

## Problem Set 2, Microeconomics 2.

March 28, 2003. Due: March 31

### Problem 1

Consider the signaling problem discussed in class. Assume that  $\theta_L = 1$ ,  $\theta_H = 2$ ,  $c(e, \theta_L) = e^2$  and  $c(e, \theta_H) = \frac{e^2}{k}$ , where  $k > 1$ .

(i) Find the separating equilibrium with the lowest education level and compute the utilities of both types in this equilibrium.

(ii) Consider a social planner who does not know  $\theta$  but can offer workers a menu of education and wages that allows for cross subsidies across the two types. The constraints that the social planner faces are: (a) Incentive compatibility: if  $(w_L, 0) \neq (w_H, e_H)$  is the pair of wages and education suggested to the two workers, the low type must be better off choosing  $(w_L, 0)$  and the high type must be better off choosing  $(w_H, e_H)$ ; (b) Budget Balance:  $\lambda w_H + (1 - \lambda) w_L = \lambda \theta_H + (1 - \lambda) \theta_L = 1 + \lambda$ . Assume that the utility that you computed in part (i) for the high type is higher than  $1 + \lambda$  (the utility corresponding to the prohibition of signaling). Show that there is no menu that the social planner can choose that satisfies (a) and (b), and that leads to a Pareto improvement relative to the outcome computed in part (i).

(iii) Provide an example where in fact the social planner can find a menu that satisfies (a) and (b), and that leads to a Pareto improvement relative to the outcome computed in part (i).

### Problem 2 (Prices as signals of quality).

Assume that a firm is a monopolist that produces a good of quality  $\theta$ . The firm knows the quality, the consumers do not.

If quality were observable, the firm would face a demand curve  $Q = \theta - P$ . Quality can be either high ( $\theta = 2$ ) or low ( $\theta = 1$ ). The marginal cost to the firm of producing a low quality good is zero. The marginal cost of producing a high quality good is  $c > 0$ . There is no fixed cost. The firm chooses

prices as a function of  $\theta$ . Denote by  $\mu(P)$  be the probability that consumers assign to the good being high quality given a price  $P$ . The demand curve is:  $Q = 2\mu(P) + (1 - \mu(P)) - P$

- (i) First find the full information optimal prices and compute profits.
- (ii) Find a  $c^*$  such that for  $c > c^*$ , the full information optimal prices constitute a separating equilibrium.
- (iii) Assume now that  $c < c^*$ . What is the price that will be chosen by a low quality firm in a separating equilibrium? Find the lowest price charged by the high quality firm in a separating equilibrium. Does a separating equilibrium always exist?

**Problem 3**

Problem 13.C.5 in MWG. Note that in this problem  $P$  cannot depend on advertising level.

**Problem 4** (This problem is hard)

A decision maker must choose some decision  $y \in [0, 1]$ . His payoff depends on the decision and on the state of the world  $\theta$ :  $U(y, \theta) = -(y - \theta)^2$ . The decision maker can base his decision on a message  $m$  sent at no cost by an expert who knows  $\theta$ . The expert's payoff function is  $V(y, \theta) = -(y - (\theta + \frac{1}{10}))^2$ . Assume that the decision maker believes that  $\theta$  is uniformly distributed on  $[0, 1]$ . In **stage 1** the expert sends a message  $m \in [0, 1]$ . In **stage 2** the decision maker chooses  $y$ . Let  $m^*(\theta)$  denote an equilibrium message as a function of  $\theta$ .

(i) Show that there is no fully revealing equilibrium, e.g.  $m^*(\theta) = \theta$  is not an equilibrium.

(ii) Show that there is an equilibrium where the expert communication strategy is the following: if  $\theta \in [0, \frac{3}{10}]$  then the expert sends one message and if  $\theta \in [\frac{3}{10}, 1]$  the sender sends another message.

More formally,  $m^*(\theta) = A$  for  $\theta \in [0, \frac{3}{10}]$  and  $m^*(\theta) = B$  for  $\theta \in [\frac{3}{10}, 1]$  for some  $A \neq B$ . What does the decision maker choose after message  $A$ ? What does the decision maker choose after message  $B$ ?

Compute the ex-ante (before  $\theta$  is known) expected payoff of the expert and of the of the decision maker in this equilibrium.

(iii) Suppose now that the decision maker delegates the decision to the expert and describe the equilibrium choice in this case. Compute the ex-ante expected payoff of the expert and of the decision maker in this equilibrium. Comment.