rest of the economy is in the steady state. It is straightforward to verify numerically that the uniqueness result holds when the aggregate economy exhibits small percentage fluctuations around the steady state of the type we analyzed in the text.

Lotteries

We have shown that despite the non-convex bargaining set, the bargaining problem we posed has a unique global optimum over the space of deterministic wage contracts. However, the non-convex bargaining set opens up the possibility that a lottery contract might dominate a deterministic one. Here we show that there are indeed gains from a lottery contract: But under reasonable conditions, these gains are negligible and easily offset by small transactions costs of running the lottery.

We proceed as we did in the previous section and consider the problem of a firm and its workers that are contemplating a deviation from the steady state equilibrium. Whereas before we restricted attention to alternative deterministic wage contracts, we now permit the deviating parties to consider lotteries. The bargaining set for the lottery case is obtained by randomizing over a feasible set of contract wages that lie in the bargaining set for the deterministic case portrayed in Figure 4. By construction, of course, the randomized bargaining set is convex, meaning that this problem satisfies the Nash axioms.

In general lottery contracts are difficult to characterize. However, because the bargaining set for the deterministic case is smoothly non-convex, the problem simplifies. It should be clear from Figure 4 that for any feasible set of contract wages $[\underline{w}^r, \overline{w}^r]$, the randomized bargaining set is largest for a lottery over the two endpoints of the deterministic bargaining set frontier. In particular, each of the possible wage outcomes defines a pair consisting of the surplus outcomes for the firm and worker. For the low wage outcome $\underline{w}^r$, the pair is $(H(\underline{w}^r), J(\underline{w}^r))$ and for the high wage outcome $\overline{w}^r$, it is $(H(\overline{w}^r), J(\overline{w}^r))$, with $H(\underline{w}^r) < H(\overline{w}^r)$ and $J(\underline{w}^r) > J(\overline{w}^r)$. The relevant randomized bargaining set frontier is thus the set of pairs of possible convex combinations of worker and firm surpluses evaluated at the low and the high wage outcome, obtained from varying the probability of the high wage outcome π from zero to unity.

Figure 5 illustrates the randomized bargaining set for the lottery case and shows how the bargaining outcome is improved. The optimum for the lottery case, point L, is at higher Nash product level set than is point D, the outcome for the deterministic case. Given $\underline{w}^r$ and $\overline{w}^r$, point L defines an optimal value of the lottery probability π. It should also be clear from Figure 5 that it is optimal to set the lottery endpoints $\underline{w}^r$ and $\overline{w}^r$ as far apart as is feasible. Doing so maximizes the size of the randomized bargaining set.

We now proceed to explicitly solve the bargaining problem and then evaluate the gains quantitatively. In doing so, we take account of two practical considerations in running the lottery. The first involves the feasible values for $\underline{w}^r$ and $\overline{w}^r$. Absent any uncertainty, the size of the randomized bargaining set is maximized by having the lottery endpoints correspond to the endpoints of the deterministic bargaining set frontier. That is, in this instance it is optimal to set $\underline{w}^r$ equal to the worker reservation wage $\underline{w}$, defined by equation (B17), and to set $\overline{w}^r$ equal to the firm reservation value, defined by equation (B18). This is the case that Figure 5 portrays.

With aggregate uncertainty, however, the reservation wages fluctuate around their steady state values, as the analysis in section 5 makes clear. What this means is that the lottery wages must lie somewhere in the interior of the steady state bargaining set. Otherwise, there is the danger that over the life of the contract the wage could move outside the bargaining set, leading to a costly breakdown of the employment relation. Indeed, Figures 3 and 4 suggest that for the contract wage to stay in the bargaining set, the lottery endpoints must narrow considerably relative to the steady state worker and firm reservation wages.

To get an explicit sense of how this consideration affects the gains from a lottery, we restrict the lottery endpoints to be of the following form:

$$\underline{w}^r = \underline{w}(1+\underline{r}) \tag{B19}$$

$$\overline{w}^r = \overline{w}(1-\overline{r}) \tag{B20}$$

where $\underline{r}$ and $\overline{r}$ are adjustment factors that move the lottery endpoints inside the bargaining set, with $\underline{r}$, $\overline{r} > 0$. In the limiting case of $\underline{r} = \overline{r} = 0$, the lottery endpoints are exactly equal to the endpoints of the bargaining set. As we describe below, we choose $\underline{r}$ and $\overline{r}$ based on a quantitative evaluation of the fluctuations in the reservations wages induced by aggregate uncertainty. To be clear, while we use the stochastic model to calibrate the restrictions on the lottery, for expositional purposes we analyze the impact in the deterministic model. Since the aggregate uncertainty is only first order, the deterministic model should provide a good sense of the net gains from the lottery and the implications of the restrictions on the lottery endpoints.

The second consideration we introduce is to allow for costs of running the lottery. As a practical matter, one could imagine costs of ensuring that the lottery outcome is observed by a third party that could enforce the outcome. One could well imagine that doing so requires payment of costs up from legal fees and other kinds of costs. Accordingly, we assume that there is a fixed cost for each party that is amortized as a fixed cost per period over the life of the contract. Let c_H and c_J be the fixed per period costs for the worker and firm, respectively. Note that there will be an indirect effects of the transactions costs as well a direct effect. The indirect effect materializes due to the impact of the costs on firm hiring and on the firm and worker reservation wages. In what follows, we ask how large these costs have to be as a fraction of output per worker to eliminate the lottery gains.

The bargaining problem under lotteries is to choose a probability of the high wage outcome, π, to maximize the Nash product, given by

$$[EH]^{\eta}\,[EJ]^{1-\eta} \tag{B21}$$

subject to

$$\begin{aligned} EH &= (1-\pi)\,H\left(\underline{w}^r\right) + \pi H\left(\overline{w}^r\right) && \text{(B22)}\\ EJ &= (1-\pi)\,J\left(\underline{w}^r\right) + \pi J\left(\overline{w}^r\right) && \text{(B23)} \end{aligned}$$

$$\begin{aligned} H\left(\overline{w}^r\right) &= \overline{w}^r - b - c_H + \beta[\rho\lambda H(\overline{w}^r) + \rho\,(1-\lambda)\,\tilde{H} - \tilde{p}\tilde{H}] && \text{(B24)}\\ H\left(\underline{w}^r\right) &= \underline{w}^r - b - c_H + \beta[\rho\lambda H(\underline{w}^r) + \rho\,(1-\lambda)\,\tilde{H} - \tilde{p}\tilde{H}] && \text{(B25)} \end{aligned}$$

$$\begin{aligned} J\left(\overline{w}^r\right) &= \tilde{a} - \overline{w}^r - c_J - (\kappa/2)\,x^2\left(\overline{w}^r\right) + [\rho + x(\overline{w}^r)]\,\beta\left[\lambda J(\overline{w}^r) + (1-\lambda)\tilde{J}\right] && \text{(B26)}\\ J\left(\underline{w}^r\right) &= \tilde{a} - \underline{w}^r - c_J - (\kappa/2)\,x^2\left(\underline{w}^r\right) + [\rho + x(\underline{w}^r)]\,\beta\left[\lambda J(\underline{w}^r) + (1-\lambda)\tilde{J}\right] && \text{(B27)} \end{aligned}$$

with

$$\begin{aligned} \kappa x(\overline{w}^r) &= \beta\left[\lambda J(\overline{w}^r) + (1-\lambda)\tilde{J}\right] && \text{(B28)}\\ \kappa x(\underline{w}^r) &= \beta\left[\lambda J(\underline{w}^r) + (1-\lambda)\tilde{J}\right] && \text{(B29)} \end{aligned}$$

and subject to the equations for the lottery endpoints, given by (B19) and (B20).

The optimal lottery probability is then given by

$$\pi = \frac{\eta \left[H\left(\overline{w}^r\right) - H\left(\underline{w}^r\right)\right] J\left(\underline{w}^r\right) - (1-\eta)\left[J\left(\underline{w}^r\right) - J\left(\overline{w}^r\right)\right] H\left(\underline{w}^r\right)}{\left[H\left(\overline{w}^r\right) - H\left(\underline{w}^r\right)\right]\left[J\left(\underline{w}^r\right) - J\left(\overline{w}^r\right)\right]} \tag{B30}$$

Observe that in the benchmark case where the $\underline{r}$ and $\overline{r}$ adjustment factors are zero: $H\left(\underline{w}^r\right) = H\left(\underline{w}\right) = 0$ and $J\left(\overline{w}^r\right) = J\left(\overline{w}\right) = 0$, implying that (B30) simplifies to

$$\pi = \eta \tag{B31}$$

To calibrate $\underline{r}$ and $\overline{r}$, we simulate the model with aggregate uncertainty to assess how far within the steady state bargaining set the lottery endpoints must lie to ensure the wage does not move outside the bargaining set over the period of the contract. Since with Calvo structure contracts vary in length ex post, we consider a range of plausible outcomes. We start with contracts that last 3 quarters, the mean outcome. We then simulate the model for a long time series and ask what is the minimum value of $\underline{w}^r$ and the maximum value of $\overline{w}^r$ that ensures the wage stays in the bargaining set. These values in conjunction with the steady state endpoints of the bargaining set, $\underline{w}$ and $\overline{w}$, then imply values of $\underline{r}$ and $\overline{r}$, as equations (B19) and (B20) suggest. In this instance $\underline{r} = 0.14$ and $\overline{r} = 0.06$, implying that the low wage lottery outcome must lie fourteen percent above the worker's reservation value while the high wage outcome must lie six percent below the firm's reservation value. We repeat this exercise for contracts that ex post last 4, 5 and 6 quarters.

Table A shows the gains in surplus from the lottery under different assumptions about the restrictions on the lottery endpoints. It also reports the lottery cost, as a percent of output per worker, that eliminates the gains in terms of Nash product. We assume that this cost is equally divided between the worker and the firm, though the results are not sensitive to alternative ways of dividing it.

Table A: Gains from lotteries

Lottery endpoint restriction	Percent gain in Nash product, NP	Percent gain in worker surplus, H	Percent gain in firm surplus, J	Lottery cost eliminating gains, percent of output
None	7.10	6.04	8.18	1.33
3Q: $\underline{r} = 0.14$, $\overline{r} = 0.06$	1.33	3.23	-0.53	0.26
4Q: $\underline{r} = 0.16$, $\overline{r} = 0.07$	0.85	3.00	-1.25	0.16
5Q: $\underline{r} = 0.18$, $\overline{r} = 0.08$	0.45	2.83	-1.87	0.08
6Q: $\underline{r} = 0.20$, $\overline{r} = 0.09$	0.14	2.72	-2.38	0.01

As the table shows, in the baseline case where the lottery payoffs correspond to the steady state worker and firm reservation wages, the lottery generates a 7.10 percent gain in surplus relative to the deterministic case, with a 6.04 percent increase for the worker and an 8.18 percent increase for the firm. This expected gain, however, is erased if the associated transactions cost equals 1.33 percent of output per worker. If we restrict the lottery payoffs to the bounds for the bargaining set implied by the stochastic case for a contract lasting three quarters, then the gain in surplus drops to 1.13 percent. Further, the firm actually loses. The lottery cost required to eliminate the gains from lotteries falls to 0.26 percent of output per worker. The gains decline steadily as we consider restrictions based on longer contracts ex post. For the case of six quarters, the gain in surplus is only 0.14 percent and the transactions costs required to offset it are just 0.01 percent of output per worker. Keep in mind that ex ante, there is an 18 percent chance the contract will last at least six quarters, meaning that the firm will likely want to consider the restrictions on the lottery endpoints for contracts of a much longer duration. Thus, the constraints on the lottery endpoints implied by the six quarter constraint likely understate the constraints on the lottery endpoints that the firm would have to impose in the stochastic case.

More generally, while the lottery has appeal a priori due to the non-convex bargaining set, the spread in the lottery payoffs is restricted by the need to stay in the bargaining set. We have presented a conservative estimate of how much the lottery payoffs must be shrunk to stay within the bargaining set. We have found that even under these conservative estimates, the gains from the lottery are virtually nil.

Appendix C: Solving for the Contract Wage

Staggered Nash bargaining

- Consider a renegotiating firm and its workers. Given that next period's wage w' equals this period wage with probability λ and next period's target wage $w^{*\prime}$ with probability $1-\lambda$, we can write the worker surplus $H(w^*, \mathbf{s})$ and the firm surplus $J(w^*, \mathbf{s})$ as

$$H(w^*, \mathbf{s}) = w^* - b + \beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\left[\rho\lambda H\left(w^*, \mathbf{s}'\right) + \rho(1-\lambda) H\left(w^{*\prime}, \mathbf{s}'\right) - p\bar{H}_x\left(\mathbf{s}'\right)\right] |\mathbf{s}\right\} \quad \text{(C1)}$$

$$J(w^*, \mathbf{s}) = a - w^* - \frac{\kappa}{2} x(w^*, \mathbf{s})^2 + \left[\rho + x(w^*, \mathbf{s})\right] \beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\left[\lambda J\left(w^*, \mathbf{s}'\right) + (1-\lambda) J\left(w^{*\prime}, \mathbf{s}'\right)\right] |\mathbf{s}\right\} \quad \text{(C2)}$$

where

$$\kappa x(w^*, \mathbf{s}) = \beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\left[\lambda J\left(w^*, \mathbf{s}'\right) + (1-\lambda) J(w^{*\prime}, \mathbf{s}')\right] |\mathbf{s}\right\} \quad \text{(C3)}$$

- The first order condition for Nash bargaining is

$$\chi(w^*, \mathbf{s}) J(w^*, \mathbf{s}) = \left[1 - \chi(w^*, \mathbf{s})\right] H(w^*, \mathbf{s}) \quad \text{(C4)}$$

with

$$\chi(w^*, \mathbf{s}) = \frac{\eta}{\eta + (1-\eta)\mu(w^*, \mathbf{s}) / \epsilon(\mathbf{s})} \quad \text{(C5)}$$

where

$$\epsilon(\mathbf{s}) = 1 + \rho\beta\lambda E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\epsilon\left(\mathbf{s}'\right) |\mathbf{s}\right\} \quad \text{(C6)}$$

$$\mu(w^*, \mathbf{s}) = 1 + \left[\rho + x(w^*, \mathbf{s})\right] \beta\lambda E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\mu\left(w^*, \mathbf{s}'\right) |\mathbf{s}\right\} \quad \text{(C7)}$$

Contract wage

Worker surplus

- Write the worker surplus as

$$\begin{aligned} H(w^*, \mathbf{s}) &= w^* - \left[b + \beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right) p\bar{H}_x\left(\mathbf{s}'\right) |\mathbf{s}\right\}\right] + \rho\beta E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right) H\left(w^{*\prime}, \mathbf{s}'\right) |\mathbf{s}\right\} \\ &\quad + \rho\beta\lambda E\left\{\Lambda\left(\mathbf{s}, \mathbf{s}'\right)\left[H\left(w^*, \mathbf{s}'\right) - H\left(w^{*\prime}, \mathbf{s}'\right)\right] |\mathbf{s}\right\} \end{aligned}$$

- Write the term $E\{H(w^*, \mathbf{s}') - H(w^{*\prime}, \mathbf{s}') |\mathbf{s}\}$ in the second row of the equation above as

$$E\left\{H\left(w^*, \mathbf{s}'\right) - H\left(w^{*\prime}, \mathbf{s}'\right) |\mathbf{s}\right\} = E\left\{\left(w^* - w^{*\prime}\right) |\mathbf{s}\right\} + \rho\beta\lambda E\left\{\Lambda\left(\mathbf{s}', \mathbf{s}''\right)\left[H\left(w^*, \mathbf{s}''\right) - H\left(w^{*\prime}, \mathbf{s}''\right)\right] |\mathbf{s}\right\}$$

- Loglinearizing and iterating forward

$$E\left[\widehat{H}\left(w^{*},\mathbf{s}'\right)-\widehat{H}\left(w^{*\prime},\mathbf{s}'\right)\right]=\left(\tilde{w}/\tilde{H}\right)\tilde{\epsilon}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)$$

where we simplify the expectation notation by dropping the conditioning to the state $\mathbf{s}$.

- Loglinearizing the worker surplus and substituting the expression just found gives

$$\begin{aligned}\widehat{H}\left(w^{*},\mathbf{s}\right) &= \left(\tilde{w}/\tilde{H}\right)\left[\widehat{w}^{*}+\left(\rho\beta\lambda\right)\tilde{\epsilon}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\right]-\beta\tilde{p}E\left[\widehat{p}+\widehat{\widetilde{H}}_{x}\left(\mathbf{s}'\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right] \\ &\quad +\rho\beta E\left[\widehat{H}\left(w^{*\prime},\mathbf{s}'\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right]\end{aligned}$$

Firm surplus

- Combining (C2) and (C3) we obtain

$$J\left(w^{*},\mathbf{s}\right)=a-w^{*}+\frac{\kappa}{2}x\left(w^{*},\mathbf{s}\right)^{2}+\rho\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\left[\lambda J\left(w^{*},\mathbf{s}'\right)+\left(1-\lambda\right)J\left(w^{*\prime},\mathbf{s}'\right)\right]|\mathbf{s}\right\}$$

- Write the firm surplus as

$$\begin{aligned}J\left(w^{*},\mathbf{s}\right) &= a-w^{*}+\frac{\kappa}{2}x\left(w^{*},\mathbf{s}\right)^{2}+\rho\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)J\left(w^{*\prime},\mathbf{s}'\right)|\mathbf{s}\right\} \\ &\quad +\rho\beta\lambda E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\left[J\left(w^{*},\mathbf{s}'\right)-J\left(w^{*\prime},\mathbf{s}'\right)\right]|\mathbf{s}\right\}\end{aligned}$$

with

$$\kappa x\left(w^{*},\mathbf{s}\right)=\beta E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)J\left(w^{*\prime},\mathbf{s}'\right)|\mathbf{s}\right\}+\beta\lambda E\left\{\Lambda\left(\mathbf{s},\mathbf{s}'\right)\left[J\left(w^{*},\mathbf{s}'\right)-J\left(w^{*\prime},\mathbf{s}'\right)\right]|\mathbf{s}\right\}$$

- Write the term $E\left\{J\left(w^{*},\mathbf{s}'\right)-J\left(w^{*\prime},\mathbf{s}'\right)|\mathbf{s}\right\}$ in the expression for $J\left(w^{*},\mathbf{s}\right)$ as

$$\begin{aligned}E\left\{J\left(w^{*},\mathbf{s}'\right)-J\left(w^{*\prime},\mathbf{s}'\right)|\mathbf{s}\right\} &= -E\left\{\left(w^{*}-w^{*\prime}\right)|\mathbf{s}\right\}+\frac{\kappa}{2}E\left\{\left[x\left(w^{*},\mathbf{s}'\right)^{2}-x\left(w^{*\prime},\mathbf{s}'\right)^{2}\right]|\mathbf{s}\right\} \\ &\quad +\rho\beta\lambda E\left\{\Lambda\left(\mathbf{s}',\mathbf{s}''\right)\left[J\left(w^{*},\mathbf{s}''\right)-J\left(w^{*\prime},\mathbf{s}''\right)\right]|\mathbf{s}\right\}\end{aligned}$$

with

$$\kappa E\left\{x\left(w^{*},\mathbf{s}'\right)-x\left(w^{*\prime},\mathbf{s}'\right)|\mathbf{s}\right\}=\beta\lambda E\left\{\Lambda\left(\mathbf{s}',\mathbf{s}''\right)\left[J\left(w^{*},\mathbf{s}''\right)-J\left(w^{*\prime},\mathbf{s}''\right)\right]|\mathbf{s}\right\}$$

- Loglinearizing

$$\begin{aligned}E\left[\widehat{J}\left(w^{*},\mathbf{s}'\right)-\widehat{J}\left(w^{*\prime},\mathbf{s}'\right)\right] &= -\left(\tilde{w}/\tilde{J}\right)E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)+\left(\kappa\tilde{x}^{2}/\tilde{J}\right)E\left[\widehat{x}\left(w^{*},\mathbf{s}'\right)-\widehat{x}\left(w^{*\prime},\mathbf{s}'\right)\right] \\ &\quad +\rho\beta\lambda E\left[\widehat{J}\left(w^{*},\mathbf{s}''\right)-\widehat{J}\left(w^{*\prime},\mathbf{s}''\right)\right]\end{aligned}$$

with

$$E\left[\widehat{x}\left(w^{*},\mathbf{s}'\right)-\widehat{x}\left(w^{*\prime},\mathbf{s}'\right)\right]=\lambda E\left[\widehat{J}\left(w^{*},\mathbf{s}''\right)-\widehat{J}\left(w^{*\prime},\mathbf{s}''\right)\right]$$

- Substituting and interating forward

$$E\left[\widehat{J}\left(w^{*},\mathbf{s}'\right)-\widehat{J}\left(w^{*\prime},\mathbf{s}'\right)\right]=-\left(\tilde{w}/\tilde{J}\right)\tilde{\mu}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)$$

and

$$E\left[\widehat{x}\left(w^{*},\mathbf{s}'\right)-\widehat{x}\left(w^{*\prime},\mathbf{s}'\right)\right]=-\left(\tilde{w}/\tilde{J}\right)\lambda\tilde{\mu}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)$$

- Loglinearizing the firm surplus and substituting

$$\begin{aligned}\widehat{J}\left(w^{*},\mathbf{s}\right) &= \left(\tilde{a}/\tilde{J}\right)\widehat{a}-\left(\tilde{w}/\tilde{J}\right)\left[\widehat{w}^{*}+\left(\rho\beta\lambda\right)\tilde{\mu}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\right]+\left(\kappa\tilde{x}^{2}/\tilde{J}\right)\widehat{x}\left(w^{*},\mathbf{s}\right)\\ &\quad+\rho\beta E\left[\widehat{J}\left(w^{*\prime},\mathbf{s}'\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right]\end{aligned}$$

Contract wage

- The loglinear version of the Nash first order condition, equation (C4), is

$$\widehat{J}\left(w^{*},\mathbf{s}\right)+\left(1-\tilde{\chi}\right)^{-1}\widehat{\chi}\left(w^{*},\mathbf{s}\right)=\widehat{H}\left(w^{*},\mathbf{s}\right)$$

- Substituting the loglinear expressions for $\widehat{J}\left(w^{*},\mathbf{s}\right)$ and $\widehat{H}\left(w^{*},\mathbf{s}\right)$, using also the Nash foc next period, yields

$$\begin{aligned}&\left(\tilde{a}/\tilde{J}\right)\widehat{a}-\left(\tilde{w}/\tilde{J}\right)\left[\widehat{w}^{*}+\left(\rho\beta\lambda\right)\tilde{\mu}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\right]+\left(\kappa\tilde{x}^{2}/\tilde{J}\right)\widehat{x}\left(w^{*},\mathbf{s}\right)\\ =\;&\left(\tilde{w}/\tilde{H}\right)\left[\widehat{w}^{*}+\left(\rho\beta\lambda\right)\tilde{\epsilon}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\right]-\beta\tilde{p}E\left[\widehat{p}+\widehat{\tilde{H}}_{x}\left(\mathbf{s}'\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right]\\ &-\left(1-\tilde{\chi}\right)^{-1}E\left[\widehat{\chi}\left(w^{*},\mathbf{s}\right)-\rho\beta\widehat{\chi}\left(w^{*\prime},\mathbf{s}'\right)\right]\end{aligned}$$

- Rearranging and collecting terms

$$\begin{aligned}&\tilde{w}\widehat{w}^{*}+\left(\rho\beta\lambda\right)\psi\tilde{w}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\\ =\;&\tilde{\chi}\tilde{a}\widehat{a}+\tilde{\chi}\left(\kappa\tilde{x}^{2}\right)\widehat{x}\left(w^{*},\mathbf{s}\right)+\left(1-\tilde{\chi}\right)\beta\tilde{p}\tilde{H}E\left[\widehat{p}+\widehat{\tilde{H}}_{x}\left(\mathbf{s}'\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}'\right)\right]\\ &+\tilde{\chi}\left(1-\tilde{\chi}\right)^{-1}\tilde{J}E\left[\widehat{\chi}\left(w^{*},\mathbf{s}\right)-\rho\beta\widehat{\chi}\left(w^{*\prime},\mathbf{s}'\right)\right]\end{aligned}$$

where

$$\psi=\tilde{\chi}\tilde{\mu}+\left(1-\tilde{\chi}\right)\tilde{\epsilon}=\frac{\tilde{\mu}\tilde{\epsilon}}{\eta\tilde{\epsilon}+\left(1-\eta\right)\tilde{\mu}}$$

- Finally, we can write

$$\widehat{w}^* = (1-\tau)\,\widehat{w}^o\,(w^*, \mathbf{s}) + \tau E \widehat{w}^{*\prime}$$

where τ is given by

$$\tau = \frac{(\rho\beta\lambda)\,\psi}{1+(\rho\beta\lambda)\,\psi}$$

and where $\widehat{w}^o\,(w^*, \mathbf{s})$ is the target wage, given by

$$\widehat{w}^o\,(w^*, \mathbf{s}) = \tilde{\chi}\varphi_a \widehat{a} + \tilde{\chi}\varphi_x \widehat{x}\,(w^*, \mathbf{s}) + (1-\tilde{\chi})\,\varphi_p E\left[\widehat{p} + \widehat{\bar{H}}_x\left(\mathbf{s}'\right) + \widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right)\right] + \widehat{\Phi}\left(w^*, w^{*\prime}, \mathbf{s}, \mathbf{s}'\right)$$

with

$$\widehat{\Phi}\left(w^*, w^{*\prime}, \mathbf{s}, \mathbf{s}'\right) = E\left[\widehat{\chi}\,(w^*, \mathbf{s}) - \rho\beta\widehat{\chi}\left(w^{*\prime}, \mathbf{s}'\right)\right]$$

and

$$\varphi_a = \tilde{a}\tilde{w}^{-1} \qquad \varphi_x = \kappa\tilde{x}^2\tilde{w}^{-1} \qquad \varphi_p = \tilde{p}\beta\tilde{H}\tilde{w}^{-1} \qquad \varphi_\chi = \tilde{\chi}\,(1-\tilde{\chi})^{-1}\,(\kappa\tilde{x}/\beta)\,\tilde{w}^{-1}$$

Firm and worker discount factors

- The loglinear worker discount factor is

$$\widehat{\epsilon}\,(\mathbf{s}) = \rho\beta\lambda E\left[\widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right) + \widehat{\epsilon}\left(\mathbf{s}'\right)\right]$$

- We now proceed to find a loglinear recursive expression for the firm discount factor.
- Loglinearizing equation (C7) and iterating forward

$$\begin{aligned}\widehat{\mu}\,(w^*, \mathbf{s}) &= (\tilde{x}\beta\lambda)\,\widehat{x}\,(w^*, \mathbf{s}) + (\beta\lambda)\,E\left[\widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right) + \widehat{\mu}\left(w^*, \mathbf{s}'\right)\right] \\ &= (\tilde{x}\beta\lambda)\,E\left[\widehat{x}\,(w^*, \mathbf{s}) + (\beta\lambda)\,\widehat{x}\left(w^*, \mathbf{s}'\right) + (\beta\lambda)^2\,\widehat{x}\left(w^*, \mathbf{s}''\right) + ...\right] \\ &\quad + (\beta\lambda)\,E\left[\widehat{\Lambda}\left(\mathbf{s}, \mathbf{s}'\right) + (\beta\lambda)\,\widehat{\Lambda}\left(\mathbf{s}', \mathbf{s}''\right) + (\beta\lambda)^2\,\widehat{\Lambda}\left(\mathbf{s}'', \mathbf{s}'''\right) + ...\right]\end{aligned}$$

- Recall from previous section

$$E\left[\widehat{x}\left(w^*, \mathbf{s}'\right) - \widehat{x}\left(w^{*\prime}, \mathbf{s}'\right)\right] = -\varkappa_w\lambda\tilde{\mu}E\left(\widehat{w}^* - \widehat{w}^{*\prime}\right)$$

$$E\left[\widehat{x}\left(w^*, \mathbf{s}''\right) - \widehat{x}\left(w^{*\prime\prime}, \mathbf{s}''\right)\right] = -\varkappa_w\lambda\tilde{\mu}E\left(\widehat{w}^* - \widehat{w}^{*\prime\prime}\right)$$

and so on, where $\varkappa_w = \tilde{w}/\tilde{J}$.

- Substituting and rearranging

$$\begin{aligned}\widehat{\mu}\left(w^{*},\mathbf{s}\right) &= \left(\tilde{x}\beta\lambda\right)E\left[\widehat{x}\left(w^{*},\mathbf{s}\right)+\left(\beta\lambda\right)\widehat{x}\left(w^{*\prime},\mathbf{s}^{\prime}\right)+\left(\beta\lambda\right)^{2}\widehat{x}\left(w^{*\prime\prime},\mathbf{s}^{\prime\prime}\right)+...\right]\\ &-\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left(\beta\lambda\right)\tilde{\mu}\widehat{w}^{*}\\ &+\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)E\left[\left(\beta\lambda\right)\widehat{w}^{*\prime}+\left(\beta\lambda\right)^{2}\widehat{w}^{*\prime\prime}+...\right]\\ &+\left(\beta\lambda\right)E\left[\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)+\left(\beta\lambda\right)\widehat{\Lambda}\left(\mathbf{s}^{\prime},\mathbf{s}^{\prime\prime}\right)+\left(\beta\lambda\right)^{2}\widehat{\Lambda}\left(\mathbf{s}^{\prime\prime},\mathbf{s}^{\prime\prime\prime}\right)+...\right]\end{aligned}$$

- Write recursively as

$$\begin{aligned}\widehat{\mu}\left(w^{*},\mathbf{s}\right)+\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left(\beta\lambda\right)\tilde{\mu}\widehat{w}^{*} &= \left(\tilde{x}\beta\lambda\right)\widehat{x}\left(w^{*},\mathbf{s}\right)+\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left(\beta\lambda\right)E\widehat{w}^{*\prime}+\left(\beta\lambda\right)E\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)\\ &+\left(\beta\lambda\right)E\left[\widehat{\mu}\left(w^{*\prime},\mathbf{s}^{\prime}\right)+\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left(\beta\lambda\right)\tilde{\mu}\widehat{w}^{*\prime}\right]\end{aligned}$$

- Finally, rearrange as

$$\begin{aligned}\widehat{\mu}\left(w^{*},\mathbf{s}\right) &= \left(\tilde{x}\beta\lambda\right)\widehat{x}\left(w^{*},\mathbf{s}\right)-\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left(\beta\lambda\right)\tilde{\mu}E\left(\widehat{w}^{*}-\widehat{w}^{*\prime}\right)\\ &+\left(\beta\lambda\right)E\left[\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)+\widehat{\mu}\left(w^{*\prime},\mathbf{s}^{\prime}\right)\right]\end{aligned}$$

The spillover effects

- The target wage is

$$\begin{aligned}\widehat{w}^{o}\left(w^{*},\mathbf{s}\right) &= \tilde{\chi}\varphi_{a}\widehat{a}+\tilde{\chi}\varphi_{x}\widehat{x}\left(w^{*},\mathbf{s}\right)+\left(1-\tilde{\chi}\right)\varphi_{p}E\left[\widehat{p}\left(\mathbf{s}\right)+\widehat{\overline{H}}_{x}\left(\mathbf{s}^{\prime}\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)\right]\\ &+\varphi_{\chi}E\left[\widehat{\chi}\left(w^{*},\mathbf{s}\right)-\rho\beta\widehat{\chi}\left(w^{*\prime},\mathbf{s}^{\prime}\right)\right]\end{aligned}$$

- Let's find expressions for $\widehat{x}\left(w^{*},\mathbf{s}\right)$, $\widehat{\chi}\left(w^{*},\mathbf{s}\right)$, $\widehat{\chi}\left(w^{*\prime},\mathbf{s}^{\prime}\right)$ and $\widehat{\overline{H}}_{x}\left(\mathbf{s}^{\prime}\right)$ in terms of gaps between contract and average wages.
- Applying the same procedure as above

$$\widehat{x}\left(w^{*},\mathbf{s}\right)-\widehat{x}\left(\bar{w},\mathbf{s}\right)=-\varkappa_{w}\lambda\tilde{\mu}\left(\widehat{w}^{*}-\widehat{\bar{w}}\right)$$

  where $\widehat{x}\left(\bar{w},\mathbf{s}\right)$ is the average hiring rate.

- Consider the non recursive loglinear expressions for $\widehat{\mu}\left(w^{*},\mathbf{s}\right)$ and $\widehat{\mu}\left(\bar{w},\mathbf{s}\right)$

$$\widehat{\mu}\left(w^{*},\mathbf{s}\right)=\left(\tilde{x}\beta\lambda\right)\widehat{x}\left(w^{*},\mathbf{s}\right)+\left(\beta\lambda\right)E\left[\widehat{\mu}\left(w^{*},\mathbf{s}^{\prime}\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)\right]$$

$$\widehat{\mu}\left(\bar{w},\mathbf{s}\right)=\left(\tilde{x}\beta\lambda\right)\widehat{x}\left(\bar{w},\mathbf{s}\right)+\left(\beta\lambda\right)E\left[\widehat{\mu}\left(\bar{w},\mathbf{s}^{\prime}\right)+\widehat{\Lambda}\left(\mathbf{s},\mathbf{s}^{\prime}\right)\right]$$

Taking differences, substituting and iterating forward

$$\begin{aligned}\widehat{\mu}(w^*,\mathbf{s})-\widehat{\mu}(\bar{w},\mathbf{s}) &= (\tilde{x}\beta\lambda)\left[\widehat{x}(w^*,\mathbf{s})-\widehat{x}(\bar{w},\mathbf{s})\right]+(\beta\lambda)E\left[\widehat{\mu}(w^*,\mathbf{s}')-\widehat{\mu}(\bar{w},\mathbf{s}')\right]\\ &= -(\tilde{x}\beta\lambda)(\varkappa_w\lambda\tilde{\mu})\left(\widehat{w}^*-\widehat{\bar{w}}\right)+(\beta\lambda)E\left[\widehat{\mu}(w^*,\mathbf{s}')-\widehat{\mu}(\bar{w},\mathbf{s}')\right]\\ &= -(\tilde{x}\beta\lambda)(\varkappa_w\lambda\tilde{\mu})\tilde{\mu}\left(\widehat{w}^*-\widehat{\bar{w}}\right)\end{aligned}$$

- Now we have

$$\widehat{\chi}(w^*,\mathbf{s})=(1-\tilde{\chi})\left[\widehat{\epsilon}(\mathbf{s})-\widehat{\mu}(w^*,\mathbf{s})\right]$$

- Taking differences

$$\begin{aligned}\widehat{\chi}(w^*,\mathbf{s})-\widehat{\chi}(\bar{w},\mathbf{s}) &= -(1-\tilde{\chi})\left[\widehat{\mu}(w^*,\mathbf{s})-\widehat{\mu}(\bar{w},\mathbf{s})\right]\\ &= (1-\tilde{\chi})(\tilde{x}\beta\lambda)(\varkappa_w\lambda\tilde{\mu})\tilde{\mu}\left(\widehat{w}^*-\widehat{\bar{w}}\right)\end{aligned}$$

Similarly

$$E\left[\widehat{\chi}(w^{*\prime},\mathbf{s}')-\widehat{\chi}(\bar{w}',\mathbf{s}')\right]=(1-\tilde{\chi})(\tilde{x}\beta\lambda)(\varkappa_w\lambda\tilde{\mu})\tilde{\mu}E\left(\widehat{w}^{*\prime}-\widehat{\bar{w}}'\right)$$

- Using the results in previous section

$$E\left[\widehat{H}(w^{*\prime},\mathbf{s}')-\widehat{H}(\bar{w}',\mathbf{s}')\right]=(1-\tilde{\chi})\tilde{\chi}^{-1}\varkappa_w\tilde{\epsilon}E\left(\widehat{w}^{*\prime}-\widehat{\bar{w}}'\right)$$

$$E\left[\widehat{J}(w^{*\prime},\mathbf{s}')-\widehat{J}(\bar{w}',\mathbf{s}')\right]=-\varkappa_w\tilde{\mu}E\left(\widehat{w}^{*\prime}-\widehat{\bar{w}}'\right)$$

- Start from the Nash foc next period

$$E\widehat{J}(w^{*\prime},\mathbf{s}')+(1-\tilde{\chi})^{-1}E\widehat{\chi}(w^{*\prime},\mathbf{s}')=E\widehat{H}(w^{*\prime},\mathbf{s}')$$

Substitute and rearrange to obtain

$$E\widehat{J}(\bar{w}',\mathbf{s}')+(1-\tilde{\chi})^{-1}E\widehat{\chi}(\bar{w}',\mathbf{s}')=E\widehat{H}(\bar{w}',\mathbf{s}')+\gamma E\left(\widehat{w}^{*\prime}-\widehat{\bar{w}}'\right)$$

with

$$\gamma=\left[1-\eta(\tilde{x}\beta\lambda)(\lambda\tilde{\mu})\right]\tilde{\mu}\eta^{-1}\varkappa_w$$

- Using finally

$$\widehat{x}(\bar{w},\mathbf{s})=E\left[\widehat{J}(\bar{w}',\mathbf{s}')+\widehat{\Lambda}(\mathbf{s},\mathbf{s}')\right]$$

we have

$$E\left[\widehat{H}(\bar{w}',\mathbf{s}')+\widehat{\Lambda}(\mathbf{s},\mathbf{s}')\right]=\widehat{x}(\bar{w},\mathbf{s})-\gamma E\left(\widehat{w}^{*\prime}-\widehat{\bar{w}}'\right)+(1-\tilde{\chi})^{-1}E\widehat{\chi}(\bar{w}',\mathbf{s}')$$

where $\widehat{H}(\bar{w}',\mathbf{s}')=\widehat{\bar{H}}_x(\mathbf{s}')$.

- Substituting in the target wage and rearranging we obtain

$$\widehat{w}^{o}\left(w^{*},\mathbf{s}\right)=\widehat{\overline{w}}^{o}+\frac{\tau_{1}}{1-\tau}E\left(\widehat{\overline{w}}'-\widehat{w}^{*\prime}\right)+\frac{\tau_{2}}{1-\tau}\left(\widehat{\overline{w}}-\widehat{w}^{*}\right)$$

with

$$\widehat{\overline{w}}^{o}=\tilde{\chi}\varphi_{a}\widehat{a}+\left[\tilde{\chi}\varphi_{x}+\left(1-\tilde{\chi}\right)\varphi_{p}\right]\widehat{x}\left(\bar{w},\mathbf{s}\right)+\tilde{\chi}\varphi_{p}\widehat{p}\left(\mathbf{s}\right)+\widehat{\overline{\Phi}}$$

where

$$\widehat{\overline{\Phi}}=\varphi_{\chi}E\left[\widehat{\chi}\left(\bar{w},\mathbf{s}\right)-\left(\rho-\tilde{p}\right)\beta\widehat{\chi}\left(\bar{w}',\mathbf{s}'\right)\right]$$

and where

$$\tau_{1}=\left[\varphi_{\chi}\left(1-\tilde{\chi}\right)\left(\tilde{x}\beta\lambda\right)\left(\varkappa_{w}\lambda\tilde{\mu}\right)\tilde{\mu}\left(\rho\beta\right)+\left(1-\tilde{\chi}\right)\varphi_{p}\gamma\right]\left(1-\tau\right)$$
$$\tau_{2}=\left(\varkappa_{w}\lambda\tilde{\mu}\right)\left[\tilde{\chi}\varphi_{x}-\varphi_{\chi}\left(1-\tilde{\chi}\right)\left(\tilde{x}\beta\lambda\right)\tilde{\mu}\right]\left(1-\tau\right)$$

- We next present the complete loglinear model. To simplify notation we use time indexes and, for any variable z, write $\widehat{z}\left(\bar{w},\mathbf{s}\right)$ as $\widehat{\overline{z}}_{t}$.

Appendix D: Complete Loglinear Model

- Technology

$$\widehat{\widetilde{y}}_t = \widehat{z}_t + \alpha\widehat{\widetilde{k}}_t + (1-\alpha)\,\widehat{\widetilde{n}}_t \tag{D1}$$

- Resource constraint

$$\widehat{\widetilde{y}}_t = y_c\widehat{\widetilde{c}}_t + y_i\widehat{\widetilde{\imath}}_t + (1-y_c-y_i)\left(2\widehat{\widetilde{x}}_t + \widehat{\widetilde{n}}_t\right) \tag{D2}$$

where $y_c = \tilde{c}/\tilde{y}$ and $y_i = \tilde{\imath}/\tilde{y}$.

- Matching

$$\widehat{\widetilde{m}}_t = \sigma\widehat{\widetilde{u}}_t + (1-\sigma)\,\widehat{\widetilde{v}}_t \tag{D3}$$

- Employment dynamics

$$\widehat{\widetilde{n}}_{t+1} = \widehat{\widetilde{n}}_t + (1-\rho)\,\widehat{\widetilde{x}}_t \tag{D4}$$

- Transition probabilities

$$\widehat{q}_t = \widehat{\widetilde{m}}_t - \widehat{\widetilde{v}}_t \tag{D5}$$

$$\widehat{p}_t = \widehat{\widetilde{m}}_t - \widehat{\widetilde{u}}_t \tag{D5}$$

- Unemployment

$$\widehat{\widetilde{u}}_t = -\left(\tilde{n}/\tilde{u}\right)\widehat{\widetilde{n}}_t \tag{D7}$$

- Capital dynamics

$$\widehat{\widetilde{k}}_{t+1} = (1-\delta)\,\widehat{\widetilde{k}}_t + \delta\widehat{\widetilde{\imath}}_t \tag{D8}$$

- Aggregate vacancies

$$\widehat{\widetilde{x}}_t = \widehat{q}_t + \widehat{\widetilde{v}}_t - \widehat{\widetilde{n}}_t \tag{D9}$$

- Consumption and savings

$$0 = E_t\widehat{\Lambda}_{t,t+1} + (1-\beta\,(1-\delta))\,E_t\widehat{r}_{t+1} \tag{D10}$$

- Marginal utility

$$E_t\widehat{\Lambda}_{t,t+1} = \widehat{\widetilde{c}}_t - E_t\widehat{\widetilde{c}}_{t+1} \tag{D11}$$

- Aggregate hiring

$$\widehat{\widetilde{x}}_t = E_t\left\{\varkappa_a\widehat{a}_{t+1} - \varkappa_w\widehat{\widetilde{w}}_{t+1} + \widehat{\Lambda}_{t,t+1} + \beta\widehat{\widetilde{x}}_{t+1}\right\} \tag{D12}$$

where $\varkappa_a = \tilde{a}\varkappa$, $\varkappa_w = \tilde{w}\varkappa$ and $\varkappa = \beta\left(\kappa\tilde{x}\right)^{-1}$

- Marginal product of labor

$$\widehat{a}_t = \widehat{\widehat{y}}_t - \widehat{\widehat{n}}_t \tag{D13}$$

- Capital renting

$$\widehat{r}_t = \widehat{\widehat{y}}_t - \widehat{\widehat{k}}_t \tag{D14}$$

- Effective bargaining power

$$\widehat{\widehat{\chi}}_t = -(1-\tilde{\chi})\left(\widehat{\widehat{\mu}}_t - \widehat{\widehat{\epsilon}}_t\right) \tag{D15}$$

with

$$\widehat{\widehat{\epsilon}}_t = \rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\widehat{\epsilon}}_{t+1}\right) \tag{D16}$$

$$\widehat{\widehat{\mu}}_t = (\tilde{x}\lambda\beta)\,\widehat{\widehat{x}}_t - (\tilde{x}\beta\lambda)(\varkappa_w\lambda\tilde{\mu})(\beta\lambda)\,\tilde{\mu}E\left(\widehat{\widehat{w}}_t - \widehat{\widehat{w}}_{t+1}\right) + \lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\widehat{\mu}}_{t+1}\right) \tag{D17}$$

- Spillover-free target wage

$$\widehat{\widehat{w}}_t^o = \tilde{\chi}\varphi_a\widehat{a}_t + (1-\tilde{\chi})\,\varphi_p\widehat{p}_t + \left[\tilde{\chi}\varphi_x + (1-\tilde{\chi})\,\varphi_p\right]\widehat{\widehat{x}}_t + \varphi_\chi\left(\widehat{\widehat{\chi}}_t - \beta\,(\rho-p)\,E_t\widehat{\widehat{\chi}}_{t+1}\right) \tag{D18}$$

where

$$\varphi_a = \tilde{a}\tilde{w}^{-1} \quad \varphi_x = \kappa\tilde{x}^2\tilde{w}^{-1} \quad \varphi_p = \tilde{p}\beta\tilde{H}\tilde{w}^{-1} \quad \varphi_\chi = \tilde{\chi}\,(1-\tilde{\chi})^{-1}\,(\kappa\tilde{x}/\beta)\,\tilde{w}^{-1}$$

- Aggregate wages

$$\widehat{\widehat{w}}_t = \gamma_b\widehat{\widehat{w}}_{t-1} + \gamma_o\widehat{\widehat{w}}_t^o + \gamma_f E_t\widehat{\widehat{w}}_{t+1} \tag{D19}$$

where

$$\gamma_b = (1+\tau_2)\,\phi^{-1} \quad \gamma_o = \varsigma\phi^{-1} \quad \gamma_f = \left(\tau\lambda^{-1} - \tau_1\right)\phi^{-1}$$
$$\phi = (1+\tau_2) + \varsigma + \left(\tau\lambda^{-1} - \tau_1\right) \qquad \varsigma = (1-\lambda)(1-\tau)\,\lambda^{-1}$$

$$\begin{aligned}\tau_1 &= \left[\varphi_\chi\,(1-\tilde{\chi})\,(\tilde{x}\beta\lambda)\,(\varkappa_w\lambda\tilde{\mu})\,\tilde{\mu}\,(\rho\beta) + (1-\tilde{\chi})\,\varphi_p\gamma\right](1-\tau)\\ \tau_2 &= (\varkappa_w\lambda\tilde{\mu})\left[\tilde{\chi}\varphi_x - \varphi_\chi\,(1-\tilde{\chi})\,(\tilde{x}\beta\lambda)\,\tilde{\mu}\right](1-\tau)\end{aligned}$$

- Technology process

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \varepsilon_t^z \tag{D20}$$

Figure 1: Impulse responses to a technology shock

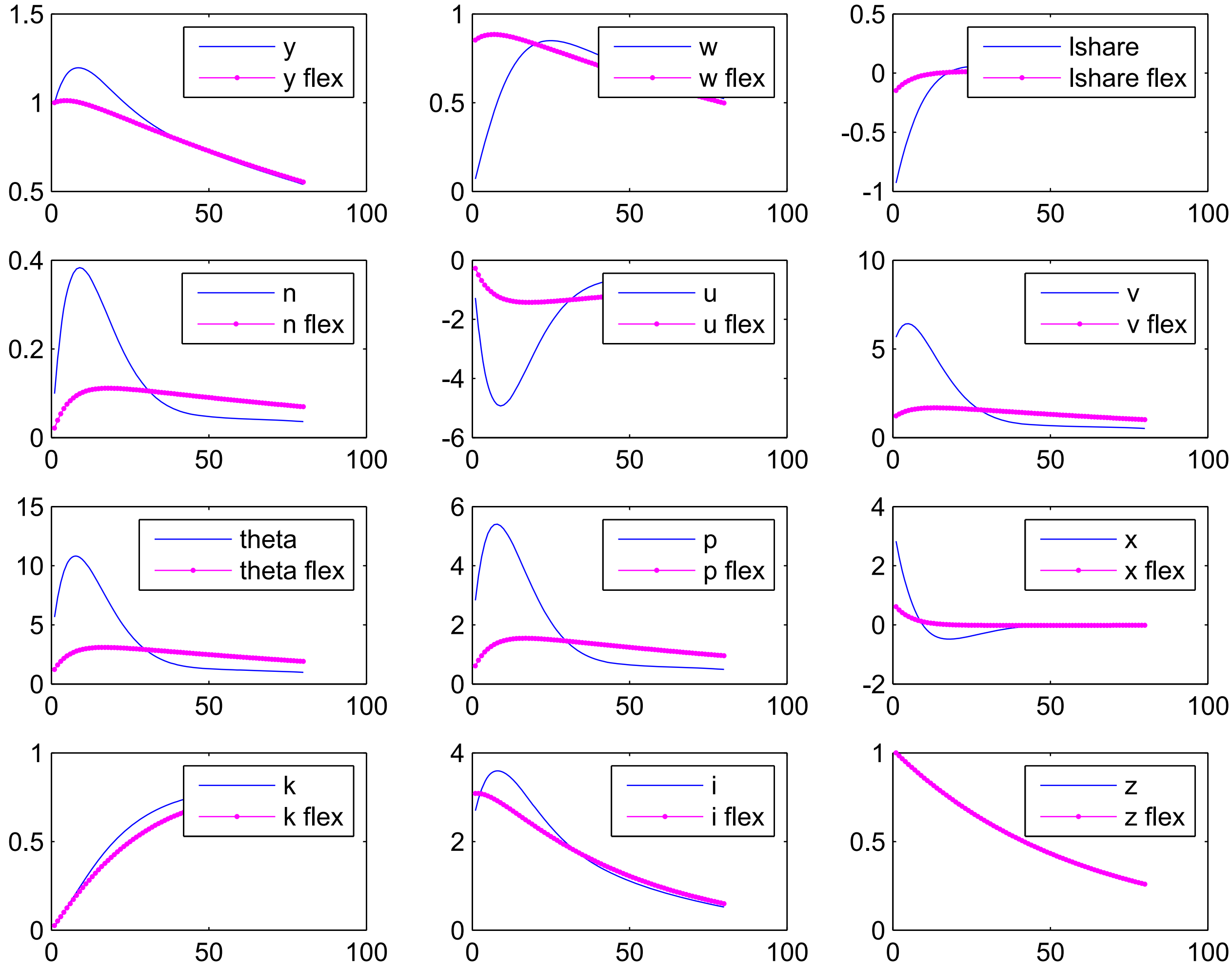


Figure 2: Simulated bargaining set versus contract wage of 9 months ex-post duration

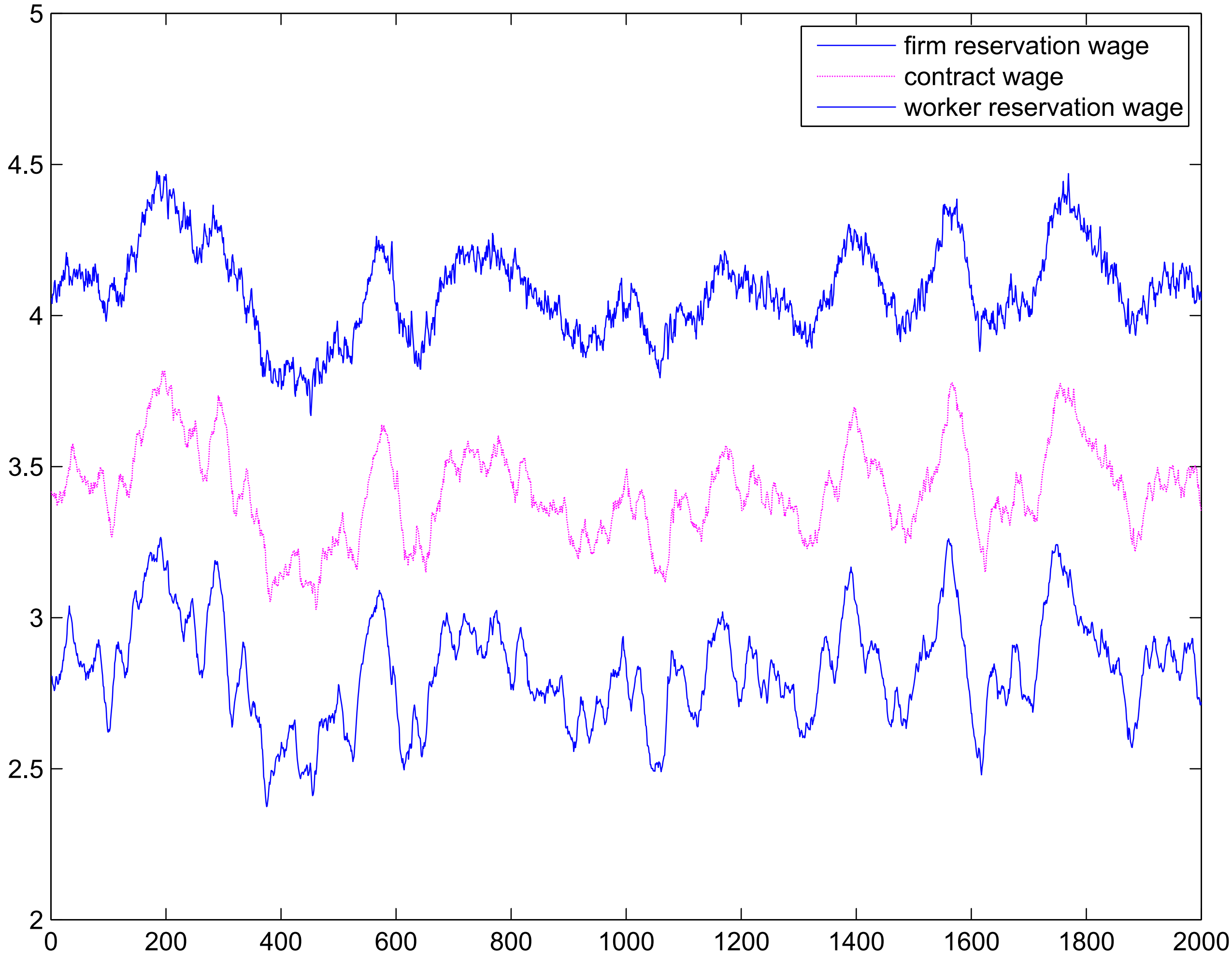
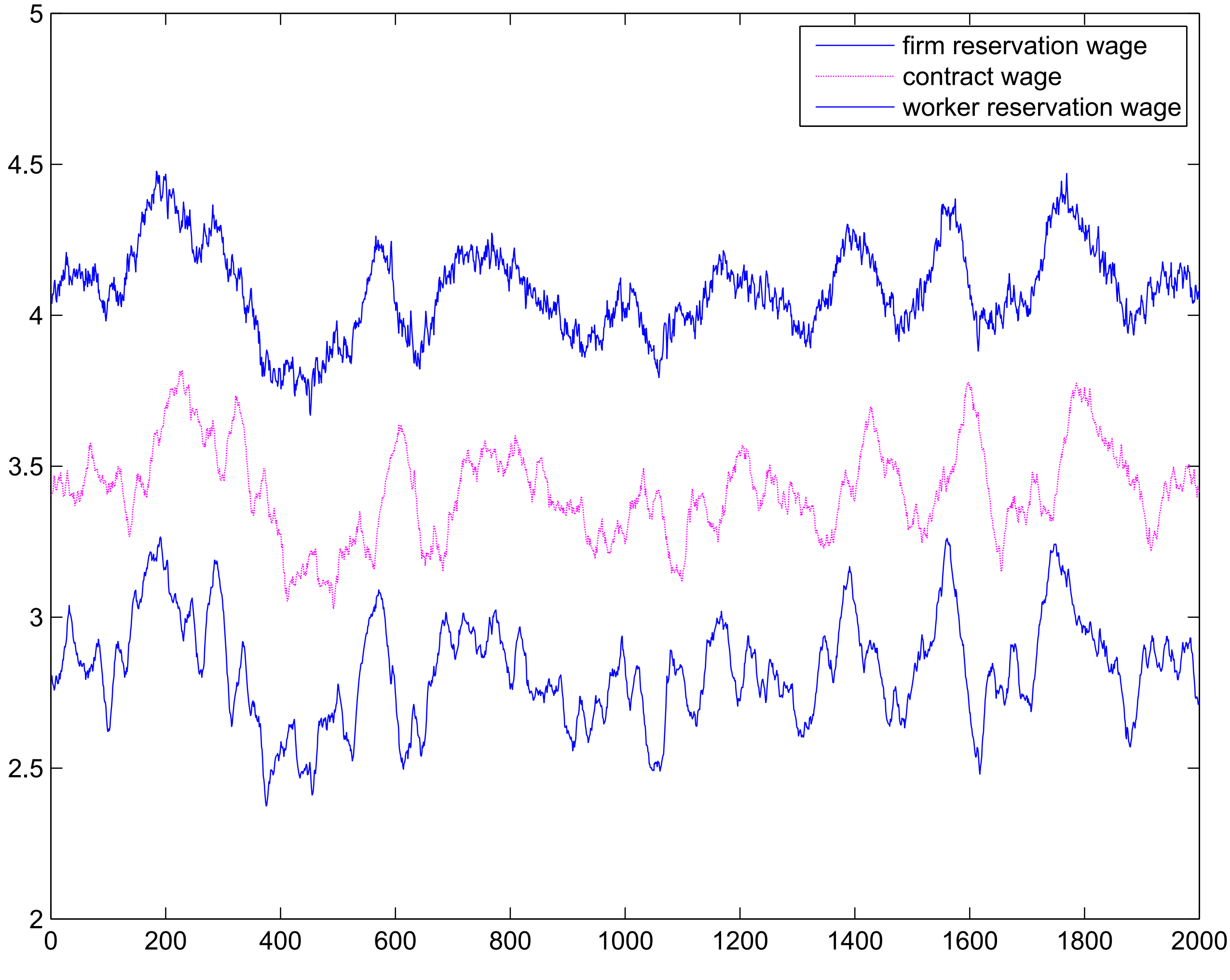


Figure 3: Simulated bargaining set versus contract wage of 40 months ex-post duration



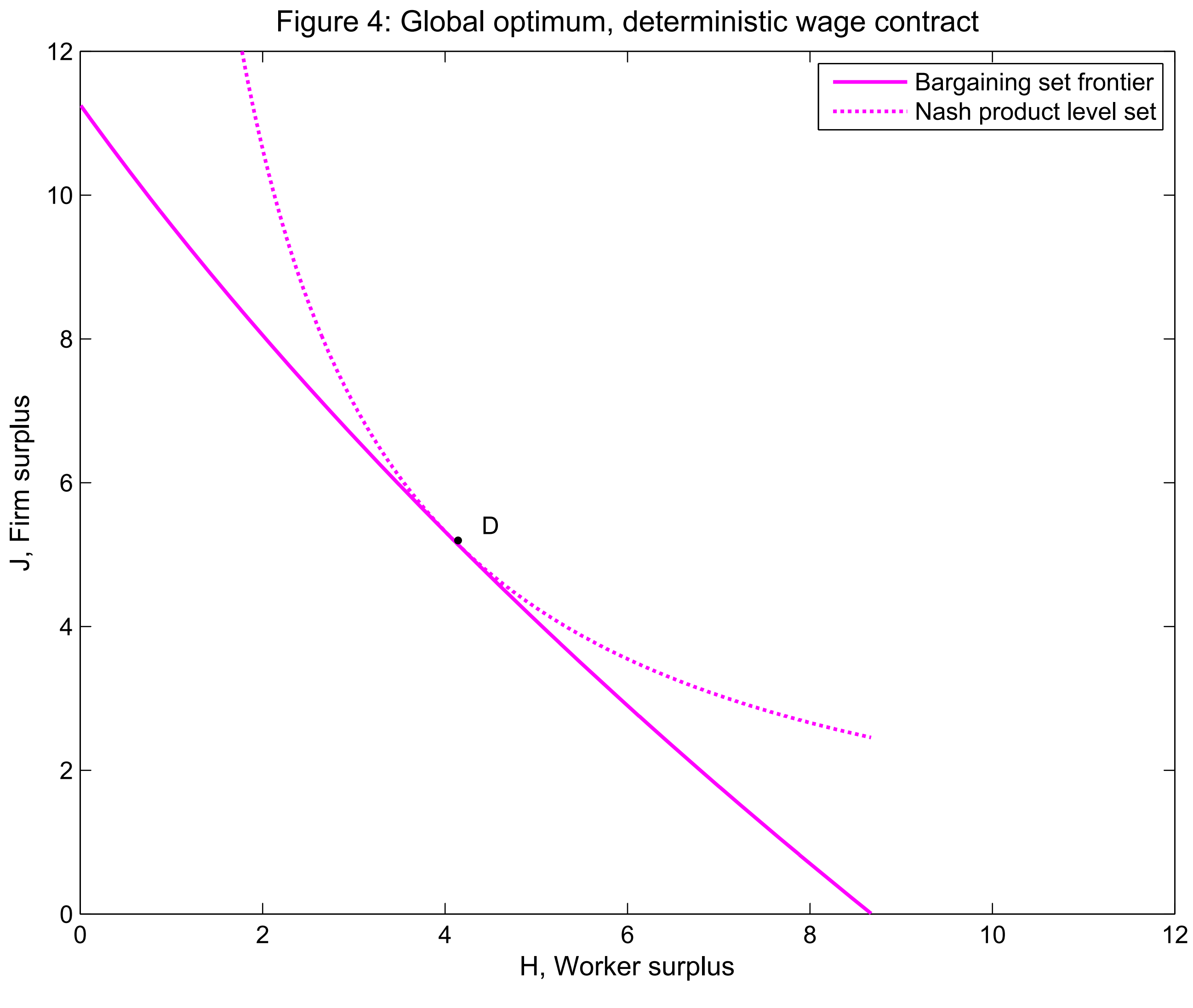


Figure 4: Global optimum, deterministic wage contract
Bargaining set frontier
Nash product level set
D
J, Firm surplus
H, Worker surplus
0
2
4
6
8
10
12

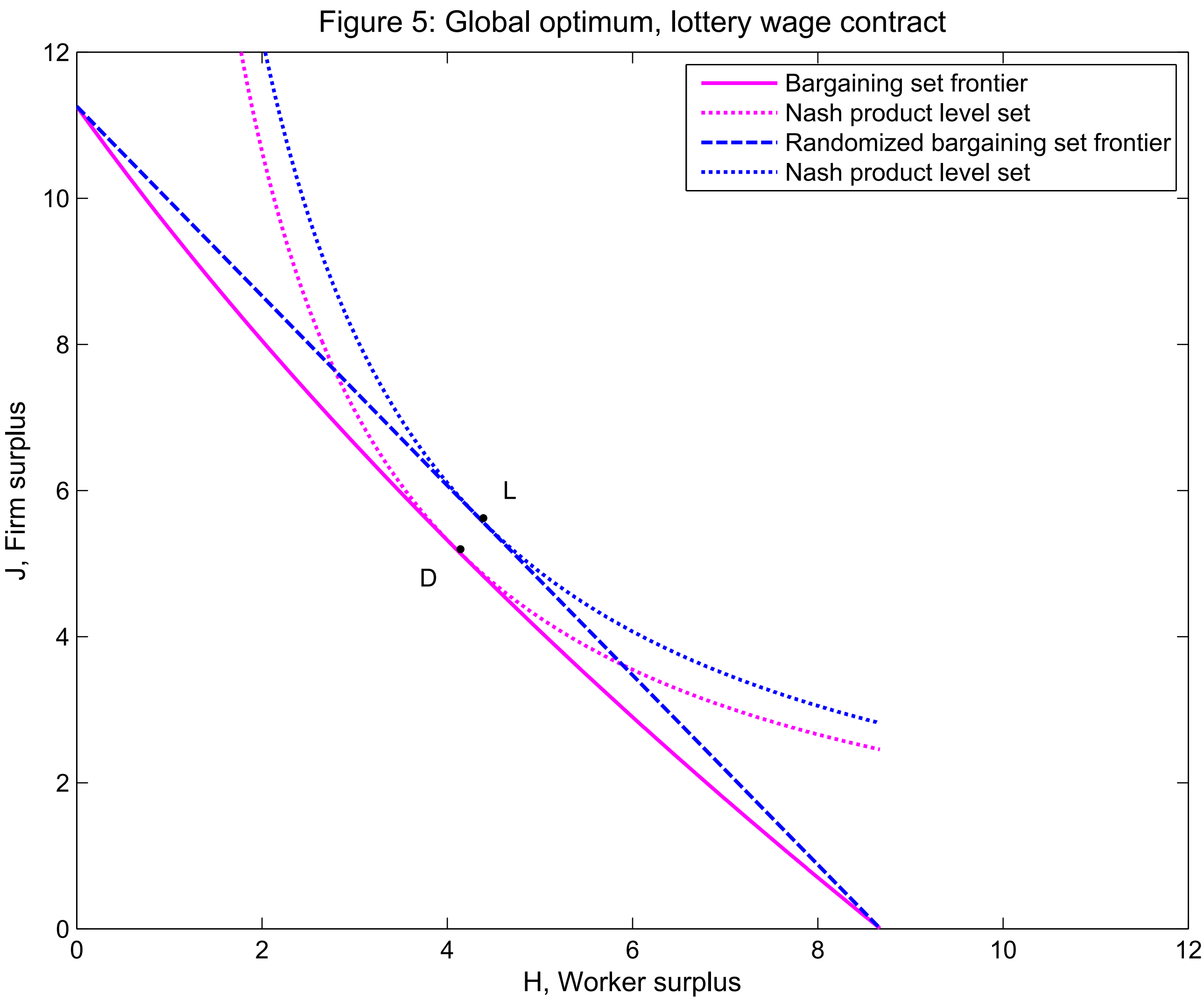


Figure 5: Global optimum, lottery wage contract
Bargaining set frontier
Nash product level set
Randomized bargaining set frontier
Nash product level set
L
D
J, Firm surplus
H, Worker surplus
0
2
4
6
8
10
12