

Problem Set 3, Microeconomics 2.

April 4, 2003. Due: April 7, 2003

Problem 1

Problem 13.D.1 MWG

Problem 2

Problem 13.D.2 MWG

Problem 3

Consider the following principal-agent model. Both the principal and the agent are risk neutral. There are two possible outcomes: success (s, f) or failure. The effort e chosen by the agent determines the probability of success. For simplicity, assume that the probability of success is e . Effort is not observable by the principal. The principal observes whether the outcome was s or f and chooses an incentive scheme: a pair w_s, w_f of wages contingent on the outcome. Assume that there is limited liability, i.e. there is a lower bound on wages. In particular, assume that both wages must be nonnegative ($w_i \geq 0, i = s, f$). The payoff function of the agent is: $U(e, w_s, w_f) = ew_s + (1 - e)w_f - c(e)$, where $c(e)$ is increasing, differentiable and convex.

(i) Suppose that the principal wants to implement an effort e^* . Assume first that $c(e) = \frac{c}{2}e^2$ and obtain exact solutions for the optimal incentive scheme, i.e. the incentive scheme that minimizes the expected wage bill of the principal. Consider next the case of a general cost function and characterize the optimal incentive scheme.

(ii) Go back to the case of a quadratic cost function and assume that the principal's payoff is $V(e, w_s, w_f) = e - k(ew_s + (1 - e)w_f)$. Obtain the optimal solution for the principal (in addition to what you obtained in part (i) you now need to obtain the optimal effort).

(iii) Assume now that there is an additional outcome that we call h (for high success). The probability of outcome h given e is e^2 . The incentive scheme is now made up of three elements: w_h, w_s, w_f and the utility of the agent is given by $U(e, w) = w_h e^2 + w_s e + w_f (1 - e - e^2) - c(e)$. Assume that $c'''(e) \geq 0$ (yes, the third derivative) for every $e \in [0, 1]$. Suppose that the principal wants to implement effort e^* and characterize the optimal incentive scheme. Note: make sure that you check second order conditions. You may want to start by looking at the quadratic case first ($c(e) = \frac{c}{2}e^2$).

(iv) (Only if you are curious. Still the case of three outcomes.) Assume that $c(e) = e^{\frac{3}{2}}$. This means that, although c is convex, $c'''(e) < 0$. Characterize the optimal incentive scheme for implementing e^* (nothing more, nothing less).

Problem 4

Consider the following principal-agent problem. The observable outcome is the profit π of the principal. The agent chooses effort $e \in [0, \infty)$. Effort is not observable by the principal. Assume that the density of profits conditional on effort ($f(\pi|e)$) is uniform on $[e, e + 1]$. The agent's payoff is $U(w, e) = v(w) - c(e)$ where v is strictly increasing, strictly concave, and unbounded below, and c is convex. The principal is risk neutral and maximizes expected profits minus the expected wage bill. Assume that the agent's reservation utility is zero.

(i) Assume that the principal wants to implement effort $e^* > 0$ and characterize the optimal incentive scheme. Obtain an exact expression for the expected wage bill required to implement e^* in the optimal incentive scheme. Hint 1: pointwise maximization may not be the best approach. Hint 2: you have to use the fact that some outcomes are impossible for some efforts.

(ii) Assume now that $v(w) = \log(w)$ and $c(e) = \frac{e^2}{10}$ and obtain the optimal effort choice for the principal.