The Political Economy of Debt and Entitlements*

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Abstract

This paper presents a dynamic political-economic model of total government obligations. Its focus is on the interplay between debt and entitlements. In our model, both are tools by which temporarily powerful groups can extract resources from groups that will be powerful in the future: debt transfers resources across periods; entitlements directly target the future allocation of resources. We prove the following results. First, fiscal rules can have perverse effects: if entitlements are unconstrained, and there are capital market frictions, debt limits robustly lead to an increase in total government obligations and to worse outcomes for all groups. Analogous results hold for entitlement limits. Second, debt and entitlements respond in opposite ways to political instability and, in contrast with prior literature, political instability may even reduce debt when entitlements are endogenous. Finally, we identify a possible explanation for the joint growth of debt and entitlements.

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1 Introduction

A leading explanation for the determinants of government debt was proposed by Persson and Svensson (1989) and by Alesina and Tabellini (1990). According to this explanation, when political power changes over time, the currently powerful groups have a systematic incentive to over-borrow and over-consume in the present, because their group is otherwise unable to commit to the allocation of future resources.

This view of debt as a "commitment tool" has been very influential in the politico-economic literature. Many contemporary theoretical models of government debt embed, at their core, the above-mentioned force (see, e.g.: Battaglini and Coate 2008; Song, Storesletten, and Zilibotti 2012; Halac and Yared 2014). This same idea inspired an influential empirical literature (see., e.g., Grilli *et al.* 1991; Pettersson-Lidbom 2001).

What is missing from this now-standard approach is an account of how *entitlements*, such as pensions and health care, are determined. Like debt, entitlements also allow current generations to pre-commit resources available for future resorce allocation. Furthermore, entitlements are major factors in fiscal sustainability for many countries.[1] Therefore, it is interesting to explore a politico-economic model in which debt and entitlement levels are *jointly determined* in equilibrium.[2]

Our aim is to study the interplay between debt and entitlements and thus to investigate the *political economy of total government obligations.* To maximize comparability with the literature that builds on Alesina and Tabellini (1990), we adhere closely to their basic framework. We make a single departure from the canonical framework of Alesina and Tabellini (1990): we allow for the coexistence of debt and entitlements.

The key ingredients of our model are the following. In each period, a political process determines spending on a public good as well as private goods for two groups. Political power changes over time, as for instance in an intergenerational setting. The currently powerful groups (we call these "group A") can use debt to leverage future resources to finance higher current consumption. In addition, group A can set entitlements, i.e., "pre-commit" some fraction of future resources to a desired allocation. Thus, both debt and

[1] In the U.S. entitlements have grown rapidly since the 1960s and have overtaken discretionary spending. The Steuerle and Roeper index of Fiscal Democracy measures the percentage of (projected) revenues **not** claimed by permanent programs currently in place. In the US, this index dropped from 65% in 1962 to a range between 0 and 20 percent in the period 1998-2012; it is forecast to stay in this range through 2022, and there is no expectation of improvement in the more distant future. See Steuerle (2014). Evans, Kotlikoff, and Phillips (2012) provide another measure of fiscal sustainability –the so-called *duration to game over*. In the case of the US, this measure also points to the high (or even unsustainable) fiscal burden of entitlement programs. A similar pattern in the growth of entitlements holds across OECD countries.

[2] Leading academics have long pointed out that debt and entitlements should be recognized as a combined burden from an accounting perspective (see Kotlikoff and Burns, 2004). Accordingly, the European Commission has recently developed a forward-looking measure of fiscal sustainability, the "intertemporal net worth," that captures government obligations much more broadly than simple measures of debt. But despite this recognition in the policy world and at the level of measurement, there is no academic work that studies the political forces that jointly shape debt and entitlements.

entitlements are tools for temporarily powerful groups to extract resources from groups that will be more powerful in the future (we call these "group B"). Note the stark assumption that entitlements *for generation A* cannot be changed once generation A is old.[3] Albeit stark, this assumption is not counterfactual[4] and it has the advantage of being symmetric to the assumption, which is standard in this literature, that there is no default on government debt.

We first characterize the equilibrium allocation when there are no limits to debt and entitlements. This presents the first contrast with the conventional no-entitlements framework. While absent entitlements debt is always positive, in the presence of endogenous entitlements debt levels are reduced to an extent that group A may choose to run a surplus. Nevertheless, we show that the possibility of entitlements leads to an increase of total government obligations compared to the case of zero entitlements. Furthermore, entitlements allow group A to smooth its private consumption over time but they crowd out period-2 public consumption.

We then present our main results, which concern the analysis of fiscal rules. A key determinant of the consequences of fiscal rules is the extent of capital market frictions. In a world with frictionless capital markets, debt limits have no effects because agents can undo such effects by increasing entitlements and borrowing privately. This is not to say that debt and entitlements are neutral, as contraints on total obbligations have real effects on intergenerational allocations of resources. When frictions are sizable, then debt limits are partially effective in that they reduce government obligations, but less than one for one because the group in power in the first period substitutes partially toward entitlements. When capital market frictions are in an intermediate range such that agents actively participate in capital markets, we show that fiscal rules have perverse effects. If entitlements are unconstrained, debt limits robustly lead to an increase in total government obligations and to worse outcomes for all groups. Analogous results hold for entitlement limits.

We then explore how equilibrium levels of debt and entitlements respond to changes in the persistence of political power, or its opposite, government instability. This exploration is motivated by prior literature that provides compelling reasons to think that debt decreases as political power becomes more persistent.[5] This result can be reproduced in our

[3] Nothing in our setting would prevent generation B's entitlements from being set at a lower level than generation A's, as typically happens in a pension reform, say, although our model is silent regarding generation B's entitlement choice.

[4] The entitlements of current beneficiaries have proved difficult to change without the current beneficiaries' consent. In the vast majority of US states, for example, changing future benefits for current employees is extremely difficult; see Munnell and Quinby (2012). And across the world, even when pension laws have been revised, the benefits of current retirees have generally been protected. Typically, what is reformed are the benefits of future beneficiaries. We discuss the mechanisms that protect entitlements in Section 9.2, but in our model we simply assume that the entitlements of current beneficiaries cannot be changed without the current beneficiaries' consent.

[5] The logic is that debt is issued by the currently politically powerful only when they fear losing power

environment when we do not allow for entitlements. However, we show that predictions become more nuanced if entitlements are endogenous. Specifically, we show that debt and entitlements move in opposite directions when persistence increases. Furthermore, debt sometimes increases with persistence–and even more surprisingly, the total fiscal burden (debt plus entitlements) may increase with persistence. This is a rich set of empirical implications that could in principle be tested in cross-country data, as was done for the Alesina-Tabellini model by Ozler and Tabellini (1991), Crain and Tollison (1993), Franzese (2002), and Lambertini (2003).

Finally, despite the fact that debt and entitlements are *substitute* tools of intergenerational redistribution, we show that an increase in preference polarization may lead to an increase in both debt and entitlements.

2 Related Literature

Some papers explain debt as the outcome of a struggle between different groups in the population who want to gain more control over resources. The reason debt is accumulated is that the group that is in power today may not be in power tomorrow, and debt is a way to take advantage of this temporary power. For instance, Cukierman and Meltzer (1989) and Song, Storesletten, and Zilibotti (2012) argue that debt is a tool used to redistribute resources across generations. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) argue that debt represents a way to tie the hands of future governments that will have different preferences from the current one. In Tabellini and Alesina (1990), voters choose the composition of public spending in an environment where the median voter theorem applies. If the median voter remains the same in both periods, the equilibrium involves budget balance. If the median voter tomorrow has different preferences, the current median voter may choose to run a budget deficit to take advantage of his temporary power and tie the hands of the future government. The equilibrium may also involve a budget surplus because there is an "insurance" component that links the two periods as well: a surplus tends to equalize the median voter's utility in the two periods. Tabellini and Alesina (1990) detail conditions such that deficits will be incurred and show that increased polarization leads to larger deficits.

Browning (1975) and Boadway and Wildasin (1989) have studied voting models of pensions in which age is the only dimension of heterogeneity. Conde-Ruiz and Galasso (2005) study a two-dimensional voting model in which pensions coexist with a welfare state. Thus they allow for voting on both intragenerational and intergenerational redistribution. They argue that pensions are particularly stable because the elderly are a relatively homogeneous voting group, and the pension system is supported by a broad coalition including the low-income young.

in the future (see the seminal work of Alesina and Tabellini 1990).

Tabellini (1991) also illustrates how debt and social security differ as distributional instruments in an overlapping generations environment. In contrast with our model, the main force concerns the difference in default between the two instruments.

Battaglini and Coate (2008) present a dynamic model of taxation and debt where a rich policy space is considered within a legislative bargaining environment. Velasco (1996) discusses a model where government resources are "common property" with which interest groups can finance their own consumption. Deficits arise in his model because of a dynamic "common pools" problem. Lizzeri (1999) presents a model of debt as a tool of redistributive politics.

The dynamic public finance literature (e.g., Golosov, Tsyvinski and Werning 2006) provides a setup that is suited to the normative study of debt and entitlements, although this question has not been a main substantive focus of this literature so far.[6]

This paper is also related to work on legislative bargaining with endogenous status quo. Kalandrakis (2004) studies a classic divide-the-dollar problem where the division agreed to in one period is the status quo for the next period. Bowen, Chen and Eraslan (2014) study a model in which two parties decide unanimously how to allocate a given budget to spending on a public good and private transfers. The focus is on the comparison between two political institutions: discretionary vs. mandatory public good spending (private transfers are discretionary in both cases).[7] When the public good is discretionary (mandatory), the status quo level of the public good is zero (the one from the previous period). By contrast, we focus on the interplay between debt and entitlements.

A very different approach to understanding public debt is explored by Azzimonti et al. (2014). They propose a multi-country model with incomplete markets, and they show that governments may choose higher public debt when financial systems are more integrated. They thus offer an explanation of the rise in debt as driven by an increase in financial integration. Related to our discussion of the comovement of entitlements and debt, they also show that debt increases with the level of idiosyncratic risk.

3 Model

In this section we present the model. Section 9 provides a discussion of some of the assumptions.

Our model only has two periods. The two-period (finite-horizon) model facilitates comparison with some of the prior work done in the literature on debt and outlines how some basic forces are changed by the presence of entitlements.

[6] See also Stantchveva (2014) and (2016). Regarding the latter, one could think of entitlements as promised spending on education and health.

[7] In a modified version of this model–with two periods and no private transfers–Bowen et al. (2015) allow the party making the take-it-or-leave-it offer to choose whether the public good is discretionary or mandatory. The focus is on the efficiency of the public good provision under various budgetary institutions.

The model has three groups: group A, which represents a single generation that works in period 1 and is retired in period 2; group B1 which are the retired in period 1; and group B2 which are the workers in period 2. Thus, we set up a two-period overlapping generation model where, in each period, workers hold political power.

3.1 Demography and economy

In each period t there is an endowment of 1 that can be allocated to private goods for the either of the group x_A^t and x_B^t who are alive in that period, or to a public good g^t. Part of the period-1 endowmend can be privately saved. Preferences in each period are given by:

$$u_i\left(x_i^t, x_j^t, g^t\right) = h\left(x_i^t\right) + v\left(g^t\right),$$

where $h(\cdot)$ and $v(\cdot)$ are concave, and twice continuously differentiable. We also assume that both the private and the public goods are sufficiently valuable that $h'(0) = \infty = v'(0)$, implying that it is not optimal for one group to spend all the resources on its own private good or on the public good. Utility is additive across periods and there is no discounting. There are no capital markets to either privately save or borrow.[8]

The resource constraints in periods 1 and 2 are given by:

$$\begin{aligned} x_A^1 + x_B^1 + g^1 &= 1 + d - s && (1) \\ x_A^2 + x_B^2 + g^2 &= 1 - d + \varphi(s), && (2) \end{aligned}$$

where d represents debt or surplus, s represents group 1's private savings (which could be negative), and $\varphi(s)$ represents group A's credit or debt due in period 2. We assume that $\varphi(s) \leq s$ with $\varphi(0) = 0$. This assumption allows for a capital market frictions, that is, an unfavorable wedge between the private interest rate and the interest rate on the public debt. The wedge vanishes if $\varphi(s) \equiv s$. This assumption is, we believe, uncontroversial, if $s < 0$ so that agents borrow. This is the primary focus of our analysis. In the case in which $s > 0$, one possible interpretation of the friction is a tax on returns to savings.

We assume $x_A^t, x_B^t, g^t \geq 0$.

We assume no default on debt, but we revisit this assumption in Section 9.2.4.

3.2 Political structure and entitlements

Except for Section 7, throughout the paper the political structure is such that group A decides the allocation in period 1; group B decides the allocation in period 2 subject to debt and entitlements, as specified below.

[8] In Section 4.1 we argue that the main results are unchanged if we allow for private savings as long as there is some intertemporal friction such as a positive tax on capital.

In period 1, group A chooses the quintuple:

$$\left(x_A^1, x_B^1, g^1, d, E, s\right),$$

subject to the resource constraint (1). E is a nonnegative number that represents group A's entitlements in the future. In period 2, group B chooses the triple:

$$\left(x_A^2, x_B^2, g^2\right),$$

subject to the resource constraint (2) and to the following additional constraint:

$$x_A^2 = E + \varphi\left(s\right).$$

This constraint indicates that group B2 never transfers to group A more than the minimal possible amount, namely the level of entitlements (E) so that group A's private consumption sums to E plus group A's private savings.

4 Benchmarks and Preliminary Discussion

In this section we highlight some features of the setup that are conceptually important to frame the results in this paper.

4.1 Private Savings Frictions and Ricardian Equivalence

If $\varphi\left(s\right) \equiv -\infty$ for any $s \neq 0$, then saving/borrowing frictions are so large that it is impractical for group A to save or borrow privately. If furthermore entitlements are absent (or restricted to equal zero), then the only way for group A to save (borrow) is by running a government surplus (debt). In this scenario our model becomes essentially equivalent to early political economy models of debt such as Alesina and Tabellini (1990) and Tabellini and Alesina (1990): it can be shown that Group A runs up debt in period 1 and that public good provision decreases between periods 1 and 2.

If $\varphi\left(s\right) \equiv s$, i.e., there are no capital market frictions, then a Ricardian-equivalence-type result arises, in the following sense. Suppose that group 1 selects a triple d, E, and s in order to generate a certain allocation $\left(x_A^1, g^1, x_A^2, g^2\right)$ for itself. If subsequently d was exogenously shifted to $d' \neq d$, then group A would still be able to achieve $\left(x_A^1, g^1, x_A^2, g^2\right)$ by opportunely adapting E and s. A direct implication of this observation is that debt limits are neutral if $\varphi\left(s\right) \equiv s$. Clearly, analogous considerations apply to entitlement limits if there are no frictions and debt is unconstrained.

4.2 Absent entitlements, debt limits redistribute across generations

The above Ricardian equivalence property of irrelevance of debt limits requires flexibility in choosing entitlements (and no borrowing frictions). If for instance, entitlements are constrained to be zero, then restricting group A's ability to create debt redistributes utility from the first period groups (A and B1) to group B2. In this sense, changing a debt limit can lead to different allocations on some redistributive frontier but cannot lead to either a Pareto-improvement or worsening. This feature is shared with early models of sovereign debt such as Alesina and Tabellini (1990) and Tabellini and Alesina (1990). In those models debt accumulation is viewed negatively because from a utilitarian perspective since they allow the current group to extract resources from a group that will have power in the future. We will see below that this issue is more subtle in the presence of entitlements.

5 Equilibrium Analysis Without Fiscal Rules

Absent fiscal rules it is without loss of generality to set $s^* = 0$ because, absent debt limits, it is optimal for group A to neither save nor borrow (strictly optimal if there are even small saving/borrowing frictions). This is because private savings/borrowings are redundant for group A. To see this, suppose that group 1 selects a triple d, E, and s in order to generate a desired allocation $\left(x_A^1, g^1, x_A^2, g^2\right)$. If subsequently s was exogenously shifted to $s' = 0$, then group A would still be able to achieve $\left(x_A^1, g^1, x_A^2, g^2\right)$ by opportunely adapting d and E (notice that this adaptation may not be feasible if there are debt limits). This redundancy of private savings is a manifestation of the Ricardian equivalence property highligted in Section 4.1.

We start with some definitions and preliminary analysis. We denote by c the portion of the second period budget that has been already been committed (either in debt or entitlements) in period 1.

Definition 1 ***(second-period policies)*** *Define second-period policy choices conditional on a budget commitment of c as the set $X(c), G(c)$ that solves:*

$$\max_{(x,g)} h(x) + v(g) \;\; s.t. \;\; x + g \leq 1 - c.$$

$X(c)$ represents the amount of private good that a group (either A or B, whichever has the power to choose the allocation) would allocate itself in period 2, subject to the constraint that a fraction c of period 2's endowment has been reserved for other purposes. $G(c)$ represents the corresponding amount of public good.

Lemma 1 ***(well-behaved second-period policies)*** *Second period policy choices* $X(c)$ *and* $G(c)$ *are single-valued differentiable functions that are decreasing in c. Thus, increasing the fraction of the second period budget which is committed lowers private and public consumption in the second period.*

Proof. See Appendix A. ■

In period 2, group B is in power. We can use Definition 1 to describe group B's allocation choice.

Corollary 1 ***(second period equilibrium allocation)*** *Assume the second period starts with pre-defined commitments d of debt and E of entitlements. Then in period 2 group B allocates exactly E to group A's private good, allocates* $X(d+E)$ *in private good to itself, and allocates* $G(d+E)$ *to the public good.*

Given period-2 policy choices, we can move to consideration of optimal policies in the first period.

Definition 2 ***(first-period policies)*** *Define first-period policy choices as the set* (x^*, g^*, d^*, E^*) *that solves:*

$$\max_{(x,g,d,E)} h(x) + v(g) + h(E) + v(G(d+E)) \;\; s.t. \;\; x + g \leq 1 + d. \tag{3}$$

The four-tuple (x^*, g^*, d^*, E^*) maximizes group A's lifetime payoff. This payoff is partly accrued in period 1 (the first two addends in equation (3)) and partly in period 2 (the last two addends in equation (3)). However, in period 2 group A does not directly control the allocation; therefore, its private consumption in period 2 is given by the amount E it chose in entitlements in the first period, and its amount of public consumption is determined by whatever amount group B chooses to provide given the (uncommitted) resources available in the second period.

In what follows, we assume that $v(G(\cdot))$ is concave. This is a technical assumption that guarantees concavity of the problem faced by group A. Because $G(\cdot)$ is endogenous, it is helpful to provide sufficient conditions on the primitives that ensure the desired property. Lemma 2 in the appendix provides these conditions. It is easy to see that the optimal allocation is interior: Group A does not fully commit period 2's budget: $d^* + E^* < 1$. The reason is that when government obligations are too high, second period public good provision becomes very small, and the high marginal utility of public consumption requires that this provision stay bounded away from zero.

The next discusses two properties of the equilibrium of the model when entitlements are allowed.

Proposition 1 ***(Equilibrium characterization)*** *In equilibrium:*

1. *Total government obligations are always positive and larger than in the case without entitlements:* $d^* + E^* > d^*_{E=0} > d^*$;
2. *Group A may choose to run a surplus; for example, if* $h(x) = v(x)$ *have a CRRA form* $(x)^{1-\rho} / (1-\rho)$, *then group A runs a surplus if and only if* $\rho > 1$.

Proof. See Appendix A. ■

First, consider the important literature that has highlighted the role of debt as an instrument to leverage temporary power (e.g., Alesina and Tabellini 1990, Tabellini and Alesina 1990, Persson and Svensson 1989).[9] If, consistent with this literature, entitlements were left out of our model (i.e., implicitly set to zero), Proposition 11 part 2 indicates that the equilibrium level of debt would be larger than if entitlements were accounted for by the model. That is, by abstracting from the presence of entitlements, there is a risk of over-estimating the amount of debt that is created in an effort to take advantage of temporary power. Note however, that from Proposition 1 part 1 a model that abstracts from entitlements would underestimate the total level of government obligations (i.e., the sum of debt and entitlements).

Part 1 of Proposition 1 shows that allowing for entitlements increases government obligations. In our model both types of government obligations arise because in period 2 group A lacks political control and understands that a fraction of any uncommitted dollar will be diverted from public consumption to group B's private consumption. Absent entitlements, the only way to pre-commit period 2 dollars is to consume them today (by issuing debt). But, due to the concavity of the utility function, group A would prefer to allocate (at least some of) these period 2 dollars to its private consumption in period 2. This is exactly what entitlements allow group A to do. This additional commitment channel raises the value of committing period 2 dollars and therefore leads to larger government obligations.

Parts 2 of Proposition 1 highlights the differences with the no-entitlements benchmark. The presence of entitlements allows for better intertemporal resource allocation (part **??**) and lessens the proclivity to accumulate debt (part 2).[10]

The two key first-order conditions that determine debt and entitlements are obtained by differentiating the objective function (3) with respect to E and d respectively:

$$h'(E) = -v'(G(d+E))\, G'(d+E), \tag{4}$$

$$h'(x) = -v'(G(d+E))\, G'(d+E). \tag{5}$$

[9] This determinant of debt is a major component of recent developments in the political economy theory of public debt (see, e.g., Battaglini and Coate 2008, Battaglini 2011, and Azzimonti et al. 2015).

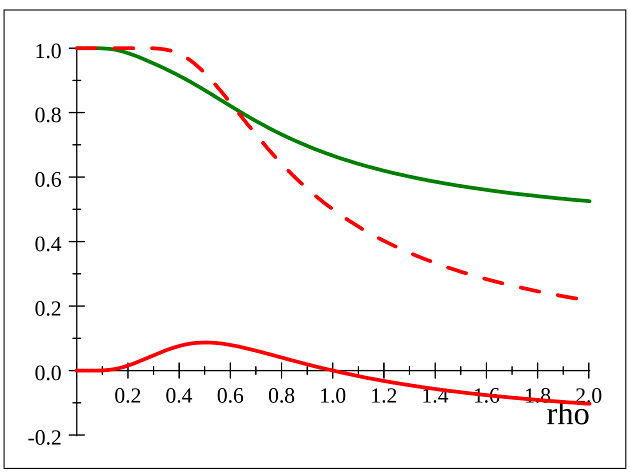
[10] We wish to emphasize that the possibility of surplus is not of interest in itself. We view it as a useful contrast with the no entitlement benchmark. Of course, surplus is also a possibility in other political economy models of debt (e.g., Persson and Svensson 1989). However, this is due to differences in the utility functions between groups.

These equations illustrate the different roles of debt and entitlements. Group A uses debt to smooth consumption over time and entitlements to smooth consumption over types of goods in period 2. As in the model with a single "commitment instrument," the crowdout effect gives an incentive to group A to increase government obligations; this effect pushes toward both higher debt and higher entitlements. The smoothing effect, however, is quantitatively different in terms of how it affects debt, and qualitatively different in terms of how it affects entitlements.

Regarding debt, the smoothing effect is amplified by the presence of entitlements and in fact still remains strong even when debt is zero: part of period 2 resources are pre-committed to entitlements, and thus for any given level of debt, the imbalance in public consumption between periods 1 and 2 is even more pronounced. To smooth public consumption across periods, group A may therefore choose to run a surplus.

Regarding entitlements, the smoothing effect can be either positive or negative because entitlements may be either lower or higher than public consumption depending on the degree of risk aversion and on the relative importance of public consumption. In contrast, in the case of debt, the sign of the smoothing effect is always negative because public consumption is always lower in the second period.

To illustrate more explicitly how these forces balance to produce equilibrium debt and entitlements, let us consider CRRA preferences (that is, $h(x) = (x)^{1-\rho} / (1-\rho)$ and $v(x) = \alpha h(x)$) and a low value of $\alpha = .5$, which represents an environment with relatively high distributive conflict since the value of public consumption is relatively low.[11] Figure 5 shows how equilibrium magnitudes vary with ρ, and contrasts them with the case in which entitlements are exogenously set to zero.



Entitlements (Green); Debt with entitlement (Plain Red); Debt without entitlements (Dashed Red).

Consider first the case without entitlements. The dashed red line depicts equilibrium debt when entitlements are not allowed. In this case, debt is always positive. When $\rho \to 0$

[11] A case with lower distributive conflict, where $\alpha = 1.5$, is discussed in Appendix B.

(to the limiting case of linear preferences), the conflict among the two groups becomes extreme because group B would spend the entire budget on its private consumption, leaving nothing for the public good. The crowdout effect is thus maximal, and group A chooses maximal debt. As ρ increases, the smoothing effect starts to matter more and more and it become more important to devote resources to second period public consumption, so debt falls.

Suppose now that entitlements are allowed (represented by the solid lines). Just as in the case with no entitlements, when $\rho \to 0$ there is an extreme conflict of views in period 2 and the crowdout effect is maximal. However, the consequence is very different: in the limit there is no debt and full entitlements. Entitlements are a superior way to capture the second period resources as long as $\rho > 0$. Just as before, when ρ increases the smoothing effect implies that it becomes more desirable to devote part of the budget to the public good, so entitlements drop. Debt responds non-monotonically to an increase in ρ. While resources become available for the public good over both periods, the amount invested in the public good is different in the two periods; for low values of ρ, the crowdout effect dominates, and for high values of ρ the smoothing effect dominates.

6 Fiscal Rules

We now discuss the consequences of fiscal rules for equilibrium allocations and for the welfare of the three groups in our economy. We begin by discussing debt limits, then we consider limits to entitlements. Finally, we discuss limits to the overall level of government obligations.

Debt limits restrict group A's policy space in period 1. Therefore, tightening a binding debt limits necessarily reduces group A's overall utility. Moreover, it is easy to see that group B1's utility also decreases as a consequence of the tightened debt limit because group A, being "poorer" chooses to provide a lower level of public good in period 1. But, a naive intuition suggests that group B2 should benefit from the protection provided by debt limits. Our analysis shows that the effects of fiscal rules crucially depend on the level of financial frictions relative to how tight the debt limit is. This is because the effect of a tightening of a fiscal rule depends on whether group A privately borrows at the initial debt limit.

The first necessary step for this question requires understanding how, changing a debt limit $\overline{d}$ affects total obligations $\overline{d}+E\left(\overline{d}\right)$. Of course, tightening a binding debt limit reduces the marginal cost of entitlements because it frees up resources in period 2. Therefore, group A optimally responds by increasing entitlements: there is a strategic substitution between debt and entitlements. The key question then, is the quantitative response of entitlements to the debt limit.

We already saw in the previous Section that there are no effects of tightening the debt limit in the extreme important benchmark of no financial frictions ($\varphi(s) \equiv s$): in this case, group A would increase entitlements and private borrowing by the same amount to restore its allocation to the desired status quo ante level.

In the other extreme case in which credit market frictions are sufficiently high that group A is at a corner in its borrowing decisions, then it can be shown that debt ceilings are partially effective, i.e., tightening a binding debt ceiling $\overline{d}$ reduces $\overline{d} + E\left(\overline{d}\right)$, the sum of debt and entitlements in equilibrium. The intuition for this result is the following. Suppose that, in response to the tighter debt ceiling, group A increased entitlements to keep total obligations $\overline{d} + E\left(\overline{d}\right)$ constant. Then, the marginal cost of obligations, given by the crowdout effect of reduced public consumption in period 2, is unchanged. However, the marginal benefit of these obligations is now lower: since the debt limit was already binding, group A's preferred composition of obligations favored transfering resources toward first period consumption. Thus, increasing private second period consumption, which is the effect of increased entitlements has lower marginal benefit for group A. Thus, restoring second period optimality requires reducing total obligations.

Note that the case with no savings, **and** no entitlements mirrors the effects of debt limits in the Alesina and Tabellini (1990) model.[12]

We now consider the case in which capital market distortions are at an intermediate level, so that, given a binding debt limit, group A chooses to borrow in equilibrium. This case is more complex, so we specialize to the class of Constant Relative Risk Aversion (CRRA) utility functions We show that, in fact, government obligation **increase** as a consequence of a tightening of the debt limit. A direct consequence of this result is that, in this case, all groups suffer when debt limits are tightened.

Fix $h(x) = (x)^{1-\rho} / (1-\rho)$ and $v(x) = \alpha h(x)$. The parameter α captures the value that all groups place on public consumption. Set

$$\varphi(s) = \begin{cases} s\overline{\tau} & \text{for } s < 0 \\ s\underline{\tau} & \text{for } s > 0 \end{cases} \quad \text{where } 0 < \underline{\tau} < 1 < \overline{\tau}.$$

This functional form satisfies $\varphi(s) \leq s$, meaning that both saving and borrowing is less advantageous for group A than for the government. Within this framework, the following result applies. Define first $j(\alpha, \rho) \equiv \dfrac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\overline{\tau}}\right)^{\frac{1}{\rho}} - \left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right) - 1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\overline{\tau}}\right)^{\frac{1}{\rho}} + \left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right) + 1}$.

Proposition 2 *Fix a binding debt limit $\overline{\mathrm{d}}$ such that $\overline{\mathrm{d}} < j(\alpha, \rho)$. Then, total government obligations inherited by group B2 are decreasing in $\overline{\mathrm{d}}$, and so tightening the debt limit harms group B2 as well as harming groups A and B1.*

[12] In their model political decisions are over two different types of public goods, so capital market distortions do not play an important role.

The condition on the debt limit $\overline{d}$ ensures that savings are negative in equilibrium, that is, that in period 1 group A finds it optimal to borrow a positive amount privately, in addition to issuing government debt. The intuition for why group B2's utility is reduced by tightening binding debt limits is as follows. As the debt limit becomes tighter, group A partly compensates by borrowing more and setting higher entitlements. The net effect on period 2 obligations (and hence group B's welfare) is in principle ambiguous, and indeed, absent borrowing frictions the decrease in government debt is exactly offset by group A's increase in entitlements (Ricardian equivalence). With borrowing frictions, however, group A's increased borrowing leads to a greater period-2 wealth loss, and so group A is forced to rely more on entitlements in period 2. This substitution adversely affect group B's obligations in period 2.

Proposition 2 identifies a Pareto-harmful effect of debt limits. This is a novel effect relative to the existing literature, which views debt limits as constraints on the ability of temporarily-powerful group to expropriate future generations. This novel effect arises due to the presence of entitlements.

We now consider the possibility of entitlement limits $\overline{E}$ and discuss the consequences for total government obligations $d\left(\overline{E}\right)+\overline{E}$. As for the case of debt limits, there are three cases depnding on the severity of credit market frictions. If there are no frictions, then there is no effect of entitlement limits on total obligations because debt and savings adjust to restore the pre-existing allocation. If financial frictions are very severe, then entitlement limits are partially effective: total obligations fall when entitlement limits are tightened. However, in this case, the consequence of an entilement limit is a reduction of private consumption for group A in period 2 and an increase in private and public consumption in period 1. Therefore, in contrast to the case of debt limits with severe credit market frictions, both groups B1 and B2 can benefit from a tighter entitlement limit. The next result presents the mirror image of Proposition 2 for the case of entitlement limits when preferences are CRRA, capital market frictions are intermediate, and group A actively saves in equilibrium. Define $i\left(\alpha,\rho\right)\equiv\frac{2\underline{\tau}^{\frac{1}{\rho}}}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(1+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+\underline{\tau}^{\frac{1}{\rho}}}$

Proposition 3 *Fix a binding entitlement limit $\overline{E}$ such that $\overline{E}<i\left(\alpha,\rho\right)$. Then, total government obligations inherited by group B2 decrease in $\overline{E}$, and first period spending on the public good increases in $\overline{E}$ so tightening the entitlement limit harms group B1 and B2 as well as harming group A.*

The condition on entitlement limits ensures that savings are positive.

The logic for the perverse effect of tightening entitlement limits on the total level of government obligations is almost the same as for Proposition 2.

Most of the discussion of fiscal rules so far has relied on the substitution between entitlements and debt when the use of one of these tools is restricted. This suggests that the most effective way to limit the fiscal burden on the period 2 group is to restrict the overall level of government obligations directly. Leaving aside potential concerns of enforceability of such limits, it is correct that these limits would be effective in our model. Such limits would have a straightforward intergenerational redistributive effect along the lines of debt limits in the Tabellini and Alesina model, as discussed in Section 4.2.

7 Persistence in Power

An important question in prior literature (e.g., Alesina and Tabellini 1990) concerns the response of debt to government instability.[13] The idea is that instability increases the intensity of the conflict between groups in charge in different periods, so debt may respond to this. In fact, Alesina and Tabellini (1990) show that, under mild conditions on preferences, debt increases as the political system becomes more unstable (measured by the probability that the government remains in charge). The question that we wish to address is whether this result still holds once we allow for entitlements and, more generally, what is the effect of political instability on total government obligations.

We follow Alesina and Tabellini and assume that, conditional on being in charge in the first period, group A stays in charge with probability π, whereas power changes hands (group B takes over) with probability $1-\pi$. Thus, π is a measure of persistence of the political system, while $1-\pi$ is a measure of the instability of the political system.

In the event that group A persists in power, one needs to specify how entitlements constrain group A's period-2 decision. At issue is whether group A, if in power in period 2, might be forced to allocate at least E to its own private good, even if it prefers to reduce some of its own entitlements in favor of financing more public goods. We assume that this is not the case; that is, the entitlements set by group A in period 1 do not bind group A itself in the event that group A persists in power in period 2. This is because we feel that any generation should be able to give up some pre-existing entitlements easily if they wish.[14]

Proposition 4 ***(effects of increasing the probability*** π ***that group A stays in power in period 2).*** *In equilibrium:*

1. *absent entitlements, that is, if* $E \equiv 0$*, debt is decreasing in* π*.*

2. *debt and entitlements move in opposite directions when* π *varies;*

3. *total government obligations move in the same direction as debt when* π *varies;*

[13] This question was investigated empirically by Ozler and Tabellini (1991), Crain and Tollison (1993), Hallerberg and Von Hagen (2000), Franzese (2002), and Lambertini (2003).

[14] For some assumptions on preferences, this assumption is not binding. Furthermore, for more general preferences, it is possible to relax this assumption without affecting the results qualitatively.

4. *debt and entitlements are monotonic in* π; *debt (entitlements) is monotonically decreasing (increasing) if equilibrium debt is positive at* $\pi = 0$, *and debt (entitlements) is monotonically increasing (decreasing) if equilibrium debt is negative at* $\pi = 0$.

Proof. See Appendix D. ■

The intuition behind part 1 is the following. When there are no entitlements–i.e., $E = 0$–the direct effect is that when group A remains in charge tomorrow, debt is harmful for that group because it reduces both private and public consumption (while it only reduces public consumption if B is in charge tomorrow). Thus, more political persistence (higher π) leads to a reduction in debt.

The intuition for Proposition 4 parts 2 and 3 is as follows. Given that entitlements are relevant only if group B is in power, their level does not depend directly on π. Rather, the effect of a change of π on entitlements is indirect–through the effect of π on debt. Given that debt and entitlements are substitutes, they move in opposite directions when π changes (part 2). Given that the elasticity of entitlements to debt is larger than -1, total government obligations move in the same direction as debt when π changes (part 3).

In order to gain an intuition for part 4, note that when group A stays in power, debt (or surplus) introduces an intertemporal distortion. Group A would prefer debt to be zero in this event. By increasing π, the probability of that distortion increases; hence debt must get closer to zero. The monotonicity for entitlements follows from the monotonicity proved in part 2.

Proposition 4 speaks to the testable implications sought by the literature following Alesina and Tabellini (1990). This literature (see footnote 7) seeks to explain debt accumulation using power persistence as the explanatory variable. Our analysis suggests that entitlements are an important moderating variable in this relationship. For example, the evolution of debt should be different in a jurisdiction that has the ability to incur debt but not to alter entitlements (e.g., many European cities), compared to a jurisdiction that has the ability to alter both debt and entitlements (national governments, for example). By the same token, the evolution of entitlements should be different in jurisdictions in which debt is severely constrained (e.g., US states) compared to national governments.

8 Preference polarization

Tabellini and Alesina (1990) explore how a polarization in preferences for public policy leads to an increase in debt. In our context, this same polarization can be modeled as a decreased valuation for the public good compared to private goods. The idea is that, as preference polarization increases, there are fewer goods whose enjoyment society as a whole shares. So, for example, an increase in income inequality might cause the young rich and the young poor to diverge in the type of goods that they prefer. To model polarization we introduce a parameter α that governs how much, within each cohort, everyone likes the public good relative to the private goods.

For the purposes of this analysis we focus on the case of CRRA preferences with $h(x) = (x)^{1-\rho} / (1-\rho)$ and $v(x) = \alpha h(x)$. The parameter α captures the value that all groups place on public consumption and lower values of α imply more disagreement over the distribution of resources since both groups wish to shift consumption toward their private good.

Proposition 5 ***(growth in debt and entitlements)*** *Suppose $h(x) = (x)^{1-\rho} / (1-\rho)$ and $v(x) = \alpha h(x)$, where $\alpha > 0$ is a parameter that captures the degree of redistributive conflict. Then, as α decreases: for $\rho < 1$ (respectively: $\rho > 1$), debt and entitlements increase jointly if and only if α is smaller (respectively: larger) than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.*

Proof. See Appendix E. ■

A reduction in α has the following effects on debt and entitlements. As α falls, there is a direct effect of a reduction in the value that group A places on public consumption. This is a force in favor of increasing debt and entitlements because both can lead to increases in private consumption for group A. Of course, the reduction in α also reduces group B's value for public consumption, implying that group B contributes less to the public good in the second period both in total and at the margin, changing both the crowdout and the smoothing effects. A larger crowdout effect also pushes toward an increase in debt and entitlements. However, the smoothing effect can become larger pushing in the opposite direction. Figure 1 illustrates the region of parameters for which debt and entitlements increase with conflict.

We have also considered two other ways to define increasing intergenerational conflict. First, we have considered a case in which α decreases over time in the same way for both groups. In this case we have shown that debt and entitlements increase with conflict for all values of ρ. Second, we have considered a case in which α is smaller for the new generation: group B has lower α than group A. In this case there is always comovement between debt and entitlements, but debt and entitlements increase in conflict if and only if $\rho < 1$.[15]

This proposition can be contrasted with Alesina and Tabellini (1990). In their model, an increase in disagreement always leads to an increase in debt. Absent entitlements, our model delivers the same result. When entitlements are endogenous, the results are more subtle: entitlements always increase with conflict, but this is not always the case for debt.

[15] Interestingly, the latter version of the model is close to the model used by Persson and Svensson (1989) to study debt. For the case without entitlements we can replicate their results in our version of the model.

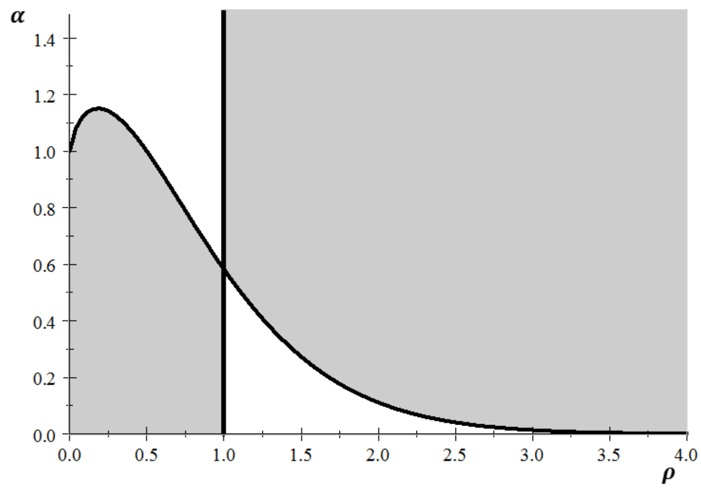


Figure 1: Shaded areas give the combinations of α and ρ such that debt and entitlements co-move when conflict increases.

9 Discussion of Modeling Assumptions

9.0.1 Conflict of Interest: Age or Wealth?

Interpreting groups as workers and retirees means that the conflict of interest in our model is generational. A different perspective could be that the key conflict for public policy is not workers vs. pensioners, but rather rich vs. poor. For some public policies this alternative view is likely correct (in the case of taxes, for example). However, welfare policy often appears to go beyond the rich-poor cleavage. Indeed, Bonoli (2000), p. 5 argues:

> "the main political cleavage in social policy-making seems to be shifting from the left-right axis to an opposition between governments, to a large extent regardless of their political orientation, and a pro-welfare coalition of interest groups, which is often led by the labour movement. This has long been the case in France where the Socialist governments of the 1980s clashed with the unions on a number of occasions. As new left-of-centre governments have been voted into power in Europe, this shift in the dominant cleavage in the politics of social policy has become more evident. In Germany, Italy and, to a lesser extent, Britain, the left-of-centre governments of the late 1990s are committed to continue reforming their welfare states, and the main confrontation is between themselves and the labour movement. [...] in most OECD countries a relatively small fraction of public spending is means-tested."

Thus, there is clearly a generational dimension to distributional conflict. However, it may still be interesting to extend the model to allow for additional forms of heterogeneity. The simplest extension of our model that accommodates additional forms of heterogeneity is the following. Assume that in period 1 there are two homogeneous groups, as in our basic model. In period 2 individual agents in each group receive income that is subject to idiosyncratic uncertainty yielding a non degenerate income distribution within groups. The polity becomes more complex because groups are no longer homogeneous in period 2 and some choices need to made about how preferences are aggregated within group. One simple way to think about it is to maintain the assumption that group B is in power, and assume that the median voter in group B is decisive, as if this is the median of the majority party in congress. However, other alternative ways to aggregate preferences could be explored.

A force pushing for an increase in entitlements in such a variation of the model is the fact that these now have an additional social insurance role for group 1 agents who face income risk in the second period. The challenge is to understand how debt responds. There are two effects. First, an increase in entitlements will initially reduce public good provision in the second period, and therefore lead to a reduction in debt (this is our substitution effect). Second, there is a countervailing force: group B becomes more unequal, and public good provision serves a redistributive function. We conjecture that for some preference profiles it should be possible to obtain an overall increase in public good provision in the second period, and therefore an increase in debt.

9.0.2 Workers, Not Retirees, Hold the Political Power

Throughout most of this paper workers, not retirees, hold political power. (The assumption is relaxed in Section 7, where we assume that group A stays in power with probability π.) Is this a natural assumption?

In a sense, the assumption that workers have all the political power is natural: there are more workers than pensioners, and thus the median voter must be a worker, not a pensioner. But what of the idea that–because there are a lot of older voters and they are more likely to vote–older voters should have a disproportionate influence on policy? The idea has some merit, but the fact is that while older voters are more likely to vote, the median voter's age is far lower than pensionable age (see, e.g., Galasso and Profeta 2004 and Galasso 2008). In the US, for example, the median voter's age in the 2008 election was about 45. The fact that the median voter is squarely of working age holds true across all democracies, including demographically older democracies.

If the median voter is a worker, what are the consequences for welfare policy? According to the median-voter model, all the political power should rest with the workers. Assuming that workers' interests regarding welfare policy are relatively homogenous, it should not matter whether the retiree population is getting older or larger (provided retirees do not exceed 50%): in a median-voter model, welfare policy should only reflect the preference of workers. Consistent with this hypothesis, Vanhuysse (2012) finds that, while there is a lot of cross-national variation in the pro-elderly bias of welfare spending in the OECD, "population aging actually cannot explain very much of this pro-elderly bias variance. For instance, countries such as Denmark, Finland and Sweden are demographically old societies, yet they boast among the lowest pro-elderly spending biases in the OECD world." For the US, Mulligan and Sala-i-Martin (1999) share the view that this pro-elderly bias cannot be solely explained by demographics. These findings are consistent with our modeling assumption that welfare policy (i.e., entitlements) is chosen by workers and protected by pensioners.

9.1 Modeling Entitlements

We model entitlements as spending floors on private consumption only. This choice was made for tractability. We also explored a different, more flexible specification in which group A can create entitlements on public goods E_g as well as on their private consumption. In this specification, there is a "budget constraint" for commitment: $E + E_g \leq \overline{E}$. The logic is that there is a limit $\overline{E}$ to the intergenerational commitments that a court system will enforce or a political system will protect. Under plausible parameter values, we found that group A would choose $E_g = 0$: that is, group A would use whatever commitment power is available on private, not public, goods.[16] This finding supports our modeling choice of simply assuming no ability to commit on public good provision.

It might appear that the model gives generation A a lot of "intergenerational power" by allowing them to set their own entitlements in period 2. But if our model is interpreted as a building block of an overlapping generation model, then it does not take much power to set entitlements. This is because in period 1 there are only two groups who are alive: the worker generation who will benefits from entitlements in the next period; and the retiree generation who will not be around in the next period. Therefore, there is no conflict of interest regarding entitlements between the generations who are alive in period 1. This observation suggests that *setting* entitlements may not in reality require a lot of power. In contrast, *defending* entitlements in period 2 would require power.

We assume that retirees can fully defend whatever entitlements they previously arranged for themselves. Of course, such an extreme form of commitment is not necessary to get the flavor of our results. All that is required is that new generations cannot easily renegotiate the older generations' entitlements. Surely generations that "fight for their right" to entitlements must believe that these entitlements cannot easily be renegotiated, otherwise the political fights would hardly be worthwhile. But is this belief factually correct? Have pensioners been able to defend their entitlements? We take up this issue in the following section.

[16] The intuition is as follows: given that the level of public good is positive in period 2, it is too costly to set E_g such that it affects the level of public good provided in that period.

9.2 What Are the Sources of the Commitment to Entitlements?

The promises of the welfare state have been resilient beyond expectations. In the 1980s, some leading political scientists expected the welfare state to retrench.[17] In the US and the UK, Reagan and Thatcher had brought conservative, anti-welfare agendas to power; union membership was declining across OECD countries and the "working class" was decreasing; and government debt was shooting up, creating fiscal pressure. And yet, despite this unfavorable environment, there was no retrenchment. Instead, the welfare state continued to expand (see OECD 2012, p.5). The welfare state also survived cataclysmic events, such as the fall of communist regimes (see Vanhuysse 2006, p. 77-8).

In this section we explore the sources of the commitment to these promises. In our view, the forces discussed in this subsection can, when taken together, explain the remarkable durability of welfare commitments. An important qualification is in order: while the mechanisms mentioned below suffice to *defend* the status quo (entitlements), they do not create sufficient power to *improve upon* the status quo. In other words, these mechanisms would not allow the pensioners to increase the generosity of existing policies.[18] This is an important observation because, in our model, we do not contemplate such renegotiation of entitlements.

9.2.1 Entitlements Defended Through Political Institutions

In the seminal book *Dismantling the Welfare State*, Pierson (1994) called attention to the remarkable durability of the welfare state, founding the literature on the "new politics of the welfare state."[19] Pierson (1994, pp. 1-2) argues that

> "retrenchment is a distinctive and difficult political enterprise. It is in no sense a simple mirror image of welfare state expansion.[...] Retrenchment advocates must operate on a terrain that the welfare state itself has fundamentally transformed. Welfare states have created their own constituencies. If citizens dislike paying taxes, they nonetheless remain fiercely attached to public social provision. That social programs provide concentrated and direct benefits while imposing diffuse and often indirect costs is an important source of their continuing political viability. Voters' tendency to react more strongly to losses than to equivalent gains also gives these programs strength."

In other words, Pierson argues that entitlements could be defended because welfare policies created constituencies that became entrenched, and because the benefits of entitle-

[17] For an intellectual history, see Myles and Guadagno (2002) and Gingrich (2015).

[18] The economic literature on pensions recognizes the importance of the status quo, and studies how this importance differs across political institutions (see, e.g., Hansson and Stuart 1989, and Azariadis and Galasso 2002). It also explores how lobbying helps explain how the elderly defend their entitlements (see, e.g., Grossman and Helpman 1998 and Mulligan and Sala-i-Martin 1999).

[19] For a review of this literature see Gingrich (2015).

ments are relatively concentrated (according to our model, on the old), but their costs are diffuse.[20] In a similar vein, Pierson (1994, p. 42) points out that undoing pay-as-you-go pension entitlements would impose concentrated costs on the "switch generation," which would need to fund two pensions. Bonoli (2000, p. 5) identifies some institutional features that make it easier to defend entitlements:

> "The degree of influence that pro-welfare interest groups have on policy depends to a large extent on the opportunities provided by the political institutions. Absence of veto points means that governments will be able to go much further in the restructuring of their welfare state. In contrast, political systems that offer veto points will find it more difficult to adapt their welfare states and pension systems to a changing economic and demographic environment."

Thus, Bonoli argues that universalism contributes to status-quo bias: countries where change requires the consensus of broad coalitions find it difficult to move away from a high-welfare status quo.[21]

If commitments were defended exclusively through political power, we would expect groups with more political power to be better able to defend their entitlements. Instead, when entitlements have been retrenched (typically, for pension benefits), the adjustments have been made gradually and with grandfather clauses designed to *protect senior citizens in proportion to their seniority*, and thus *in inverse proportion to their electoral/political power*.[22,23] This observation suggests that commitments to entitlements are not defended through political power alone.

[20] In this vein, Vanhuysse (2006) argues that the cohesion of threatened workers and pensioners was instrumental in preserving the commitment to pensions in certain Eastern European countries during the transition from Communism.

[21] Bonoli's leading example is Switzerland, but the same argument could apply to France, Germany, and Italy. In the same vein, Orenstein (2000, p. 2) argues that countries with more "veto actors"–social and institutional actors with an effective veto over reform–engaged in less radical reform. According to Orenstein, Poland and Hungary have generated less radical change than Kazakhstan, partly because they had more representative political systems, to which more associations, interest groups, and "proposal actors" have access.

[22] Pension retrenchment has taken two main forms: so-called *parametric reforms* (e.g., increases in retirement age or decreases in benefits) and so-called *systemic reforms* (that is, moving from public, defined-benefit systems to private, defined-contribution systems). Parametric reforms have been more common in Europe (OECD 2013), whereas systemic reforms have been more common in Latin America (Mesa-Lago and Márquez 2007).

[23] Regardless of the form retrenchments have taken, a broadly shared principle has held true: current pensioners have been grandfathered in. Indeed, current pensioners have been automatically protected against increases to the pensionable age (pensioners cannot be recalled to work), and they have also typically been protected against decreases in benefit levels (with the occasional exception of cost-of-living adjustments). Some academic authors actually promote grandfathering of current pensioners as welfare-improving (see, e.g., Conesa and Garriga 2008, Aubuchon et al. 2011). As concerns current workers, decrease in pension benefits have typically been phased in gradually; for example, increases in the retirement age have typically been larger for workers who were younger at the time of the reform. Quoting from Arpaia et al. (2009), emphasis ours: "Almost all countries increased the statutory retirement age, *the majority opting for a smooth transition towards higher retirement ages.* [...] The age of eligibility to a state pension

9.2.2 Judicial Protection of Entitlements

Judicial recourse has been a powerful protection against the renegotiation of welfare policies. There are many examples in which the courts have prohibited legislatures from impairing entitlements. In Illinois, for instance, the *New York Times* reports that:

> "All seven members of the state's highest court found that a pension overhaul lawmakers had agreed to almost a year and a half ago violated the Illinois Constitution. The changes would have curtailed future cost-of-living adjustments for workers, raised the age of retirement for some and put a cap on pensions for those with the highest salaries. But under the state Constitution, benefits promised as part of a pension system for public workers "shall not be diminished or impaired." (Davey 2015)

Illinois is the norm, not the exception, among US states. In their report on legal constraints on changing public pensions, Munnell and Quinby (2012) indicate that "[f]or the vast majority of states, changing future benefits for current employees is extremely difficult."[24] Outside the US, courts have invalidated welfare reforms in, e.g., Turkey (2006),[25] Latvia (2009),[26] Portugal (2014),[27] and Italy (2015).[28]

9.2.3 Commitment to Entitlements Arising from Threat of Breakdown in the Social Contract

The economic literature identifies conditions under which the younger generation does not renege on entitlements that benefit the older generation because they know that this would lead to tomorrow's younger generation following suit and reneging on the entitlements that benefit them. By reneging on pensions today, the young generation would deprive itself of pensions tomorrow. When the economy is dynamically inefficient, they are better off not reneging. A pay-as-you-go pension system (or any similar form of entitlements) may

was *progressively* increased from 65 to 67 in Denmark, Sweden and Germany, in the latter *with a very long phasing-in period.* [...] The retirement age was also *progressively* increased in the Czech Republic (2003) [...], in Hungary (1997) up to 62, Slovenia (1999) and Romania (2000)." If, as argued in Section 9.0.2, the median voter is a 45 year-old worker, then the effect of gradual phase-ins has been to provide a higher level of protection to citizens who are farther away from the median voter.

[24] Similarly, Monahan (2013, p. 5) states that "Changes to a participant's benefit once she has retired will be extremely difficult to make in any state." At the US federal level, interestingly, the entitlement to Social Security is not legally enforceable. "Congress has the power legislatively to promise to pay individuals a certain level of Social Security benefits, and to provide legal evidence of Congress's 'guarantee' of the obligation of the federal government to provide for the payment of such benefits in the future. While Congress may decide to take whatever measures necessary to fulfill such an obligation, courts would be unlikely to find that Congress's unilateral promise constitutes a contract which could not be modified in the future." Page ii, Lanza and Nicola (2014).

[25] Stewart (2008).

[26] See Social Protection – Human Rights (2009).

[27] See Fox (2014).

[28] See Merler (2015).

thus be part of an equilibrium, hence sustained without any institutional or legal defense of the pension system (see, e.g., Samuelson 1958 and Aaron 1966, and the discussion in Weil 2008). Browning (1975) shows that, because workers see past contributions as a sunk cost, a pay-as-you-go pension system can be political sustainable even when the economy is dynamically inefficient.

9.2.4 Allowing for Default

In our model we assume that there is no default on either debt or entitlements. This assumption is in line with most of the prior literature on the political economy of debt. However, a novel question arises in our context: the possibility that default may impact debt and entitlements differentially. This is a rich question with many possible angles. For instance, an important difference between debt and entitlements arises because debt is partly owed to outsiders (sovereign debt), while entitlements are only "owed" to a specific group of voters. This difference potentially generates different political incentives to default. We believe that this is an important difference, and we plan to purse it in follow-up work. Yet, it requires a major departure from the model that we have worked with so far. Tabellini (1991) also points out that there is an additional potential difference among the coalitions supporting default, even if he focuses on domestic debt and does not focus on the comparative statics of default.

Here we discuss some preliminary analysis of the consequences of default for the size of debt and entitlements. To fix ideas, let us begin by assuming that there are exogenous probabilities δ and η with which debt repayments and entitlement payments are reduced by a fixed amount λ (the default size). In equilibrium, for investors to be willing to lend, the interest rate on debt has to be adjusted to reflect this probability of default. This, of course, affects the willingness of group A to take on debt. This market discipline effect is absent in the case of entitlements. In fact, we believe that we can construct scenarios in which debt decreases with the probability and size of default, while entitlements are increasing in the same quantities. Note that these are statements about the effect of default on one given obligation–say entitlements–on the endogenous size of that obligation. The effect of an increase in the default probability for entitlements on the equilibrium size of debt is complex and may well be positive: if lenders expect pensions to be reduced, they may be more willing to lend.

A richer model of default incorporates an endogenous default response by group B to the size of debt and entitlements. A particularly simple way to do this is to assume that there is a default technology for debt $F(d, d^r, I^r)$ and one for entitlements $H\left(E, E^r, I^E\right)$. These reflect the cost for group B to change the amount (entitlements) from d to d^r (E to E^r). In order to introduce uncertainty about these possibilities, we could add some shocks to the size of the endowment available in period 2. In turn group A may, at some cost, build institutions I^E and I^d in period 1 that raise the cost of defaulting on these promises in the subsequent period. We conjecture that in equilibrium, group A would over-entitle itself relative to the target level of desired entitlements and build institutions to protect debt and entitlements in anticipation of partial default on both.

10 Conclusions

Entitlements are a key determinant of fiscal sustainability beyond the level of sovereign debt. Despite the policy relevance of entitlements, the political economy literature has not yet focused on the interplay between debt and entitlements. And yet there is a lot we can learn from taking a closer look at the interplay between these two quantities.

In this paper we have presented a very simple politico-economic model, as close as possible to Alesina and Tabellini (1990), where entitlements and debt are jointly determined. The main findings are the following. Debt and entitlement are strategic substitutes: constraining debt increases entitlements (and vice versa). From a welfare perspective, it is good to constrain entitlements, but not too strictly; and constraining debt may be detrimental even in the absence of shocks to be smoothed – an unintended consequence of fiscal rules. Debt and entitlements move in opposite directions in response to political instability. Surprisingly, debt, and even the expected total fiscal burden can go down in response to political instability when entitlements are endogenous. We have proposed that the joint growth of debt and entitlements could be attributed to increases with redistributive conflict.

We view this paper as an instructive first step in a larger research program: to build state-of-the art theoretical politico-economic models which explore the forces that shape total government obligations, that is, the sum of debt and entitlements.

Appendix A: Proofs and Additional Example for Section 5

Proof of Lemma 1. Uniqueness follows directly from the concavity of the problem, and differentiability from the implicit function theorem. Using the constraint to substitute for x and taking the first order conditions with respect to c we have:

$$-h'(1-c-g)+v'(g)=0. \tag{6}$$

Replacing g with $G(c)$ and differentiating yields:

$$v''(G(c))\,G'(c)=-h''(1-c-G(c))\,(1+G'(c)),$$

hence

$$G'(c)=-\frac{h''(1-c-G(c))}{v''(G(c))+h''(1-c-G(c))}\in(-1,0).$$

Because at the optimum the constrain holds with equality, we have:

$$X(c)+G(c)=1-c$$

Differentiating, we have $X'(c)=-1-G'(c)\in(-1,0)$. ■

Lemma 2 ***(sufficient conditions for concavity)*** *$v(G(\cdot))$ is concave if $G(x)$ is concave.*

1. *$G(x)$ is concave if and only if $\frac{v''\left([v']^{-1}(x)\right)}{h''\left([h']^{-1}(x)\right)}$ is nonincreasing in x.*
2. *(symmetric case) $G(x)$ is concave if $h(x)=v(x)$.*
3. *(proportional CRRA functions) $G(x)$ is concave if $v(x)$ is CRRA and $h(x)=\alpha v(x)$ for $\alpha>0$.*
4. *(CRRA functions with different curvatures) Suppose $v(x)=x^p/p$ and $h(x)=x^q/q$, with $p,q<1$. Then $G(x)$ is strictly concave if and only if $p<q$.*

Proof.
Part 1. $X(c), G(c)$ solve:

$$\max_{(x,g)} h(x)+v(g)\ \text{ s.t. }\ x+g\leq 1-c.$$

$G(c)$ is concave in c iff $G(1-c)$ is concave in c. So, let's make the change of variables $k=1-c$ and write the following auxiliary problem:

$$\max_{(x,g)} h(x)+v(g)\ \text{ s.t. }\ x+g\leq k.$$

Denote the solutions to the auxiliary problem by $\widetilde{X}(k), \widetilde{G}(k)$. Let us derive necessary and sufficient conditions for $\widetilde{G}(k)$ to be globally concave.

Form the auxiliary problem's Lagrangian to get the first order conditions:

$$h'\left(\widetilde{X}(k)\right) = \lambda(k) = v'\left(\widetilde{G}(k)\right). \tag{7}$$

Differentiate with respect to k :

$$h''\left(\widetilde{X}(k)\right)\widetilde{X}'(k) = \lambda'(k) = v''\left(\widetilde{G}(k)\right)\widetilde{G}'(k).$$

Note that $\lambda'(k) < 0$. Use (7) to substitute for $\widetilde{X}(k)$ and $\widetilde{G}(k)$:

$$h''\left([h']^{-1}(\lambda(k))\right)\widetilde{X}'(k) = \lambda'(k) = v''\left([v']^{-1}(\lambda(k))\right)\widetilde{G}'(k).$$

Eliminate $\lambda'(k)$ to get:

$$\frac{\widetilde{X}'(k)}{\widetilde{G}'(k)} = \frac{v''\left([v']^{-1}(\lambda(k))\right)}{h''\left([h']^{-1}(\lambda(k))\right)}.$$

Since the constraint $x + g \leq k$ must hold with equality, we must have $\widetilde{X}'(k) + \widetilde{G}'(k) = 1$, whence our equation can be rewritten as follows:

$$\frac{1}{\widetilde{G}'(k)} - 1 = \frac{v''\left([v']^{-1}(\lambda(k))\right)}{h''\left([h']^{-1}(\lambda(k))\right)}.$$

Therefore $\widetilde{G}'(k)$ is decreasing in k if and only if:

$$\frac{v''\left([v']^{-1}(\lambda(k))\right)}{h''\left([h']^{-1}(\lambda(k))\right)} \text{ is nondecreasing in } k.$$

Since $\lambda(k)$ is decreasing in k (recall that $\lambda'(k) < 0$), the above condition is equivalent to:

$$\frac{v''\left([v']^{-1}(x)\right)}{h''\left([h']^{-1}(x)\right)} \text{ nonincreasing in } x.$$

Part 2. In this case we can see directly that $\widetilde{G}(c)$ is linear. Indeed, symmetry and concavity guarantee that $\widetilde{X}(c) = \widetilde{G}(c) = c/2$. Thus $\widetilde{G}(c)$ is (weakly) concave.

Part 3. Consider now $\widetilde{v}(x) = \alpha v(x)$. Then:

$$\begin{aligned} \widetilde{v}'(x) &= \alpha v'(x) \\ \widetilde{v}''(x) &= \alpha v''(x) \\ [\widetilde{v}']^{-1}(x) &= [v']^{-1}\left(\frac{x}{\alpha}\right). \end{aligned}$$

When $v(x) = x^p/p$ we get:

$$\begin{aligned} v'(x) &= x^{p-1} \\ v''(x) &= (p-1)\, x^{p-2} \\ [v']^{-1}(x) &= (x)^{1/(p-1)} . \end{aligned}$$

Thus:

$$\begin{aligned} \widetilde{v}''(x) &= \alpha (p-1)\, x^{p-2} \\ [\widetilde{v}']^{-1}(x) &= [v']^{-1}\left(\frac{x}{\alpha}\right) = \left(\frac{x}{\alpha}\right)^{1/(p-1)} = \left(\frac{1}{\alpha}\right)^{1/(p-1)} [v']^{-1}(x) . \end{aligned}$$

So

$$\begin{aligned} & \widetilde{v}''\left([\widetilde{v}']^{-1}(x)\right) \\ = \; & \alpha (p-1) \left([\widetilde{v}']^{-1}(x)\right)^{p-2} \\ = \; & \alpha (p-1) \left(\left(\frac{1}{\alpha}\right)^{1/(p-1)} [v']^{-1}(x)\right)^{p-2} \\ = \; & \left(\frac{1}{\alpha}\right)^{(p-2)/(p-1)} \alpha (p-1) \left([v']^{-1}(x)\right)^{p-2} \\ = \; & \left(\frac{1}{\alpha}\right)^{(p-2)/(p-1)} \alpha v''\left([v']^{-1}(x)\right) \\ = \; & \alpha^{\frac{1}{p-1}} v''\left([v']^{-1}(x)\right) . \end{aligned}$$

Thus

$$\frac{\widetilde{v}''\left([\widetilde{v}']^{-1}(x)\right)}{v''\left([v']^{-1}(x)\right)} = \alpha^{\frac{1}{p-1}} \text{ independent of } x.$$

Thus the condition in part 1 of the lemma is verified trivially.

Part 4. Given the functional forms of $v(x)$ and $h(x)$ we get:

$$\begin{aligned} v''\left([v']^{-1}(x)\right) &= (p-1)\,(x)^{(p-2)/(p-1)} , \\ h''\left([h']^{-1}(x)\right) &= (q-1)\,(x)^{(q-2)/(q-1)} , \end{aligned}$$

so that

$$\frac{v''\left([v']^{-1}(x)\right)}{h''\left([h']^{-1}(x)\right)} = \left(\frac{p-1}{q-1}\right) x^{\frac{(p-2)}{(p-1)} - \frac{(q-2)}{(q-1)}} .$$

Because $p, q < 1$ the term in parentheses is positive. Therefore, the RHS is decreasing in x if and only if

$$\begin{aligned}\frac{(p-2)}{(p-1)} &< \frac{(q-2)}{(q-1)} \\ \frac{1}{(p-1)} &> \frac{1}{(q-1)}.\end{aligned}$$

Because $p, q < 1$ this equation is equivalent to $q > p$. ■

Proof of Proposition 1.

Part 1. Let us first prove that $d^* + E^* > 0$. Suppose, by contradiction, that $d^* + E^* \leq 0$. Given that $E^* > 0$, we have from Lemma 1 and the proof of part **??** of this Proposition, that public good provision must be higher in period 2 than in period 1 (i.e. $G(d^* + E^*) \geq g^*$). This implies that $v'(g^*) > -v'(G(d^* + E^*))\, G'(d^* + E^*)$. Therefore, group A could increase its lifetime payoff by increasing public good provision in period 1. To do so, group A just has to increase debt (or reduce surplus). So, $d^* + E^* \leq 0$ cannot be true. We now prove that $d^* + E^* > d^*_{E=0}$. From Proposition 11 part 4, we know that, for $\overline{E} < E^*$, increasing $\overline{E}$ increases $d\left(\overline{E}\right) + \overline{E}$. Thus, to get the result, it suffices to note that, in $\overline{E} = 0$, $d\left(\overline{E}\right) + \overline{E} = d^*_{E=0}$, and that, in $\overline{E} = E^*$, $d\left(\overline{E}\right) + \overline{E} = d^* + E^*$.

Part 2. If, by contradiction, group A were to choose $d^* + E^* = 1$ then period 2's budget constraint implies the no public good could be provided. Because of Inada conditions for group A's utility function, this could never be an optimal choice for group A.

Part 3. Fix any d (for example, $d = d^*$) and consider the vector (x, g, E) that solves problem (3) conditional on the debt level being set at d. The conditional problem is separable in the sense that the x and g that solve the conditional problem (3) are the solutions to the following simpler problem which does not involve E:

$$\max_{(x,g)} h(x) + v(g) \text{ s.t. } x + g \leq 1 + d.$$

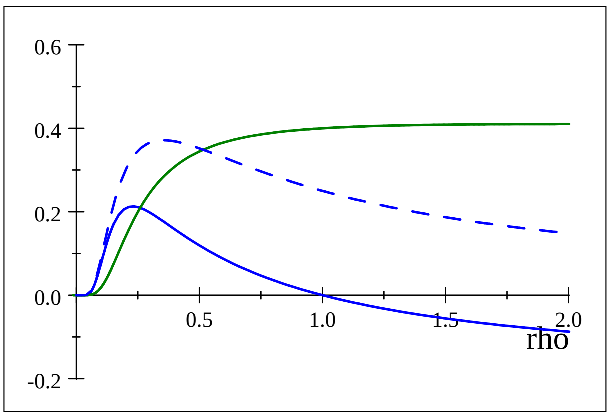
This problem was introduced in Definition 1, and so the g that solves the conditional problem (3) must be exactly $G(-d)$. The solution to the unconditional problem (3) is then $g^* = G(-d^*)$. Now from Lemma 1 we know that $G(\cdot)$ is a decreasing function, so if d^* is positive then $g^* = G(-d^*) > G(d^* + E^*)$. The inequality is the desired conclusion.

Part 4. Suppose, by contradiction, that the marginal value of group A's private consumption was larger in period 2. Then group A could reduce debt (or increase surplus) by one unit, and simultaneously increase entitlements by one unit. This operation would not change group B's optimization problem in period 2, hence public good provision in period 2 would be unchanged; and it would increase group A's lifetime utility.

Part 5. See Proposition 6 part 3 in the Appendix F. ■

Additional Example

In the text we showed how debt and entitlements respond to changes in ρ for a case in which there is relatively high conflict between groups, i.e., $\alpha = 0.5$. We now consider a case with relatively low conflict, i.e., $\alpha = 1.5$. When $\rho \to 0$, there is essentially no conflict between groups because group B spends the entire budget on the public good. Thus, in this case, the optimal debt (entitlements) level goes to zero as ρ goes to zero. When ρ increases, conflict starts mattering, but the effect differs for debt and entitlements. For debt, and the crowdout effect first dominates so debt rises until it is overtaken by the smoothing effect and debt drops. For entitlements, both effects pull in the same direction. Because group B starts allocating resources to its private consumption, the crowdout effect pulls entitlements up. Because of the change in the concavity of the utility function, group A wants to balance private and public consumption more in period 2. Given that we start from zero private consumption (and full public consumption), this requires an increase in entitlements.



Entitlements (Green); Debt with entitlement (Plain Blue); Debt without entitlements (Dashed Blue).

11 Proofs of Section 6

Lemma 3 *The second period equilibrium allocation, as a function of d and E, is:*

$$\begin{aligned} x_B(d, E, \alpha) &= \frac{1}{1+\alpha^{\frac{1}{\rho}}}(1-d-E), \\ G_B(d, E, \alpha) &= \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}(1-d-E). \end{aligned}$$

Proof. Group B maximizes the following expression:

$$U_B = (x_B)^{1-\rho} + \alpha (G_B)^{1-\rho} = \frac{(1-d-E-G_B)^{1-\rho}}{1-\rho} + \alpha \frac{(G_B)^{1-\rho}}{1-\rho}.$$

This maximization immediately delivers the desired expressions for $x_B(d, E, \alpha)$ and $G_B(d, E, \alpha)$ ■

Let us now turn to period 1. Here we focus on the case in which $s \neq 0$ in equilibrium. As we will see, this is guaraneed when d, $\bar{\tau}$, and $\underline{\tau}$ are such that $d < \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$ or $d > \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$.

Given the second period equilibrium allocation and $s \neq 0$, group A's problem in period 1 is

$$\begin{aligned}
&\max_{g,d,E} \frac{\left(1+d-g^1-s\right)^{1-\rho}}{1-\rho}+\alpha\frac{(g)^{1-\rho}}{1-\rho}+\frac{(s\tau+E)^{1-\rho}}{1-\rho}+\alpha\frac{\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}(1-d-E)\right)^{1-\rho}}{1-\rho} \\
\text{s.t. } s \;=\; &\arg\max_{s} \frac{(1+d-g-s)^{1-\rho}}{1-\rho}+\alpha\frac{(g)^{1-\rho}}{1-\rho}+\frac{(s\tau+E)^{1-\rho}}{1-\rho}+\alpha\frac{\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}(1-d-E)\right)^{1-\rho}}{1-\rho}
\end{aligned}$$

Lemma 4 *The equilibrium level of savings as a function of* d, E, *and* g *is:*

$$s(d,E,g^1)=\frac{1}{\left(\tau+\tau^{\frac{1}{\rho}}\right)}\left(\tau^{\frac{1}{\rho}}(1+d-g)-E\right).$$

Proof. The FOC gives:

$$\begin{aligned}
(1+d-g-s)^{-\rho} &= \tau\,(\tau s+E)^{-\rho} \\
(\tau s+E) &= \tau^{\frac{1}{\rho}}(1+d-g-s) \\
s(d,E,g) &= \frac{1}{\left(\tau+\tau^{\frac{1}{\rho}}\right)}\left(\tau^{\frac{1}{\rho}}(1+d-g)-E\right).
\end{aligned}$$

■

Using the previous Lemma, we can rewrite group A's problem:

$$\max_{g,d,E}\frac{\left(1+\tau^{\frac{1-\rho}{\rho}}\right)\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{1-\rho}}{1-\rho}+\alpha\frac{(g)^{1-\rho}}{1-\rho}+\alpha\frac{\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}(1-d-E)\right)^{1-\rho}}{1-\rho}$$

Lemma 5 *Fix* $d < \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$ *or* $d > \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$.

Then the first period equilibrium allocation, as a function of d, is:

$$g^{*}(d)=\frac{\alpha^{1/\rho}(d(\tau-1)+\tau+1)\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left((\tau)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)},$$

$$E^{*}(d)=1-d-\frac{\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}(d(\tau-1)+\tau+1)}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left((\tau)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}$$

and

$$s^{*}(d)=1+d-\frac{\left(\alpha^{\frac{1}{\rho}}+1\right)(d(\tau-1)+\tau+1)\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left((\tau)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}$$

with $\tau=\underline{\tau}$ *if* $\dfrac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\underline{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}<d$ *and* $\tau=\bar{\tau}$ *if* $d<\dfrac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$.

Proof. The first order conditions give

$$\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{-\rho}\left(-\frac{\tau}{\tau+\tau^{\frac{1}{\rho}}}\right)+\alpha\left(g^{1}\right)^{-\rho}+\left(\frac{\tau^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{-\rho}\left(-\tau\frac{\tau^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}\right)=0$$

$$\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{-\rho}=\alpha\left(g^{1}\right)^{-\rho}$$

$$g\left(1+\alpha^{\frac{1}{\rho}}\frac{\tau}{\tau+\tau^{\frac{1}{\rho}}}\right)=\frac{\alpha^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau)$$

$$\frac{1}{\left(1+\alpha^{\frac{1}{\rho}}\frac{\tau}{\tau+\tau^{\frac{1}{\rho}}}\right)}\frac{\alpha^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau)=g$$

$$\frac{\alpha^{\frac{1}{\rho}}}{\tau+\alpha^{\frac{1}{\rho}}\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau)=g$$

and

$$\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{-\rho}\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\right)+\left(\frac{\tau^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}(\tau+E+d\tau-g\tau)\right)^{-\rho}\left(\frac{\tau^{\frac{1}{\rho}}}{\tau+\tau^{\frac{1}{\rho}}}\right)$$

$$=\quad\alpha\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-\bar{d}-E\right)\right)^{-\rho}$$

which rewrites as:

$$
\begin{aligned}
&\frac{1}{\tau}\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+E+d\tau-g\tau\right)\right)^{-\rho} = \left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-\bar{d}-E\right)\right)^{-\rho} \\
&\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-\bar{d}-E\right)\right) = \left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\left(\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+E+d\tau-g\tau\right)\right) \\
&\frac{\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-d\right)-\left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+d\tau-g\tau\right)}{\left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}+\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}} = E
\end{aligned}
$$

Combining the FOCs, we get:

$$
\begin{aligned}
g^* &= \frac{\alpha^{\frac{1}{\rho}}}{\tau+\alpha^{\frac{1}{\rho}}\tau+\tau^{\frac{1}{\rho}}}\left(\tau+\frac{\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-\bar{d}\right)-\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+\bar{d}\tau-g\tau\right)}{\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\right)}+\bar{d}\tau\right) \\
&= \frac{\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}
\end{aligned}
$$

which simplifies to

$$
g^*\left(d\right) = \frac{\alpha^{1/\rho}(d(\tau-1)+\tau+1)\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\left(\tau\right)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}
$$

and thus

$$
\begin{aligned}
E^*\left(d\right) &= \frac{1}{\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}+\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}}\left[\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1-d\right)\right. \\
&\quad \left.-\left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+d\tau-\frac{\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}\tau\right)\right]
\end{aligned}
$$

which simplifies to:

$$
E^*\left(d\right) = 1-d-\frac{\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}(d(\tau-1)+\tau+1)}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\left(\tau\right)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}.
$$

Thus:

$$
\begin{aligned}
s^{*} \quad = \quad & \frac{1}{\left(\tau+\tau^{\frac{1}{\rho}}\right)}\left[\tau^{\frac{1}{\rho}}\left(1+\bar{d}-\frac{\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}\right)\right. \\
& -\frac{\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}(1-d)-\left(\frac{\alpha^{\frac{1}{\rho}+1}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}\left(\tau+d\tau-\frac{\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}\tau\right)}{\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}}\frac{1}{\tau+\tau^{\frac{1}{\rho}}}+\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}}
\end{aligned}
$$

which simplifies to:

$$
\begin{aligned}
s^{*} \quad = \quad & \frac{1}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}\left[d\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\right. \\
& -\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}} \\
& \left.+d\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\right]
\end{aligned}
$$

and then to:

$$
s^{*}(d)=1+d-\frac{\left(\alpha^{\frac{1}{\rho}}+1\right)(d(\tau-1)+\tau+1)\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left((\tau)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}
$$

$s^{*}(d)$ is positive iff:

$$
\begin{aligned}
0 \quad < \quad & \frac{1}{\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\tau\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}} \\
& \times\left[d\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right. \\
& \left.+d\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+d\alpha^{\frac{1}{\rho}}\tau^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}\right],
\end{aligned}
$$

which rewrites as, successively:

$$\begin{aligned}
& d\left(\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\right) \\
> \; & \alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{2}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\left(\alpha\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}; \\
& d\left(\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+1\right) \\
> \; & \left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-1; \\
d \; > \; & \frac{\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}-1}{\left(\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+1\right)}; \\
d \; > \; & \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}
\end{aligned}$$

Thus $d < \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$ guarantees that there is an equilibrium with $s = s^* < 0$, and $\frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\tau}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1} < d$ guarantees that there is an equilibrium with $s = s^* > 0$.

■

Proof of Proposition 2

Proof. From the previous proposition, we have that

$$E^*(d) + d = 1 - \frac{\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}(d(\tau-1)+\tau+1)}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left((\tau)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}$$

$$
\begin{aligned}
\frac{\partial\left(E+d\right)}{\partial d} &= \frac{\partial\left(1-\frac{\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\left(d(\tau-1)+\tau+1\right)}{1+\left(\alpha^{\frac{1}{\rho}}+1\right)\left(\left(\tau\right)^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)}\right)}{\partial d} \\
&= -\left(\tau-1\right)\left(\alpha^{\frac{1}{\rho}}+1\right)\frac{\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\alpha^{\frac{1}{\rho}}\tau^{1-\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}+\tau^{\frac{1}{\rho}(\rho-1)}+1}
\end{aligned}
$$

which is negative since $d < \frac{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}-\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)-1}{\left(1+\alpha^{\frac{1}{\rho}}\right)\left(\left(\frac{1}{\bar{\tau}}\right)^{\frac{1}{\rho}}+\left(\frac{\alpha^{\frac{1}{\rho}}}{\alpha^{\frac{1}{\rho}}+1}\right)^{\frac{1}{\rho}}\right)+1}$ guarantees that the equilibrium is such that $s^* < 0$ and $\tau = \bar{\tau} > 1$. This implies that group B2 is worse off as d becomes smaller. It is obvious that group A is worse off when more constrained. Finally, group B1's utility is derived from consuming $g^*\left(d\right)$, which is increasing in d since $\tau = \bar{\tau} > 1$; therefore group B1's utility is worse off as d decreases. ■

Appendix D: Proofs of Section 7

The next proof makes use of the following property:

Lemma 6 *Let $f(x,y)$ be a concave function. Denote $y^*\left(x\right) = \arg\max_x f(x,y)$ and $F\left(x\right) = f\left(x, y^*\left(x\right)\right)$. Then $F\left(x\right)$ is concave.*

Proof. The first derivative of F is:

$$
\frac{\partial}{\partial x}F\left(x\right) = f_x + f_y \cdot y^{*\prime}\left(x\right).
$$

The second derivative of F is:

$$
\begin{aligned}
\frac{\partial^2}{\partial x^2}F\left(x\right) &= f_{xx} + f_{xy}\cdot y^{*\prime}\left(x\right) + \left[f_{yx} + f_{yy}\cdot y^{*\prime}\left(x\right)\right]y^{*\prime}\left(x\right) + f_y \cdot y^{*\prime\prime}\left(x\right) \\
&= \begin{bmatrix} 1 & y^{*\prime}\left(x\right)\end{bmatrix}\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy}\end{bmatrix}\begin{bmatrix} 1 \\ y^{*\prime}\left(x\right)\end{bmatrix} + f_y \cdot y^{*\prime\prime}\left(x\right).
\end{aligned}
$$

The last addend is zero because $f_y = 0$ by definition of $y^*\left(x\right)$. So the above is a quadratic form of the Hessian, and the quadratic form is negative by concavity of $f(x,y)$. Hence $\frac{\partial^2}{\partial x^2}F\left(x\right) < 0$. ■

Proof of Proposition 4.

Part 1. Substitute $E\left(d\right) \equiv 0$ into equation (9) below and repeat the argument that follows. Proposition 10 part 1 ensures that at $\pi = 0$ we have $d^* > 0$. The desired conclusion then follows from part 4.

Part 2. If group A persists in power it faces the following period-2 allocation problem:

$$
\max_{x,g} h\left(x\right) + v\left(g\right) \text{ s.t. } x + g \leq 1 - d.
$$

The solution to this problem is given by $X(d)$, $G(d)$ (refer to Lemma 1). Using these expressions we can write group A's allocation problem as follows:

1.
$$\max_{d,E} U(d) + \pi\left[h(1-d-G(d)) + v(G(d))\right] + (1-\pi)\left[h(E) + v(G(d+E))\right], \tag{8}$$

where $U(d)$ is defined in (22). Fix d; due to multiplicative separability of E and π, the optimal level of entitlements $E(d)$ is independent of π. This means that any effect of varying π on E^*, the equilibrium level of entitlements, must be channeled through the choice of debt, that is:
$$\frac{\partial E^*}{\partial \pi} = \frac{\partial E(d)}{\partial d}\frac{\partial d^*}{\partial \pi}.$$
Because $E(d)$ is independent of π, Proposition 11 part 1 applies, ensuring that $\partial E(d)/\partial d < 0$. Thus we have proved that equilibrium debt and entitlement move in opposite directions when π varies.

Part 3. It follows directly from the previous step that
$$\frac{\partial d^*}{\partial \pi} + \frac{\partial E^*}{\partial \pi} = \frac{\partial d^*}{\partial \pi}\left(1 + \frac{\partial E(d)}{\partial d}\right).$$

It thus remains to prove that $1 + \frac{\partial E(d)}{\partial d} > 0$. Differentiating equation (4) implicitly, we obtain:
$$\frac{\partial E}{\partial d} = -\frac{v''(G_B(d+E))(G_B'(d+E))^2 + v'(G_B(d+E))G_B''(d+E)}{h''(E) + v''(G_B(d+E))(G_B'(d+E))^2 + v'(G_B(d+E))G_B''(d+E)}$$

From the concavity of $v(G_B(\cdot))$ and the concavity of $h(\cdot)$, we have that this is larger than -1.

Part 4. We first need to prove that the objective function in equation (8) is concave. Rewrite this by distributing $U(d)$ in the two brackets. This yields
$$\begin{aligned}&\pi\left[h(1+d-g^*(d)) + v(g^*(d)) + h(1-d-G(d)) + v(G(d))\right] + \\ &(1-\pi)\left[h(1+d-g^*(d)) + v(g^*(d)) + h(E(d)) + v(G(d+E(d)))\right].\end{aligned} \tag{9}$$

We prove that the functions in the two brackets are concave in d. Taking the second derivative of the first bracket gives:
$$\begin{aligned}&\pi[h''(1+d-g(d))(1-g'(d))^2 - h'(1+d-g(d))g''(d) + v''(g(d))(g'(d))^2 + v'(g(d))g''(d) + \\ &h''(1-d-G(d))(1+G'(d))^2 - h'(1-d-G(d))G''(d) + v''(G(d))(G'(d))^2 + v'(G(d))G''(d)]\end{aligned}$$

By definition of $g^*(d)$ we have $h'(1+d-g^*(d)) = v'(g^*(d))$, and by definition of $G(d)$, we have $h'(1-d-G(d)) = v'(G(d))$. Therefore, the expression boils down to
$$\pi[h''(1+d-g(d))(1-g'(d))^2 + v''(g(d))(g'(d))^2 + h''(1-d-G(d))(1+G'(d))^2 + v''(G(d))(G'(d))^2]$$

From the concavity of $h(\cdot)$, $v(\cdot)$, and $v(G(\cdot))$, we have that this is necessarily negative. For the function in the second bracket, first note that the following function is concave in (d, E):
$$h(1+d-g^*(d)) + v(g^*(d)) + h(E) + v(G(d+E)).$$

We know from the proof of Proposition 11 that $h(1+d-g^*(d))+v(g^*(d))$ is concave in d and thus (d,E). Furthermore, we have that (i) $h(E)$ is a concave function of (d,E) (by assumption), and (ii) because $d+E$ is a concave function of (d,E), and $v(G(\cdot))$ is concave (by assumption), then $v(G(d+E))$ is also concave in (d,E). Given that $E(d)$ solves problem 20, we can apply Lemma 6, to show that the function in the second bracket is concave in d.

We can now prove that $d^*(\pi)$, the value of d that maximizes 9 for a given π, is monotonic in π. By the concavity of the functions in the two brackets of 9, we have that

$$\frac{\partial}{\partial d}\left[h(1+d-g^*(d))+v(g^*(d))+h(1-d-G(d))+v(G(d))\right], \text{ and}$$
$$\frac{\partial}{\partial d}\left[h(1+d-g^*(d))+v(g^*(d))+h(E(d))+v(G(d+E(d)))\right]$$

are both decreasing functions. Denoting by d_1 and d_2, respectively, the values at which these two functions are equal to 0, we have that $d^*(\pi)$ lies in the interval with extremes d_1 and d_2, where d_1 may be larger or smaller than d_2. In that interval, one of the two functions must be negative, and the other one positive. Thus, increasing π moves $d^*(\pi)$ monotonically towards d_1. To conclude the proof, note that we have $d^*(1)=0$ (when A is the social planner, smoothing requires no debt or surplus); thus if debt started out positive at $\pi=0$ then it must monotonically decrease with π; and if debt started out negative at $\pi=0$ then it must monotonically increase in π. Together with part 2 of this Proposition, this shows that entitlements are monotonic in π (in the opposite direction as debt). ■

12 Appendix E: Proofs of Section 8

Proof of Proposition 5. From Proposition 8 in Appendix F. we have that: E^* is decreasing in α for any $\rho>0$; for $\rho<1$, d^* is decreasing in α if and only if α is smaller than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$; for $\rho>1$, d^* is decreasing in α if and only if α is greater than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$. ■

Appendix F: CRRA Preferences

In this Appendix we provide closed-form solutions for the equilibrium under the assumption that groups have CRRA preferences, that is, when:

$$u_i\left(x_i^t, x_j^t, g^t\right)=\frac{\left(x_i^t\right)^{1-\rho_i}}{1-\rho_i}+\alpha_i\frac{\left(g^t\right)^{1-\rho_i}}{1-\rho_i},$$

with $\rho_i>0$.

Note that we allow the possibility that $\rho_A\neq\rho_B$ and $\alpha_A\neq\alpha_B$. This type of heterogeneity is not contemplated in the main model, but this extension poses no difficulties. To simplify notation, $\rho_A = \rho$ and $\alpha_A = \alpha$.

Period 2

Group B's problem is

$$\max_{g} \frac{(1-d-E-g)^{1-\rho_B}}{1-\rho_B} + \alpha_B \frac{g^{1-\rho_B}}{1-\rho_B}.$$

The first order with respect to g read:

$$(1-d-E-g)^{-\rho_B} = \alpha_B g^{-\rho_B},$$

and solving for g yields the public good allocation in period 2:

$$G(d+E) = \frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-d-E). \tag{10}$$

Group B's private consumption in period 2, $X(d+E)$, can be recovered from the budget constraint $G(d+E) + X(d+E) = 1-d-E$.

Note that this implies $v(G(d+E)) = \frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-d-E)\right)^{1-\rho_B}}{1-\rho_B}$.

Period 1

Group A's problem is:

$$\max_{g,E,d} \frac{(1+d-g)^{1-\rho}}{1-\rho} + \alpha\frac{g^{1-\rho}}{1-\rho} + \frac{E^{1-\rho}}{1-\rho} + \alpha\frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-d-E)\right)^{1-\rho}}{1-\rho}.$$

Let $g^*(\overline{d})$ and $E\left(\overline{d}\right)$ denote the optimal choice of period-1 public good and entitlements, respectively, conditional on a debt level $\overline{d}$.

Lemma 7 ***(period-1 equilibrium allocations conditional on a given debt level in the CRRA case)***

1. *The equilibrium allocation of period-1 public good conditional on a debt level $\overline{d}$, is $g^*(\overline{d}) = \left(1+\overline{d}\right)\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}$;*

2. The equilibrium allocation of entitlements conditional on a debt level $\overline{d}$, is $E(\overline{d}) = \left(1-\overline{d}\right) / \left[1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}\right]$.

Proof.

Part 1. The function $g^*(\overline{d})$ solves the following problem:

$$g^*(\overline{d}) = \arg\max_g \frac{\left(1+\overline{d}-g\right)^{1-\rho}}{1-\rho} + \alpha\frac{g^{1-\rho}}{1-\rho}.$$

The first order with respect to g reads:

$$\left(1+\overline{d}-g\right)^{-\rho_B} = \alpha g^{-\rho},$$

and solving for g yields:

$$g^*(\overline{d}) = \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1+\overline{d}\right)$$

Part 2. The function $E(\overline{d})$ solves the following problem:

$$E(\overline{d}) = \arg\max_E \frac{E^{1-\rho}}{1-\rho} + \alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{1-\rho}\frac{\left(1-\overline{d}-E\right)^{1-\rho}}{1-\rho}.$$

The first order with respect to E read:

$$\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{1-\rho}\left(1-\overline{d}-E\right)^{-\rho} = E^{-\rho},$$

and solving for E yields, after some algebra:

$$E(\overline{d}) = \frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}\left(1-\overline{d}\right). \tag{11}$$

■

Lemma 8 *The equilibrium level of debt in the CRRA case is determined by the following condition:*

$$\frac{(1-d)}{(1+d)} = \left(1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{\left(1+\alpha^{\frac{1}{\rho}}\right)}. \tag{12}$$

Proof. Substitute the expressions for $g^*(\overline{d})$ and $E(\overline{d})$ into group A's period-1 problem:

$$
\begin{aligned}
&\max_{d} \frac{\left(1+d-K\left(1+d\right)\right)^{1-\rho}}{1-\rho}+\alpha\frac{\left(K\left(1+d\right)\right)^{1-\rho}}{1-\rho}+\frac{\left(Z\left(1-d\right)\right)^{1-\rho}}{1-\rho}+\alpha\frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-d-Z\left(1-d\right)\right)\right)^{1-\rho}}{1-\rho} \\
=\ &\max_{d} \frac{\left(1+d\right)^{1-\rho}}{1-\rho}\left[\left(1-K\right)^{1-\rho}+\alpha K^{1-\rho}\right]+\frac{\left(1-d\right)^{1-\rho}}{1-\rho}\left[Z^{1-\rho}+\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-Z\right)\right)^{1-\rho}\right],
\end{aligned}
\tag{13}
$$

where we have denoted $K=\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}$ and $Z=1/\left[\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}+1\right]$. The problem is concave in d. The first order conditions with respect to d are:

$$
\left(1+d\right)^{-\rho}\left[\left(1-K\right)^{1-\rho}+\alpha K^{1-\rho}\right]=\left(1-d\right)^{-\rho}\left[Z^{1-\rho}+\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-Z\right)\right)^{1-\rho}\right],
$$

or equivalently:

$$
\frac{\left(1-d\right)}{\left(1+d\right)}=\left\{\left[Z^{1-\rho}+\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-Z\right)\right)^{1-\rho}\right]\left[\frac{1}{\left(1-K\right)^{1-\rho}+\alpha K^{1-\rho}}\right]\right\}^{1/\rho}.
$$

After much algebra, the first term in square brackets simplifies to:

$$
\left(\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}+1\right)^{\rho},
$$

and the second term in square brackets simplifies to:

$$
\left(1+\alpha^{\frac{1}{\rho}}\right)^{-\rho}.
$$

Substituting back into the first order conditions we get equation (12). ■

Proposition 6 ***(properties of the equilibrium in the CRRA case).*** *In the CRRA case (with the same α):*

1. *the equilibrium level of debt is strictly interior: $d^* \in (-1,1)$;*
2. *$d^*=0$ when $\rho=1$;*
3. *$d^*>0$ if and only if $\rho<1$;*
4. *the equilibrium level of entitlements is strictly positive, $E^*>0$;*
5. *in equilibrium, total obligations, that is, debt and entitlements, are related by the following proportion: $\left(1+d^*\right)/E^*=\left(1+\alpha^{\frac{1}{\rho}}\right)$.*

Proof.

Part 1. The RHS of (12) is nonnegative. The LHS of (12) goes from $+\infty$ at $d = -1$ to 0 at $d = 1$; and it is a decreasing function of d on $[-1, 1]$. Therefore, the equilibrium level of debt $d^* \in (-1, 1)$.

Part 2. The RHS of (12) equals 1 when $\rho = 1$, hence $d^* = 0$ when $\rho = 1$.

Part 3. $d^* > 0$ if and only if the RHS of (12) is smaller than 1, that is, if and only if:

$$\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}} < 1,$$

which is the case if and only if $1 - \rho > 0$ or $\rho < 1$.

Part 4. $E^* = E(d^*)$. The conclusion follows from expression (11) and the fact that $d^* < 1$.

Part 5. Combine expressions (11) and (12) to get:

$$\begin{aligned}
E(d^*) &= \frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}(1-d^*) \\
&= \frac{1+d^*}{\left(1+\alpha^{\frac{1}{\rho}}\right)}.
\end{aligned}$$

■

Proposition 7 *In the CRRA case (with the same α) we have: $d^* = \frac{1-h(z)}{1+h(z)}$ and $E^* = \frac{2}{[1+h(z)](1+z)}$ where $z = \alpha^{\frac{1}{\rho}}$ and $h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}$.*

Proof. From Lemma 8, we know that

$$\frac{(1-d)}{(1+d)} = \left(1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{\left(1+\alpha^{\frac{1}{\rho}}\right)}.$$

Make the change of variable: $\alpha^{\frac{1}{\rho}} = z > 0$. Because $\rho > 0$ by assumption, z is a monotonic function of α. Then

$$\begin{aligned}
\frac{(1-d)}{(1+d)} &= \left(1+z\left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{(1+z)} \\
&= \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}.
\end{aligned}$$

Denote:

$$h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}.$$

Then we have:

$$\begin{aligned}\frac{(1-d)}{(1+d)} &= h(z), \\ d^{*} &= \frac{1-h(z)}{1+h(z)}.\end{aligned} \tag{14}$$

Take the expression for $E(d)$ from Lemma 7, and substitute z to get:

$$E(d) = \frac{(1-d)}{(1+z)\,h(z)}.$$

Substitute from (14) to get:

$$E^{*} = \frac{2}{[1+h(z)]\,(1+z)}$$

■

Proposition 8 ***(comparative statics with respect to*** α*) In the CRRA case (with the same* α*) :*

1. *E^{*} is decreasing in α for any $\rho > 0$.*
2. *d^{*} is decreasing in α: for $0 < \rho < 1$ if and only if α is larger than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$; for $\rho > 1$ if and only if α is smaller than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.*

Proof.
Part 1. E^{*} is increasing in z if and only if $[1+h(z)]\,(1+z)$ is decreasing in z. First, compute:

$$h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}.$$

$$h'(z) = -\frac{1}{(1+z)^{2}} + \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)^{2}}.$$

Then

$$\begin{aligned} & \frac{\partial}{\partial z}[1+h(z)]\,(1+z) \\ = \; & h'(z)\,(1+z) + 1 + h(z) \\ = \; & \left[-\frac{1}{(1+z)^{2}} + \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)^{2}}\right](1+z) + 1 + \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}} \\ = \; & \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)} + 1 + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}} > 0. \end{aligned}$$

Hence E^* is decreasing in z, and hence in α, for any $\rho > 0$.
Part 2. We have:

$$\begin{aligned} d^* &= \frac{1-h(z)}{1+h(z)} \\ \frac{\partial}{\partial z} d^* &= \frac{-h'(z)(1+h(z)) - (1-h(z))h'(z)}{(1+h(z))^2} \end{aligned}$$

This has the same sign as its numerator:

$$\begin{aligned} & -h'(z)(1+h(z)) - (1-h(z))h'(z) \\ = & -2h'(z) \\ = & \ 2\frac{1}{(1+z)^2}\left[1 - \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\right]. \end{aligned}$$

This expression is positive iff

$$\rho > \left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}.$$

So debt is increasing in z if and only if $\rho > \left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}$. In the case $0 < \rho < 1$, this condition rewrites as:

$$\begin{aligned} 0 &< \rho < \left(\frac{\alpha^{1/\rho}}{1+\alpha^{1/\rho}}\right)^{\frac{1-\rho}{\rho}} \\ -\rho^{\frac{\rho}{1-\rho}}\alpha^{1/\rho} &< \rho^{\frac{\rho}{1-\rho}} < \alpha^{1/\rho} - \rho^{\frac{\rho}{1-\rho}}\alpha^{1/\rho} \\ -\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}\alpha &< \left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho} < \alpha \end{aligned}$$

So if $0 < \rho < 1$ then debt is decreasing in α for α larger than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$. In the case $1 < \rho$, the condition rewrites as:

$$\begin{aligned} 0 &< \rho < \left(\frac{\alpha^{1/\rho}}{1+\alpha^{1/\rho}}\right)^{\frac{1-\rho}{\rho}} \\ \infty &> \rho^{\frac{\rho}{1-\rho}}\left(1+\alpha^{1/\rho}\right) > \alpha^{1/\rho} \\ \infty &> \left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho} > \alpha \end{aligned}$$

So if $1 < \rho$ then debt is decreasing in α for α smaller than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$. ■

Proposition 9 ***(welfare properties of a constrained equilibrium in the CRRA case).*** *In the CRRA case with $\alpha_B = \alpha$ and $\rho_B = \rho$:*

1. a zero-debt cap is binding if and only if $\rho < 1$*;*

2. relaxing a binding zero-debt cap is Pareto-optimal if:

$$\left(\alpha^{\frac{1}{\rho}}\right)^{\frac{2\rho-1}{\rho}}\left(1+\alpha^{\frac{1}{\rho}}\right)^{\frac{1-\rho}{\rho}}+\left(\alpha^{\frac{1}{\rho}}\right)^{\frac{\rho+1}{\rho}}>\left(1+\alpha^{\frac{1}{\rho}}\right)^{2};$$

3. if $\alpha = 1$ *then relaxing a zero-debt cap is Pareto-optimal if* $\rho < \log 2/\log 6 \simeq 0.39$.

4. if $\rho = 1-\varepsilon$ *then relaxing a binding zero-debt cap is never Pareto-optimal for any* α *provided that* ε *is sufficiently small.*

Proof.

Part 1. Expression (13) describes group A's period-1 allocation problem as a function of d alone. In the case $\rho < 1$ the unconstrained optimum d^* exceeds 0 (see Proposition 6 part 3). A zero-debt cap rule out d^*. Because problem (13) is concave in d, the constrained optimum is for group A to choose debt right up to the ceiling. Thus a zero-debt cap is binding. If $\rho > 1$ Proposition 6 part 3 indicates that a zero-debt cap is not binding.

Part 2. In the CRRA case, we can leverage (10) and Lemma (7) to write:

$$\begin{aligned}
-G'\left(\overline{d}+E(\overline{d})\right) &= \frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}};\\
\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} &= \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}};\\
E'\left(\overline{d}\right) &= -\frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}.
\end{aligned}$$

Recall the condition Proposition 12 states that if a debt ceiling $\overline{d}$ is binding for group A, then relaxing the debt ceiling increases group B's lifetime utility if

$$-G'\left(\overline{d}+E(\overline{d})\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} > 1+E'\left(\overline{d}\right).$$

Substitute to get:

$$\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}} > 1-\frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}$$

Set $\alpha_B = \alpha$ and $\rho_B = \rho$, then manipulate the previous equation to get:

$$\left(\alpha^{\frac{1}{\rho}}\right)^{\frac{2\rho-1}{\rho}}\left(1+\alpha^{\frac{1}{\rho}}\right)^{\frac{1-\rho}{\rho}}+\left(\alpha^{\frac{1}{\rho}}\right)^{2}>\left(1+\alpha^{\frac{1}{\rho}}\right)^{2}. \tag{15}$$

Part 3. When $\alpha = 1$ condition (15) specializes to:

$$2^{\frac{1-\rho}{\rho}} + 1 > 4.$$

After some manipulation, this condition is seen to be equivalent to:

$$\rho < \frac{\log 2}{\log 6}.$$

Part 4. When $\rho = 1$ condition (15) specializes to:

$$\alpha + \alpha^2 > (1 + \alpha)^2 .$$

This condition never holds for any $\alpha > 0$. Therefore, if $\rho = 1 - \varepsilon$ condition (15) must also fail for ε small enough. ■

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13 Scratch material

Before proceeding to discuss the characterization of the first period allocation, and of debt and entitlements, it is useful to obtain a benchmark result in which, as in the prior literature, we do not allow for entitlements. We will then contrast this with the case in which entitlements are allowed. A useful feature of our model is that, absent entitlements, it behaves similarly to prior literature on debt, notably the literature following Alesina and Tabellini (1990). In this literature, debt arises whenever the currently powerful generation fears the loss of political power in the future.

Proposition 10 ***(No entitlement benchmark)*** *Fix $E \equiv 0$. Then in equilibrium:*

1. *Group A runs up debt in period 1, $d^*_{E=0} > 0$;*

2. *Group A's private consumption decreases between periods 1 and 2;*

3. *Public good provision decreases between periods 1 and 2.*

Proof. See Appendix A. ■

Proof of Proposition 10.

Part 1. With the constraint $E^* \equiv 0$, group A's problem (3) specializes to:

$$\max_{(x,g,d)} h(x) + v(g) + v(G(d)) \text{ s.t. } x + g \leq 1 + d.$$

Form the Lagrangian and take first-order conditions to get:

$$v'(g^*_{E=0}) = -v'(G(d^*_{E=0}))\, G'(d^*_{E=0}) = h'(x^*_{E=0}). \tag{16}$$

The proof of Lemma 1 shows that $G'(c) \in (-1, 0)$, hence the first equality in (16) implies $v'(G(d^*_{E=0})) > v'(g^*_{E=0})$, which in turn implies:

$$G(d^*_{E=0}) < g^*_{E=0}. \tag{17}$$

Now take the second equality in (16), substitute from equation (6) in the proof of Lemma 1, and again use $G'(c) \in (-1, 0)$ to write:

$$h'(1 - d^*_{E=0} - G(d^*_{E=0})) > h'(x^*_{E=0}),$$

whence:

$$1 - d^*_{E=0} - G(d^*_{E=0}) < x^*_{E=0}. \tag{18}$$

Adding up (17) and (18) yields:

$$1 - d^*_{E=0} < x^*_{E=0} + g^*_{E=0}.$$

The last line can be rewritten as $X\left(d^*_{E=0}\right) + G\left(d^*_{E=0}\right) < x^*_{E=0} + g^*_{E=0}$, which implies that more resources are allocated to private and public good consumption in in period 1 than in period 2 (the implication follows because group 2 in period 1, as well as group 1 in period 2, are allocated zero private consumption). So it must be $d^*_{E=0} > 0$.

Part 2. In equilibrium group 1 is not allocated any private good in period 2.

Part 3. See (17). ■

To gain some intuition for this result, it is useful to examine the first order condition determining debt. Equation (19) below is obtained by differentiating the objective function (3) with respect to d (and fixing $E = 0$):

$$h'(x) = -v'(G(d))\, G'(d). \tag{19}$$

If group A were in charge in both periods, then the term $G'(d)$ would not appear. This term captures the fact that an extra dollar left uncommitted to period 2 only increases public consumption by $G'(d) < 1$, the marginal amount chosen by group B, with the remainder going to group B's private consumption. We call the presence of this term the *crowdout effect*. The crowdout effect gives an incentive to increase debt. There is also a *smoothing effect* that works as follows: because of concavity in the utility function, group A wants to smooth consumption over time. If public consumption is smaller in period 2 (which must be the case when debt is positive), then the smoothing effect gives an incentive to decrease debt. The balance of the crowdout effect and the smoothing effect determines the equilibrium level of debt.

Part 2 is immediate: group B has no incentive to allocate resources to group A's private consumption since it does not benefit from doing so.

Part 3 follows directly from part 1: since debt is positive, there are fewer resources effectively available for consumption, hence less public good, in period 2 than in period 1.

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14 Strategic Substitutes Property

We now discuss the relation between debt and entitlements. The next proposition establishes that the effects of tightening a debt ceiling are partly (but not fully) offset by a strategic adjustment in entitlements–and viceversa, that the effects of constraining entitlements are partly (but not fully) offset by a strategic increase in debt.

Proposition 11

1. ***(strategic substitutes property)*** *Fix debt at a level $\overline{d}$. The entitlement level $E\left(\overline{d}\right)$ that maximizes group A's lifetime utility conditional on $\overline{d}$ is a decreasing function of $\overline{d}$;*

2. ***(strategic substitutes property)*** *Fix entitlements at a level $\overline{E}$. The debt level $d\left(\overline{E}\right)$ that maximizes group A's lifetime utility conditional on $\overline{E}$ is a decreasing function of $\overline{E}$;*

3. ***(debt ceilings are partially effective)*** *Tightening a binding debt ceiling $\overline{d}$ reduces $\overline{d} + E\left(\overline{d}\right)$, the sum of debt and entitlements in equilibrium;*

4. ***(entitlement caps are partially effective)*** *Tightening a binding cap on entitlements $\overline{E}$ reduces $d\left(\overline{E}\right) + \overline{E}$, the sum of debt and entitlements in equilibrium.*

Proof. See Appendix B. ■

Proof of Proposition 11.

Part 1. Fix any d and consider the vector (x, g, E) that solves problem (3) conditional on the debt level being set at d. The entitlement level that solves the conditional problem is the solution to the following simpler problem which does not involve x or g:

$$\max_{E} h(E) + v(G(d+E)). \tag{20}$$

The first order conditions read:

$$h'(E) = -v'(G(d+E))\, G'(d+E). \tag{21}$$

The LHS is an decreasing function of E. Because $v(G(d+E))$ is concave in E, its first derivative with respect to E, $v'(G(\cdot))\, G'(\cdot)$, is a decreasing function of E. The RHS is its opposite, and therefore an increasing function of E. Now increase d. The LHS function stays unchanged. The RHS function shifts up. Therefore the two functions now cross at a lower level of E.

Part 2. Fix any E and consider the vector (x, g, d) that solves problem (3) conditional on the entitlement level being set at E. The debt level that solves the conditional problem is the solution to the following simpler problem:

$$\max_{(x,g,d)} h(x) + v(g) + v(G(d+E)) \;\text{ s.t. }\; x + g \leq 1 + d.$$

Define the following value function:

$$U(d) = \max_{(x,g)} h(x) + v(g) \;\text{ s.t. }\; x + g - 1 \leq d. \tag{22}$$

Because $h(\cdot)$ and $v(\cdot)$ are concave, $U(d)$ is concave in d. Our problem can then be rewritten as follows:

$$\max_{d} U(d) + v(G(d+E)). \tag{23}$$

This problem is isomorphic to (20), and the rest of the proof follows the argument in part 1.

Part 3. Consider any $\overleftarrow{d} < \overline{d}$. Suppose by contradiction that $\overleftarrow{d} + E\left(\overleftarrow{d}\right) > \overline{d} + E\left(\overline{d}\right)$. Then, because the RHS of (21) is an increasing function of $d + E$, we have:

$$-v'\left(G\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right)\right) G'\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right) > -v'\left(G\left(\overline{d} + E\left(\overline{d}\right)\right)\right) G'\left(\overline{d} + E\left(\overline{d}\right)\right). \tag{24}$$

Now, we know from part part 1 that $E\left(\overleftarrow{d}\right) > E\left(\overline{d}\right)$. Because $h'(\cdot)$ is a decreasing function it follows that:

$$h'\left(E\left(\overleftarrow{d}\right)\right) < h'\left(E\left(\overline{d}\right)\right). \tag{25}$$

By definition of $E\left(\overline{d}\right)$, the RHS of (24) must equal the RHS of (25). But then it follows that:

$$h'\left(E\left(\overleftarrow{d}\right)\right) < -v'\left(G\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right)\right) G'\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right),$$

which contradicts the definition of $E\left(\overleftarrow{d}\right)$.

Part 4. Problem (23) is isomorphic to (20), and the rest of the proof follows the argument in part 3. ■

To understand the mechanism at work in part 1 (which also applies to part 2), imagine that group A is constrained and can only run debt up to $\overline{d}$. This may be because of a fiscal rule or for other reasons. Imagine now that the fiscal rule is relaxed. The relaxation causes a reduction in group B's fiscal capacity in the second period, and therefore a reduction in public-good spending in that period. This reduction raises the marginal cost of entitlements for group A.

Parts 3 and 4 show that, despite the partial crowding out, constraining either debts or entitlements still reduces the total obligations that group A bequeaths to group B.

Despite its simplicity, Proposition 11 has a number of important implications.

First, consider the important literature that has highlighted the role of debt as an instrument that perpetuates temporary power (e.g., Alesina and Tabellini 1990, Tabellini and Alesina 1990, Persson and Svensson 1989).[29] If, consistent with this literature, entitlements were left out of our model (i.e., implicitly set to zero), Proposition 11 part 2 indicates that the equilibrium level of debt would be larger than if entitlements were accounted for by the model. That is, by abstracting from the presence of entitlements, there is a risk of over-estimating the amount of debt that is created in an effort to take advantage of temporary power. Note however, that from Proposition 1 part 1 a model that abstracts from entitlements would underestimate the total level of government obligations (i.e., the sum of debt and entitlements).

Second, Proposition 11 has consequences for the evaluation of fiscal constitutions. For instance, the implementation of fiscal rules or debt ceilings may have the unintended (and difficult-to-measure) consequence of increasing entitlements, thus partially offsetting the reduction in debt. By the same token, implementing pension reforms will make it harder to stabilize government debt. This latter trade-off seems confirmed by the current structure of the EU's *Stability and Growth Pact*, a fiscal rule that binds EU states. In 2005, it was agreed that the spending ceiling enacted by the Pact would be relaxed for countries that implemented structural reforms. As in our Proposition 11, structural reform and deficit

[29] This determinant of debt is a major component of recent developments in the political economy theory of public debt (see, e.g., Battaglini and Coate 2008, Battaglini 2011, and Azzimonti et al. 2015).

reduction are treated as strategic substitutes.

Third, the result yields a testable implication: entitlements should be larger where balanced-budget rules are more stringent.[30] This speaks to the literature exploring the effects of fiscal rules (Poterba 1996, Alesina and Perotti 1999, Badinger and Reuter 2015).

15 Welfare Effects of Constraining Debt or Entitlements

Absent entitlements, in our model, debt is harmful from the perspective of utilitarian social welfare: there is no insurance motive (no shock, recession, war, or natural disaster) in the model that could make debt desirable from the perspective of a utilitarian social planner. In this section we study the welfare effects of fiscal rules constraining debt or entitlements in our setting, where policies are not set by the social planner but emerge endogenously from the political process.

We start with debt. The following proposition shows that, due to the endogenous response of entitlements to tightening debt constraints, the welfare effect of introducing such constraints can be quite different than in a model without entitlements.

Proposition 12 ***(welfare effects of constraining debt)***.

1. *Suppose debt is unconstrained and $h(x) = v(x)$ have a CRRA form $(x)^{1-\rho}/(1-\rho)$. There is a $\hat{\rho}$ such that introducing a barely-binding debt cap leaves group A indifferent at the margin; group B is made strictly better off if $\rho > \hat{\rho}$ and strictly worse off if $\rho < \hat{\rho}$;*

2. *For some parameter configurations, relaxing a binding constraint on debt is a strict Pareto improvement: for example, if $h(x) = v(x)$ have a CRRA form $(x)^{1-\rho}/(1-\rho)$, then relaxing a binding constraint on debt makes both groups A and B better off at the margin if $\rho < \hat{\rho}$.*

Proof. See Appendix C. ■

Proof of Proposition 12. The proof starts with a foundational result. Let $g^*(\overline{d})$ and $E\left(\overline{d}\right)$ denote the optimal choice of period-1 public good and entitlements, respectively, conditional on a debt level $\overline{d}$. Suppose a debt ceiling $\overline{d}$ is binding for group A. We now show that relaxing the debt ceiling increases group B's lifetime utility if

$$-G'\left(\overline{d} + E(\overline{d})\right) \frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} > 1 + E'\left(\overline{d}\right).$$

Let $x^*(\overline{d})$ and $g^*\left(\overline{d}\right)$ denote the optimal choice of private and public goods, respectively, in the first period conditional on a debt level $\overline{d}$. Similarly, $X(\overline{d} + E)$ and $G\left(\overline{d} + E\right)$ denote the optimal

[30] A suggestive piece of prima facie evidence comes from the interaction between entitlements (proxied by the percentage of pensions unfunded)and the stringency of balanced-budget rules in US states (as measured by Hou and Smith 2006). The correlation between the two is indeed positive (0.173).

choice of private and public goods, respectively, by group B in the second period conditional on a debt level $\overline{d}$ and an entitlements level E.

(i) Period-1 social return to relaxing debt limit.

Suppose a binding debt cap is increased by one dollar (so the budget available to group A increases). The variation in period-1 utility for group A is:

$$\frac{\partial W_A^1}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right).$$

The variation in period-1 utility for group B is:

$$\frac{\partial W_B^1}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}}$$

(ii) Period-2 social return to relaxing debt limit.

Let $E\left(\overline{d}\right)$ denote the solution to problem (20) for $d = \overline{d}$. The period-2 total variation in utility for group A is given by

$$\frac{\partial W_A^2}{\partial \overline{d}} = E'(\overline{d})h'\left(E(\overline{d})\right) + \left(1 + E'\left(\overline{d}\right)\right)G'\left(\overline{d} + E(\overline{d})\right)v'\left(G\left(\overline{d} + E(\overline{d})\right)\right).$$

Use the first-order conditions (21) to simplify this expression:

$$\frac{\partial W_A^2}{\partial d} = G'\left(\overline{d} + E(\overline{d})\right)v'\left(G\left(\overline{d} + E(\overline{d})\right)\right).$$

The total variation in utility for group B is given by

$$\frac{\partial W_B^2}{\partial \overline{d}} = \left(1 + E'\left(\overline{d}\right)\right)\left[X'(\overline{d} + E(\overline{d}))h'\left(X(\overline{d} + E(\overline{d}))\right) + G'\left(\overline{d} + E(\overline{d})\right)v'\left(G\left(\overline{d} + E(\overline{d})\right)\right)\right].$$

The first order conditions for group B require that $h'\left(X(\overline{d})\right) = v'\left(G\left(d + E(\overline{d})\right)\right)$, whence

$$\frac{\partial W_B^2}{\partial \overline{d}} = \left(1 + E'\left(\overline{d}\right)\right)\cdot\left[X'(\overline{d} + E(\overline{d})) + G'\left(\overline{d} + E(\overline{d})\right)\right]\cdot v'\left(G\left(\overline{d} + E(\overline{d})\right)\right).$$

From the proof of Lemma 1 we have that $X'(\cdot) + G'(\cdot) = -1$. Substitute into group B's total variation to get:

$$\frac{\partial W_B^2}{\partial \overline{d}} = -\left(1 + E'\left(\overline{d}\right)\right)\cdot v'\left(G\left(\overline{d} + E(\overline{d})\right)\right).$$

(iii) Conditions for improvement in group B's lifetime utility:

By assumption the debt limit is binding for group A, which implies:

$$\frac{\partial W_A^1}{\partial \overline{d}} + \frac{\partial W_A^2}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right) + v'\left(G\left(\overline{d} + E(\overline{d})\right)\right)G'\left(\overline{d} + E(\overline{d})\right) > 0. \tag{26}$$

Group B's lifetime utility is given by:

$$\begin{aligned}\frac{\partial W_B^1}{\partial \overline{d}}+\frac{\partial W_B^2}{\partial \overline{d}} &= v'\left(g^*\left(\overline{d}\right)\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}}-v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)\left(1+E'\left(\overline{d}\right)\right) \\ &> v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)\left[-G'\left(\overline{d}+E(\overline{d})\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}}-\left(1+E'\left(\overline{d}\right)\right)\right], \end{aligned} \tag{27}$$

where the inequality follows from (26). This expression has the same sign as the expression in brackets. The desired statement follows (remember that we now work under the assumption that $h(x) = v(x)$ have a CRRA form $(x)^{1-\rho}/(1-\rho)$).

Part 1. If debt is unconstrained, that is, $\overline{d} = d^*$, then the inequality in (26) is replaced by an equality. Consequently, the inequality in (27) is replaced by an equality. Thus the variation in group B's lifetime utility from forcing group A to incur slightly more debt than they would choose to is exactly equal to the RHS of (27). If $\rho > (\log 2/\log 6)$, Proposition 9 part 3 ensures that the bracketed term in expression (27) is negative for any d. Hence group B suffers from relaxing the binding constraint. Group A is indifferent at the margin because we assumed that the constraint was barely binding. Conversely, therefore, tightening any barely-binding constraint on debt makes group A no worse off and group B strictly better off at the margin. The case where $\rho < (\log 2/\log 6)$ is exactly symmetric.

Part 2. If debt is constrained at $\overline{d} < d^*$, then the variation in group B's lifetime utility from allowing group A to incur slightly more debt is bounded below by the RHS of (27). Since $\rho < (\log 2/\log 6)$, Proposition 9 part 3 ensures that the bracketed term in expression (27) is positive for any d. Hence group B benefits from relaxing the binding constraint. Group A benefits too because we assumed that the constraint was binding. Conversely, therefore, tightening any binding constraint on debt strictly hurts both groups. ■

The intuition of part 1 of Proposition 12 is as follows. A marginal constraint on debt transfers resources from period 1 to period 2. This increases group B's utility in period 2 (total obligations go down, thus available resources to group B go up), but decreases it in period 1 (debt goes down, hence public good provision). To understand the net welfare effect, note that (i) group B's marginal utility of consumption is higher in period 2 than in period 1 (because public good provision is higher in the second period), and (ii) due to the endogenous reaction of entitlements to a change in debt, only a portion of the resources transferred to period 2 benefit group B. Thus, the net welfare effect for group B is positive if the difference in marginal utility across periods is sufficient to compensate for the "loss" in resources.

Proposition 12 part 2 is rather intriguing. Despite the fact that debt is used by group A as a tool to expropriate group B, in equilibrium, allowing more debt can be socially beneficial. A central driver for Proposition 12 part 2 is the fact that, in our model, resources are used less efficiently in period 2 than in period 1. Indeed, group A chooses the level of entitlements, taking into account that only a fraction of the remaining budget in period 2 will be devoted to the public good by group B. As a consequence, group A entitles itself excessively (from a social perspective). By decreasing the budget in period 2, debt helps reduce that inefficiency. However, of course, increasing debt also has a cost: it leads to a decrease in other types of consumption in period 2. This is particularly costly from a utilitarian perspective because group 2 receives lower consumption to start with and public consumption is lower in period 2 to start with (Proposition 1). If the cost of this lack of consumption smoothing is low, i.e., if the intertemporal elasticity of substitution is relatively small, than the benefit can be larger than the cost even for group B. However, if this cost is large, then debt is harmful for group B and can be harmful for utilitarian social welfare as well.

The effect of a constraint on entitlements is more straightforward. In our model, entitlements have both negative and positive features from the perspective of utilitarian social welfare: entitlements allow group A to expropriate group B, but they are also a tool for group A to guarantee itself some consumption in period 2, thereby allowing for some consumption smoothing across periods for group A. Accordingly, the following proposition shows that it is good to constrain entitlements a bit, but not too much. If one believes that the real-world status quo is one in which entitlements have been relatively unconstrained thus far, then part 1 of Proposition 12 reassures us that a bit of reform might indeed be a good thing.

Proposition 13 ***(welfare effects of constraining entitlements).***

1. *There exists a constraint on entitlements that increases utilitarian social welfare;*
2. *Eliminating entitlements altogether decreases utilitarian welfare relative to any allocation with positive entitlements.*

Proof. See Appendix C. ■

Proof of Proposition 13.

Part 1. Group A's allocation problem can be described as a constrained optimization problem. If a constraint on entitlements barely binds, its Lagrange multiplier will be close to zero. In the limit, it will be exactly zero. Thus the marginal impact on group A's lifetime payoff of introducing a constraint on entitlements is zero. Let's now consider the effects on group B's lifetime payoff. Proposition 11 part 4 ensures that tightening a cap on entitlements reduces the total obligations that group A bequeaths to group B. This means that group B is strictly better off in period 2. Also, group B is strictly better off in period 1 because constraining entitlements leads group A to increase debt (Proposition 11 part 2), and part of the additional period-1 resources will be allocated to the public good in period 1, from which group B benefits.

Part 2. Follows from the Inada condition on group A's utility for the private good. ■