Topic 2

Incorporating Financial Frictions in DSGE Models

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Overview

Conventional Model with Perfect Capital Markets:

1. Arbitrage between return to capital and riskless rate

$$E_t \beta \Lambda_{t,t+1} R_{kt+1} = E_t \beta \Lambda_{t,t+1} R_{t+1}$$

   where $\beta \Lambda_{t,t+1}$ is the household's stochastic discount factor

2. FInancial structure irrelevant.

Overview (con't)

With capital market frictions:

1. External finance premium $\Rightarrow$

$$E_t \beta \Lambda_{t,t+1} R_{kt+1} > E_t \beta \Lambda_{t,t+1} R_{t+1}$$

2. Premium depends inversely on borrower balance sheets $\Rightarrow$

3. If borrower balance sheets move procyclically, external finance premium move countercyclically:

   $\Rightarrow$ feedback betweeen financial and real sectors ("financial accelerator,")

   $\Rightarrow$ disturbances originating in the financial sector can have real effects.

Bernanke/Gertler/Gilchrist Financial Accelerator Model

Dynamic General Equilibirum Framework with

1. Money
2. Imperfect Competition
3. Nominal Price Rigidities (Calvo staggered price setting.)
4. Financial Accelerator as in Bernanke/Gertler(1989), featuring asset price mechanism in Kiyotaki and Moore (1997)

Sectors

1. Households
2. Business Sector
   (a) entrepreneur/firms
   (b) capital producers
   (c) retailers
3. Central Bank

Households

- Objective

$$\max E_t \sum_{i=0}^{\infty} \beta^i [\log (C_{t+i}) + a_m \log(\frac{M_{t+i}}{P_{t+i}}) - a_n \frac{1}{1+\gamma_n} L_{t+i}^{1+\gamma_n}] \qquad (1)$$

subject to

$$C_t = \frac{W_t}{P_t} L_t + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{\frac{1}{1+i_t} B_t - B_{t-1}}{P_t} \qquad (2)$$

- As in Woodford (2003), we restrict attention to the cashless limit of the economy (the limit as $a_m \to 0$).

Decision Rules

- labor supply

$$\frac{W_t}{P_t} = a_n L_{t+i}^{\gamma_n} / (\frac{1}{C_t}) \tag{3}$$

- consumption/saving;

$$\frac{1}{C\,t} = E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \beta \frac{1}{C\,t+1} \right\} \tag{4}$$

Entrepreneurs/Firms

- Produce wholesale output

- Competitive, risk neutral, face capital market frictions.

- A measure unity in the market at any time.

- i.i.d survival probability θ : The expected horizon is accordingly $\frac{1}{1-\theta}$. $1-\theta$ enter to replace exiting entrpreneurs.

- Exiting entrepreneurs make a small transfer to new entrepreneurs and then consume the rest.

Production Technology

The production technology is given by

$$Y_t = \omega_t A_t \left(K_t\right)^{\alpha} \left(L_t\right)^{(1-\alpha)}. \tag{5}$$

where ω_t is i.i.d with

$$E\{\omega_t\} = 1$$

Labor Demand

F.O.N.C.

$$\frac{W_t}{P_{wt}} = (1 - \alpha)\frac{Y_t}{L_t}$$

Capital Demand

- Gross Return to Capital

$$E_t\left\{R_{kt+1}\right\} = E_t\left\{\frac{\frac{P_{w+1}}{P_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t}\right\}$$

- Opportunity Cost

$$E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\right\}$$

Capital Demand (con't)

Under perfect markets, capital demand given by

$$E_t\left\{R_{kt+1}\right\} = E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\right\}$$

With imperfect markets:

$$E_t\left\{R_{kt+1}\right\} > E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\right\}$$

Capital Demand (con't)

The finance of capital is divided between net worth and debt:

$$Q_t K_{t+1} = N_t + \frac{B_t}{P_t}.$$

Costly State Verification

Assume:

(i) costly state verification and limited liability

(ii) one period contracts

(iii) payouts based only on firm-specific contingencies

- $\Longrightarrow$:

1. Debt with costly default is optimal

2. Agency costs of external finance (expected default costs)

3. Collateral reduces expected default costs

Optimal Choice of Capital

$$Q_t K_{t+1} = \upsilon\left(\frac{E_t\left\{R_{kt+1}\right\}}{E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\right\}}\right) N_t$$

Optimal Choice of Capital(con't)

Aggregate Demand for Capital (Inverting the previous equation)

$$E_t\left\{R_{kt+1}\right\} = (1+\chi_t)E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\right\}$$

with

$$\chi_t = \chi\left(\frac{Q_t K_{t+1}}{N_t}\right)$$

and

$$\chi'(\cdot) > 0,\ \chi(0) = 0,\ \chi(\infty) = \infty$$

Evolution of Net Worth

$$N_t = \theta V_t + (1-\theta)D$$

where

$$V_t = (1-m_t)R_{kt}Q_{t-1}K_t - \left[(1+i_{t-1})\frac{P_{t-1}}{P_t}\right]\frac{B_t}{P_{t-1}}$$

with

$$R_{kt} = \frac{\frac{P_{wt}}{P_t}\alpha\frac{Y_{tt}}{K_{tt}} + (1-\delta)Q_t}{Q_{t-1}}$$

$$m_t = \mu G(\omega^*_{t-1})$$

Evolution of Net Worth (con't)

- Main Sources of Net Worth Fluctutions

  Unexpected movements in Q_t and P_t

- Irving Fisher's debt-deflation hypothesis: unanticipated declines in price level raies real debt burdens.

The Role of Leverage

Given $Q_{t-1}K_t = N_{t-1} + \frac{B_{t-1}}{P_{t-1}t}$

$$V_t = \{[(1 - m_t)R_{kt} - R_t]\phi_{t-1} + R_t\}N_{t-1}$$

with

$$\phi_{t-1} = \frac{Q_{t-1}K_t}{N_t}$$

$$R_t = (1 + i_{t-1})\,\frac{P_{t-1}}{P_t}$$

- The sensitivity of net worth to unanticipated returns is increasing in the leverage ratio ϕ_{t-1}..

Capital Producers

- Capital Producers are competitive. They produce new capital and sell at the price Q_t.

- Evolution of capital

$$K_{t+1} = \Phi(\frac{I_t}{K_t})K_t + (1-\delta)K_t$$

$$\Phi' > 0, \Phi'' < 0, \Phi(\frac{I}{K}) = \frac{I}{K}$$

Optimal Choice of Investment

$$E_{t-1}\{Q_t - [\Phi'(\frac{I_t}{K_t})]^{-1}\} = 0$$

i.e.,Q is increasing $\frac{I_t}{K_t}$ as in Tobin's Q theory

Note: Marginal product of capital used in producing new capital goods is zero within a local region of the steady state. See BGG.

Retailers

- Buy wholesale output and sell as differentiated product

- Set prices on a staggered basis as in Calvo (1983)

$$\frac{P_t}{P_{t-1}} \approx (\mu \frac{P_t^w}{P_t})^{\lambda} E_t (\frac{P_{t+1}}{P_t})^{\beta}$$

in loglinear form

$$\pi_t = \lambda(p_{wt} - p_t) + \beta E_t \pi_{t+1}$$

- Note: $p_t - p_{wt}$ is the log price markup.

Resource Constraint

Let $C_t^e \equiv$ entrepreneurial consumption and $M_t \equiv$ total monitoring costs:

$$Y_t = C_t + C_t^e + I_t + G_t + M_t$$

with

$$C_t^e = (1 - \phi)(V_t - D)$$

$$M_t = m_t R_t Q_{t-1} K_t$$

Monetary and Fiscal Policy

Monetary Rule:

$$i_t = \rho i_{t-1} + (1-\rho)[\gamma_\pi \pi_t + \gamma_y (y_t - y_t^n)] + \varepsilon_t^{rn}$$

$$i_t = r_{t+1} - E_t \pi_{t+1}$$

Fiscal Policy:

Gov't spending exoxgenous and finance by lum sum taxes.

Investment, Finance and Monetary Policy in BGG

$$I_t/K_t = \phi(Q_t) \tag{6}$$

$$E_t R^k_{t+1} = (1 + \chi\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right))\left\{(1 + i_t)\frac{P_t}{P_{t+1}}\right\} \tag{7}$$

where

$$E_t R^k_{t+1} = E_t \left\{ \frac{\frac{P_{w+1}}{P_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t} \right\} \tag{8}$$

Investment, Finance and Monetary Policy in BGG (con't)

Note:

$$N_t = \theta\{(1 - m_t)R_{kt}Q_{t-1}K_t - (1 + i_{t-1})\frac{P_{t-1}}{P_t}\frac{B_t}{P_{t-1}}\} + (1 - \theta)D$$

Thus:

i. Positive feedback between asset prices and investment (financial accelerator)

ii. Strength depends positively on leverage ratio ratio Q_tK_{t+1}/N_t.

iii. Monetary Policy has additional impact via balance sheets

LOG-LINEARIZED BGG MODEL

Aggregate demand

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}inv_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c_t^e + ...$$

$$c_t = -\sigma r_{t+1} + E_t c_{t+1}$$

$$c_t^e = \frac{1-\phi}{\phi}n_{t+1}$$

$$(inv_t - k_t) = \varphi q_t$$

$$E_t r_{kt+1} = (1-\vartheta) E_t (p_{wt+1} - p_{t+1} + y_{t+1} - k_{t+1}) + \vartheta E_t q_{t+1} - q_t$$

$$E_t r_{kt+1} - r_{t+1} = -\upsilon (n_t - q_t - k_{t+1})$$

LOG-LINEARIZED BGG MODEL (con't)

Aggregate supply

$$y_t = a_t + \alpha k_t + (1 - \alpha) l_t$$

$$y_t - l_t = \mu_t + \gamma_l l_t + c_t$$

$$\pi_t = \kappa(p_{wt} - p_t) + \beta E_t \pi_{t+1}$$

LOG-LINEARIZED BGG MODEL (con't)

Evolution of state variables

$$k_{t+1} = \delta inv_t + (1-\delta)k_t$$

$$n_t = \frac{\theta RK}{N}[r_t^k - r_t] + \theta R(r_t + n_{t-1})$$

with

$$r_r = i_{t-1} - \pi_{t-1}$$

LOG-LINEARIZED BGG MODEL (con't)

Monetary Policy Rule

$$i_t = \rho i_{t-1} + (1-\rho)[\gamma_\pi \pi_t + \gamma_y (y_t - y_t^n)] + \varepsilon_t^{rn}$$

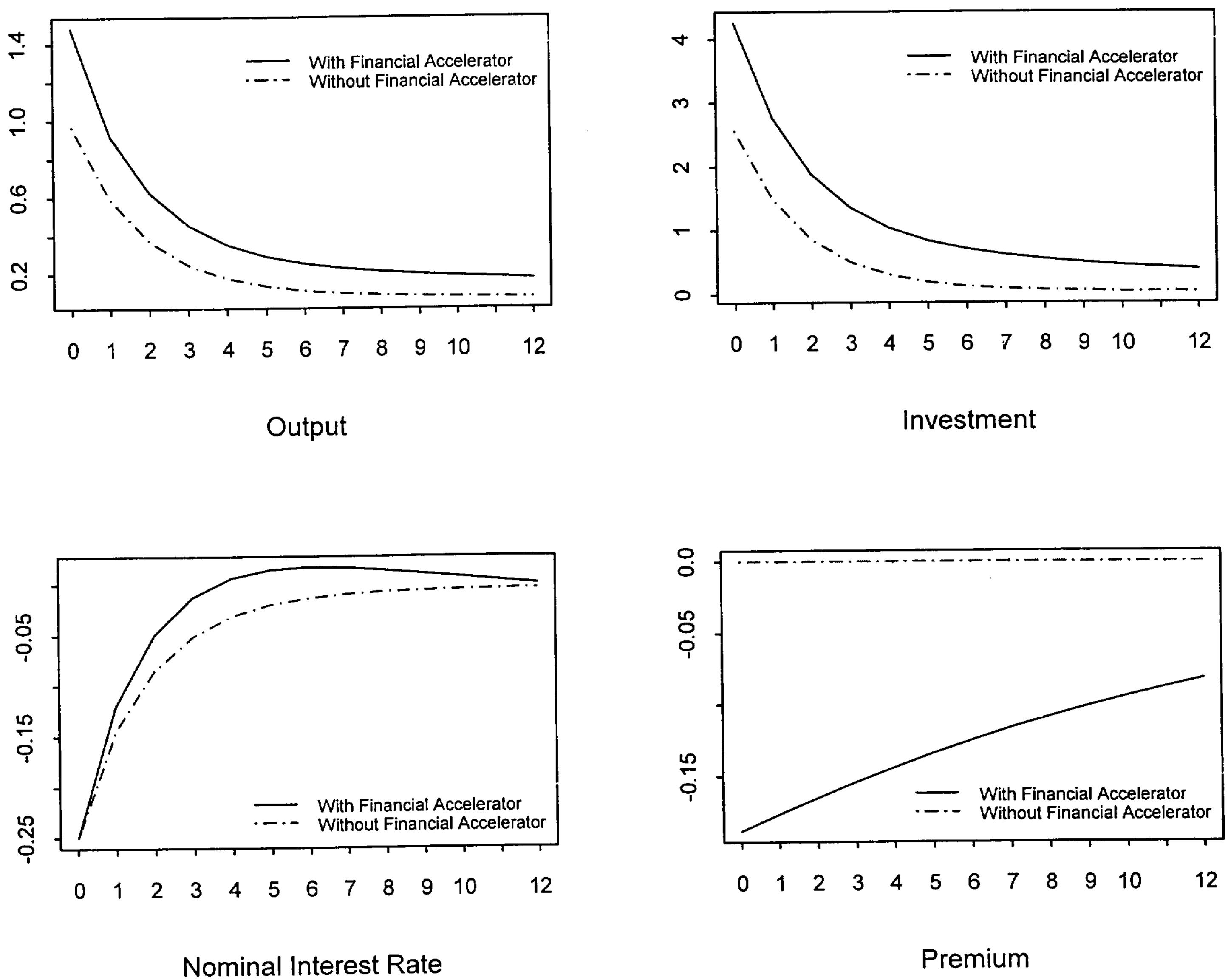
$$i_t = r_{t+1} - E_t \pi_{t+1}$$

Calibrating Financial Sector Parameters

Choose (i) survival probability θ, (ii) monitoring costs μ, and (iii) the moments of the idiosyncratic shock to match evidence on:

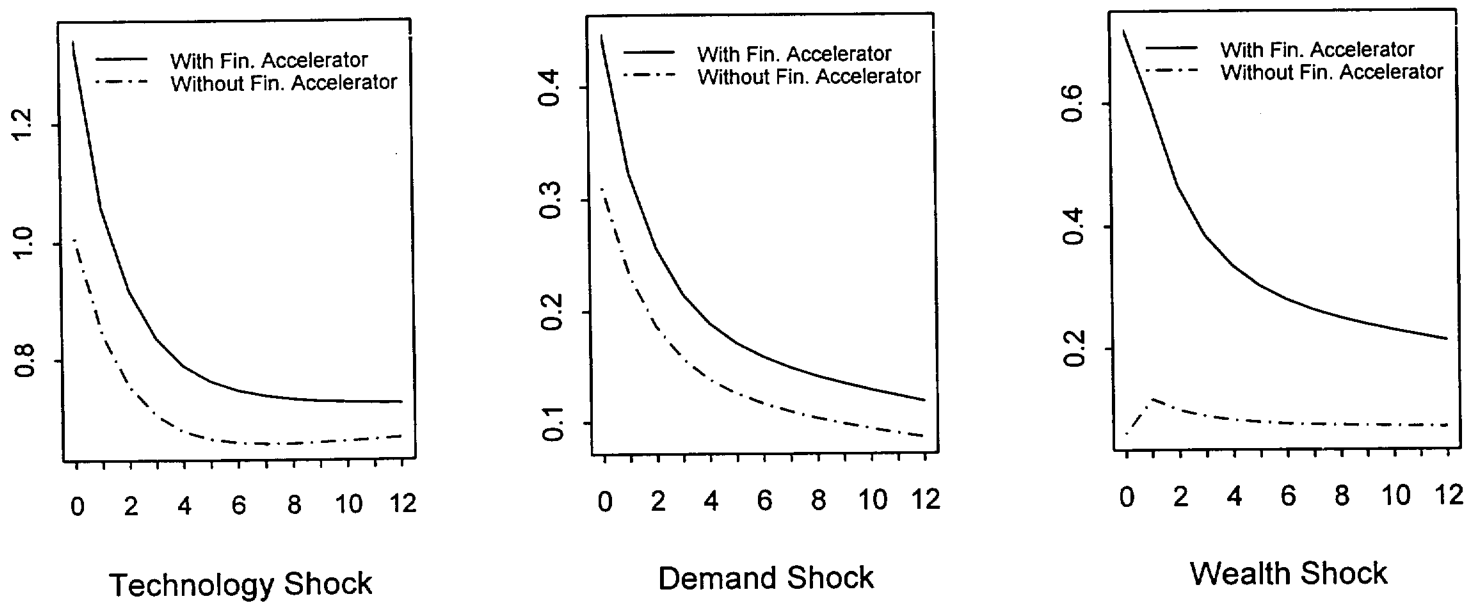
1. Steady state external finance premium: R_k/R..
2. Steady state leverage ration QK/N
3. Annual business failure rate.

Figure 3: Monetary Shock - No Investment Delay



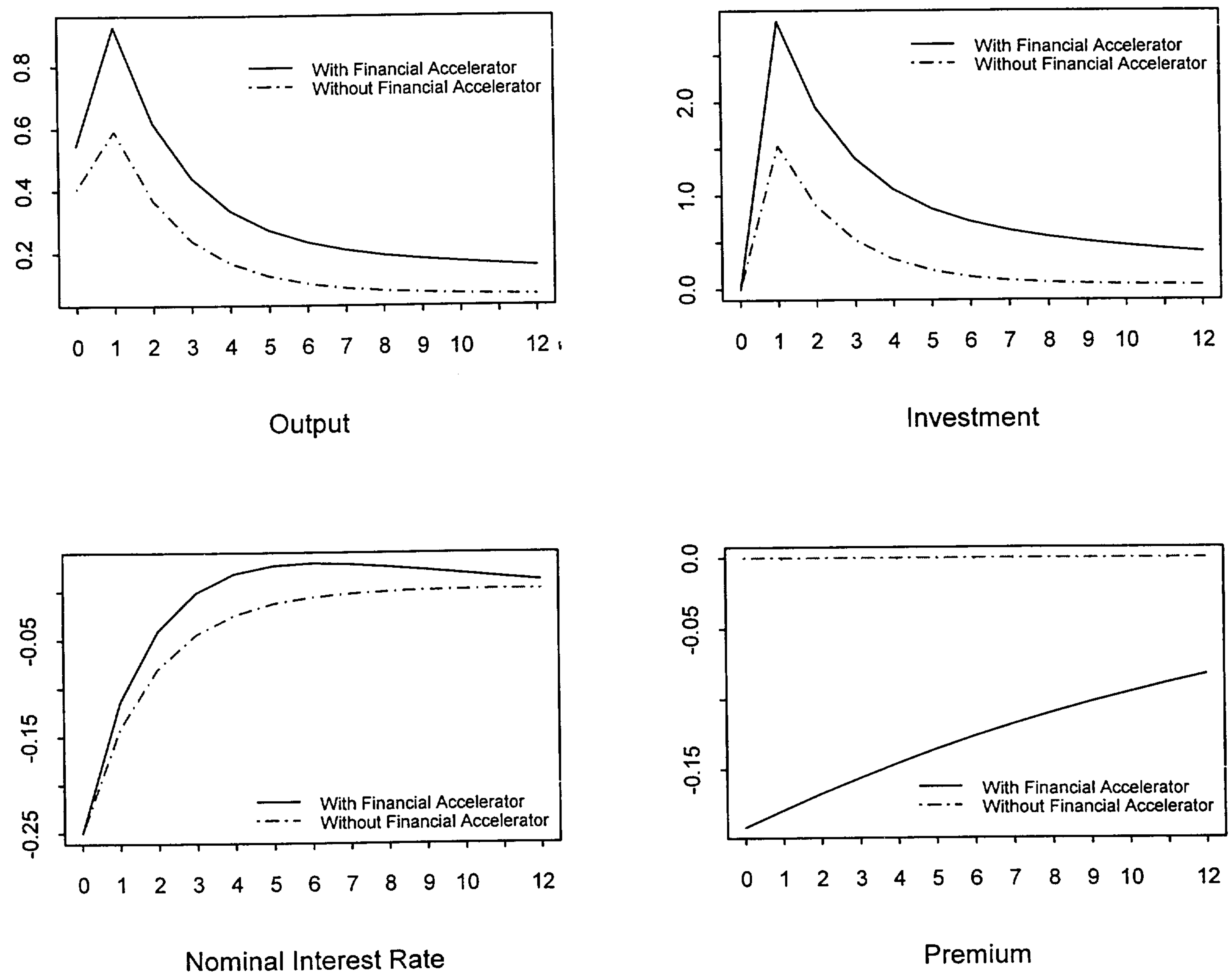
All Panels: Time Horizon in Quarters

Figure 4: Output Response - Alternative Shocks



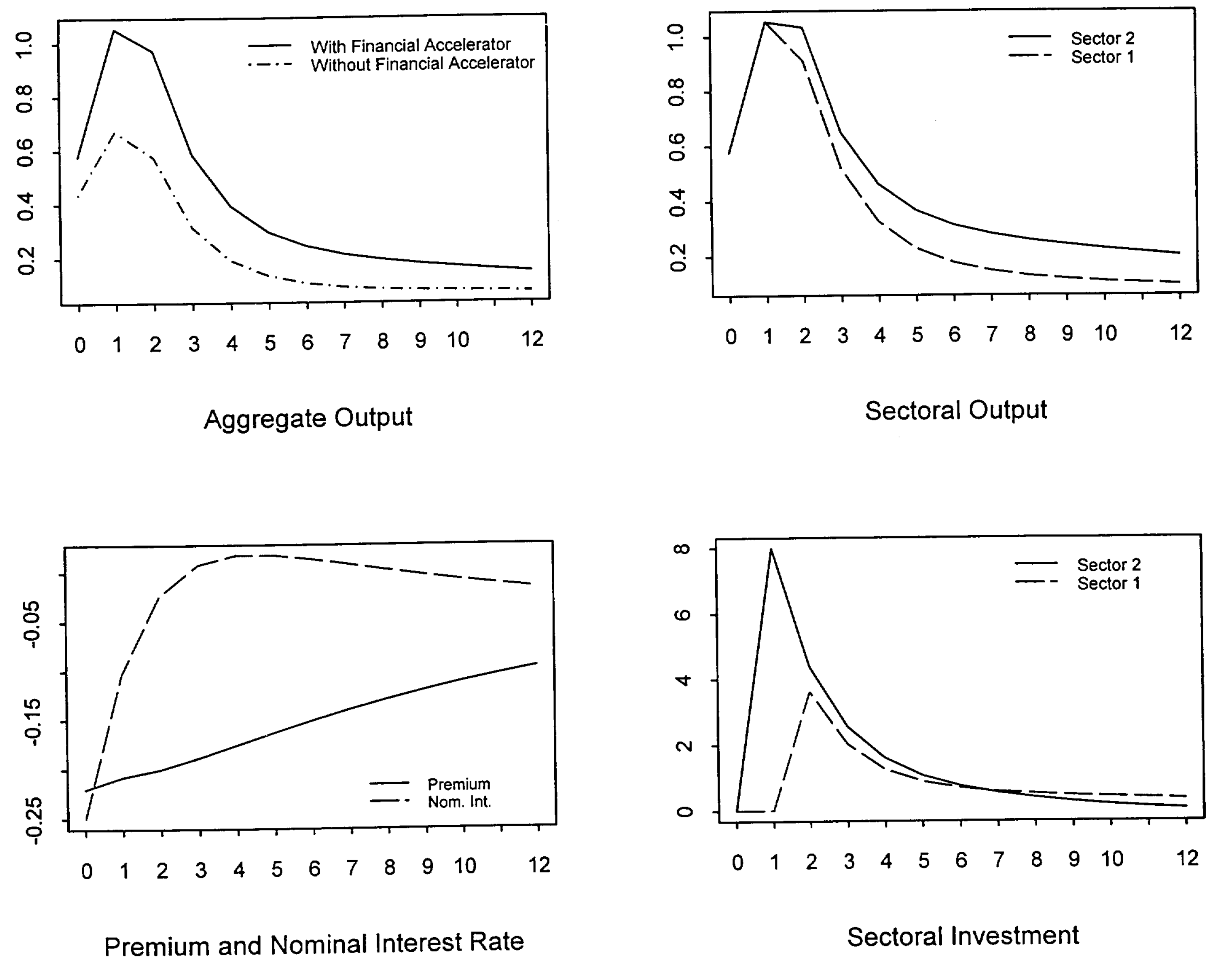
All Panels: Time Horizon in Quarters

Figure 5: Monetary Shock - One Period Investment Delay



All Panels: Time Horizon in Quarters

Figure 6: Monetary Shock - Multisector Model with Investment Delays



All Panels: Time Horizon in Quarters; Panels 2-4: Model with Financial Accelerator.