Storable good monopoly: the role of commitment[1]

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Abstract

We study dynamic monopoly pricing of storable goods in an environment where demand changes over time.

The literature on durables has focused on incentives to delay purchases. Our analysis focuses on a different intertemporal demand incentive. The key force on the consumer side is advance purchases or stockpiling. In the case of storable goods the stockpiling motive has been documented in recent empirical literature. We show that, in this environment, if the monopolist cannot commit, then prices are higher in all periods, and social welfare is lower, than in the case in which the monopolist can commit. This is in contrast with the analysis in the literature on the Coase conjecture.

1 Introduction

A large literature in industrial organization examines the dynamics of firm behavior. Most of this literature ignores consumers' intertemporal incentives. For many goods, such as durable and storable goods, however, intertemporal incentives on the consumer side are potentially important. A specific kind of intertemporal demand incentive generated by durability has attracted a lot of attention: the incentive for consumers to postpone purchases in the expectation of better deals in the future. This effect emerges most starkly in the Coase conjecture (Coase 1972, Bulow 1982, Gul, Sonneschein, and Wilson 1986) where durability and intertemporal demand incentives combine to generate a striking prediction: if consumers are patient, or transactions can occur quickly, and the monopolist lacks the ability to commit, the power of the monopolist to extract surplus is completely undermined, and the monopoly distortion disappears. Although many papers in this literature deliver less extreme outcomes (e.g., Sobel 1991), a consistent picture has emerged: when goods are durable, monopoly power may not be as bad as in the textbook model because lack of commitment undermines monopoly power. In principle, this literature has powerful policy implications concerning the desirability of regulation and antitrust policy in durable goods industries.

In this paper we show that an alternative - empirically relevant - intertemporal demand incentive can lead to exactly the opposite conclusions concerning the consequences of commitment. Specifically, we consider an environment in which consumers have the incentive to store in the expectation of higher future prices. This is a demand anticipation motive rather than the demand postponement incentive which is the focus of the durable goods literature. Recent empirical studies (e.g., Pesendorfer 2002 Hendel and Nevo 2004a) have documented that purchasing patterns respond to the timing of price changes in a way consistent with demand anticipation.[1] This evidence pertains mostly to groceries, but there are many other goods for which it is plausible to think that demand anticipation is relevant (e.g. oil and various intermediate goods, see Hall and Rust 2000 for a detailed study).

We study monopoly pricing in the presence of demand anticipation. Specifically, in our model, goods are storable and demand varies deterministically over time. We compare the equilibrium when the monopolist can commit to the equilibrium when the monopolist cannot commit. We show that lack of commitment leads to higher prices in all periods, lower welfare, and higher wasteful storage, than if the monopolist could commit. Thus, the monopolist's inability to commit worsens distortions in the presence of demand anticipation. This result is in stark contrast with the results in the literature on the Coase conjecture.

To gain an initial intuition for our main result, consider a world with two periods where consumers' demand goes up in the second period. Suppose that the marginal cost of storage is constant and is smaller than the difference between the static monopoly prices. If the monopolist charged the static monopoly price in each period, consumers would have an incentive to purchase in the first period for consumption in the second period. In the equilibrium with commitment the monopolist counters this demand anticipation motive by announcing current and future prices to ensure that consumers do not store. The intuition is that the monopolist makes sure that consumers pay in the second period what they would otherwise spend for storage, allowing the monopolist to capture the

[1] See also Aguirregabiria (1999), Erdem, Imai, and Keane (2003), Hendel and Nevo (2004b).

increased surplus. Relative to static monopoly prices, under commitment, the monopolist raises the first period price and lowers the second period price: the possibility of storage reduces the price increase.

Suppose now that the monopolist cannot commit to future prices. Note first that the commitment solution is no longer an equilibrium. If consumers did not store in the first period the monopolist would have an incentive to raise the price toward the static monopoly price in the second period. This points to the direction in which lack of commitment pushes the monopolist, namely, to raise the second period price relative to the case of commitment. This increase in second period prices leads to positive storage. By affecting storage, every increase in the second period price leads to a loss in the profit margin proportional to the difference between second and first period prices. When the monopolist can commit, this effect is taken into account, leading to overall lower prices and storage, and higher profits and welfare. When the monopolist is unable to commit to a moderate second period price, it resorts to a high first period price as the only available tool to lower wasteful storage. The first period price (as well as the second period price) is then higher than under commitment.

The analysis of this paper may lead to a more cautious evaluation of contracts that enhance a firm's commitment ability: the policy advice that emerges from the literature on the Coase conjecture is to be suspicious of contractual arrangements, such as rental or leasing contracts, that enhance commitment since these may restore monopoly power, and lead to higher prices and lower welfare. In contrast, in our model, enhancing a monopolist's ability to commit may lead to lower prices and reduce wasteful storage.

In Section 2 we present a simple two period example to highlight the basic logic of the result. In Section 3.1 we present the general model, in Section 3.2 we state the main result of the paper and provide an outline of the logic of the result. While most proofs are in the Appendix, in Sections 3.3 and 3.4 we offer a more formal analysis of equilibrium in the two commitment scenarios. Section 4 discusses the related literature. Section 5 discusses the robustness of our results and argues that demand anticipation may be relevant in durable goods markets, and our analysis may also apply to such markets.

2 A Linear Example

We first present a simple two period example with linear demand to convey the intuition for the basic effect. In order to highlight the role of storability and commitment we compare monopoly prices in three cases. We first present the unconstrained monopoly problem, namely, static monopoly prices without storage by consumers. We then solve the monopoly problem allowing for storage and commitment to future prices. This involves a no-arbitrage constraint that limits the difference between the prices in the two periods. Finally, we study the case where consumers can store but the monopolist cannot commit. In this last case the monopolist suffers from both the storability (no-arbitrage) and no-commitment (or perfection) constraints.

Assume that there are no costs of production throughout and that the interest rate is zero. The demand in the first period is

$$D_1(p_1) = 100 - p_1.$$

The demand in the second period is

$$D_2(p_2) = 200 - p_2.$$

With no constraints the firm maximizes

$$p_1 D_1(p_1) + p_2 D_2(p_2) \tag{1}$$

by charging prices $p_1^m = 50$, $p_2^m = 100$, and earning $50\,(50) + 100\,(100) = 12,500$.

Assume that consumers can store the product in the first period for consumption in the second period: storing S units costs $c(S) = 10S$.

Since prices cannot differ by more than the marginal cost of storage, the possibility of storage effectively constrains the firm's intertemporal pricing problem. Plugging the no arbitrage constraint $p_2 = p_1 + 10$, the firm's objective function under commitment becomes:

$$V^C\,(p_1) = p_1 D_1(p_1) + (p_1 + 10)\,D_2(p_1 + 10). \tag{2}$$

Profits are maximized at $p_1^c = 70$ and $p_2^c = p_1^c + 10 = 80$ leading to profits of $(50 + 20)\,(50 - 20) + (100 - 20)\,(100 + 20) = 12,500 - 400 - 400 = 11,700$, with a loss of 800 (400 in each period) caused by the inability of the firm to equate the marginal revenues across the two periods.

The solution under commitment involves no storage. Every stored unit causes a loss of 10 to the monopolist since it implies a sale at p_1 instead of p_2. By slightly lowering p_2 the monopolist can ensure that no storage takes place, giving rise to a discontinuous increase in profits. Thus, it cannot be the case that storage is positive.

Finally, we come to the case in which the firm cannot commit to future prices. How does the objective function change now to incorporate the commitment constraint? In period 2 the firm will face a residual demand curve of $D_2(p_2) - S = 200 - p_2 - S$. Profits $p_2\,(D_2(p_2) - S)$ are maximized at $p_2\,(S) = (200 - S)\,/2$.

Since under commitment $S = 0$ and $p_2 = 80$, we can immediately see that the commitment solution is not an equilibrium absent commitment: given $S = 0$, the monopolist would set $p_2\,(0) = 100$.

Equilibrium storage is determined so as to induce the monopolist to raise the price in the second period by 10. Given p_1, if the agents store too little, then the firm responds by choosing $p_2 > p_1 + 10$, but then a larger storage would be optimal. If agents store too much, then the firm responds by choosing $p_2 < p_1 + 10$, against which the agents would prefer to store nothing. Thus, equilibrium storage $S(p_1)$ must induce $p_2\,(S\,(p_1)) = p_1 + 10$. Using the optimal second period price $p_2\,(S) = (200 - S)\,/2$ and $p_2 = p_1 + 10$ we find equilibrium storage: $S(p_1) = \max\{0, 180 - 2p_1\}$.

We can write the new optimization problem in either of two ways. First, we can simply recognize that sales increase in the first period by the inventory amount and decrease in the second period by the same amount. So the objective function with both arbitrage and perfection constraint incorporated is

$$V^{NC}\,(p_1) = p_1\,(D_1\,(p_1) + S(p_1)) + (p_1 + 10)\,(D_2\,(p_1 + 10) - S(p_1))\,. \tag{3}$$

This can be simplified by canceling the term $S(p_1)p_1$ to obtain

$$V^{NC}(p_1) = p_1 D_1(p_1) + (p_1 + 10)(D_2(p_1 + 10)) - 10S(p_1). \tag{4}$$

This expression could also be obtained directly by recognizing that the cost of holding storage is ultimately a cost that comes out of the seller's pocket since consumers break even in the storage decision. By comparing equation (4) with equation (2), we see that the third term is exactly the cost of the lack of commitment.

Solving the maximization problem in equation (4) yields $p_1^{nc} = 75$ and so $p_2^{nc} = 85$. Equilibrium storage is 30. Plugging into (4), profits become $(50 + 25)(50 - 25) + (100 - 15)(100 + 15) - 10(30) = 12,500 - 625 - 225 - 300 = 11,350$. So the perfection constraint costs an extra 350. There is a cost of 50 because prices are set too high (both prices rise by five and while this puts the second period price closer to the unconstrained optimum, it makes the first period price, which was already too high, even higher). In addition, there is a cost of 300 because of storage, as 30 of the units consumed in the second period are sold at p_1^{nc} instead of p_2^{nc}.

All the analysis in this Section was undertaken for cases in which, absent commitment, in equilibrium inventory is positive. However, if in the example the demand increase was less pronounced, for example $D_2(p_2) = 140 - p_2$, then prices would be 55 and 65 without the perfection constraint and 60 and 70 with it. However, while the distortion causes both a profit and a welfare loss the cost of inventory is just enough so that the firm chooses the first period price in a way that will cause no inventory to be held. As a consequence the second period price is equal to the uncostrained monopoly price, while the first period price is higher.

If we made the difference in the demands smaller a similar result would occur, until we reduced second period demand to $120 - p_2$ or less, at which point the storability constraint would not bind because the cost of inventory is too high for storage to affect the monopolist's optimal behavior.

3 General Analysis

We now present the general model to show that the basic force highlighted in the example is quite general. We first state our basic assumptions. We provide an outline of the logic of the result, and then go into more detail on the characterization of equilibrium under both commitment and no-commitment.

3.1 The model

A monopolist faces demand for a storable good in each of T periods. For simplicity we assume that costs of production are zero and that there is no discounting.

Demand in each period t comes from a unit measure of identical consumers[2] whose utility is quasi-linear in the consumption of the good, x_t, and money m_t: $U_t(x_t, m_t) = u_t(x_t) + m_t$. We assume that each u_t is continuously differentiable.

[2] The assumption of identical consumers is made for convenience. The main result is unaffected by heterogeneity. Furthermore, if storage is undertaken by competitive arbitrageurs, the equilibria we characterize only depend on the sequence of aggregate market demands, independently of the sequence of individual preferences that generate it.

We assume that the cost of storage is linear: $c(S) = cS$.[3] Storage can be equivalently thought of as undertaken by consumers or by competitive arbitrageurs.

At each date t, given any sequence of prices $p_t, ..., p_T$ and the current inventory S_{t-1}, consumers choose purchases q_t, consumption levels x_t and storage levels S_t to maximize $\sum_{i=t}^{T}[U_i(x_i, m_i) - q_i p_i - cS_i]$ subject to $q_i = x_i + S_i - S_{i-1}$.

Let $D_t(p_t)$ be the static demand function associated with U_t, i.e., the maximizer of $u_t(q) - qp_t$. Denote revenue in period t by $R_t(p_t) = D_t(p_t)p_t$, marginal revenue in period t by $MR_t(p_t)$, and the static monopoly price at period t (the maximizer of $R_t(p)$) by p_t^m.

Assumption 1. *For each t, $R_t(\cdot)$ is concave and t-*times differentiable.[4]

Assumption 2. $D_t(\max_{\tau}\{p_\tau^m\}) > 0$ *for every t.*

Assumption 3. $c < \max_{1 \le t \le T-1}\{p_{t+1}^m - p_t^m\}$.

Assumption 2 ensures that in equilibrium consumers consume a positive quantity at every date.[5] Assumption 3 is a non triviality assumption that ensures that storage matters.

In order to characterize the equilibrium, optimal storage decisions must be defined for all possible prices. Period t storage decisions can in principle depend on the sequence of all future prices. However, in equilibrium, the storage decision of the consumer at period t only depends on the prices at periods t and $t+1$. Specifically, if

(i) $p_{t+1} - p_t < c$ then $S_t = 0$;

(ii) $p_{t+1} - p_t = c$, then consumers are indifferent. As we will see, in this case S_t is defined in equilibrium by taking into account the monopolist's response to storage.

We will show below that cases (i) and (ii) are the only possible outcomes in equilibrium. If instead (out of equilibrium) $p_{t+1} - p_t > c$, then S_t is chosen to satisfy demand from period $t+1$ up to the next period in which consumers purchase, namely, until the next period τ in which the price p_τ is lower than the cost of buying now and storing until period τ $(p_t + c(t - \tau))$. The details for this case are provided in the appendix.

3.2 Main Result and Argument Outline

We first state the main result of the paper comparing commitment and no-commitment. We then provide an outline of the argument and intuition for the result within the context of increasing

[3] This assumption allows for a sharper characterization but it is not essential for the main result. We briefly discuss the case of convex cost of storage in the Concluding Remarks. Depreciation is an alternative cost of storage which is also wasteful if production costs are positive. We investigated the case of depreciation and our main result is unchanged.

[4] Although concavity of revenue may seem uncontroversial, it does rule out some important cases. For a discussion of interesting consequences of nonconcavities in revenues, see for instance Johnson and Myatt (1993). For most of the analysis we only need revenues to be continuously differentiable. Higher order differentiability is only required to prove differentiability of equilibrium storage without commitment, which is a convenient, but not essential, element of our proofs.

[5] If Assumption 2 is not satisfied, the monopolist may not offer positive quantities in some periods. Dealing with this possibility is straightforward but tedious. Nothing of substance is affected by this assumption.

demand. Under increasing demand and small c, the problems are easy to characterize. In the next sections we show the generality of the result. The details of the proof are in the appendix.

We will use superscript c on equilibrium variables under commitment and superscript nc for those under no-commitment.

Proposition 1 *For all t, $p_t^c \leq p_t^{nc}$ and $S_t^c \leq S_t^{nc}$: prices and storage are uniformly lower under commitment. Profits and consumer surplus are higher under commitment.*

We now provide a brief outline of the argument proving that equilibrium prices are always lower under commitment.

As a first step, it is useful to start by going back to the example in Section 2 and obtain some more general insights for the two period problem. By comparing equations (2) and (4) we obtain

$$V^{NC}(p_1) = V^{C}(p_1) - S(p_1)\,c. \tag{5}$$

It is easy to see that this equation does not depend in any way on the specific demand functions. Thus, equation (5) holds generally, implying, as we saw in Section 2, that the seller's profits in the two commitment scenarios only differ by $S(p)\,c$, which can be interpreted as the cost of the perfection or credibility constraint. As shown later, storage $S(p_1)$ is a decreasing function.[6] Thus, the firm that is unable to commit (who maximizes V^{NC}) has an incentive to raise p_1 beyond p_1^c (the optimal price under commitment); namely, the incentive to reduce storage. Hence, equation (5) implies that $p_1^{nc} > p_1^c$. Since, as we saw in Section 2, prices rise by c per period in both scenarios, this also implies that p_2^{nc} is higher than p_2^c.[7]

We now discuss the relation between marginal revenues in the two scenarios because it will be convenient in comparing prices in the two scenarios in the general case. Absent storage, the monopolist would choose p_1^m and p_2^m so that $MR_1(p_1^m) = MR_2(p_2^m) = 0$. If instead storage is binding, the storage constraint implies $p_2 = p_1 + c$. Under commitment, optimal pricing is formally identical to price discrimination under costly arbitrage. The first period price p_1 is set so that that all units that are purchased in period 1 are used exclusively for consumption in period 1, namely so that $S = 0$. This requires $p_2 = p_1 + c$. The optimal price, p_1^c maximizes the sum of revenues $TR_1(p_1) + TR_2(p_1 + c)$. Thus, $MR_1(p_1^c) + MR_2(p_1^c + c) = 0$. This is a necessary condition in the general case (T periods and arbitrary demand sequences), we will prove this formally in Lemma 2.

Without commitment, the monopolist chooses $p_2(S)$ to maximize $(D_2(p_2) - S)\,p_2$. Since $p_2(0) = p_2^m$, we cannot have an equilibrium with zero storage, unless c is high.[8] However, storage is costly for the monopolist because it involves selling units for second period consumption at a price

[6] This will be proved in the appendix but it is natural to expect that storage declines when the price of buying for storage goes up.

[7] For the case of T periods with always binding storage constraints, it can be easilty shown that

$$V^{NC}(p_1) = V^{C}(p_1) - \sum_{\tau=1}^{T-1} S_\tau\left(p_1 + (\tau - 1)\,c\right)c.$$

Since each $S_{\tau-1}$ function is decreasing, we obtain that $p_1^{nc} > p_1^c$, and hence, all prices are higher without commitment.

[8] This is roughly because $p_1 = p_2^m - c$ which, for small c the first period price is very far from the optimal first period price. Thus, the monopolist is better off lowering p_1 and accepting some storage.

of p_1 instead of p_2. Consumers' incentives to store increase in the expected p_2. Thus, the monopolist would like to guarantee a low enough p_2. However, absent commitment such a guarantee would not be credible and consumers expect the seller to choose the price that is optimal for a given S. Thus, lacking ability to assure a low p_2, the monopolist is left with p_1 as the only tool to discourage storage. From equation (5) and the previous discussion concerning commitment, we see that the marginal value of raising p_1 is given by $MR_1(p_1) + MR_2(p_1 + c) - c\frac{dS_1}{dp_1}$. Since, as shown below, $\frac{dS_1}{dp_1} < 0$, at an optimal price without commitment, $MR_1(p_1^{nc}) + MR_2(p_1^{nc} + c) = c\frac{dS_1}{dp_1} < 0$. This, combined with the previous discussion for the case of commitment implies that

$$MR_1(p_1^{nc}) + MR_2(p_1^{nc} + c) < MR_1(p_1^{c}) + MR_2(p_1^{c} + c)$$

Since MR_t are decreasing functions, this inequality implies that absent commitment prices are higher in both periods.

In the general T period model, similar reasoning will allow us to conclude that implications of the necessary conditions for equilibrium are:

$$\sum_{t=1}^{T} MR_t(p_t^{c}) = 0 \tag{6}$$

and

$$\sum_{t=1}^{T} MR_t(p_t^{nc}) = c\sum_{t=1}^{T} \left.\frac{dS_t}{dp_t}\right|_{p_t=p_t^{nc}} < 0. \tag{7}$$

Thus,

$$\sum_{t=1}^{T} MR_t(p_t^{nc}) < \sum_{t=1}^{T} MR_t(p_t^{c}). \tag{8}$$

In one special case, this inequality immediately allows us to conclude that prices under commitment are always lower than without commitment. This is the case in which p_t^m is increasing and storage constraints are always binding (which is guaranteed if there is a sufficiently small storage cost). In this case, $p_t = p_1 + (t-1)c$. Thus, inequality (8) can be rewritten as

$$\sum_{t=1}^{T} MR_t(p_1^{nc} + (t-1)c) < \sum_{t=1}^{T} MR_t(p_1^{c} + (t-1)c).$$

Since MR_t is decreasing for each t, it must be the case that $p_1^c < p_1^{nc}$. Since in both scenarios prices increase by c per period, prices must be lower under commitment in all periods.

In the general case, when prices may go down (due to demand reductions), and hence storage is not always binding, the problem is more delicate. We are still able to conclude that $\sum_{t=1}^{T} MR_t(p_t^{nc}) < \sum_{t=1}^{T} MR_t(p_t^{c})$. The problem is that, in equilibrium, storage constraints may be binding for different sets of periods in the two scenarios.

Section 3.3 and 3.4 provide the essential steps for a characterization of equilibrium under the alternative commitment scenarios. These sections prove that equation (6) holds in general in the commitment scenario and equation (7) holds in general in the no commitment scenario. The proof of Proposition 1 is in the Appendix.

3.3 Equilibrium conditions under commitment

Intuition suggests that, under commitment, we can view the monopolist's problem as a maximization subject to a "storage-proof" constraint: the sequence of prices must be chosen to ensure zero storage; i.e., prices increases by no more than c per period. The first Lemma shows that this intuition is correct.

Lemma 1 *In an equilibrium with commitment $p^c_{t+1} \leq p^c_t + c$. Furthermore, $S^c_t = 0$ for all t.*

The proof is in the Appendix but the logic of this Lemma is similar to the one discussed in the Example in Section 2. Storage S_t involves purchases at price p_t for consumption in subsequent period t' at effective prices of $p_t + (t' - t)\,c$. But the monopolist can sell the same units in period t' at a higher profit by choosing a price of $p_{t'} = p_t + (t' - t)\,c$ thereby capturing the storage cost.

Because of Lemma 1, the commitment problem boils down to the choice of a price sequence $\{p_t\}_{t=1}^T$ to maximize

$$\sum_{t=1}^{T} D_t(p_t)p_t \tag{9}$$

subject to

$$p_{t+1} \leq p_t + c \;\text{ for all }\; t = 1, ..., T-1. \tag{10}$$

Lemma 2 *There is a unique sequence of equilibrium prices under commitment. This sequence is identified by the first order conditions of the maximization of (9) subject to (10). Furthermore a necessary condition for the price sequence $\{p^c_t\}_{t=1}^T$ to be the equilibrium under commitment is given by equation (6)*

$$\sum_{t=1}^{T} MR_t(p^c_t) = 0. \tag{11}$$

We emphasize that we have only provided necessary conditions for equilibrium. These are enough for the purpose of making a comparison between the two scenarios. However, condition (6) is not sufficient for an equilibrium. Many price sequences satisfy (6). For instance, consider the demand functions discussed in Section 2: $D_1(p_1) = 100 - p_1$ and $D_2(p_2) = 200 - p_2$. Prices $p_1 = 80$ and $p_2 = 70$ satisfy condition (6) but they do not constitute an equilibrium with commitment. In a WebAppendix we provide a simple algorithm to construct the equilibrium.

3.4 Equilibrium conditions without commitment

We now study equilibrium conditions without commitment. We do not explicitly construct the entire equilibrium sequence, although it is not difficult to do so. We will instead obtain necessary conditions for equilibrium. This is all that is required in order to compare the two scenarios.

Consider an equilibrium price sequence $\{p^{nc}_t\}_{t=1}^T$ without commitment. We will break the price sequence into monotonic subsequences. If between period t and period $t+1$, the "storage-proof" constraint is not binding (that is, the equilibrium price either decreases, or increases by less than c), then we can think of the equilibrium as being made of two *locally* independent sequences:

the sequence up to t and the sequence following t. Let $(T_1, ..., T_m)$ be a sequence of dates, with $1 \leq T_1 \leq ... \leq T_m \leq T$ such that for each i, $p^{nc}_{T_i} + c < p^{nc}_{T_i+1}$, i.e., storage is not binding between periods T_i and period $T_i + 1$.[9]

Note that $S_{T_i} = 0$ for every i, and that for each subsequence $\{T_i + 1, ..., T_{i+1}\}$ we can construct the local necessary equilibrium conditions by working backwards from period T_{i+1} to $T_i + 1$ without worrying about the rest of the game. We will then piece together all these conditions in order to make a comparison with the case of commitment.

We now consider one specific time interval $\{T_{i-1} + 1, ..., T_i\}$ in which storage binds for at least two periods.[10] We look at the case in which $S_t > 0$ along the subsequence. As in the example in Section 2 this will be the case when c is sufficiently small. It is easy to deal with the case in which $S_t = 0$ even along a nontrivial subsequence.

At any period t given an inherited storage S_{t-1} the firm's problem is characterized by the following recursive problem:

$$V_t(S_{t-1}) = \max_{p_t} \{ (D_t(p_t) - S_{t-1} + S_t(p_t)) p_t + V_{t+1}(S_t(p_t)) \} \tag{12}$$

Because period T_i is effectively a terminal period, we have:

$$V_{T_i}(S_{T_i - 1}) = \max_{p_{T_i}} [D_{T_i}(p_{T_i}) - S_{T_i - 1}] p_{T_i}.$$

The first term on the right-hand side of equation (12) is the revenue from period t sales which includes storage S_t for consumption in period $t + 1$ and the forgone sales due to storage from the previous period S_{t-1}; the second term $V_{t+1}(S_t(p_t))$ represents the revenues from future sales which are affected by p_t via storage S_t.

The maximization in equation (12) determines a sequence of optimal prices $p_t(S_{t-1})$. To complete the picture, we must describe the law of motion of the state variable S_t. Given the price at period t, in the equilibrium of the continuation game, storage $S_t(p_t)$ is determined so as to induce the monopolist to choose $p_{t+1}(S_t) = p_t + c$. Thus, $S_t(p_t)$ is such that

$$p_{t+1}(S_t(p_t)) = p_t + c \tag{13}$$

i.e., the optimal price at period $t+1$ given S_t is exactly c higher than p_t. Because the price increases by c every period, consumers are indifferent between any amount of storage, and therefore, they are willing to store $S_t(p_t)$.

The first order condition for an optimum at period t is then given by

$$MR_t(p^{nc}_t) - S_{t-1} + S_t(p^{nc}_t) + p_t \left. \frac{dS_t}{dp_t} \right|_{p^{nc}_t} + \frac{dV_{t+1}(S_t)}{dS} \left. \frac{dS_t}{dp_t} \right|_{p^{nc}_t} = 0 \tag{14}$$

[9] It can of course be the case that for some i's, $T_i + 1 = T_{i+1}$. In this case, the equilibrium price at T_i is the static monopoly price: $p^{nc}_{T_i} = p^m_{T_i}$. Furthermore, it is also possible that $T_1 = T_m = T$ in which case the storage constraint is always binding.

[10] Such a subsequence must exist by Assumption 3. Otherwise storage is never binding.

Since, by the envelope theorem, $\frac{dV_{t+1}(S_t)}{dS} = -p_{t+1}(S_t(p_t))$, and in equilibrium, $p_{t+1}(S_t(p_t)) - p_t = c$, equation (14) can be rewritten as

$$MR_t(p_t^{nc}) = S_{t-1} - S_t(p_t^{nc}) + c \left.\frac{dS_t(p)}{dp}\right|_{p_t^{nc}}. \tag{15}$$

(For periods T_{i-1}, and T_i, recall that $S_{T_{i-1}} = S_{T_i} = 0$.)

We have now concluded our characterization of the necessary conditions for equilibrium along a price subsequence with binding storage constraints. Note that, by summing equations (15) for all t between $T_{i-1}+1$ and T_i we obtain:

$$\sum_{t=T_{i-1}+1}^{T_i} MR_t(p_t^{nc}) = c \sum_{t=T_{i-1}+1}^{T_i} \left.\frac{dS_t(p)}{dp}\right|_{p_t^{nc}} \tag{16}$$

The following Lemma shows that $\frac{dS_t(p)}{dp} \leq 0$ allowing us to conclude that along a price subsequence with binding storage constraints, equation (7) holds.

Lemma 3 *Consider any p_t such that $S_t(p_t) > 0$. $S_t(p_t)$ is t-times differentiable at p_t. Furthermore, $\left.\frac{dS_t(p_t)}{dp_t}\right|_{p_t^{nc}} \leq 0$.*

This Lemma is proved in the Appendix. It is also easy to see that if instead storage constraints are nonbinding between periods t and $t+1$, the price in period t has to equal the unconstrained monopoly price $p_t^{nc} = p_t^m$ implying that $MR_t(p_t^{nc}) = 0$. Furthermore, when storage is nonbinding, we must have $\left.\frac{dS_t(p)}{dp}\right|_{p_t^{nc}} = 0$. This means that if we sum over *all* periods, equation (16) continues to hold, and we obtain $\sum_{t=1}^{T} MR_t(p_t^{nc}) < 0$. This allows us to compare prices in the two commitment scenarios.

4 Related Literature

Anton and Das Varma (2005) study a two period duopoly model in which consumers can store first period purchases. They study the impact of storability on the intertemporal price path. They find that prices increase over time if consumers are patient and storage is affordable. The low initial prices are a consequence of the firms' incentive to capture future market share from their rival. In contrast to the duopoly case, the demand shifting incentives do not show up under monopoly or competition. Under these market structures there is no incentive to capture future market share, so the price dynamics are absent. Jeuland and Narasimhan (1985) present a model in which storability may allow a monopolist to price discriminate among consumers because of a negative correlation between demand and cost of storage. Hong, McAfee and Nayyar (2000) is a competitive industry model, where consumers are assumed to chose a store based on the price of a single item and can store up to one unit.

There is a vast literature on durable goods. For a recent survey see Waldman (2003). The literature that is most related to our paper is the one on the Coase conjecture. This literature started with Coase (1972). Stokey (1981), Bulow (1982), and Gul, Sonnenschein, and Wilson (1986) are some of the early papers that provided a formal analysis of Coase's conjecture.

Sobel (1991) (see also Conlisk, Gerstner, and Sobel 1984, Sobel 1984, and Board 2005) describes a model of a market with a durable good monopolist in which, at every date a mass of new consumers enter. Consumers have unit demands and two possible valuations for the good. Sobel (1991) characterizes the set of equilibria under the assumption that the monopolist cannot commit. Board (2004) assumes that the monopolist commits and allows for a more general time path of entry of consumers. An important feature of the analysis in this strand of the literature is the possibility of price cycles, namely sales. We focus on a different effect (demand anticipation) and we obtain different results on the effect of commitment.

Nichols and Zeckhauser (1977) study the role of stockpiling by the government (e.g., strategic oil reserves). The government, as a large player, counters the ability of a monopolistic foreign seller that charges high prices in periods of high demand. They show that the government can affect the seller's behavior in welfare improving ways by undoing price discrimination across the two periods, and by increasing output through indirect subsidization.[11]

5 Concluding Remarks

We have shown that when a good is storable and demand changes deterministically over time, a monopolist always charges higher prices when it lacks ability to commit.

One assumption that we have maintained throughout is linearity of storage costs. In a WebAppendix we provide an analysis of the case with convex cost of storage. When the cost of storage is strictly convex, there are two main differences relative to the linear case: (i) storage is typically positive under commitment as well; (ii) There is no longer a simple arbitrage condition linking prices across periods and we no longer obtain as crisp a comparison between commitment and no commitment. However, the main result is roubust. We show that prices cannot be uniformly higher under commitment, and we performed extensive numerical computations with specific functional forms for demand and cost of storage. In all these computations, our result generalizes.

We have focused most of our analysis on storable goods, i.e. goods that are perishable in consumption but can be stored for future consumption (e.g., canned foods, laundry detergent, soft drinks, gasoline...). This provides a particularly stark scenario because in this case, the only intertemporal demand incentive is the demand anticipation motive. In contrast, the previous literature on durable goods has focused on environments where the only intertemporal demand incentive is demand postponement, thereby ruling out the incentives to anticipate purchases. However, demand anticipation may also arise in the case of durables: for instance, consumers may be willing to buy summer clothes in the Fall if the price is sufficiently low. It is easy to provide examples of plausible environments in which, just as in this paper, demand anticipation leads to a reversal of

[11] We have explored the effect of government stockpiling under all possible assumptions on commitment by the seller and by the government. Stockpiling by the government improves welfare in all scenarios except if the seller commits and the government does not.

the Coase result even in the case of durables. More generally, it is likely that in the case of durables, demand anticipation and demand postponement incentives can be present at different times in the same market due, for example, to demand seasonality. In these circumstances the overall effect of commitment on prices and welfare is difficult to assess since it depends on the exact nature of the cycle. Our analysis suggests that in environments with demand fluctuations, which are common in many markets, Coase's stark predictions may be significantly altered.

Appendix

Deferred details on consumers' optimal storage.

We first obtain optimal storage for a given anticipated sequence of prices $\{p_t\}_{t=1}^{T}$ and then adapt the construction to allow for unanticipated (off-equilibrium) prices. Note that optimal storage can be multi-valued when consumers are indifferent, in which case, equilibrium storage is pinned down by equilibrium conditions involving optimization by the firm. Note also that we can bound the set of relevant prices to be between 0 and $max_t\{p_t^m\}$.

Consider a sequence $\{p_t\}_{t=1}^{T}$ of prices expected by consumers, and initial storage $S_0 = 0$. For every period t define $\Upsilon_t = \{\tau | \min_{1\leq\tau\leq t}\{p_\tau + (t-\tau)c\}\}$ as the set of periods that minimize the cost of purchasing for period t consumption. If Υ_t has more than one element, then the consumer is indifferent between purchases in any period in Υ_t. By the definition of Υ_t, period t consumption is $D_t^* \equiv D_t(p_\tau + (t-\tau)c)$ for any $\tau \in \Upsilon_t$. By considering all sets Υ_t for $t = \tau+1, ..., T$ we can find S_τ recursively from period 1 onwards. For $\tau = 1, ..., T-1$,

$$S_\tau \in \left[\sum_{j=\tau+1}^{T} I(\Upsilon_j, \leq \tau)(1 - I(\Upsilon_j, > \tau))D_j^*, \sum_{j=\tau+1}^{T} I_\tau(\Upsilon_j, \leq \tau)D_j^*\right] \tag{17}$$

where $I(\Upsilon_j, \leq \tau)$ is an indicator function, that takes on value 1 (0 otherwise) if there is a period $k \leq \tau$ such that $k \in \Upsilon_j$, i.e., period j consumption could be optimally purchased in period $k \leq \tau$. The maximal quantity that the consumer stores in period τ (the upper bound of the interval in equation 17) is obtained by adding over all periods j up to T the quantity stored in prior periods for consumption in period j whenever there is a least cost way to purchase that quantity in periods prior to (and including) τ. The indicator $I(\Upsilon_j, > \tau)$ takes on value 1 (0 otherwise) if there is a period $k > \tau$ such that $k \in \Upsilon_j$. To obtain the lowest quantity that the consumer stores in period τ (the lower bound of the interval in equation 17), we resolve consumer indifference in favor of buying later. Specifically, the lower bound is attained if the consumer chooses never to purchase in periods prior to (and including) τ for consumption in period j whenever there is a $k > \tau$ such that $k \in \Upsilon_j$. In some cases the lower bound and the upper bound are identical so that the interval collapses to a single point. In this case, optimal storage is uniquely pinned down.

The description of storage above suffices for the case of commitment. However, for the no-commitment case, we need to specify storage when unanticipated price choices by the monopolist, and associated changes in conjectured future prices imply that the consumer finds himself with more or less inventory than optimal relative to this new price sequence. One possibility is that S_{t-1} is larger than the upper bound in 17. If $S_{t-1} \geq \sum_{j=t}^{T} D_j^*$, then the consumer does not need to buy again, and storage follows the optimal allocation of current inventory to future periods, which is obtained as the solution of the following maximization:

$$\max_{\{x_k\}} \sum_{k=t}^{T} (u_k(x_k) - c(k-t)x_k) \tag{18}$$
$$s.t. \ \sum_{k=t}^{T} x_k \leq S_{t-1}$$

If instead $S_{t-1} < \sum_{j=t}^{T} D_j^*$ the consumer has to buy at some future date. To find the first period k with positive purchases, the consumer solves a modified version of the problem discussed before equation (17) starting in period t. The modification regards the definition of the set Υ_j, to exclude purchases prior to t: $\Upsilon_j = \{\tau | \underset{t \leq \tau \leq j}{Min} \{p_\tau + (j-t)c\}\}$. Denote by UB_t^j (LB_t^j) the upper (lower) bound for S_j in equation (17) for the problem starting at t. Then, let k be the lowest j such that $UB_t^j > S_{t-1} - \sum_{l=t}^{j} D_l^*$. Prior to k, $S_j = S_{t-1} - \sum_{l=t}^{j-1} D_l^*$. From k onwards storage in determined as in equation (17); namely, storage is (and will remain) within the prescribed bounds. If, for some t, S_{t-1} is lower than the lower bound in equation (17), consider the set Υ_j for all $j > t$ (as modified in the previous paragraph). The consumer purchases as soon as $LB_t^j > S_{t-1} - \sum_{l=t}^{j} D_l^*$. Up to j the consumer behaves as prescribed by the maximization in equation (18). After j storage follows the bounds in equation (17) for the problem starting in period t.

Proof of Lemma 1

Proof. We first prove that, if $p_{t+1}^c \leq p_t^c + c$ for all $t = 1, ..., T-1$, then $S_t^c = 0$ for all t. Consider an equilibrium price sequence $\sigma^c = \{p_t^c\}_{t=1}^T$ such that $p_{t+1}^c \leq p_t^c + c$ for all t, and assume by way of contradiction that storage is positive in some periods. Let τ be the last period such that storage is positive. Note that, because $p_{t+1}^c \leq p_t^c + c$ for all t, if consumers choose $S_\tau > 0$ it must be that $p_{\tau+1}^c = p_\tau^c + c$. We now show that the following price sequence $\sigma = \{p_t\}_{t=1}^T$ is a profitable deviation:

$$\begin{aligned} p_t &= p_t^c & &\text{for } t = 1, ..., \tau \\ p_t &= p_t^c - (t-\tau)\varepsilon & &\text{for } t = \tau+1, ..., T \end{aligned}$$

Observe that, under sequence σ, $p_{t+1} < p_t + c$ for $t = \tau, ..., T-1$ so that $S_t = 0$ for $t = \tau, ..., T-1$. Thus, the two price sequences σ^c and σ generate different profits only after period τ. Specifically,

$$\pi(\sigma) - \pi(\sigma^c) = \sum_{t=\tau}^{T} D_t(p_t)p_t - \left[\sum_{t=\tau}^{T} D_t(p_t^c)p_t^c - cS_\tau^c \right].$$

Since revenues are continuous, and p_t and p_t^c differ by $(t-\tau)\varepsilon$, $\left(\sum_{t=\tau}^{T} D_t(p_t)p_t - \sum_{t=\tau}^{T} D_t(p_t^c)p_t^c \right)$ is negligible. On the other hand, cS_t^c is not negligible. This term appears because the lower price in period $\tau+1$ in the deviating sequence leads consumers to purchase in period $\tau+1$ instead of storing in period τ. Thus, $\pi(\sigma)-\pi(\sigma^c)>0$, a contradiction.

The argument to prove that, in an equilibrium with commitment, $p^c_{t+1}\leq p^c_t+c$ for all t follows a similar logic. Consider an equilibrium price sequence $\sigma^c=\{p^c_t\}^T_{t=1}$ and assume by way of contradiction that $p^c_{t+1}>p^c_t+c$ for some period. Let τ be the last period such that $p^c_\tau\leq p^c_{\tau-1}+c$ and $p^c_{\tau+1}>p^c_\tau+c$, and let $\tau'>\tau$ be the latest period such that $p^c_t>p^c_\tau+(t-\tau)c$ for all $t=\tau+1,...,\tau'$. Consumers' optimal storage decisions imply that $S_\tau=\sum\limits_{t=\tau+1}^{\tau'}D_t(p^c_\tau+(t-\tau)c)$. We now construct a profitable deviation: a price sequence $\sigma=\{p_t\}^T_{t=1}$:

$$\begin{aligned} p_t &= p^c_t && \text{for } t=1,...,\tau \\ p_t &= p^c_\tau+(t-\tau)(c-\epsilon) && \text{for } t=\tau+1,...,T. \end{aligned}$$

Note that, under price sequence σ, $S_\tau=0$ for $\tau=t,...,T-1$. Moreover, by the way we defined τ', $p^c_t-p_t\leq(t-\tau)\epsilon$. Finally, the price sequence σ affects profits only after period τ. Thus,

$$\pi(\sigma)-\pi(\sigma^c)=\sum_{t=\tau}^{\tau'}\left(D_t\left(p_t\right)p_t-p^c_\tau D_t\left(p^c_\tau+(t-\tau)c\right)\right)+\sum_{t=\tau'+1}^{T}\left(D_t(p_t)p_t-D_t(p^c_t)p^c_t\right) \tag{20}$$

Because revenues are continuous, and because p_t and p^c_t are very close for $t>\tau'+1$, the second sum on the right-hand side of equation (20) is negligible. In contrast, the first sum on the right-hand side of equation (20) is not negligible: it is approximately equal to $\sum\limits_{t=\tau+1}^{\tau'}D\left(p^c_\tau+(t-\tau)c\right)c$. Thus, $\pi(\sigma)-\pi(\sigma^c)>0$, a contradiction. ■

Proof of Lemma 2

Proof. Because, by assumption, the revenue functions $R_t(p)=D_t(p)p$ are concave in p, consequence, the constrained problem of maximizing $\sum\limits_{t=1}^{T}D_t(p_t)p_t$ under the linear constraints $p_{t+1}\leq p_t+c$ for all $t=1,...,T-1$ has a unique solution. To verify that $\sum\limits_{t=1}^{T}MR_t(p^c_t)=0$, consider the Lagrangian function $L(p_1,...,p_T,\lambda_1,...,\lambda_{T-1})=\sum\limits_{t=1}^{T}D_t(p_t)p_t-\sum\limits_{t=1}^{T-1}\lambda_t(p_{t+1}-p_t-c)$, where $\lambda_1,...,\lambda_{T-1}$ are the Lagrange multipliers. The associated first order conditions are: $MR_1(p^c_1)+\lambda_1=0$, $MR_2(p^c_2)-\lambda_1+\lambda_2=0$, ..., $MR_T(p^c_T)-\lambda_{T-1}=0$. If we sum these over all t's, the multipliers cancel out and the statement is verified. ■

Proof of Lemma 3

Proof. Recall that assumption 1 guarantees that for all t, period-t revenues are t-times differentiable. The proof of differentiability proceeds by (backward) induction. Consider first the final period of an increasing sequence T_i. For $p_{T_i-1}<p^m_{T_i}-c$, it must be that $S_{T_{i-1}}\left(p_{T_i-1}\right)>0$ otherwise the monopolist would choose $p_{T_i} = p^m_{T_i}$ which is not consistent with equilibrium. For $p_{T_i-1} < p^m_{T_i} - c$ (i.e., whenever storage is positive in equilibrium) $S_{T_{i-1}}(p_{T_i-1})$ is defined by equation (15):

$$MR_{T_i}(p_{T_i-1} + c) = S_{T_{i-1}}(p_{T_i-1}).$$

Thus, $S_{T_{i-1}}$ has the same order of differentiablity as $MR_{T_i}(p_{T_i-1}+c)$: it is $T_i - 1$ times differentiable as long as period T_i revenues R_{T_i} are differentiable T_i times.[12] Now assume that S_{t+1} is differentiable $t+1$ times. Using the fact that, in an equilibrium of the subgame following period t, $p_{t+1} = p_t + c$, we can rewrite equation (15) for period $t+1$ as

$$S_t(p_t) \equiv MR_{t+1}(p_t + c) + S_{t+1}(p_t + c) - c\, \frac{dS_{t+1}(p_{t+1})}{dp_{t+1}}\bigg|_{p_t+c}. \tag{21}$$

Thus, S_t is t-times differentiable as long as revenues R_{t+1} are $t+1$ times differentiable.

We now show that $\frac{dS_t(p_t)}{dp_t} \leq 0$. Using equation (21), we can write:

$$\frac{dS_t(p_t)}{dp_t} = \frac{d}{dp_t}\left(MR_{t+1}(p_t + c) + S_{t+1}(p_t + c) - c\, \frac{dS_{t+1}(p_{t+1})}{dp_{t+1}}\bigg|_{p_t+c}\right). \tag{22}$$

Since the term in parenthesis in the right-hand side of equation (22) is the first derivative of the period $t+1$ objective function (see equation 15), we see that the expression on the right-hand side of equation (22) is the second derivative with respect to p_{t+1} of the period $t+1$ objective function. Thus, because of necessary conditions for optimality, the right-hand side of equation (22) cannot be positive. We therefore conclude that $\frac{dS_t(p_t)}{dp_t} \leq 0$. Furthermore, for any period T_i, the inequality is strict: recall for $t = T_i$, $MR_{T_i}(p_{T_i-1} + c) = S_{T_{i-1}}(p_{T_i-1})$, so $\frac{dS_{T_i-1}(p)}{dp_{T_i-1}} = MR'_{T_i}\left(p^{nc}_{T_i}\right) < 0$ because revenues are concave. ■

Proof of Proposition 1

We begin by obtaining an additional feature of the equilibrium price sequence. If between period t and period $t+1$, the equilibrium price either decreases, or increases by less than c, then we can think of the equilibrium as being made of two (locally) independent sequences: the sequence up to t and the sequence following t. In other words, if the storage constraint is not binding in period t, it is as if periods before t and period after t are independent. The following Lemma generalizes this idea and makes it precise.

Lemma 4 *Let $T_1, T_2, ..., T_n$ be such that $1 \leq T_i \leq T-1$, and $p^c_{T_i+1} < p^c_{T_i} + c$. Then,*

$$\sum_{t=1}^{T_1} MR_t(p^c_t) = \sum_{t=T_1+1}^{T_2} MR_t(p^c_t) = ... = \sum_{t=T_n+1}^{T} MR_t(p^c_t) = 0.$$

[12] Note however that $S_1(p_1)$ may not be differentiable at $p_1 = p^m_2 - c$ because for any $p_1 > p^m_1 - c$, $S_1(p_1) = \frac{dS(p_1)}{dp_1} = 0$, while for $p_t < p^m_{t+1} - c$, $\frac{dS_{t1}(p_1)}{dp_1}$ is typically bounded away from zero. Notice however that the limits from both the right and the left of $\frac{dS_1(p_1)}{dp_1}$ exist and are both nonpositive.

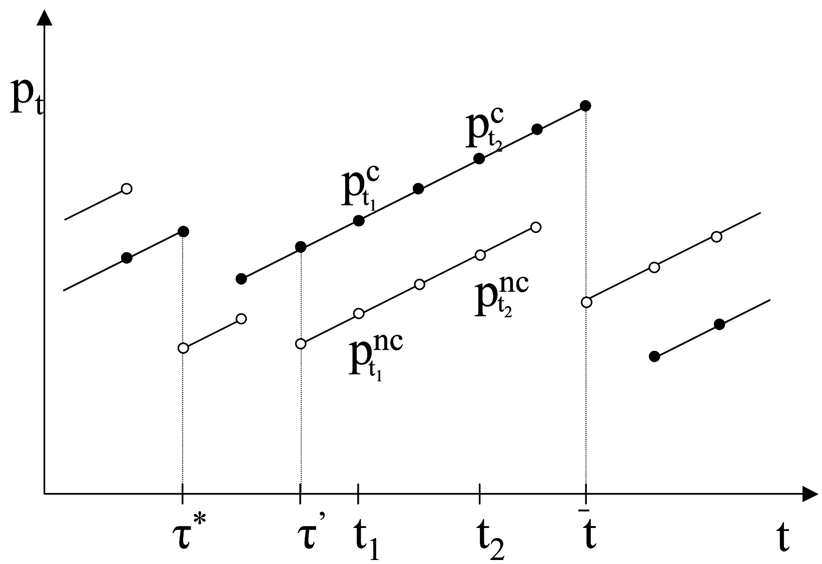

Figure 1:

Proof. Suppose by way of contradiction that $\sum_{t=T_i+1}^{T_{i+1}} MR_t(p_t^c) < 0$. Then the monopolist could increase his profits by decreasing prices slightly for all $t = T_i + 1, ..., T_{i+1}$. Similarly, if $\sum_{t=T_i+1}^{T_{i+1}} MR_t(p_t^c) > 0$, the monopolist could then increase his profits by slightly increasing prices for all $t = T_i + 1, ..., T_{i+1}$. ■

The next Lemma links the subsequences of binding storage constraints in the no-commitment case.

Lemma 5 *Let $\{p_t^{nc}\}_{t=1}^T$ be an equilibrium of the game without commitment. Then, for any possible pair of dates t_1, t_2,*

$$\sum_{t=t_1}^{t_2} MR_t(p_t^{nc}) = c \sum_{t=t_1}^{t_2} \left. \frac{dS_t(p)}{dp} \right|_{p_t^{nc}} + S_{t_1-1} - S_{t_2}. \tag{23}$$

Proof. (15) This is obtained by summing equations (15) over all $t = t_1 ... t_2$. ■

We are now ready to prove the Proposition.

Proof. Assume by way of contradiction that there exists a sequence of periods $t = t_1, ..., t_2$ (with, possibly, $t_1 = t_2$) for which $p_t^{nc} < p_t^c$ for all $t = t_1, ..., t_2$ (see Figure). We show that this implies that there is a profitable deviation under commitment.

Let $\tau' \leq t_1 - 1$ be the earliest period such that $p_{t+1}^{nc} = p_t^{nc} + c$ for all $t = \tau', ..., t_1 - 1$, i.e., τ' is the first period of a no-commitment sequence with binding storage constraint. Clearly, this implies that, unless $\tau' = 1$, $p_{\tau'}^{nc} < p_{\tau'-1}^{nc} + c$, and $p_t^{nc} = p_{t_1}^{nc} - (t_1 - t)c$ for all $t = \tau', ..., t_1 - 1$.

Let τ^* be the earliest period such that $p_t^{nc} < p_t^c$ for all $t = \tau^*, ..., t_1$. This means that, unless $\tau^* = 1$, $p_{\tau^*-1}^{nc} \geq p_{\tau^*-1}^c$. Let $\underline{t} = \min\{\tau^*, \tau'\}$ and let $\overline{t} \geq t_1$ be the latest period such that $p_{t+1}^c = p_t^c + c$ for all $t = t_1, ..., \overline{t}$, i.e., $\overline{t}$ is the latest period of a commitment sequence with binding storage constraint. This means that, unless $\overline{t} = T$, $p_t^c = p_{t_1}^c + (t - t_1)c$ for all $t = t_1, ..., \overline{t}$ and $p_{\overline{t}+1}^c < p_{\overline{t}}^c + c$.[13]

We now establish that $p_t^{nc} < p_t^c$ for all $t = \underline{t}, ..., \overline{t}$. We break this step into two parts.

Consider first the periods $t = t_1, ..., \overline{t}$. The constraints $p_{t+1}^{nc} \leq p_t^{nc} + c$ imply that $p_t^{nc} \leq p_{t_1}^{nc} + (t - t_1)c$. By contradiction, $p_{t_1}^{nc} < p_{t_1}^c$ and, by construction, $p_t^c = p_{t_1}^c + (t - t_1)c$. These three inequalities imply that $p_t^{nc} < p_t^c$ for all $t = t_1, ..., \overline{t}$.

Consider now the periods $t = \underline{t}, ..., t_1 - 1$. If $\underline{t} = \tau^*$, $p_t^{nc} < p_t^c$ by the construction. If instead $\underline{t} = \tau'$ we can repeat a similar argument to the one offered above. By construction, $p_t^{nc} = p_{t_1}^{nc} - (t_1 - t)c$ for all $t = \tau', ..., t_1 - 1$. Because $p_{t+1}^c \leq p_t^c + c$ for all t we obtain that $p_t^c \geq p_{t_1}^c - (t_1 - t)c$ for all $t = \tau', ..., t_1 - 1$. Finally, because $p_{t_1}^{nc} < p_{t_1}^c$ by contradiction, $p_t^{nc} < p_t^c$ for all $t = \underline{t}, ..., t_1 - 1$.

Observe now that, unless $\underline{t} = 1$, $p_{\underline{t}}^{nc} < p_{\underline{t}-1}^{nc} + c$. This is obviously true if $\underline{t} = \tau'$. If instead $\underline{t} = \tau^*$, this is implied by inequality $p_{\tau^*}^{nc} < p_{\tau^*}^c$, the storage constraint $p_{\tau^*}^c \leq p_{\tau^*-1}^c + c$, and inequality $p_{\tau^*-1}^c \leq p_{\tau^*-1}^{nc}$.

We now establish that $\sum_{t=\underline{t}}^{\overline{t}} MR_t(p_t^c) < 0$. The equilibrium condition without commitment (23) implies that:

$$\sum_{t=\underline{t}}^{\overline{t}} MR_t(p_t^{nc}) = c \sum_{t=\underline{t}}^{\overline{t}} \left. \frac{dS_t(p)}{dp} \right|_{p_t^{nc}} + S_{\underline{t}-1} - S_{\overline{t}}.$$

Because either $p_{\underline{t}}^{nc} < p_{\underline{t}-1}^{nc} + c$ or $\underline{t} = 1$, storage $S_{\underline{t}-1} = 0$. This and the fact that $\left. \frac{dS_\tau(p)}{dp} \right|_{p_\tau^{nc}} \leq 0$ for all t imply that

$$\sum_{t=\underline{t}}^{\overline{t}} MR_t(p_t^{nc}) \leq 0. \tag{24}$$

Because for all t, $MR_t(p_t)$ are decreasing in p_t and because $p_t^{nc} < p_t^c$ for all $t = \underline{t}, ..., \overline{t}$, from inequality (24) we obtain

$$\sum_{t=\underline{t}}^{\overline{t}} MR_t(p_t^c) < 0. \tag{25}$$

Finally, we have that either $\overline{t} = T$ or $p_{\overline{t}+1}^c < p_{\overline{t}}^c + c$. We can then use Lemma 4 to obtain

$$\sum_{t=1}^{\overline{t}} MR_t(p_t^c) = 0. \tag{26}$$

As a consequence, if $\underline{t} = 1$, conditions (25) and (26) are a contradiction. If instead $\underline{t} \geq 2$, condition (26) and inequality (25) imply that

$$\sum_{t=1}^{\underline{t}-1} MR_t(p_t^c) > 0. \tag{27}$$

[13] It is possible that $\overline{t} \leq t_2$.

Inequalities (25) and (27) imply that the monopolist could increase his profit by marginally increasing prices p_t^c (by the same amount) for all $t = 1, ..., \underline{t} - 1$, and by marginally decreasing them (by the same amount) for all $t = \underline{t}, ..., \overline{t}$. This contradicts the hypothesis that the sequence $\{p_\tau^c\}_{\tau=1}^T$ is an equilibrium with commitment. ■

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7 Supplemental Appendix

7.1 Convex Cost of Storage

Assume that the cost of storage is a twice continuously differentiable function $c(S)$ with $c'(S) > 0$, $c''(S) > 0$, and $c(0) = c'(0) = 0$.

Consider any fixed sequence of prices $\{p_t\}_{t=1}^{T}$. Suppose that the buyer begins date t with a stock S_{t-1} of the good. Let $S_t(p_1, ..., p_T)$ be the optimal storage choice by the consumer. The following Lemma provides a simple characterization of the solution of the buyer's problem.

Lemma 6 *Assume that $p_t \leq p_T^m$ for all t. Then, at date $t \leq T-1$ the consumer stores quantity S_t that solves*

$$c'(S_t) = \max\{0, p_{t+1} - p_t\}, \tag{28}$$

consumes

$$x_t = D_t(p_t),$$

and purchases $b_t = D_t(p_t) + S_t - S_{t-1}$ units.

By Lemma 6, we can write the consumer's optimal storage decision at period t as a function of period t and period $t+1$ prices only. Denote by $S_t(p_t, p_{t+1})$ the optimal storage decisions at period t as defined by equation (28).

7.1.1 Commitment

By Lemma 6, under commitment the monopolist chooses a sequence of prices $\{p_t\}_{t=1}^{T}$ to maximize

$$\sum_{t=1}^{T} [D_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1})]p_t \tag{29}$$

with $S_0(p_0, p_1) = S_T(p_T, p_{T+1}) = 0$ and $S_t(p_t, p_{t+1})$ defined by equation (28).

The first order conditions at period t is:

$$MR_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1}) - \frac{\partial S_{t-1}(p_{t-1}, p_t)}{\partial p_t}(p_t - p_{t-1}) - \frac{\partial S_t(p_t, p_{t+1})}{\partial p_t}(p_{t+1} - p_t) = 0. \tag{30}$$

Notice first that $S_t(p_t, p_{t+1})$ might not be differentiable in p_t when $p_t = p_{t+1}$. This is because, for a fixed p_{t+1}, $\frac{\partial S_t(p_t, p_{t+1})}{\partial p_t} < 0$ if $p_t < p_{t+1}$ and $\frac{\partial S_t(p_t, p_{t+1})}{\partial p_t} = 0$ if $p_t > p_{t+1}$. Similarly, $S_t(p_t, p_{t+1})$ might not be differentiable in p_{t+1} when $p_{t+1} = p_t$. Notice however that the limits from both the right and the left of $S_t(p_t, p_{t+1})$ exists and are both nonpositive. Finally, notice that if $p_t \neq p_{t+1}$

$$\frac{\partial S_t(p_t, p_{t+1})}{\partial p_t} = -\frac{\partial S_t(p_t, p_{t+1})}{\partial p_{t+1}}.$$

Summing (30) over t and recalling that $S_0(p_0, p_1) = S_T(p_T, p_{T+1}) = 0$ we obtain

$$\sum_{t=1}^{T} MR_t(p_t^c) = 0. \tag{31}$$

This equation is the counterpart of equation (7) that we obtained in the case of linear costs of storage. Note however that equation (31) is not as informative: because prices are now not necessarily rising at a constant rate c, we need T conditions to obtain each price.

7.1.2 No Commitment

The construction of the equilibrium absent commitment is quite similar to the analysis in Section 3. The main difference is that equilibrium storage $S_t(p_t)$ at date t must satisfy

$$c'\left(S_t\left(p_t\right)\right) = \max\{0, p_{t+1} - p_t\}.$$

Appropriately modifying the analysis of Section (3.4) we obtain that equilibrium is characterized, at period t, by:

$$MR_t(p_t) = S_{t-1} - S_t^{nc}(p_t) + (p_{t+1}^{nc}(p_t) - p_t)\frac{\partial S_t^{nc}(p_t)}{\partial p_t}$$

where $S_0 = S_T^{nc}(p_T) = 0$.

Summing over t we obtain

$$\sum_{t=1}^{T} MR_t(p_t^{nc}) = \sum_{t=1}^{T}\left((p_{t+1}^{nc} - p_t^{nc}) \left.\frac{\partial S_t^{nc}(p_t)}{\partial p_t}\right|_{p_t^{nc}} \right).$$

Going through similar steps as in the proof of Lemma 3 we can show that

$$\frac{\partial S_t^{nc}\left(p_t\right)}{\partial p_t} = \frac{\frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}}{1 - c''\left(S_t^{nc}\right)\frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}} \leq 0.$$

Because, as in the previous section, we can prove that $\frac{\partial S_{T-1}^{nc}(p_{T-1})}{\partial p_{T-1}} < 0$, we can conclude that $\sum_{t=1}^{T} MR_t(p_t^{nc}) < \sum_{t=1}^{T} MR_t(p_t^{c})$. Because MR_t are decreasing functions for all t, we can conclude that prices under commitment cannot be uniformly higher and there is also a sense in which they have to be lower "on average."

The problem of comparing prices at each period stems from the fact that both under commitment and without commitment the price sequence is determined by T conditions. Comparison of prices at each period implies the comparison of T conditions:

$$\begin{cases} MR_1(p_1^c) = -S_1(p_1^c, p_2^c) + (p_2^c - p_1^c)\frac{\partial S_1(p_1,p_2)}{\partial p_1} \\ \dots \\ MR_t(p_t^c) = S_{t-1}(p_{t-1}, p_t) - S_t(p_t, p_{t+1}) + \left(p_t^c - p_{t-1}^c\right)\frac{\partial S_{t-1}(p_{t-1},p_t)}{\partial p_{t-1}} + \left(p_{t+1}^c - p_t^c\right)\frac{\partial S_t(p_t,p_{t+1})}{\partial p_t} \\ \dots \\ MR_T(p_T^c) = S_{T-1}(p_{T-1}, p_T) + \left(p_T^c - p_{T-1}^c\right)\frac{\partial S_{T-1}(p_{T-1},p_T)}{\partial p_{T-1}} \end{cases}$$

for the case of commitment, and

$$\begin{cases} MR_1(p_1^{nc}) = -S_1 + (p_2(p_1^{nc}) - p_1^{nc}) \frac{\partial S_1^{nc}(p_1)}{\partial p_1} \\ ... \\ MR_t(p_t^{nc}) = S_{t-1}^{nc} - S_t^{nc} + (p_{t+1}^{nc}(p_t) - p_t) \frac{\partial S_t^{nc}(p_t)}{\partial p_t} \\ ... \\ MR_T(p_T^{nc}) = S_{T-1}^{nc} \end{cases}$$

without commitment.

7.2 Solution Algorithm

In this section we provide an algorithm to solve the problem of choosing a sequence of prices $\{p_t^c\}_{t=1}^T$ to maximize

$$\sum_{t=1}^{T} D_t(p_t) \tag{32}$$

subject to

$$p_{t+1}^c \leq p_t^c + c \text{ for all } t = 1, ..., T-1. \tag{33}$$

The solution algorithm is based on the following intuition. Consider first the case in which there is only one period, that is $T = 1$. In this case, the problem above reduces to the maximization of static monopoly profits $D_1(p_1)p_1$. The solution is $p_1^c = p_1^m$, where p_1^m is the static monopoly price. Consider now the two period problem. If $D_1(p_1)$ and $D_2(p_2)$ are such that $p_2^m > p_1^m + c$, the constraint (33) becomes binding and p_1^c is found by maximizing

$$D_1(p_1) + D_2(p_1 + c). \tag{34}$$

In particular, $p_1^m \leq p_1^c$. If instead $D_1(p_1)$ and $D_2(p_2)$ are such that $p_2^m \leq p_1^m + c$, the constraint (33) is not binding and the price that maximizes (34) is smaller than the static monopoly price p_1^m.

This observation can be generalized to many periods. Specifically, if constraint (33) is binding for the first T_1 periods, then the *argmax* of

$$\sum_{t=1}^{T_1} D_t(p + (t-1)c)$$

is greater than the *argmax* of

$$\sum_{t=1}^{\tau} D_t(p + (t-1)c)$$

for all $\tau = 1, ..., T_1$. This statement will be made more precise and proved in Lemma 7 below.

We now introduce some notation and describe the algorithm. Consider an interval of periods $t = t_1, ..., t_2$, with $t_1 \leq t_2$, and let $p(t_1, t_2)$ be the solution to the equation

$$\sum_{t=t_1}^{t_2} MR_t(p + (t - t_1)c) = 0. \tag{35}$$

Because the functions $MR_t(p_t)$ are strictly decreasing in p, this sum is also decreasing in p and $p(t_{1,}t_2)$ is unique.

The algorithm is defined by iterating on i. At the first step of the algorithm consider the prices $p(1,t)$, with $t = 1, ..., T$. Let T_1 be

$$T_1 = \arg \max_{t=1,...,T} p(1,t). \tag{36}$$

If the argmax is not unique, let T_1 be the greatest. For all $t = 1, ..., T_1$, set p_t^c according to:

$$p_t^c = p(1, T_1) + (t-1)c. \tag{37}$$

At the $i^{th}+1$ step of the algorithm, consider the interval of periods $t = T_i+1, ..., T$ and compute the prices $p(T_i + 1, t)$. Let T_{i+1} be

$$T_{i+1} = \arg \max_{t=T_i+1,...,T} p(T_i + 1, t).$$

If T_{i+1} is not unique, consider the greatest. For each $t = T_i + 1, ..., T$, set p_t^c according to

$$p_t^c = p(T_i + 1, T_{i+1}) + (t - T_i - 1)c.$$

The algorithm proceeds until, at some iteration, $T_{i+1} = T$.

Remark 1 *By construction, the algorithm delivers a unique solution.*

Before we prove the correspondence between the solution to the algorithm and the equilibrium we show that the price sequence obtained with the algorithm satisfies the constraint $p_{t+1}^c \leq p_t^c + c$. This allows us to draw an analogy between the T_i's of this section and those of Lermma (4).

Lemma 7 *The sequence of prices* $\{p_t^c\}_{t=1}^T$ *that solves the algorithm satisfies the contraint*

$$p_{t+1}^c \leq p_t^c + c$$

for all $t = 1, ..., T$.

Proof. By construction, constraints (33) are satisfied by p_t^c for all $t = T_i + 1, ..., T_{i+1}$. To prove that this is true also for $p_{T_i}^c$ and $p_{T_i+1}^c$, consider for simplicity the first and the second iteration. Suppose by way of contradiction that $p(T_1 + 1, T_2) > p_{T_1}^{nc} + c$ that is:

$$p(T_1 + 1, T_2) > p(1, T_1) + T_1 c. \tag{38}$$

Because, by definition of $p(T_1 + 1, T_2)$,

$$\sum_{t=T_1+1}^{T_2} MR_t(p(T_1 + 1, T_2) + (t - T_1 - 1)c) = 0$$

and because the functions $MR_t(p)$ are strictly decreasing, inequality (38) implies that

$$\sum_{t=1}^{T_2} MR_t(p(1,T_1)+(t-1)c) > 0. \tag{39}$$

Recalling that

$$\sum_{t=1}^{T_2} MR_t(p(1,T_2)+(t-1)c) = 0$$

by definition of $p(1,T_2)$, inequality (39) implies that $p(1,T_2) > p(1,T_1)$. This contradicts the hypothesis that $T_1 = arg \max_{t=1,...,T} p(1,t)$ and concludes the proof. ■

We now prove that the solution to the algorithm and the solution to the maximization problem are the same.

Lemma 8 *A price sequence $\{p_t^c\}_{t=1}^T$ maximizes (32) subject to (33) if and only if it is a solution of the algorithm.*

Proof. Let $\{p_t^c\}_{t=1}^T$ be a solution to the maximization of (32) subject to (33). Because both $\{p_t^c\}_{t=1}^T$ and the solution to the algorithm are unique, it is enough to show that $\{p_t^c\}_{t=1}^T$ solves the algorithm.

With the usual notation, let $(T_1, ..., T_m)$ be a sequence of dates, with $1 \leq T_1 \leq ... \leq T_m \leq T$, such that, for each i, $p_{T_i}^c + c < p_{T_i+1}^c$, i.e. storage is not binding between periods T_i and period T_{i+1}. Without loss of generality, consider the set of periods $t = T_1+1, ..., T_2$ and corresponding prices p_t^c.

We first show that $p_{T_1+1}^c = p(T_1+1, T_2)$. This follows immediately from Lemma (4)

$$\sum_{t=T_1+1}^{T_2} MR_t(p_{T_1+1}^c + (t-T_1-1)c) = 0, \tag{40}$$

and from the definition and uniqueness of $p(T_1+1, T_2)$.

We now show that $T_2 = \arg \max_{t=T_1+1,...,T} p(T_1+1, t)$, that is the algorithm breaks the solution price vector at T_2. We break this step in two parts.

Assume by way of contradiction that there exists a τ with $T_2 + 1 \leq \tau$ such that $p_{T_1+1}^c < p(T_1+1, \tau)$. Because the functions $MR_t(p_t)$ are decreasing in p_t, equality (40) implies that

$$\sum_{t=T_1+1}^{T_2} MR_t(p(T_1+1,\tau)+(t-T_1-1)c) < 0. \tag{41}$$

Because, by construction,

$$\sum_{t=T_1+1}^{\tau} MR_t(p(T_1+1,\tau)+(t-T_1-1)c) = 0,$$

inequality (41) implies that

$$\sum_{t=T_2+1}^{\tau} MR_t(p(T_1+1,\tau)+(t-T_1-1)c) > 0. \tag{42}$$

Notice now that $p_t^c \leq p_{T_1+1}^c + (t-T_1-1)c$ for all $t = T_2+1, ..., T$. This holds because $p_{t+1}^c \leq p_t^c + c$ for all $t = T_1+1, ..., T-1$. Using $p_{T_1+1}^c < p(T_1+1,\tau)$ we have that $p_t^c < p(T_1+1,\tau)+(t-T_1-1)c$ for all $t = T_2+1, ..., T$. Hence, by inequality (42),

$$\sum_{t=T_2+1}^{\tau} MR_t(p_t^c) > 0. \tag{43}$$

This means that profits could be increased by marginally increasing prices for all $t = T_2+1, ..., \tau$. This contradicts the hypothesis that $\{p_t^c\}_{t=1}^T$ is an optimal sequence of prices.

Finally, assume by way of contradiction that there exists a τ with $\tau < T_2$ such that $p_{T_1+1}^c < p(T_1+1,\tau)$. By definition of $p(T_1+1,\tau)$, this would imply that

$$\sum_{t=T_1+1}^{\tau} MR_t(p_t^c) > 0.$$

Profits could be increased by marginally increasing prices for all $t = T_1+1, ..., \tau$. This contradicts the hypothesis that $\{p_t^c\}_{t=1}^T$ is an optimal sequence of prices and concludes the proof. ■