

Problem Set 1, Microeconomics 2.

March 13, 2003. Due: March 24

Problem 1. Consider the following generalization of the model discussed in class. There is one potential buyer and one potential seller of a used car. The seller owns one car and, if he keeps it he receives a benefit of q , where q is the quality of the car. The buyer does not own a car. If he obtains a car, he receives a benefit of αq ($\alpha > 1$). The seller knows q , the buyer believes that q is uniformly distributed on $[0, 1]$.

(i) Characterize the competitive equilibria for each α and find a value of α (α^*) such that, for $\alpha < \alpha^*$, the unique competitive equilibrium involves no trade, and, for $\alpha > \alpha^*$, the unique competitive equilibrium involves 100% trade.

(ii) Assume now that the utility of the buyer is $q + \alpha$. Characterize the competitive equilibria for each α and show that for all $\alpha > 0$ in equilibrium there is never zero trade. Find a value of α (α^*) such that, for $\alpha > \alpha^*$, the unique competitive equilibrium involves 100% trade.

Problem 2. Consider the following dynamic variation of the model discussed in class. There is one potential buyer and one potential seller of a used car. At period zero the seller owns one car and, if he keeps it he receives a benefit of q per period, where q is the quality of the car. The buyer does not own a car at period zero. If he obtains a car, he receives a benefit of $\frac{5}{4}q$ per period. Note that the car does not depreciate between the two periods. There are two possible qualities: $q = 1, q = 2$. The seller knows q . The buyers believes that the two qualities are equally likely.

Suppose that there are two periods in which exchange between buyer and seller can take place, and assume that the discount factor is δ . Let P_1 be the price in period 1 and P_2 be the price in period 2. Clearly, exchange in the second period can only take place if there is no exchange in the first period.

(i) Assume that $P_1 = \frac{5}{4}(1 + \delta), P_2 = 2$. Show that there is a δ^* such that, for $\delta \leq \delta^*$, there is a competitive equilibrium such that quality $q = 1$ is sold in period 1 and quality $q = 2$ is sold in period 2. Compare total social surplus in this equilibrium with social surplus in the case in which markets are only open for one period (but the good lasts two periods).

(ii) Suppose now that $1(1 + \delta) < P_1 < \frac{5}{4}(1 + \delta), P_2 = 2$. Show that there is a $\hat{\delta}(P_1)$ such that for $\delta \leq \hat{\delta}(P_1)$, there is a competitive equilibrium such that quality $q = 1$ is sold in period 1 and quality $q = 2$ is sold in period 2. Compare $\hat{\delta}(P_1)$ with the δ^* computed in part (i) and comment.

(iii) Denote the two qualities by q_L and q_H such that $0 < q_L < q_H$. Generalize the analysis from part (i). Namely, assume that $P_1 = \frac{5}{4}q_L(1 + \delta), P_2 = q_H$ and find an expression for δ^* .

Problem 3. Consider the following alternative dynamic variation of the model discussed in class. Suppose that in period 0 there is a supplier of new cars (manufacturer) who supplies one new car inelastically (supply is one no matter what the price is). Assume that there are two potential consumers of this car and that there are two periods. Assume that there is no discounting and that consumers are risk neutral. Consumer 1 receives benefit q per period from a car of quality q . Consumer 2 receives benefit αq ($\alpha > 1$) per period from a car of quality q . Assume further that the quality of the car when new is distributed uniformly on $[1, 2]$, but the car depreciates so that, if the new car is worth q , than the used car (the car at the beginning of period 2) is only worth kq ($k < 1$). Thus, the total quality of a car of type q is $q + kq$.

The information structure is the following. Nobody observes the quality of the new car before purchase, so both consumers have a uniform ($[1, 2]$) prior on the quality of the new car. However, the consumer who purchased the new car will find out q between time 1 and 2 (he drives the car for one period). Thus, at the beginning of the second period, the consumer who purchased the new car has private information about q .

(i) Who will buy the new car in equilibrium? What are the possible equilibrium prices for a new car?

(ii) Is there any value of α such that there is a possibility of trade in the second period? In other words, can there be any trade of the used car?

(iii) Is the used market functioning inefficiently? Discuss.

Problem 4. Exercise 13.B.3 of MWG.