Problem Set 4, Microeconomics 2.

April 11, 2003. Due: April 14, 2003

Problem 1

There are two agents working at two distinct tasks. The value of each task to the principal is either \$10 or \$0. The following is the probability distribution of outcomes. If the worker works hard there is a .7 probability that the value is 10 and a .3 chance that the value is 0. If the worker shirks, then the probabilities reverse so that there is a .3 probability of the 10 outcome and a .7 probability of the 0 outcome. Moreover, there is correlation between the two tasks. If both workers work hard, then there is a .6 probability that each will produce a 10 outcome, if one shirks and the other works, there is a .25 probability that each will produce a 10 outcome.

(Start by putting together joint probability tables from the data above.)

Each worker has a utility function $U(w, a) = \sqrt{w} - a$ where $a = 0$ for shirking and $a = .8$ for working hard. Each worker has reservation utility level of 1.

The employer is risk neutral and seeks to maximize the net profit from the two workers.

(i) Suppose first that effort is observable. Would the employer prefer high or low effort? What are the wages?

(ii) Suppose now that effort is unobservable. Assume that you can write a contract with each worker that is only contingent on the outcome of the task that he works on (thus, there are two possible wages for each worker) and you must ensure that the worker is willing to sign the contract and to work the amount you ask him to choose. What is the best contract of this form for the employer?

Now suppose that you can write contracts with each worker that depend on the outcomes of both tasks (thus, there are 4 possible wages for each worker).

(iii) What contracts of this form are optimal if you want to induce both workers to shirk?

(iv) Suppose that you wish to offer contracts that will induce both to work hard. Assume that this must mean that you must meet the following constraints: Each worker must be induced to take the contract; each worker, assuming that the other worker is going to work hard, must be induced to work hard. What is the optimal contract in this case?

(v) Suppose now that you are worried that the contract offered in part (iv) might induce the following behavior: Each worker will shirk. You would feel less worried about this possibility if you knew that each one, given the other one was shirking, would prefer to work hard. If you add this constraint to part (iv) is the old solution still valid? If not, what is the new solution?

(vi) Can you do better to induce one worker to shirk and the other to work hard? How well do you do in this case?

Problem 2 (Tournament)

Assume that there are two agents. If agent i chooses effort $e_i \in [0, \infty)$, his "output" is $\pi_i = e_i + Z_i + a_i$, where Z_1 and Z_2 are random variables independently and identically distributed according to F, and a_i is the ability of agent i. Assume that a_1 and a_2 are common knowledge and that agent one is better, i.e. $a_1 > a_2$. Agents payoffs are as follows: the agent with the highest total output X_i wins a prize of value one. The loser wins nothing. Agents have a cost of effort $c(e)$ where c is increasing, convex and differentiable.

Assume that the distribution G of $Z_1 - Z_2$ has a density g which is single peaked with the mode at zero. Assume that agents choose effort before the realization of Z_i. Make your analysis throughout this problem assuming that first order conditions are necessary and sufficient but then convince yourself that extra conditions may be needed (e.g., what if F is degenerate or with very small support?).

(i) Show that for a fixed ability differential $a_1 - a_2$ there is an equilibrium where the two agents choose the same effort e.

(ii) Show that this equilibrium effort level is decreasing in the ability differential, i.e. agents work harder when they have similar abilities.

Problem 3 (Another - bigger - tournament)

Assume that there is a unit measure of agents. If agent i chooses effort $e_i \in [0, \infty)$, his "output" is $\pi_i = e_i + Z_i$. We will assume (despite the fact that this is not allowed) that the Z_i's are i.i.d with symmetric distribution F with stricly positive density on the support $[-1, 1]$, and a unique mode at zero. There is a measure $R < 1$ of identical prizes of value one that will go to the agents with the highest realized output, i.e., to the fraction R of agents with the highest output. Assume that agents are identical and have a linear cost of effort $c(e) = ce$ ($c > 0$).

(i) Assume first that agents choose effort simultaneously before the realization of Z_i. Show that there is a c^* and a c^{**} such that for $c > c^{**}$ all agents must make positive profits in equilibrium and for $c < c^*$ agents make zero profits in equilibrium. Show that, there is a $\widehat{c}$ such that, for $c < \widehat{c}$, there is no pure strategy symmetric equilibrium (by symmetric I mean that all agents choose the same effort). Characterize the equilibrium for such c's.

(ii) Assume now that each agent i observes the realization of Z_i before the choice of e_i. Argue that it does not matter whether the agent observes only his own noise term Z_i or he observes the realization of the noise terms of all other agents. Next characterize the equilibrium.

(iii) Compare the aggregate effort averaged across agents between part (i) and (ii) for $c < \widehat{c}$.