

# Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows

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## Abstract

We study the role of information in asset pricing models with long-run cash flow risk. To illustrate the importance of the information structure, we show how the implications of the long-run risk paradigm for the cross-sectional properties of stock returns are affected by information. When investors can fully distinguish short- and long-run consumption risk components of dividend growth innovations (*full information*), only exposure to long-run consumption risk generates significant risk premia, implying that high expected return stocks are long-duration assets, contrary to the historical data. By contrast, when investors observe the change in consumption and dividends each period but not the individual components of that change (*limited information*), exposure to short-run risk can generate large risk premia, so that high expected return stocks are short-duration assets while low expected return stocks are long-duration assets, as in the data.

JEL: G10, G12

## 1 Introduction

We study the role of information in asset pricing models with long-run cash flow risk. The idea that long-run cash flow risk can have important effects on asset prices is exemplified by the work of Bansal and Yaron (2004), who show that a small but persistent common component in the time-series processes of consumption and dividend growth is capable of generating large risk premia and high Sharpe ratios.<sup>1</sup>

A crucial aspect of the long-run risk theory is that the small persistent component in consumption growth must account for only a small fraction of its short-run variability. Otherwise, the model-implied annualized volatility of consumption and dividend growth is implausibly large. Yet despite its small role in consumption fluctuations, a maintained assumption in the literature is that investors can directly observe the small persistent component and distinguish its innovations from transitory shocks to consumption and dividend growth. We refer to this assumption as the *full information* assumption. While this is a natural starting place and an important case to understand, in this paper we consider a *limited information* specification in which market participants are faced with a signal extraction problem: they can observe the change in consumption and dividends each period, but they cannot observe the individual components of that change.

Information about long-run cash flow risk is likely to be limited. In finite samples it is difficult to distinguish statistically between a purely i.i.d. process and one that incorporates a small persistent component. Perhaps more important, for specifications of the dividend process that have been studied in the long-run risk literature, the distinct roles of persistent and transitory shocks cannot be separately identified from the history of consumption and dividend data, no matter how many observations are available. Thus, the full information assumption takes the amount of information investors have very seriously: market participants must not only understand that a small predictable component in cash-flow growth exists, they must also be able to decompose each period's innovation into its component sources and have complete knowledge of how the shocks to these sources vary and covary with one another, even though the data give us no guide for identifying these components

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<sup>1</sup>A growing body of theoretical and empirical work has been devoted to studying the role of long-run risk in consumption and dividend growth for explaining asset pricing behavior. See Parker (2001); Parker and Julliard (2004); Colacito and Croce (2004); Bansal, Dittmar and Kiku (2005b); Hansen, Heaton and Li (2008); Kiku (2005); Malloy, Moskowitz and Vissing-Jorgensen (2009); Bansal, Dittmar and Lundblad (2005a); Bansal, Kiku and Yaron (2006); Hansen and Sargent (2006).

separately.

As an illustration of the potential importance of the information structure, we study the implications of the long-run risk paradigm for the aggregate term structure of equity and for the cross-sectional properties of stock returns and cash flow duration. Empirical evidence indicates that assets with low ratios of price to measures of fundamental value (value stocks) have higher average returns than assets with high ratios of price to fundamental value (growth stocks) (Graham and Dodd (1934); Fama and French (1992)). At the same time, a second strand of empirical evidence finds that assets with high expected returns have shorter duration in their cash flows than do assets with low expected returns. In other words, the term structure of equity is downward sloping (Cornell (1999, 2000); Dechow, Sloan and Soliman (2004); Da (2005); van Binsbergen, Brandt and Kojen (2010)).<sup>2</sup> Duration here refers to the timing of expected future cash flows. Shorter duration means that the timing of a stock's expected cash flow fluctuations is weighted more toward the near future than toward the far future, whereas the opposite is true for a longer duration security. The long-run risk paradigm implies that assets with high average returns, such as value stocks, command a high risk premium because they are highly exposed to long-duration cash flow risk even if they have little or no exposure to short-duration risk. An unanswered question, therefore, is whether the long-run risk perspective can be reconciled with the seemingly contradictory cash flow duration evidence, which finds that short-duration assets earn high risk premia while long-duration assets earn low risk premia. We argue here that what may seem to be small changes in the information structure can have important implications for such questions.

We study a model in which the aggregate dividend growth rate is differentially exposed to two systematic risk components driven by aggregate consumption growth, in addition to a purely idiosyncratic component uncorrelated with aggregate consumption. One is a small but highly persistent (long-run) component as in Bansal and Yaron (2004), while the second is a transitory (short-run) i.i.d. component with much larger variance. The purely idiosyncratic component has the highest variance, as required to match the evidence that dividend growth is substantially more volatile than consumption growth. In addition, we follow the existing literature on long-run risk by employing the recursive utility specification developed by

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<sup>2</sup>It is important to note that all of the empirical measures of duration presented in these papers are measures that differ across asset classes solely because of differences in the *timing* of expected future cash flows and not because of differences in discount rates, which are held fixed across asset classes.

Epstein and Zin (1989, 1991) and Weil (1989). With recursive utility, investors are not indifferent to the intertemporal composition of risk, implying that the relative exposure to short- versus long-run risks has a non-trivial influence on risk premia.

In order to isolate the endogenous relation between cash-flow duration and risk premia in models with long-run consumption risk, we study the model's implications for equity strips, and for heterogeneous firms that differ only in the timing of their cash flows. This may be accomplished by recognizing that an equity claim is a portfolio of zero-coupon dividend claims (strips) with different maturities. Thus, long-duration assets (firms) can be modeled as equity with a high weight on long-maturity dividend claims relative to short-maturity dividend claims.

We find that, in long-run risk models with full information, assets with long duration will endogenously pay high risk premia, while those with short-duration will endogenously pay low risk premia, contrary to the historical data. In this case, the equity term structure slopes up rather than down and the relation between cash flow duration and risk premia goes the wrong way. By contrast, under limited information the term structure of equity slopes down: assets with long duration have low risk premia, while those with short-duration have high risk premia, consistent with the data.

The intuition for this result is straightforward. When investors can observe the long-run component in cash flows—in which a small shock today can have a large impact on long-run growth rates—the long-run is correctly inferred to be more risky than the short-run, implying that long-duration assets must in equilibrium command high risk premia, whereas short-duration assets command low risk premia. Under limited information, the opposite can occur because investors' optimal estimate of the long-run component in consumption growth is given by the solution to a signal extraction problem and is therefore influenced by shocks to the i.i.d. consumption component, which cannot be distinguished from shocks to the persistent component. This causes short-duration assets to have high risk premia under limited information. At the same time, shocks to the persistent component in consumption growth (which drive the persistent component in dividend growth) are too small to be distinguished, under limited information, from the large idiosyncratic dividend shocks that command no risk-premium. This causes long-duration assets to have low risk premia under limited information.

The main purpose of this paper is to illustrate the potential importance of the information structure for the cross-sectional implications of long-run risk models. Nevertheless, we also

show that some calibrations of the limited information models we explore can match the properties of cross-sectional data quantitatively. In these cases, the limited information specifications are consistent not only with the cash flow duration properties of value and growth stocks and a sizable value premium, they also explain the higher empirical Sharpe ratios of value stocks and the failure of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) to account for the value premium and the ability of high-minus-low factor (HML) of Fama and French (1993) to explain the value premium.

The rest of this paper is organized as follows. The next section discusses related literature not discussed above. Section 3 presents the asset pricing model, the model for cash flows, and the informational assumptions. Section 4 shows how one specification of limited information influences equilibrium asset returns and compares these results to a full information benchmark. A theme of this section is that the duration evidence for value and growth assets can be used to distinguish among long-run risk models that differ according to their information structures, cash flow properties, and primitive preference parameter values. Section 5 presents the quantitative implications of the information structure for the value premium. Section 6 concludes.

## 2 Related Literature

A number of recent papers address issues related to those studied here. Hansen and Sargent (2006) are also concerned about the agent's ability to observe the long-run risk component in aggregate cash flows. The agents in their model form decision rules that are robust to misspecification of their approximating model of cash-flows. Our paper, while similar in motivation to Hansen and Sargent (2006), differs along a number of dimensions. The most important is that Hansen and Sargent focus on the role of learning with robust preferences, while we focus on the agent's information structure. In addition, Hansen and Sargent study their model's implications for the unobservable return on a claim to aggregate consumption, whereas we investigate a levered equity claim similar to that studied in Bansal and Yaron (2004). This distinction is important because the parameters of the levered equity claim process we study are in general unidentified, implying that even an agent armed with an infinite sample of consumption and dividend data can never learn the correct cash flow model and must always solve a signal extraction problem. It is this signal extraction problem in the presence of long-run risk that our analysis is designed to focus on.

Our paper is also related to a recent literature that seeks to reconcile the cross-sectional

properties of equity returns simultaneously with the cash flow duration properties of value and growth assets. Lettau and Wachter (2007) and Lettau and Wachter (2009) use techniques from the affine term structure literature to develop a dynamic risk-based model that captures the value premium, the cash flow duration properties of value and growth portfolios, and the poor performance of the CAPM. However, Lettau and Wachter forgo modeling preferences, and instead directly specify the stochastic discount factor. This is important because models of preferences as a function of aggregate fundamentals often have difficulty matching the cross-sectional properties of stock returns. For example, Lettau and Wachter (2007) and Wachter (2006) show that the Campbell and Cochrane (Campbell and Cochrane (1999)) habit model implies that assets with greater risk premia are long duration assets, rather than short duration assets as in the data. Santos and Veronesi (2010) modify the Campbell and Cochrane model by adding cash flow risk for multiple risky securities and successfully generate a value premium for short-horizon assets. However, they also find that the cross-sectional dispersion in cash flow risk required to explain the magnitude of the premium is implausibly high.<sup>3</sup> None of these studies investigate the role of the information structure on asset prices, the focus of this paper.

### 3 The Asset Pricing Model

Consider a representative agent who maximizes utility defined over aggregate consumption. To model utility, we use the more flexible version of the power utility model developed by Epstein and Zin (1989, 1991) and Weil (1989), also employed by other researchers who study the importance of long-run risks in cash flows (Bansal and Yaron (2004), Hansen et al. (2008) and Malloy et al. (2009)).

Let  $C_t$  denote consumption and  $R_{C,t}$  denote the simple gross return on the portfolio of all invested wealth, which pays  $C_t$  as its dividend. The Epstein-Zin-Weil objective function

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<sup>3</sup>Two other papers study duration indirectly. Lustig and Van Nieuwerburgh (2006) study a model with heterogeneous agents and housing collateral constraints and find that conditional expected excess returns are hump-shaped in their measure of duration. Zhang (2005) shows that, when adjustment costs are asymmetric and the price of risk varies over time, growth assets can be less risky than assets in place (value stocks), consistent with the cash flow and return properties of value and growth assets. But the Zhang model does not account for the finding of Fama and French (1992) that value stocks do not have higher CAPM betas than growth stocks.

is defined recursively as:

$$U_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t [U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}$$

where  $\gamma$  is the coefficient of risk aversion and the composite parameter  $\theta = \frac{1-\gamma}{1-1/\Psi}$  implicitly defines the intertemporal elasticity of substitution  $\Psi$ .

Let  $P_{j,t}^D$  denote the ex-dividend price of a claim to an asset that pays a dividend stream  $\{D_{j,t}\}_{t=1}^{\infty}$  measured at the end of time  $t$ , and let  $P_t^C$  denote the ex-dividend price of a claim to the aggregate consumption stream. From the first-order condition for optimal consumption choice and the definition of returns

$$E_t [M_{t+1} R_{C,t+1}] = 1, \quad R_{C,t+1} = \frac{P_{t+1}^C + C_{t+1}}{P_t^D} \quad (1)$$

$$E_t [M_{t+1} R_{j,t+1}] = 1, \quad R_{j,t+1} = \frac{P_{j,t+1}^D + D_{j,t+1}}{P_{j,t}^D} \quad (2)$$

where  $M_{t+1}$  is the stochastic discount factor (SDF), given under Epstein-Zin-Weil utility as

$$M_{t+1} = \left( \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right)^{\theta} R_{C,t+1}^{\theta-1}. \quad (3)$$

The return on a one-period risk-free asset whose value is known with certainty at time  $t$  is given by  $R_{t+1}^f \equiv (E_t [M_{t+1}])^{-1}$ .

### 3.1 The Cash Flow Model and Informational Assumptions

Equities are modeled as claims to a dividend process, specified below. Here we describe the form of the stochastic process for aggregate dividend growth. Let  $D_t$  denote the aggregate dividend at time  $t$ , and let  $P_t^D$  denote the ex-dividend price of a claim to the asset that pays the stream  $\{D_t\}_{t=1}^{\infty}$ . We use lower case letters denote log variables, e.g.,  $\log(C_t) \equiv c_t$ .

We seek a model for equity cash flows that allows dividend growth rates to be potentially exposed to both transitory and persistent sources of consumption risk that drive  $M_{t+1}$ , as well as to purely idiosyncratic shocks that command no risk premium. Denote the conditional means of consumption and dividend growth as  $x_{c,t}$  and  $x_{d,t}$ , respectively. To model the persistent fluctuations in consumption risk, we follow Bansal and Yaron (2004) and assume that consumption and dividend growth rates contain a single, common predictable component with an autoregressive structure. In addition, we assume here that dividend growth

rates may also be exposed to transitory (i.i.d.) consumption risk. These assumptions give rise to the following dynamic system:

$$\Delta c_{t+1} = \mu_c + \underbrace{x_{c,t}}_{\text{LR risk}} + \underbrace{\sigma \varepsilon_{c,t+1}}_{\text{SR risk}} \quad (4)$$

$$\Delta d_{t+1} = \mu_d + \phi_x x_{c,t} + \phi_c \sigma \varepsilon_{c,t+1} + \sigma_d \sigma \varepsilon_{d,t+1} \quad (5)$$

$$x_{c,t} = \rho x_{c,t-1} + \sigma_{xc} \sigma \varepsilon_{xc,t} \quad (6)$$

$$\varepsilon_{c,t+1}, \varepsilon_{d,t+1}, \varepsilon_{xc,t} \sim N.i.i.d(0, 1). \quad (7)$$

Note that the conditional mean of dividend growth is proportional to the conditional mean of consumption growth  $x_{d,t} = \phi_x x_{c,t}$ , a specification that follows much of the long-run risk literature. We refer to the system (4)-(6), with the correlation structure (7), as the *true data generating process*.

### 3.1.1 Full Information

In the existing literature on long-run consumption risk (e.g., Bansal and Yaron (2004)), it is commonplace to assume that agents can directly observe the cash flow processes, including the latent conditional means  $x_{c,t}$  and  $x_{d,t}$ . We refer to this as the *full information* assumption. The term in equation (4) labeled “LR risk” captures the small long-run risk component emphasized in the literature because even small innovations in  $x_{c,t}$ , if sufficiently persistent, will have large effects on cash flows in the long-run, resulting in high risk premia when investors can observe  $x_{c,t}$ . In this paper we also allow dividend growth to be exposed to transitory consumption shocks, by introducing the component  $\sigma \varepsilon_{c,t+1}$  in (5). We refer to this component as the short-run risk component, labeled “SR risk,” since its correlation with the stochastic discount factor contributes to the systematic riskiness of the dividend claim, but its purely i.i.d. nature makes that risk short-lived. Because the innovation  $\varepsilon_{d,t+1}$  is uncorrelated with consumption growth, it does not contribute to systematic risk. The loadings  $\phi_x$  and  $\phi_c$  govern the exposure of dividend growth to long-run and short-run consumption risk, respectively.

Under the full information assumption, the latent conditional means  $x_{c,t}$  and  $x_{d,t}$  are fully observable. However, the system (4)-(7) cannot in general be identified from historical data on consumption and dividends. The full information assumption therefore implies that market participants have more information than do econometricians with historical data on

consumption and dividends.<sup>4</sup>

### 3.1.2 Limited Information

Under limited information, investors observe historical consumption and dividend data, but they do not directly observe the latent variables  $x_{c,t}$  and  $x_{d,t}$  or the innovations of the true data generating process (4)-(7). Armed with historical data on dividends and consumption, investors could in principal use Maximum Likelihood and the Kalman filter to estimate a general dynamic system in which innovations in the long-run components of consumption and dividend growth  $x_{c,t}$  and  $x_{d,t}$  have arbitrary correlations with each other and with the i.i.d. innovations of these series. As shown in the web Appendix that accompanies this paper, however, in the absence of apriori restrictions on the parameters, the general dynamic system is unidentified.<sup>5</sup> That is, more than one set of parameter values can give rise to the same value of the likelihood function and the data give no guide for choosing among these. Nevertheless, agents with access to historical data on consumption and dividend growth can directly estimate Wold representations for these series, as long as they follow covariance-stationary processes.

We assume that agents estimate the best fitting univariate Wold representations for consumption and dividend growth.<sup>6</sup> Given the true data generating process, univariate Wold representations for  $\Delta c_t$ , and  $\Delta d_t$  can be written as a pair of first-order autoregressive-moving

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<sup>4</sup>Observe that, in the model, if agents only observe historical dividend and consumption data, equilibrium asset prices will not contain any additional information about  $x_{c,t}$ . This follows because prices are determined endogenously from agents behavior given exogenously determined consumption and dividend processes.

<sup>5</sup>This appendix is available on the authors' web pages.

<sup>6</sup>The dynamic system (4)-(6) has a multivariate Wold representation given as a first-order vector autoregressive-moving average representation. This system imposes cross-equation restrictions that are not imposed in the pair of univariate representations above. As shown in the web Appendix, however, these cross-equation restrictions are rejected in finite sample, both in the model and in historical data. Moreover, in samples of the size currently available, we found that the ARMA models with fewer parameters to estimate often performed better and never performed worse than the multivariate specifications in forecasting consumption and dividend growth. For these reasons, we assume that agents employ the more parsimonious ARMA specifications for forecasting consumption and dividend growth.

average ( $ARMA(1, 1)$ ) processes:<sup>7</sup>

$$\Delta c_{t+1} = \mu_c(1 - \rho) + \rho\Delta c_t + v_{c,t+1}^A - b_c v_{c,t}^A \quad (8)$$

$$\Delta d_{t+1} = \mu_d(1 - \rho) + \rho\Delta d_t + v_{d,t+1}^A - b_d v_{d,t}^A. \quad (9)$$

The  $ARMA$  parameters are functions of the primitive parameters of the dynamic system (4)-(7) and the innovations  $v_{c,t+1}^A$  and  $v_{d,t+1}^A$ , which are in general correlated, are composites of the underlying innovations in (4)-(6). We refer to estimation of (8)-(9) as simply *limited information*.

The nature of the signal extraction problem inherent in limited information is that agents cannot directly observe the long-run risk component  $x_{c,t}$ , but must instead infer it from observable consumption and dividend data. The connection between this signal extraction problem and forecasts of consumption and dividend growth using historical data can be made explicit by noting that the Wold representations can be written as “innovations representations,” familiar from Kalman filter derivations. Let the scalar variable  $\hat{x}_{c,t}$  denote the optimal linear forecast of  $x_{c,t}$  based on the history of  $\Delta c_{t+1}$ , and let the scalar variable  $\hat{x}_{d,t}$  denote the optimal linear forecast of  $x_{d,t}$  based on the history of  $\Delta d_{t+1}$ . The pair of  $ARMA(1, 1)$  processes above may be recast in terms of univariate innovations representations for  $\Delta c_{t+1}$  and  $\Delta d_{t+1}$ , which are functions of  $\hat{x}_{c,t}$  and  $\hat{x}_{d,t}$ , respectively. As the Appendix shows, the optimal forecasts  $\hat{x}_{c,t}$  and  $\hat{x}_{d,t}$  are functions of the observable ARMA parameters and innovations. Thus, observations on  $\Delta c_{t+1}$  and  $\Delta d_{t+1}$  provide noisy signals of the latent variables  $x_{c,t}$  and  $x_{d,t}$ .

### 3.1.3 Policy Function Solution and State Variables

For the full information specification,  $x_{c,t}$  is observable and summarizes the information upon which conditional expectations are based. Since under full information investors know that  $x_{d,t} = \phi_x x_{c,t}$ ,  $x_{d,t}$  does not constitute an additional state variable. Solutions to the model’s equilibrium price-consumption and price-dividend ratios are found by iterating on the Euler equations (1) and (2), assuming that individuals observe the consumption and

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<sup>7</sup>Anderson, Hansen and Sargent (1998) study risk premia for a claim to aggregate consumption in a continuous time, robust-control asset pricing model in which consumption growth follows an  $ARMA(1, 1)$ , in effect giving the agent the same information structure for consumption growth as in (8). We note that if the true data generating process were an  $ARMA(1, 1)$ , the limited and full information specifications in our paper would coincide.

dividend processes in (4)-(6). This delivers a policy function for the price-consumption and price-dividend ratios as a function of a single state variable  $x_{c,t}$ . Under limited information, equilibrium price-consumption and price-dividend ratios are calculated assuming market participants observe only the composite shock processes given in (8) and (9), even though the data are actually generated by the dynamic system (4)-(6) with distinct short- and long-run components. The policy function for the price-consumption ratio is a function of the single state variable  $\hat{x}_{c,t}$ , and the price-dividend ratio is a function of two state variables  $\hat{x}_{c,t}$  and  $\hat{x}_{d,t}$ .<sup>8</sup> For each specification, we simulate histories for consumption and dividend growth from the true data generating process in (4)-(7), and use solutions to the policy functions to generate equilibrium paths for asset prices.<sup>9</sup> The process is iterated forward to obtain simulated histories for asset returns. The Appendix explains how we solve for these functional equations numerically on a grid of values for the state variables.

### 3.2 How Does The Information Structure Affect Equilibrium Outcomes?

The information structure affects equilibrium asset prices because it determines the set of state variables upon which expectations are based. This mechanism can be illustrated by comparing how the respective state variables under full and limited information specifications react to primitive shocks. Figure 1 compares the full information and signal extraction cases by plotting impulse responses to primitive shocks (in percent deviations from steady state) of  $x_{c,t}$ , as compared to  $\hat{x}_{c,t}$ ,  $x_{d,t}$ , as compared to  $\hat{x}_{d,t}$ , and the dividend surprise ( $\Delta d_t - E_{t-1}\Delta d_t$ ) under full and limited information. The first row displays the responses to a one-standard deviation increase in the i.i.d. consumption shock,  $\varepsilon_{c,t}$ , the second row displays the responses to a one-standard deviation increase in the idiosyncratic dividend shock,  $\varepsilon_{d,t}$  and the third row displays responses to a one-standard deviation increase in the innovation to the persistent component of consumption growth,  $\varepsilon_{xc,t}$ . In the figures, we denote all variables under full information without hats, and variables under limited information with hats.

<sup>8</sup>Under limited information, agents do not know the true data generating process or that there exists only a single long-run component. Instead, this must be inferred from the ARMA(1,1) processes. Under the ARMA(1,1) specifications, the correlation between the two estimated components  $\hat{x}_{c,t}$  and  $\hat{x}_{d,t}$  is less than one.

<sup>9</sup>A minor complication is that the policy functions for the limited information specifications are a function of the current innovation in the composite processes that appear in (8) and (9), whereas the actual innovations are generated from (4)-(6). However, the moving average representations are invertible, and their innovations can be recovered from  $\sum_i b_c^i (\Delta c_{t-i} - \rho \Delta c_{t-i-1} - \mu_c)$  and  $\sum_i b_d^i (\Delta d_{t-i} - \rho \Delta d_{t-i-1} - \mu_d)$ , respectively.

The results below are based on the following benchmark calibration of parameters set at monthly frequency:  $\gamma = 10$ ,  $\Psi = 1.5$ ,  $\delta = 0.998985$ ,  $\mu_c = \mu_d = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\sigma_d = 5.9$ . With the exception of  $\sigma_d$ , these parameter values are the same as those in the benchmark specification of Bansal and Yaron (2004). Notice that the innovation variance in  $x_{c,t}$  is small relative to the overall volatility of consumption (the standard deviation of  $\varepsilon_{xc}$  is 0.044 times the standard deviation of  $\varepsilon_c$ ), but the persistence of  $x_{c,t}$  is high. We deviate slightly from the calibration in Bansal and Yaron (2004) by setting  $\sigma_d$  equal to 5.9 rather than 4.5, in order to better match the correlation between consumption and dividends observed in the data.<sup>10</sup> The loadings  $\phi_x$  and  $\phi_c$  are set to 1 and 4, respectively, a calibration that we show below generates significant differences in risk premia between full and limited information. The qualitative results discussed in this section are not influenced by the precise values of  $\phi_x$  and  $\phi_c$ .

Figure 1 shows that a one-standard deviation increase in the i.i.d. consumption shock  $\varepsilon_{c,t}$  leads to a sharp, unexpected increase in dividend growth under both full and limited information (row 1, column 3). Under full information, the agent observes the source of the shock and understands that it has no persistence. Accordingly, expectations of future consumption growth and future dividend growth are unchanged in response to an innovation in  $\varepsilon_{c,t}$ , so the impulse responses of  $x_{c,t}$  and  $x_{d,t}$  are zero. By contrast, under limited information, agents cannot directly observe the source of the shock and do not know if it is persistent or transitory. The solution to the optimal filtering problem therefore implies that agents revise upward their expectation of future consumption growth and, to a lesser extent, future dividend growth, even though in reality the shock has no persistence. Thus, both  $\hat{x}_{c,t}$  and  $\hat{x}_{d,t}$  rise, but the former rises by much more (note the scales). In response to a transitory shock, agents with limited information revise their expectations of future consumption and dividend growth more than they would under full information.

Now consider the responses to an innovation in the persistent component of consumption,  $\varepsilon_{cx,t}$ , in the third row of Figure 1. Under full information, investors recognize that this is a shock to the persistent component of consumption and dividend growth and they accordingly revise upward their expectations of future consumption and dividend growth immediately upon observing the shock. Row 3 of Figure 1 shows that a one-standard deviation increase in  $\varepsilon_{cx,t}$  leads to a jump upward in  $x_{c,t}$  and  $x_{d,t}$ . By contrast, investors with limited information

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<sup>10</sup>Dividend growth is more volatile and less persistent than aggregate consumption growth (Cochrane (1994)). Nevertheless, the results reported in this section are not sensitive to the precise value of  $\sigma_d$ .

revise upward their expectation of future consumption and dividend growth only gradually and by much less than they do under full information. The state variable,  $\hat{x}_{c,t}$  responds sluggishly to the shock and  $\hat{x}_{d,t}$  barely responds at all. The error between both  $x_{c,t}$  and  $\hat{x}_{c,t}$  and  $x_{d,t}$  and  $\hat{x}_{d,t}$  dies out slowly over time. In response to a persistent shock, agents with limited information revise their expectations of future consumption and dividend growth less than they would under full information.

Finally, the middle row of Figure 1 shows that a purely idiosyncratic shock to dividend growth,  $\varepsilon_{d,t}$ , has no affect on expected consumption or dividend growth in full information, and has only a tiny affect on expected dividend growth under limited information. In the next section, we return to these responses as a way to build intuition for why risk premia differ depending on the information structure.

## 4 Theoretical Results

### 4.1 Long-Run Versus Short-Run Consumption Risk Exposure

To understand how the information structure affects equilibrium asset prices, it is instructive to begin by comparing economies with different aggregate dividend processes. Specifically, we study how risk premia differ when the relative exposure of dividend growth to long-run versus short-run consumption risk differs. Comparisons made by varying the loadings  $\phi_x$  and  $\phi_c$  should be thought of as comparisons among separate economies with different aggregate dividend processes, rather than comparisons among multiple risky assets in a single economy.

We begin in Table 1 by investigating the model’s implications for summary statistics on the price-dividend ratio, excess returns, and risk-free rate under limited and full information. The model output is generated by simulating 1000 samples of size 840 months, computing annual returns from monthly data, and reporting the average statistics across the 1000 simulations.<sup>11</sup> With the exception of the parameters  $\phi_c$  and  $\phi_x$  (where results for a range of values are presented), Table 1 presents results using the benchmark parameter configuration discussed above, for full information (FI), and limited information (LI). We denote the log return on the dividend claim  $r_{t+1} = \ln(R_{t+1})$  and the log return on the risk-free rate

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<sup>11</sup>The average levels of the price-dividend ratios reported below are not directly comparable to their empirical counterparts for actual firms, since unlike real firms, the firms in the model have no debt and do not retain earnings. Dividends in the model are more analogous to free cash flow than to actual dividends, implying that model price-dividend ratios should be lower than measured price-dividend ratios in historical data.

$r_{f,t+1} \equiv \ln \left( R_{t+1}^f \right)$ . We discuss comparisons between full information and limited information next.

#### 4.1.1 Limited Information v.s. Full Information

The results in Table 1 show that large differences in risk premia are possible between full and limited information. Consider first the results for full information. In this case, high exposure to long-run consumption risk is required to generate high risk premia. Economies comprised of assets with relatively low exposure  $\phi_x$  to long-run consumption risk and high exposure  $\phi_c$  to short-run consumption risk (e.g., row 2 of Table 1), have low risk premia and high price-dividend ratios, whereas economies comprised of assets with high  $\phi_x$  and low  $\phi_c$  (e.g., row 5 of Table 1), have high risk premia and low price-dividend ratios. In addition, substantial variation in risk premia can only be generated by heterogeneity in the exposure to long-run consumption risk; heterogeneity in short-run risk is inadequate. For example, when  $\phi_x = 3$  and  $\phi_c$  is increased from 2.2 to 6, the log risk premium  $E(r_i - r_f)$  increases by just one and a quarter percent, from 5.20% to 6.63% per annum.

The results under limited information are much the opposite. Economies comprised of assets with relatively low exposure to long-run consumption risk and high exposure to short-run consumption risk, (e.g., row 2 of Table 1), have high risk premia and low price-dividend ratios. Under this parameterization, the log risk premium  $E(r_i - r_f)$  is almost 8 percent per annum under limited information, while it is only 2.45 percent per annum under full information. At the same time, assets with high  $\phi_x$  and low  $\phi_c$  (e.g., row 5 of Table 1), the log risk premium under limited information is only 1.26% per annum whereas it is 5.2% per annum under full information.<sup>12</sup> Last, notice that, unlike full information, substantial variation in risk premia can be generated by heterogeneity in the exposure to short-run consumption risk. For example, when  $\phi_x = 3$  and  $\phi_c$  is increased from 2.2 to 6, the log risk premium increases by over 7 percentage points from 1.26% to 8.42% per annum. On the other hand, fixing  $\phi_c$  and varying  $\phi_x$  generates little variation in risk premia under limited information.

These findings are illustrated graphically in Figure 2, which plots annualized price-

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<sup>12</sup>The table reports values for  $\phi_c$  as low as 2.2. Smaller values for  $\phi_c$  are ruled out in the limited information calibration studied here by the requirement that the price-dividend ratio be finite. This is analogous to the requirement in the Gordon growth model that the expected stock return be greater than the expected dividend growth rate to keep the price-dividend ratio finite.

dividend ratios as a function of the ratio of long-run to short-run consumption risk exposure,  $\phi_x/\phi_c$ . For this figure, the ratio  $\phi_x/\phi_c$  is varied along the horizontal axis in such a way as to hold fixed the 15-month variance of dividend growth that is attributable to the consumption innovations. The left-most panel plots this ratio under limited information at the steady state value of  $\hat{x}_{c,t}$ , along with plus and minus two standard deviations around steady state in  $\hat{x}_{c,t}$  (holding fixed  $\hat{x}_{d,t}$  at its steady-state level). The middle panel plots the price-dividend ratio under limited information at the steady state value of  $\hat{x}_{d,t}$ , along with plus and minus two standard deviations around steady state in  $\hat{x}_{d,t}$  (holding fixed  $\hat{x}_{c,t}$  at its steady-state level). The right-most panel plots the price-dividend ratio under full information as a function of  $\phi_x/\phi_c$ , plus and minus two standard deviations around steady state in the single state variable  $x_{c,t}$ .

The plots in Figure 2 are upward sloping under limited information but downward sloping under full information. Since price-dividend ratios are high when risk premia are low, and vice versa, this shows that assets with cash flows that load heavily on the long-run component,  $x_{c,t}$ , are *more* risky under full information but *less* risky under limited information.

These results can be understood by noting that the risk premium on any asset in this economy is primarily determined by the covariance between the pricing kernel  $M_t$  and revisions in expectations (news) about future cash flow growth.<sup>13</sup> As such, cash flow shocks have two offsetting effects on the equity premium in full information as compared to limited information. First, when a positive innovation  $\varepsilon_{xc}$  to the persistent component of consumption growth occurs, investors with limited information assign some weight to the possibility that the shock is transitory (coming from  $\varepsilon_c$  or  $\varepsilon_d$ ). As a consequence, investors with limited information revise upward their expectation of future consumption and dividend growth by less than they would under full information. This generates a larger (in absolute value) negative correlation between  $M_t$  and cash flow news under full information than under limited information. Second, when a positive innovation  $\varepsilon_c$  to the short-run risk component occurs, investors with limited information assign some weight to the possibility that the shock is persistent (coming from the long-run risk component). As a consequence, investors with limited information revise upward their expectation of future consumption and dividend growth more than they would under full information. This generates a larger (in absolute

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<sup>13</sup>Revisions in expected future returns are relatively unimportant because we have not introduced mechanisms such as changing consumption and dividend volatility for generating time-varying risk premia on the asset.

value) negative correlation between  $M_t$  and cash flow news under limited information than under full information.

When  $\phi_x$  is large and  $\phi_c$  relatively small, the first effect dominates the second. In this case, the risk premium in the full information case can be substantial while the premium in the limited information case is quite small. On the other hand, when  $\phi_x$  is small and  $\phi_c$  relatively large, the second effect dominates the first. In this case, the risk premium in the limited information case can be substantial while the premium in the full information case is quite small. Notice that, when  $\phi_c$  is small and the long-run risk component has small variance, the ARMA dividend shock  $v_{d,t+1}^A$  is dominated by the volatile idiosyncratic cash flow shocks  $\varepsilon_{d,t+1}$  that carry no risk premium. Thus, under limited information, sufficiently high exposure to short-run risk is required to generate a large risk premium.

This intuition can be illustrated by examining impulse response functions. Figure 3 plots impulse responses of the stochastic discount factor (SDF) and return to the dividend claim. The SDF under full information is denoted  $M_t$ , and under limited information,  $\widehat{M}_t$ . Similarly, we denote the log return to the dividend claim under full information,  $r_{m,t}$ , and the same return under limited information,  $\widehat{r}_{m,t}$ . (Recall that both the stochastic discount factor and the market return depend on the perceived data generating process for  $\Delta c_t$  and  $\Delta d_t$ , so these will differ across full and limited information.) The responses are based on the same calibration used to produce the impulse responses of the state variables in Figures 2 and 3 ( $\phi_x = 1$ ,  $\phi_c = 6$ ), which delivers a high equity premium under limited information (7.73%) but a low equity premium under full information (2.45%).

The first row of Figure 3 shows why an innovation in the i.i.d. consumption shock  $\varepsilon_{c,t}$  leads to higher risk premia under limited information than under full information. Recall that, under limited information, investors respond to such a shock by revising upward their expectation of future consumption growth substantially, while an investor with full information makes no such revision in expected future consumption growth (Figure 1). As shown in Figure 3 (row 1), this generates a larger decline in the SDF in response to an i.i.d. consumption shock under limited information than under full information, and hence a larger (negative) correlation with the return. By contrast, an innovation in the persistent component of consumption,  $\varepsilon_{cx,t}$  (row 3, Figure 3), leads to higher risk premia under full information than under limited information. This occurs because investors with limited information revise their expectations of future consumption growth by less than they would under full information (row 3, Figure 1). As a consequence, there is both a much larger

decline in the SDF and a much larger increase in the return under full information than under limited information in response to a persistent consumption shock. Finally, the middle row of Figure 3 shows that a purely idiosyncratic shock to dividend growth,  $\varepsilon_{d,t}$ , has a negligible impact on the stochastic discount factor in either full or limited information, and so generates a negligible risk premium.

Of course, the total risk premium is influenced by all three shocks. The reason that the total risk premium is higher under limited information than under full information in this calibration is that the effect of persistent shocks is dominated by those of the i.i.d. shocks, due to the much smaller loading on the persistent component than on the i.i.d. component in the dividend process ( $\phi_x = 1$ ,  $\phi_c = 6$ ).

## 4.2 The Term Structure of Risk

### 4.2.1 Zero-Coupon Equity

To study the implications of the information structure for the endogenous relation between risk premia and cash flow duration, we investigate the properties of zero-coupon equity. The idea here is that an equity claim can be represented as a portfolio of zero-coupon dividend claims with different maturities (e.g., Lettau and Wachter (2007)). Let  $P_{n,t}$  denote the price of an asset at time  $t$  that pays the aggregate dividend  $n$  periods from now, and  $R_{n,t}$  the one-period return on zero-coupon equity that pays the aggregate dividend in  $n$  periods:

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}.$$

Zero-coupon equity claims are priced under no-arbitrage according to the following Euler equation:

$$\begin{aligned} E_t \left[ M_{t+1} R_{t+1}^{(n)} \right] &= 1 \implies \\ P_t^{(n)} &= E_t \left[ M_{t+1} P_{t+1}^{(n-1)} \right] \\ P_t^{(0)} &= D_t, \end{aligned}$$

where the process for cash flows that generates the data  $D_t$  is given by (4)-(6). Denote  $r_{n,t+1} = \ln(R_{n,t+1})$ . The Appendix provides detailed information on how the recursion above is solved numerically. Since the aggregate market is the claim to all future dividends, the market price-dividend ratio  $P_t^D/D_t = \sum_{n=1}^{\infty} P_t^{(n)}/D_t$ . Plotting  $E \left( r_{n,t+1} - r_{t+1}^f \right)$  against  $n$  produces a yield curve, or term structure, of zero-coupon dividend claims.

### 4.2.2 Limited Information v.s. Full Information

In this section, we compare the term structure of zero-coupon equity under limited information with that of the full information specification. Figure 4 plots summary statistics for log excess returns  $r_{n,t+1} - r_{f,t+1}$  as a function of maturity,  $n$ , under this parameter configuration. The analogous plots for the parameter configuration studied in Table 3 are qualitatively the same, but the spread in risk-premia between long- and short-duration equity is lower. The aggregate dividend claim is assumed to follow the process (4)-(6) with  $\phi_x = 1$  and  $\phi_c = 4$ . The plots reveal how the information structure impacts the endogenous relation between cash flow duration and risk premia, for the benchmark (Bansal-Yaron) calibration given above.

Figure 4 shows that, under limited information, the annualized log risk premium declines with maturity (top panel). The log risk premium is 5.5% per annum for equity that pays a dividend one month from now and 4.5% per annum for equity that pays a dividend 15 years from now (top panel). Thus, under limited information, short-duration assets, those with more weight in low-maturity equity, endogenously have higher expected returns and lower price-dividend ratios than long-duration assets with more weight in distant-maturity equity. A downward sloping equity yield curve is needed for the model to match empirical evidence that long-horizon equity is less risky than short-horizon equity.

By contrast, under full information, the annualized log risk premium increases with maturity. The log risk premium is 1.3% per annum for equity that pays a dividend one month from now and 2.3% per annum for equity that pays a dividend 15 years from now.

The key to the downward sloping zero-coupon equity curve under limited information, displayed in the top panel of Figure 4, is that the process for dividend growth under signal extraction appears close to i.i.d. (the estimated moving average and autoregressive roots in (8) and (9) are close to canceling). Thus, shocks are perceived only to affect dividend growth and returns in the near term, implying that only assets that pay a dividend in the near future command high risk premia. By contrast, under full information, when agents can directly observe  $x_{c,t}$ , it is understood that shocks can have a large, long-term affect on consumption and dividend growth. Accordingly, the long-run appears risky, and assets that pay a dividend in the far future command higher risk premia than those that pay a dividend in the near future. The endogenous relation between cash flow duration and risk premia goes the wrong way.

The middle panel of Figure 4 shows that in both limited and full information, volatility

increases with the horizon. But the bottom panels show that the Sharpe ratios decrease with the horizon under limited information whereas they rise with the horizon under full information. This suggests that the limited information specification is better able to explain the empirically higher Sharpe ratios of short-duration value stocks as compared to long-horizon growth stocks.

Figure 5 shows that, under limited information, the shortest-duration equity have high CAPM alphas, whereas the longest-maturity equity have smaller (in absolute value) negative alphas. This feature of limited information is consistent with the data (see Table 4, discussed below) and with the prior findings of Fama and French (1992). The bottom panel also shows that, under limited information, long-duration equity—despite its having lower expected excess returns than short-duration equity—has slightly higher CAPM betas, as in the data. By contrast, under full information, there is much less variation in the alphas with maturity and the variation goes the wrong way: alphas of short-duration assets are lower than those of long-duration assets.

The plots just discussed are based on particular values of the loadings  $\phi_c$  and  $\phi_x$  for the market portfolio. Results (not reported) indicate that these findings hold for a wide range of parameters in both the cash flow process and the utility function. Regardless of the values of  $\phi_c$  and  $\phi_x$ , the term structure of equity is always downward sloping under limited information, and always upward sloping under full information. Changing the relative loadings  $\phi_c$  and  $\phi_x$  merely changes the slope of the term structure, it does not change the sign of the slope. This is true as long as parameter values are set so that greater exposure to  $x_{c,t}$  makes the market portfolio riskier rather than providing insurance. In a long-run “insurance” model, the full information term structure slopes down, but overall risk premia are very low or even negative. These latter results will be important when we discuss the system signal extraction case below.

The results above may be related to similar zero-coupon equity plots in the literature. Hansen et al. (2008) present zero-coupon equity plots for price-dividend ratios  $P_t^{(n)}/D_t$  rather than mean excess returns, as in Figure 4. Since high price-dividend ratios correspond to low mean excess returns, the plots presented in Hansen et al. (2008) are mirror-images of those above. Their plots are based on the same Epstein-Zin-Weil model of preferences used here, but the results are formed from historical data and somewhat different parameter values. Hansen et. al. report price-dividend term zero-coupon equity structures for value and growth firms separately, whereas we plot the zero-coupon-equity curve for aggregate dividends. (The

dividend payments of value and growth firms are modeled below as time-varying shares in a sequence of aggregate dividend claims,  $\{D_t\}_{t=0}^{\infty}$ , with different maturities.) In this sense, the results in this section are not directly comparable to those in Hansen et al. (2008). But it is notable that Hansen et. al. find that the price-dividend decomposition for the growth portfolio eventually exceeds those of the value portfolio, as required by the data, only at sufficiently high levels of risk aversion. This finding is echoed in the results reported here: it is only with sufficiently high risk aversion and/or sufficiently volatile innovations to the long-run expected growth rate of consumption that we find a significant spread in average returns between short-duration value stocks and long-duration growth stocks.

### 4.3 Term Structure And Equity Premium Under Full Information

We have seen that the full information model generates an upward sloping term structure under the benchmark parameterization. This occurs because the dividend growth process is perceived to be much more persistent under system limited information than under limited information. Accordingly, assets that pay a dividend in the far future command higher risk premia than those that pay a dividend in the near future.

To understand both the slope and the level of the term structure, it is instructive to consider the role played by key model parameter values. Two parameters are especially important for governing the slope of the term structure: the exposure  $\phi_x$  of dividend growth to the persistent component of consumption growth, and the intertemporal elasticity of substitution,  $\Psi$ . The lower is  $\phi_x$ , the less persistent is dividend growth and the less upward sloping the curve. Indeed, a sufficiently small value for  $\phi_x$  can flip the slope of the zero-coupon equity curve to downward sloping. This occurs for reasons already discussed: when the dividend growth process has little persistence, only shocks to dividend growth in the near term generate significant revisions in expected future consumption and cash flow growth, and hence command a significant risk premium; short-duration assets are riskier than long-duration assets.

The intertemporal elasticity of substitution affects the slope of the term structure by affecting expected future returns, rather than expected future dividend growth. The lower is  $\Psi$ , the more expected returns increase in response to any given increase in expected consumption growth. Thus, a positive innovation in expected consumption growth does two things. First, it leads to an increase in expected future returns, which is associated with a capital loss for the asset today. Second, it leads to a decline in the stochastic discount

factor. The two combined imply a positive contemporaneous correlation between the pricing kernel and returns, making the overall risk premium on the asset low or even negative. This effect is stronger for assets that pay a dividend in the far future because shocks to expected consumption growth are persistent and cumulate over time. Consequently, the lower is the IES, the lower are risk premia on long-duration assets relative to short-duration assets, and the less upward sloping the zero-coupon-equity curve.

These properties of the model suggest one way that parameter values could be changed in order to make the full-information term structure of equity downward sloping: reduce  $\phi_x$  and  $\Psi$ . The role of these parameters on the slope of the zero-coupon-equity term structure can be illustrated by the approximate log-linear solution of the model, similar to Campbell (2003). Let  $V_t(\cdot)$  denote the conditional variance of the generic argument “.”. Define the slope of the log equity term structure (adjusted for Jensen’s inequality terms) as

$$S \equiv \lim_{n \rightarrow \infty} E_t[r_{n,t+1}^{ex} + .5V_t(r_{n,t+1}^{ex}) - (r_{1,t+1}^{ex} + .5V_t(r_{1,t+1}^{ex}))],$$

where the superscript “*ex*” denotes the excess return over the log risk-free rate. In the full information case it can be shown that

$$S = \frac{\phi_x - 1/\Psi}{1 - \rho} \sigma_{xc} \sigma^2 \left[ \gamma \rho_{cx_c} + \kappa_c \left( \frac{\gamma - 1/\Psi}{1 - \rho \kappa_c} \right) \sigma_{x_c} \right] \quad (10)$$

where  $\kappa_c \equiv \frac{\overline{PC/C}}{1 + \overline{PC/C}}$  is a positive linearization constant less than unity, and  $\rho_{cx_c}$  denotes the unconditional correlation between the i.i.d. consumption shock,  $\varepsilon_{c,t}$ , and the shock to long-run expected consumption growth,  $\varepsilon_{x_c,t}$ . For the rest of this discussion, we maintain the assumption that  $\gamma > 1/\Psi$ . If we also assume for the moment that  $\rho_{cx_c} \geq 0$ , then the term in the square brackets is positive, and it is possible to generate a downward sloping term structure of equity (that is, a negative spread,  $S < 0$ ) by setting  $\phi_x < 1/\Psi$ . When  $\Psi = 1$ , the valuation calculations in Hansen et al. (2008) can be used to obtain an exact solution for  $S$ . Under the assumptions just made, such calculations show analogously that  $\phi_x < 1$  is required to generate a downward sloping equity term structure.

Under full information, however, this strategy for obtaining a downward sloping term structure presents an important difficulty. When  $\phi_x < 1/\Psi$ , the model becomes one of long-run *insurance* rather than long-run *risk*. That is, innovations in  $x_{c,t}$  (holding other shocks fixed) generate a positive correlation between the pricing kernel and returns, so that the marginal contribution of the long-run component to the market risk premium is negative. This occurs because such a parameterization has the undesirable property that an increase

in the long-run expected consumption growth rate leads to a decline in the market price-dividend ratio. This property is immediately evident from the approximate formula for the log price-dividend ratio of the market return under full information:

$$p_t^D - d_t = \overline{pd} + \frac{\phi_x - 1/\Psi}{1 - \rho\kappa_d} x_{c,t},$$

where  $\overline{pd}$  is a constant and  $\kappa_d \equiv \frac{P^D/D}{1+P^D/D}$  is a positive linearization constant less than one. Note that the coefficient on  $x_{c,t}$  is negative whenever  $\phi_x < 1/\Psi$ .

In addition, in a long-run “insurance” model, the full information term structure slopes down, but the overall equity premium for the market is low or negative. This can be understood by examining the loglinear approximation of the market equity premium under full information, given by

$$\begin{aligned} E_t(r_{d,t+1}^{ex}) + .5V_t(r_{d,t+1}^{ex}) &= \gamma\phi_c\sigma^2 + \kappa_d \frac{(1-\rho)}{1-\rho\kappa_d} S \\ &+ \kappa_c \frac{\gamma - 1/\Psi}{1 - \rho\kappa_c} \sigma^2 \sigma_{x_c} [\phi_c \rho_{cx_c} + \rho_{dx_c} \sigma_{x_d}], \end{aligned} \quad (11)$$

where  $\rho_{dx_c}$  denotes the unconditional correlation between the idiosyncratic dividend shock,  $\varepsilon_{d,t}$ , and the shock to long-run expected consumption growth,  $\varepsilon_{x_c,t}$ . Since  $\kappa_d \frac{(1-\rho)}{1-\rho\kappa_d} > 0$ , if the slope  $S$  of the equity term structure is negative, it is difficult to generate a sizable equity premium.

Figure 6 illustrates this point by plotting the term structure of equity for a calibration in which  $\phi_x = 0.76$  (instead of  $\phi_x = 1$  as in Figure 5), and in which the IES is  $\Psi = 1$  (instead of  $\Psi = 1.5$ ); hence  $\phi_x < 1/\Psi$  and  $S < 0$ . If no other parameter values are changed, such a calibration produces a downward sloping term structure of equity, but the spread in risk premia between short- and long-horizon equity is small. One remedy is to adjust risk aversion upward and then insure that exposure  $\phi_c$  to short-run risk is sufficiently high to help increase the market equity premium (the level of the term structure). The results in Figure 6 are displayed for  $\gamma = 50$  and  $\phi_c = 3.6$ , with all other parameters set as in the calibration of Figure 5. As Figure 6 shows, under this calibration the full information model produce a downward sloping zero-coupon equity curve. However, the market risk premium is now negative.

The difficulty posed by the full information specification in generating a downward sloping term structure for equity simultaneously with a high equity premium cannot be remedied by freely setting the correlation  $\rho_{cx_c}$  between current consumption shocks and shocks to the

long-run expected consumption growth. For example, if we restrict  $\phi_x > 1/\Psi$  to avoid the implications just discussed, then (10) implies that we can obtain a downward sloping term structure by setting  $\rho_{cx_c} < 0$ . Unfortunately, (11) shows that this again makes the overall equity premium low or negative, since it makes both  $S$  and the third term of (11) negative.

In summary, in a long-run “insurance” model, the full information term structure slopes down, but the overall risk premium for the market is low or negative. By contrast, under limited information, a downward sloping term structure may be obtained simultaneously with a sizable market risk premium, as long as the asset’s dividend exposure to long-run risk is not too large, while its exposure to short-run risk is sufficiently high.

## 5 Implications for the Value Premium

It is possible to study the quantitative aspects of the link between duration and risk premia presented above by modeling firms explicitly. Here we do so by forming portfolios of individual firms that differ only in the timing of their cash flows. Long-duration growth firms are modeled as equity with relatively more weight placed on long-maturity dividend claims, while short-duration value firms are modeled as equity with relatively more weight placed on short-maturity dividend claims.<sup>14</sup>

As in pre-existing literature, this cash flow specification abstracts from many features of firm-level earnings.<sup>15</sup> Two key simplifying assumptions are that we ignore firm-specific cash flows and model only the component of cash flows relevant for prices and returns, namely that correlated with aggregate cash-flows. In addition, in order to maintain closed-form solutions in our discrete time setup, we assume that the share process is deterministic. Given these simplifications, the model is not able to match some features of cash flow data. For example, the deterministic share process implies that the cash flow growth rates of portfolios sorted according to firm’s price-dividend ratios are identical for all portfolios and equal to the aggregate cash flow growth rate. As a consequence, the model cannot capture that differences between cash flow growth rates of book-to-market sorted portfolios and investment strategies documented in Bansal et al. (2005a) and Hansen et al. (2008). Matching these moments

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<sup>14</sup>This methodology for describing the cash flows of individual securities was first employed in a continuous time setting by Menzly, Santos and Veronesi (2004), Santos and Veronesi (2004), and Santos and Veronesi (2010), and in a discrete setting by Lynch (2003) and Lettau and Wachter (2007).

<sup>15</sup>Here we follow the discrete time methodology described in Lettau and Wachter (2007). We outline only the main aspects of this approach and refer the reader to that article for further detail.

requires a more complex model with firm-specific cash flow shocks and stochastic shares. Although the simplicity of this specification lacks realism, by avoiding additional complexity we are able to highlight the endogenous link between risk premia and cash flow duration in models with long-run consumption risk, the focus of this paper.

The model we explore next specifies the life-cycle of cash flows at the *firm* level. It is tempting to conclude that one can draw inferences about firm-level cash-flows from the cash-flow properties of portfolios of stocks, or from the cash-flow properties of dynamic trading strategies based on those portfolios. Such inferences are not possible, however, because of the rebalancing required to maintain the investment strategy. For example, in the model explored here there is significant heterogeneity in *firm* cash flow growth rates, which are specified to follow a life cycle pattern. By contrast, there is no heterogeneity in the cash flow growth rates of *portfolios* of firms sorted on price-dividend ratios. The cross-sectional differences in life-cycle cash flows that drive the risk premia in our model wash out once firms are sorted into portfolios that are subject to rebalancing.<sup>16</sup>

We now turn to the life-cycle model of firm cash-flows. Consider a sequence of  $i = 1, \dots, N$  firms. (Hereafter we refer to these simply as ‘firms’ for brevity, even though they are better thought of as portfolios of firms at the same stage in their life cycle.) The  $i$ th firm pays a share,  $s_{i,t+1}$ , of the aggregate dividend  $D_{t+1}$  at time  $t+1$ . The share process is deterministic, with  $\underline{s}$  the lowest share of a firm in the economy. Firms experience a life-cycle in which this share grows deterministically at a rate  $g_s$  until reaching a peak  $s_{i,N/2+1} = (1 + g_s)^{N/2} \underline{s}$  when it shrinks deterministically at rate  $g_s$  until reaching  $s_{i,N+1} = \underline{s}$ . The cycle then repeats. Thus, firms are identical except that their life-cycles are out-of-phase, i.e., firm 1 starts at  $\underline{s}$ , firm 2 at  $(1 + g_s) \underline{s}$ , and so on. The parameter  $g_s$  is set to 1.67% per month, or 20% per year, as in Lettau and Wachter (2007). Shares are such that  $s_{i,t} \geq 0$  and  $\sum_{i=1}^N s_{i,t} = 1$  for all  $t$ .

Since each firm pays a dividend  $s_{i,t+1}D_{t+1}$ ,  $s_{i,t+2}D_{t+2}$ , ..., no arbitrage implies that the ex-dividend price of firm  $i$  at time  $t$  is given by

$$P_{i,t} = \sum_{n=1}^{\infty} s_{i,t+n} P_t^{(n)}.$$

When  $s_{i,t+1}$  is low, dividend payments are low today but will be high in the future when

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<sup>16</sup>Campbell, Polk and Voulteenahe (2005) make the same point and propose addressing this issue by using a “three-dimensional” procedure that follows portfolios for a number of years after portfolio formation while keeping the composition constant.

$n$  is large; these are long-duration assets with greater weight placed on distant-maturity dividend claims. When  $s_{i,t+1}$  is high, dividend payments are high today but will be low in the future; these are short-duration assets with greater weight placed on short-maturity dividend claims. From the downward sloping term structure plots presented above, we already know that, under limited information, firms with high price-dividend ratios and low risk premia will be those that pay a small share of the aggregate dividend today, but a greater share farther into the future. Such “growth” assets will endogenously have both high price-dividend ratios (accompanied by relatively low risk premia) and long duration in their cash flows. Conversely, firms with low price-dividend ratios and high risk premia will be those that pay a larger share of the aggregate dividend today, but a small share farther into the future. Such “value” assets will endogenously have both low price-dividend ratios (accompanied by relatively high risk premia) and short duration in their cash flows.<sup>17</sup>

To consider the quantitative properties of models with limited information, we sort firms into portfolios on the basis of price-dividend ratio. Portfolio returns are created by simulating a time-series for aggregate dividends and prices and, using the share process described above, forming 10 equally-weighted portfolios of the  $N$  firms by sorting firms into deciles based on their price-dividend ratios.<sup>18</sup> The portfolios are rebalanced every simulation year. This procedure creates portfolios of firms that display heterogeneity not only in their price-dividend ratios, but also (endogenously) in the duration of their cash flows. Table 2 reports the statistical properties of these portfolios. For comparison, Table 2 also provides updated evidence on the value premium in U.S. data. The table shows summary statistics from U.S. data for portfolios of firms sorted into deciles on the basis of book-to-market ratio, with decile 1 containing firms in the lowest 10 percent according book-to-market ratio, and decile 10 containing firms in the highest 10 percent according to book-to-market ratio. The monthly data are from the Center for Research in Securities Prices, and span the period 1947-2004.<sup>19</sup>

Before considering the quantitative properties of these portfolios, it is instructive to examine how the information structure influences the relation between portfolio expected

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<sup>17</sup>Da (2005) empirically measures equity duration in the manner modeled here, namely as the deviation of an asset’s current share in aggregate dividends from its steady-state value. Consistent with previous findings, (which were based on somewhat different methodologies for measuring equity duration), he finds that value stocks have much shorter cash flow duration than growth stocks.

<sup>18</sup>We set the number of firms to be 1020, implying a 1020 month, or 85 year life-cycle for a firm.

<sup>19</sup>We thank Kenneth French for compiling the portfolios from these data and making them available on his web page.

return and a commonly employed measure of duration, based on the Macaulay formula. Specifically, the Macaulay duration measure for firm  $i$  is given by

$$Duration_{i,t} = \frac{\sum_{n=1}^{\infty} n \cdot E_t [M_{t+n,t} D_{i,t+n}]}{P_{i,t}},$$

where  $M_{t+n,t} \equiv M_{t+1} \cdot M_{t+2} \cdots M_{t+n}$ . The top panel of Figure 7 shows the relation between the price-dividend ratio and average (across firms) duration. The monthly duration value is divided by 12 to obtain an annual value. The figure shows that, no matter what the information structure, there is a monotonically declining relation between  $P/D$  and duration: portfolios with low price-dividend ratio have high duration and vice-versa. What the bottom panel of Figure 7 shows, however, is that expected excess return is declining in duration under limited information, as in the data, while it is increasing under full information. This follows directly from the differing equity term structure slopes under limited and full information.

Figure 7 shows that the limited information model is capable of replicating the empirical finding that short-duration equity is more risky than long-duration equity. The quantitative implications of this finding for the value premium are explored in Table 2, which presents statistics for expected excess returns, Sharpe ratios, and CAPM regressions, based on a single long simulation of the true data generating process in (4)-(6).<sup>20</sup> We refer to the portfolio in the highest price-dividend decile as the growth portfolio, denoted  $G$  in the table, and the portfolio in the lowest price-dividend decile as the value portfolio, denoted  $V$  in the table.

The top panel of Table 2 shows the value premium for this framework under limited information under our benchmark parameterization. In the table, the benchmark parameterization is referred to as *Case 1*. In Case 1, the mean excess return on the extreme growth portfolio is 5.11%, while that of the extreme value portfolio is 6.86%, leaving a spread between the two of 1.75%. These numbers produce the right result qualitatively, but they fall short of replicating the magnitude of the spread in the data, equal to 5.42%. However, if one is willing to accept a slightly altered set of parameter values, then the limited information long-run risk model can explain a larger value spread. As an illustration, suppose values for the following parameters are changed as follows  $\sigma = 0.0051$ ,  $\sigma_{xc} = 0.08$ ,  $\rho = 0.983$ ,  $\Psi = 1.3$ ,  $\delta = 0.999$  and all other parameters are left at their benchmark levels ( $\gamma = 10$ ,  $\sigma_d = 5.9$ ,

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<sup>20</sup>We present these statistics only for the limited information specification. We have already seen that specifications with full information generate a value premium by making long-duration assets more risky than short-duration assets. Since the aim is to generate a value premium that implies long-duration assets are *less* risky than short-duration assets, we do not pursue that avenue here.

$\phi_c = 4, \phi_x = 1$ ). This parameterization is referred to as *Case 2* in Table 2. It is important to note that these changes do not alter the overall volatility of consumption growth; its annual standard deviation remains 2.8%, as calibrated in Bansal and Yaron (2004). Moreover, the slightly higher value for  $\rho$  is between the calibrated value used by Bansal and Yaron and that estimated by Bansal, Gallant and Tauchen (2007), which was  $\rho = 0.987$ . Under Case 2 parameter values, the second panel of Table 2 shows that the value spread is equal to 3.87%, about 70 percent of the value spread found in the data. The specification also predicts that Sharpe ratios rise when moving from growth to value portfolios, as in the data. Under the alternative parameter values, the Sharpe ratio of the extreme value portfolio is 0.27 higher than that of the extreme growth portfolio, about the same as in the data whereas the extreme value portfolio has a Sharpe ratio that is 0.26 higher than the extreme growth portfolio.

The limited information model produces almost no spread in the CAPM betas across portfolios, a pattern found in the classic results of Fama and French (1992) and in the updated data reported in Table 2. The pattern of alphas is also consistent with the data ranging from negative for growth portfolios to positive for value portfolios. In Table 2, model-based alphas under the alternative (Case 2) calibration rise from  $-0.3\%$  for the extreme growth portfolio to  $3.57\%$  for the extreme value portfolio, or a spread of 3.86. By comparison, in the post-war data the lowest B/M quintile has an alpha of  $-1.68\%$  and the highest has an alpha of  $4.19\%$ , or a spread of 5.87.

Finally, Table 3 shows the results of adding the *HML* (high-minus-low) factor of Fama and French (1993) as an additional regressor in CAPM time-series regressions of the excess portfolio returns onto the excess market return. *HML* is constructed as the return on a portfolio short in the extreme growth decile and long in the extreme value decile. Consistent with the empirical findings of Fama and French (1993), the model implies that adding *HML* as an additional factor reduces the magnitude of the positive CAPM alphas in the decile portfolios.

## 6 Conclusion

A recent strand of asset pricing literature emphasizes the potential role of long-run consumption risk for explaining salient asset pricing phenomena. A maintained assumption in the existing theoretical literature is that investors can directly observe such small long-run components and can distinguish their innovations from transitory shocks to consumption and dividend growth, an assumption we refer to as full information. In this paper we have studied

how equilibrium asset prices may be affected if market participants—like econometricians—must use consumption and dividend data to infer small long-run components in cash flows and consumption (limited information).

We find that the asset pricing implications of long-run risk models can be quite sensitive to the information investors have about the long-run. To illustrate the importance of the information structure, we study the cash flow duration perspective of value and growth assets. This is of interest because the existing long-run risk literature has focused on explaining the behavior of the aggregate market return and/or the return properties of value and growth stocks, but little attention has been given to how equilibrium returns are related to equity duration.

A key result of this study is that, under many parameter configurations, limited information causes market participants to demand a higher premium for engaging in risky assets than would be the case under full information. Specifically, assets that have small exposure to long-run consumption risk but are highly exposed to short-run, even i.i.d., consumption risk can command high risk premia under limited information but not under full information. Thus the term structure of equity can be downward sloping under limited information, as in the data, but is upward sloping under full information. These results show that limited information can be an important source of additional risk, and it can reverse the class of assets that commands a high risk premium.

In general, these patterns mean that the limited information specifications we explore are better able than their full information counterparts to reconcile the return properties of value and growth assets with their quite different cash flow duration properties. For example, in a full information world, long-duration assets can be made less risky than short-duration assets only at the expense of a negative market risk premium.

There are at least two ways in which this research could be extended. First, in order to focus on the role of information and its relation to the cross-section of average returns, we have not incorporated additional sources of time-varying risk that may also be unobservable, such as changing volatilities of cash flows. As such, our model of the excess return on the market does not display significant predictability, implying that the volatility of the market price-dividend ratio is lower than in the data. If the volatility of cash flows changes in a way that is not observed, the agent must solve a complex nonlinear filtering problem. We are currently studying this problem. Second, the specification of cash flows and informational assumptions pursued here is but one of many that could be fruitfully studied in future

work. In addition, investors may have preferences that differ from those presumed here, and the form of these preferences may interact with informational barriers in interesting ways. For example, informational barriers may be compounded by uncertainty over the cash flow model itself, possibly leading investors to have a preference for robustness, as in the work of Anderson et al. (1998), Anderson, Hansen and Sargent (2003) and Hansen and Sargent (2006). Exploring these extensions presents a challenge for future work.

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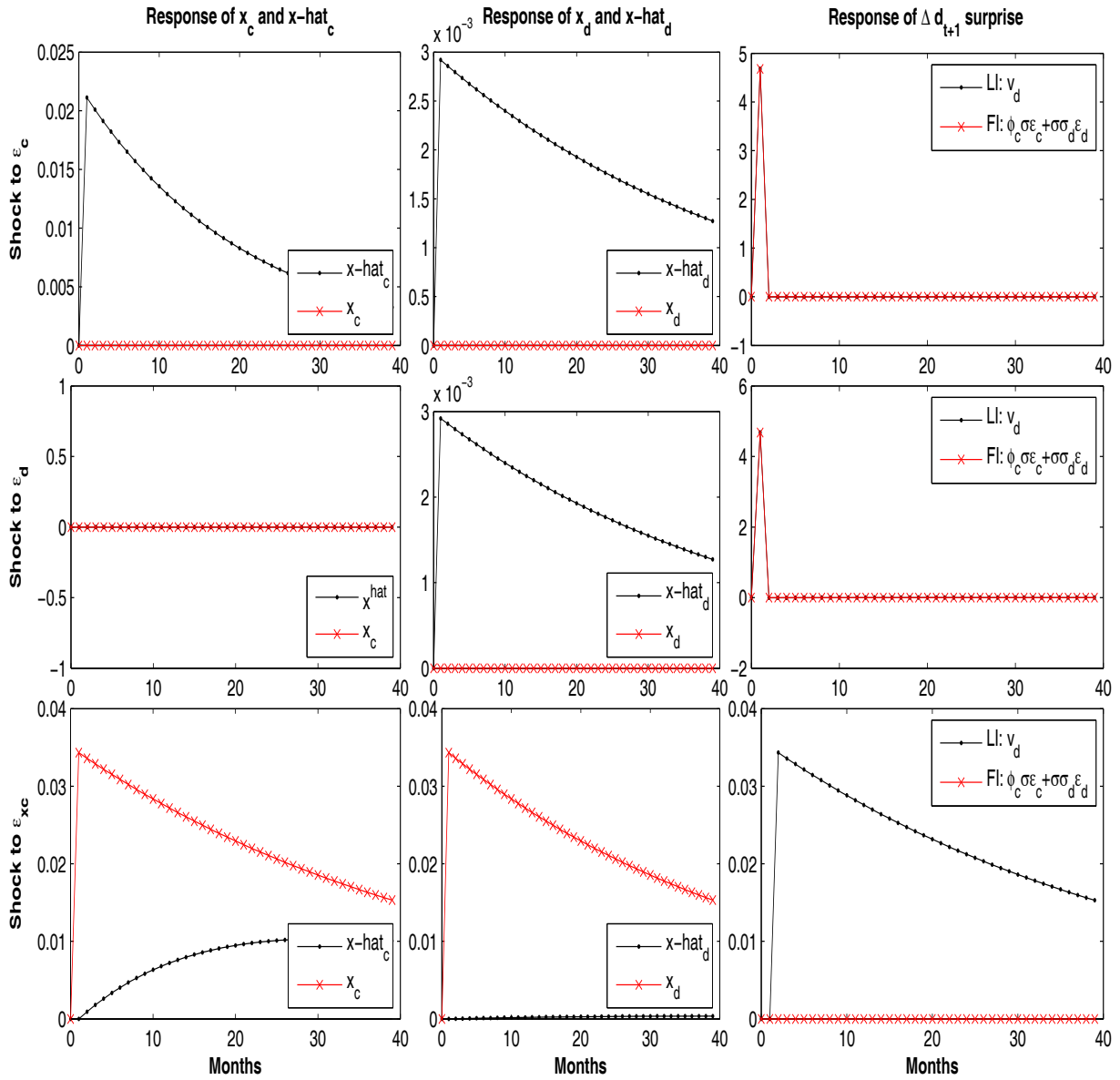
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Figure 1

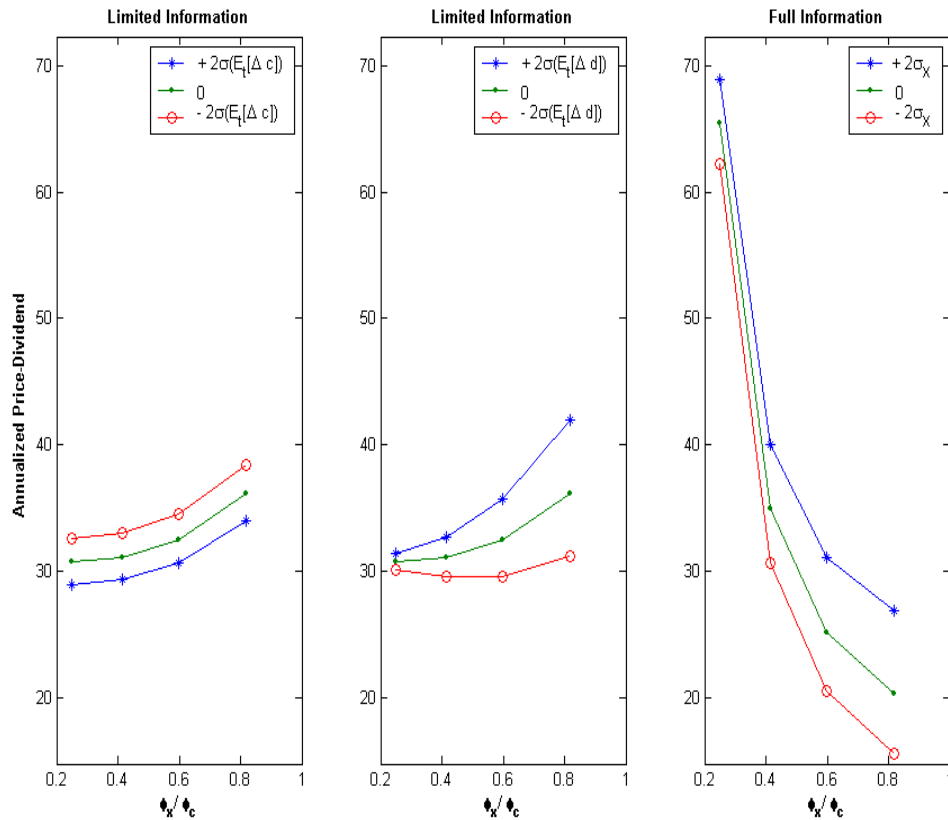
Impulse Responses of Cash Flow Forecasts and Surprises: Limited and Full Information



Notes: The figure shows the 40-month response of variables to a one-standard deviation innovation in the shock labeled at each row, with  $\phi_x = 1$ ,  $\phi_c = 6$ . The vertical axis represents monthly percent deviations of variables from steady state. Variables denoted with “hat” correspond to those from the limited information case, based on ARMA(1,1) estimations for consumption and dividend growth. Variables without a “hat” are from the full information benchmark. The responses are based on the benchmark calibration  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ .

Figure 2

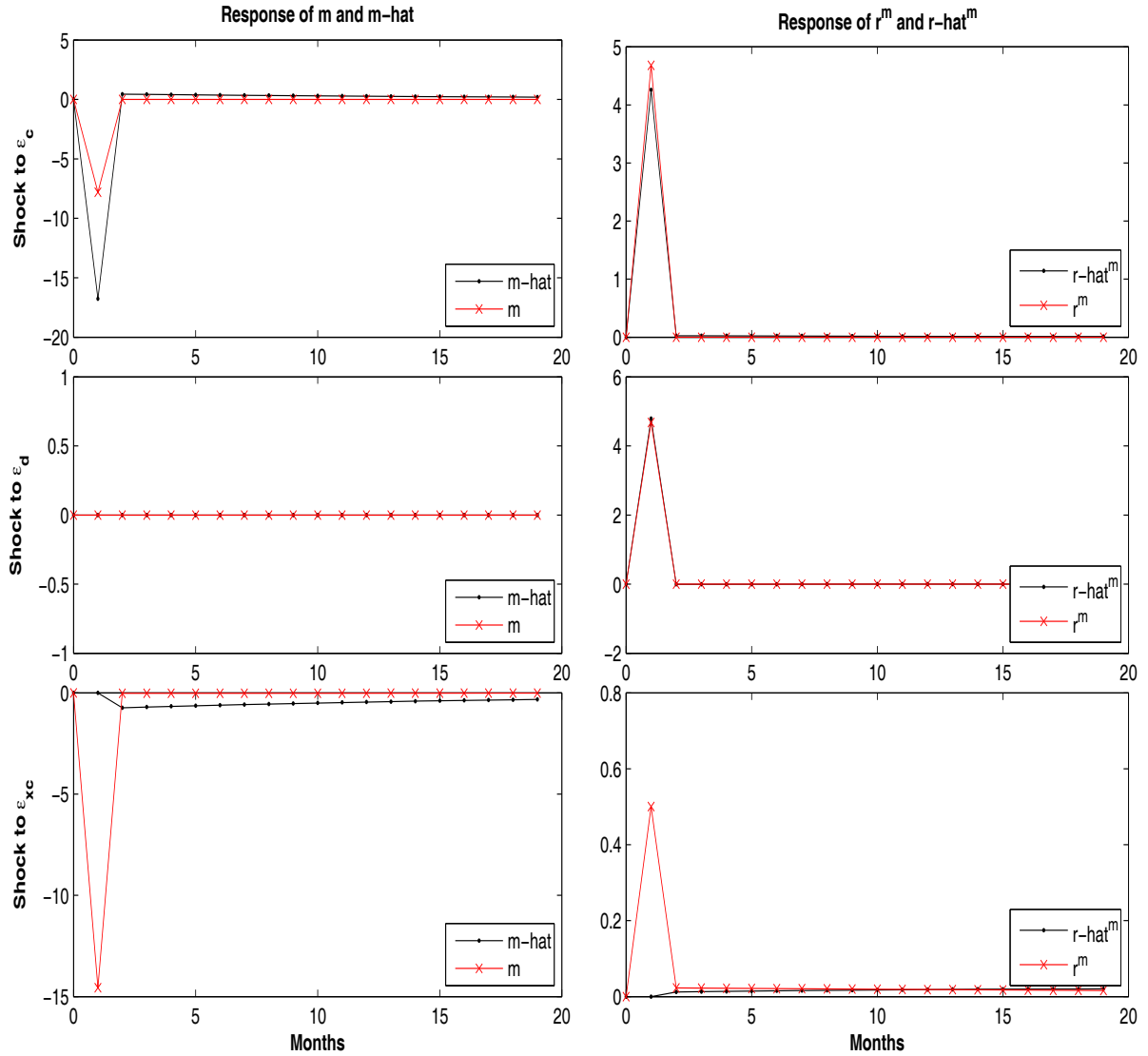
Price-Dividend Ratios: Limited and Full Information



Notes: This figure displays price-dividend ratios at steady state, and plus/minus two standard deviations of the state variable(s) around steady state, as a function of the relative exposure to long-run risk, governed by  $\phi_x$ , and to short-run risk, governed by  $\phi_c$ . Held fixed is the five-quarter variance of dividend growth attributable to the consumption innovations. Parameter values are set according to the benchmark calibration:  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ .

Figure 3

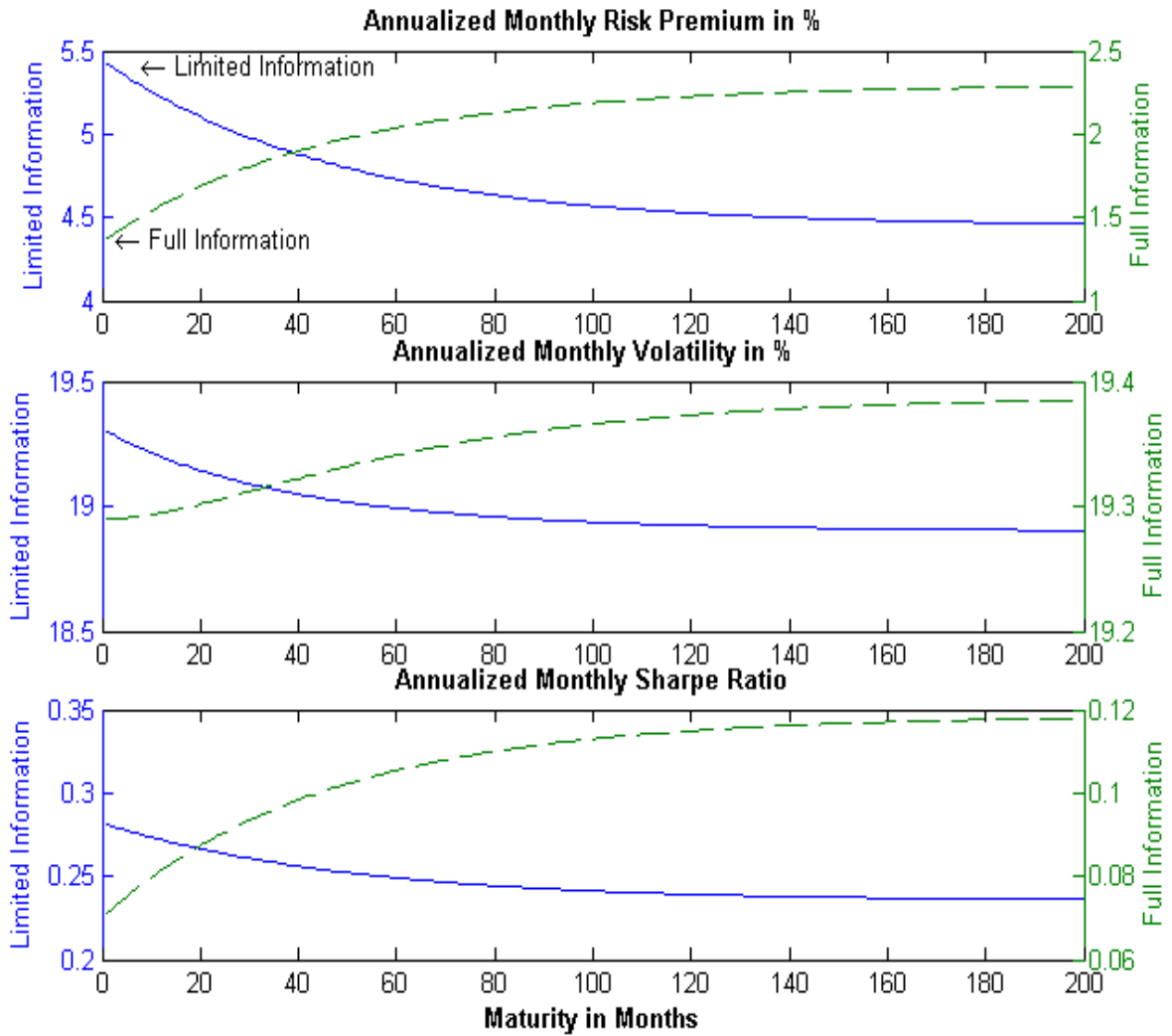
Impulse Responses of SDF and Returns: Limited and Full Information



Notes: The figure shows the 40-month response of variables to a one-standard deviation innovation in the shock labeled at each row, with  $\phi_x = 1$ ,  $\phi_c = 6$ . The vertical axis represents monthly percent deviations of variables from steady state. Variables denoted with “hat” correspond to those from the limited information case, based on ARMA(1,1) estimations for consumption and dividend growth. Variables without a “hat” are from the full information benchmark. The variable  $r^m$  denotes the return on the dividend claim;  $m$  denotes the stochastic discount factor. The responses are based on the benchmark calibration  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ .

Figure 4

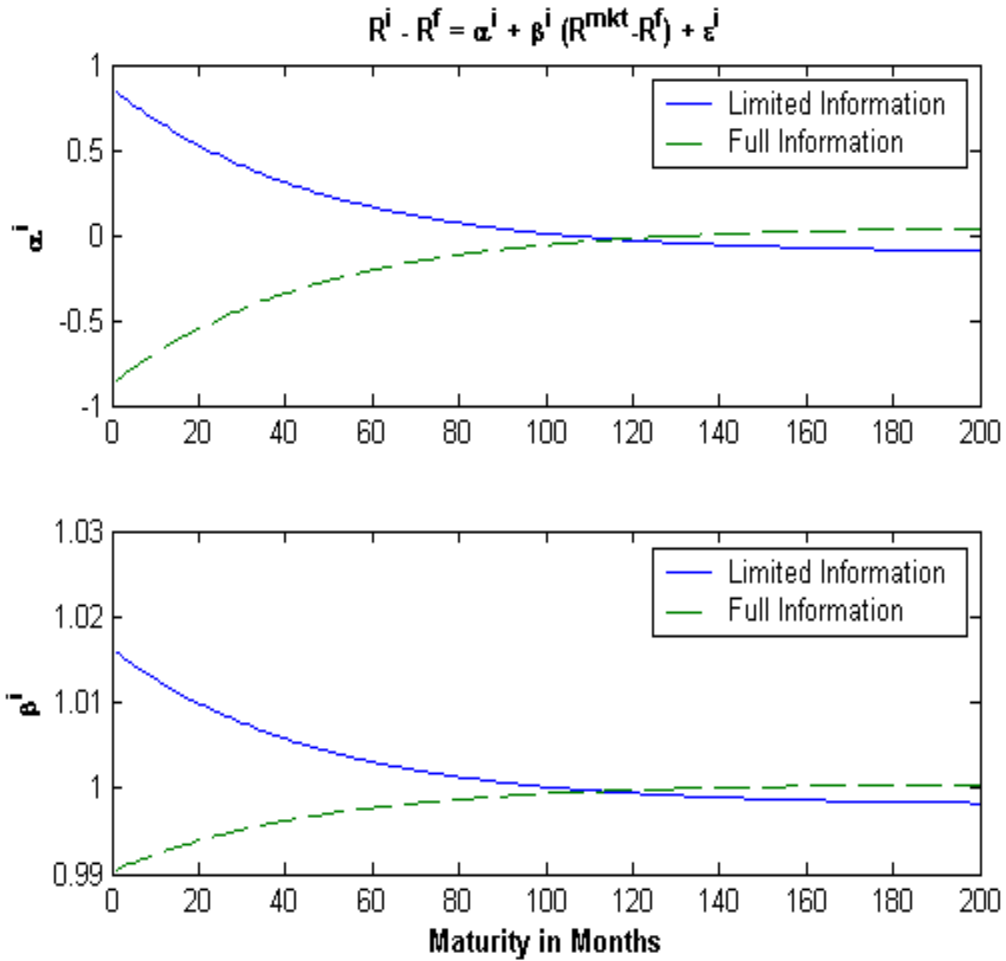
Zero-Coupon Equity: Limited and Full Information



Notes: The top panel shows log risk-premia on zero-coupon equity  $E(r_{n,t+1} - r_{t+1}^f)$  as a function of maturity,  $n$ , in months; the middle panel shows the standard deviation of excess returns on zero-coupon equity; the bottom panel shows the Sharpe ratio. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameter values are set according to the benchmark calibration  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ . The market portfolio has  $\phi_x = 1$  and  $\phi_c = 4$ .

Figure 5

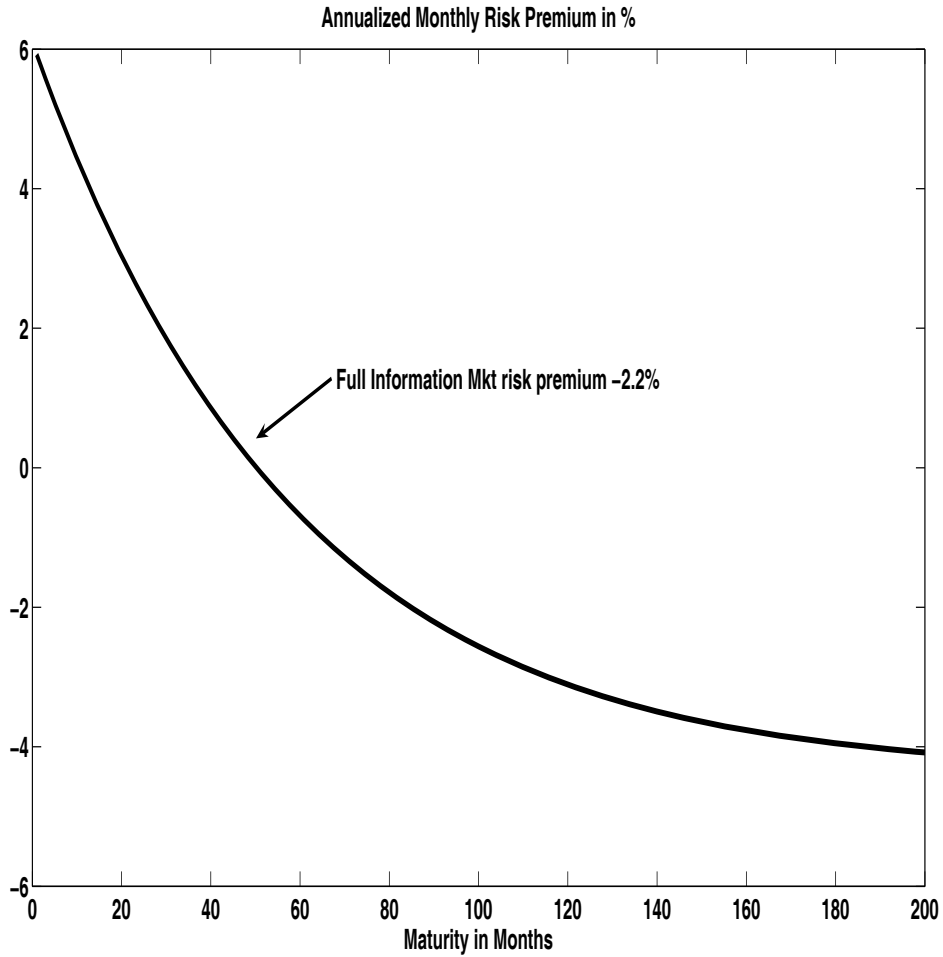
CAPM Regressions for Zero-Coupon Equity: Limited and Full Information



Notes: The top panel shows the intercept from regressions of zero-coupon equity excess returns on the excess return of the market, as a function of maturity in months; the bottom panel shows the slope coefficient from the same regression. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameter values are set according to the benchmark calibration  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ . The market portfolio has  $\phi_x = 1$  and  $\phi_c = 4$ .

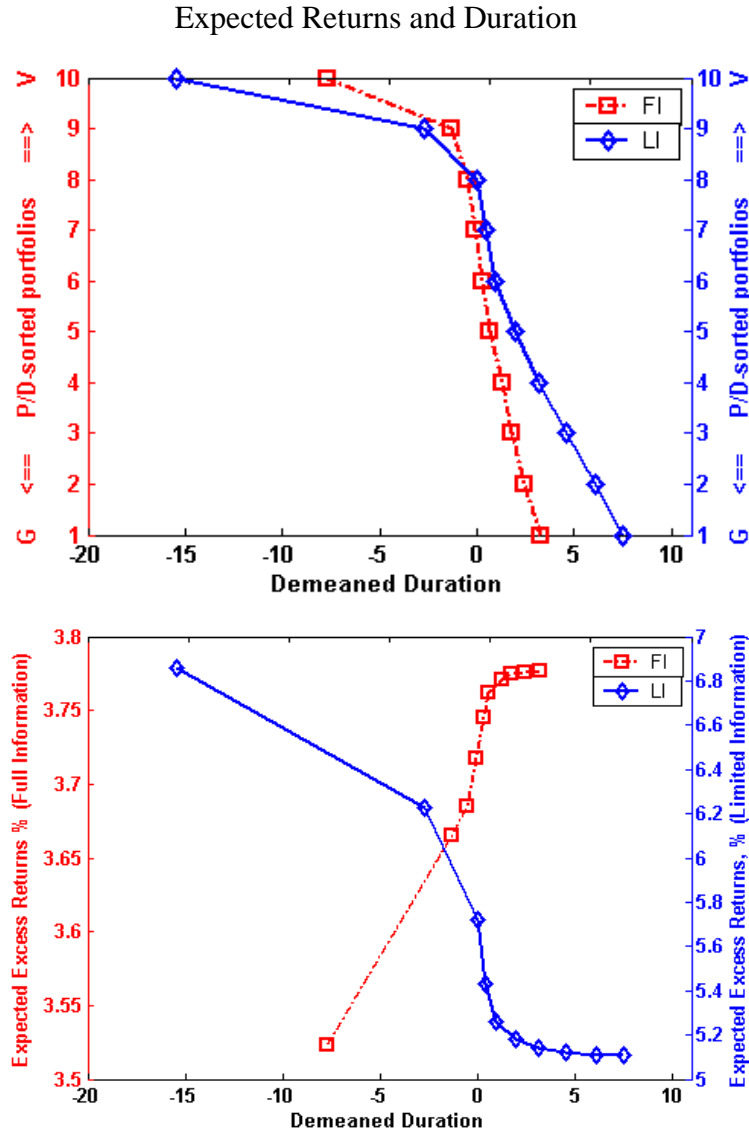
Figure 6

Zero-Coupon Equity: Long-Run Insurance Full Information Model



Notes: The figure shows log risk-premia on zero-coupon equity  $E(r_{n,t+1} - r^f_{t+1})$  as a function of maturity,  $n$ , in months. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameter values are set as follows:  $\gamma=50$ ,  $\Psi=1$ ,  $\delta=0.99327$ ,  $\mu_c = \mu_d=0.0015$ ,  $\rho=0.983$ ,  $\sigma=0.0057$ ,  $\sigma_{xc}=0.1$ ,  $\sigma_d=5.9$  and the market portfolio has  $\phi_x = 0.78$  and  $\phi_c = 3.6$ .

Figure 7



Notes: The top panel plots annualized, demeaned duration for 10 portfolios sorted on the basis of  $P/D$  ratio, under full information (FI) and limited information (LI). The duration of firm  $i$  is computed using the Macaulay formula:

$$Duration_{i,t} = \frac{\sum_{n=1}^{\infty} n \cdot E_t [M_{t+n,t} D_{i,t+n}]}{P_{i,t}},$$

where  $M_{t+n,t} \equiv M_{t+1} \cdot M_{t+2} \cdot M_{t+3} \cdots M_{t+n}$ . The bottom panel plots the endogenous relation between duration and expected excess returns for the 10 portfolios. Parameter values are set according to the benchmark calibration  $\delta = 0.998985$ ,  $\mu_d = \mu_c = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\Psi = 1.5$ ,  $\gamma = 10$ ,  $\sigma_d = 5.9$ . The market portfolio has  $\phi_x = 1$  and  $\phi_c = 4$ .

Table 1  
 Asset Pricing Implications: Full Information vs. Limited Information

Row	Model		$E(P/D)$		$E(r_i - r_f)$		$E(r_f)$		$\sigma(r_i)$	
	$\phi_x$	$\phi_c$	FI	LI	FI	LI	FI	LI	FI	LI
1	1	2.2	166	300	1.06	1.20	1.37	0.95	17.29	17.43
2	1	6	46	14	2.45	7.73	1.37	0.95	22.79	22.44
3	2	2.2	35	300	3.31	1.26	1.37	0.95	18.40	19.95
4	2	6	23	14	4.90	8.12	1.37	0.95	23.67	23.92
5	3	2.2	22	238	5.20	1.26	1.37	0.95	20.24	23.29
6	3	6	17	13	6.63	8.42	1.37	0.95	25.02	26.09

Notes: This table reports financial statistics of the model with full information (FI) and limited information signal extraction (LI), for varying degrees of exposure to the long-run and short-run risk components, governed by  $\phi_x$  and  $\phi_c$ , respectively. The other parameters are set to  $\gamma = 10$ ,  $\psi = 1.5$ ,  $\delta = 0.998985$ ,  $\mu = 0.0015$ ,  $\rho = 0.979$ ,  $\sigma = 0.0078$ ,  $\sigma_{xc} = 0.044$ ,  $\sigma_d = 5.9$ .  $E(r_i - r_f)$  denotes the annual log risk-premium, in percent;  $E(r_f)$  denotes the annual log risk-free rate, in percent, and  $\sigma(r_i)$  and  $\sigma(r_f)$  denote the standard deviations of the annual equity return and risk-free rate, respectively.  $E(P/D)$  is the annual price-dividend ratio. Statistics are averages from 1000 simulated samples of 840 monthly observations.

Table 2  
Limited Information Implications of Value and Growth Portfolios: Limited Information

		G		Growth to Value						V	V-G	
Portfolio		1	2	3	4	5	6	7	8	9	10	10-1
$E(R^i - R^f)$	Case 1	5.11	5.11	5.12	5.14	5.18	5.26	5.43	5.72	6.23	6.86	1.75
	Case 2	3.74	3.75	3.77	3.81	3.89	4.06	4.38	4.94	5.93	7.61	3.87
	Data	6.50	7.56	7.47	7.60	7.48	9.07	9.15	8.98	10.73	11.92	5.42
Sharpe Ratio	Case 1	0.25	0.25	0.25	0.25	0.25	0.25	0.26	0.27	0.30	0.32	0.08
	Case 2	0.26	0.26	0.26	0.26	0.27	0.28	0.31	0.35	0.41	0.52	0.27
	Data	0.38	0.49	0.49	0.49	0.64	0.63	0.61	0.72	0.67	0.64	0.26
CAPM: $R_t^i - R_t^f = \alpha_i + \beta_i (R_t^m - R_t^f) + \varepsilon_{it}$												
$\alpha_i$	Case 1	-0.11	-0.11	-0.10	-0.08	-0.04	0.03	0.18	0.45	0.93	1.52	1.63
	Case 2	-0.29	-0.28	-0.26	-0.22	-0.13	0.04	0.36	0.94	1.92	3.57	3.86
	Data	-1.68	-0.05	0.08	0.24	2.47	2.31	2.41	4.10	3.71	4.19	5.87
$\beta_i$	Case 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.02	0.02
	Case 2	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.01	0.00
	Data	1.10	1.02	1.01	0.96	0.89	0.90	0.86	0.87	0.92	1.00	-0.10

Notes: Results are presented for limited information specification. In each simulation year, firms are sorted into deciles based on the price-dividend ratio. Returns are calculated over the subsequent year. Intercepts and slope coefficients are from OLS time-series regressions of excess portfolio returns on the excess market return. Parameter values for case 1 are set as follows:  $\gamma = 10, \psi = 1.5, \delta = 0.999, \mu = 0.0015, \rho = 0.979, \sigma = 0.0078, \sigma_{xc} = 0.044, \sigma_d = 5.9$ . Parameter values for case 2 are set as follows:  $\gamma = 10, \psi = 1.3, \delta = 0.999, \mu = 0.0015, \rho = 0.983, \sigma = 0.0051, \sigma_{xc} = 0.08, \sigma_d = 5.9$ . The market portfolio has  $\phi_x = 1$  and  $\phi_c = 4$ . Results in rows labeled “Data” are produced as follows. Portfolios are formed by sorting firms into deciles on the boot-to-market ratio (B/M). Moments are annualized in percentages (multiplied by 1200 in the case of means and  $12/\sqrt{12}$  in the case of Sharpe ratios). Intercepts and slope coefficients are calculated from OLS time-series regressions of excess portfolio returns on the excess return on the CRSP value-weighted index. Intercepts are annualized in percentages (multiplied by 1200). The return data are monthly and span the period 1947-2004.

Table 3  
 Limited Information Implications of Value and Growth Portfolios: Limited Information

CAPM & HML: $R_t^i - R_t^f = \alpha_i + \beta_i (R_t^m - R_t^f) + \gamma_i HML_t + \varepsilon_{it}$											
	G		Growth to Value						V	V-G	
Portfolio	1	2	3	4	5	6	7	8	9	10	10-1
Case 1											
$\alpha_i$	0.36	0.36	0.35	0.33	0.28	0.21	0.08	-0.05	0.04	0.36	0.00
$\beta_i$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
$\gamma_i$	-0.29	-0.29	-0.27	-0.25	-0.20	-0.11	0.06	0.31	0.54	0.71	1.00
Case 2											
$\alpha_i$	0.64	0.63	0.61	0.56	0.47	0.29	0.02	-0.28	-0.23	0.64	0.00
$\beta_i$	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.01	0.00
$\gamma_i$	-0.24	-0.24	-0.23	-0.20	-0.16	-0.07	0.09	0.32	0.56	0.76	1.00

Notes: Results are presented for limited information specifications. In each simulation year, firms are sorted into deciles based on the price-dividend ratio. Returns are calculated over the subsequent year. Intercepts and slope coefficients are from OLS time-series regressions of excess portfolio returns on the excess market return together with *HML*. Parameter values are set as in Table 2.