A DISAGGREGATED MODEL OF REAL CASH BALANCES OF THE U.S. CORPORATE SECTOR
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Introduction

The existing models of demand for money are generally inadequate to fully specify behavior of corporate real cash balances. A new approach is needed and the model specified in this paper is an effort to fulfill this need. The model is based on the neo-classical theory of the firm; real cash balances are considered to be a function of the level of output, relative factor prices, and the opportunity cost of holding money. By specifying and measuring accurately the price and scale variables of the model it is possible to trace the influence of capital gains, interest rates, factor prices, changes in general price levels, and the size of transactions on the desired real cash holdings of a firm.

In Section I the demand for real cash balances is derived and the opportunity cost of real cash balances and user costs of labor and capital services are specified. In Section II the empirical results of the model for seventeen two-digit manufacturing industries are presented. The interindustry variations in the income and price elasticities of real cash balances are discussed and the adjustment process of the model are examined in this section. The intertemporal stability of the model and the aggregation errors and other conceptual experiments are discussed in Section III. The data used are quarterly time series for the period 1948-I to 1964-IV. The source of the data and definition of the variables of the model are explained in the appendix.

I. Specification of the Model

Real cash balances serve as productive inputs.1 They are part of the working capital of the firm facilitating its productive process, often by indirect means, such as hedging against changes in prices of capital and labor and interest rates. Holding adequate cash balances may reduce the uncertainty of meeting current payments, thus avoiding unnecessary and unprofitable liquidation of other assets. The desired real cash balances of the firm are related to the level of its operation and to the movements in the opportunity cost of money (w), the user cost of capital services (c), the price of labor services (p) and the general price level (p). Output serves as a scale variable while v, c, and w depict the "own" and cross elasticities of real cash balances to changes in these prices.2

The opportunity cost of money has three components: an interest cost, capital gains (losses) in the securities market, and a depreciation cost. The first element is the interest income foregone by holding cash rather than short-term securities. The capital gains component of v is generally due to a rise or fall in the prices of securities. The third element is a reduction in the purchasing power of money due to an increase in the general price level. We can write

\[ v = [r + \frac{p^e}{p^b} + \frac{p^e}{p}] \] (1a)

where v is the opportunity cost of money, r is the money rate of interest, \( p^b \) is the expected change in price of securities, \( p^e \) is the price of the securities, \( p \) is the level of general prices. Assuming that change in the rate of interest \( \Delta \) is a good proxy for capital gains (losses), \( \frac{p^e}{p^b} \), we can write \( v = [r - \Delta + \frac{p^e}{p}] \).

Labor is a quasi-fixed factor of production and its user cost consists of a variable component and fixed charges.3 However, due to lack of data we have used gross hourly wage rate as a proxy for w. The user cost of capital services is a modification of Jorgenson's measure; the modification is to introduce depreciation due to use as an additional element of the user cost of capital.4 That is,

\[ \bar{c} = \frac{q}{1+u} \left( \frac{(d+\delta)}{0} + \delta L \right) \frac{s - \delta}{q}, \] (1b)

where q is the price of capital goods, u is the rate of corporate tax, d is the cost of capital, \( \delta_0 \) is the rate of depreciation due to passage of time, \( \delta_1 \) is the depreciation rate due to intensive use of capital, s is the rate of utilization, and \( \delta \) is the change in price of capital goods as a measure of capital gains (losses).

Assume that the firm minimizes its expected total costs, defined as

\[ C = \bar{w}L + \bar{c}K + \bar{v}m, \] (2)

subject to a twice differentiable production function,

\[ X = F (L, K, m), \] (3)

where C is the total cost, L is the labor input measured in terms of man-hours, K is capital stock services measured as Ks where K is the capital stock and s is its rate of utilization, m is the stock of real cash balances of the firm, and X is the level of output. Minimizing (2) subject to the production function (3) suggests that the demand function for real cash balances is

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\[
m^* = f(c/\bar{w}, v, \lambda).
\]

If we allow the elements of \( v \) to appear separately in (5) we get the "unrestricted" version of our model,
\[
m^* = f'(c/\bar{w}, x, x, \hat{p}/p, x).
\]

All variables in the "RHS" of equations (4) and (5) are in terms of their expected values. Note that these demand functions are assumed to be nonhomogeneous in factor prices, i.e., a doubling of all prices, given the level of output, will lead to a different level of desired real cash balance. This assumption allows for economies of scale in purchasing inputs when their prices increase. That is, the firm can take advantage of discounts by being able to pay its bills in cash. To simplify the analysis we assume that the relationships (4) and (5) are multiplicative, i.e.,
\[
m^* = A_0 x^\alpha (c/\bar{w})^\beta,
\]
and
\[
m^* = A_1 x^\alpha (\hat{p}/p)^\omega (c/\bar{w})^\psi,
\]
where \( \rho, \alpha, \beta, \xi, \pi, \omega, \) and \( \psi \) are elasticities of \( m^* \) with respect to the explanatory variables of the model. Of particular interest are the magnitudes of \( 1/\rho, \alpha, \beta, \) and \( \xi. \) The return to scale, the own elasticities, and the cross elasticities of real cash balances can be measured from the value of these parameters. We assume the following reasonable a priori hypotheses:
\[
\begin{align*}
\frac{\partial m^*}{\partial x} & > 0, & \frac{\partial m^*}{\partial \rho} & < 0, & \frac{\partial m^*}{\partial (\hat{p}/p)} & < 0, \\
\frac{\partial m^*}{\partial \xi} & > 0, & \frac{\partial m^*}{\partial \pi} & < 0, & \frac{\partial m^*}{\partial (c/\bar{w})} & > 0.
\end{align*}
\]

The partial derivative of \( m^* \) with respect to the output of the firm, \( X, \) is certainly positive; the reasons for real cash balances to decrease with a rise in the interest rate, the general price level, and capital gains \( (\text{a decrease in } \hat{p}) \) were stated earlier and need not be repeated. The relationship of \( m^* \) with \( v, \) the opportunity cost of holding real cash balances is also apparent. The expected positive sign of the partial correlation between \( m^* \) and \( c/\bar{w} \) suggests that real cash balances should be substitutes for capital and/or labor. Moreover, the magnitude of the coefficients of \( c/\bar{w} \) may differ among industries and may take zero values, suggesting the absence of any substitution between real cash balances and other production inputs. If \( |\frac{\partial m^*}{\partial v}| < |\frac{\partial m^*}{\partial (c/\bar{w})}| \) we can infer that real cash balances are better substitutes for financial securities than for capital or labor.

Generally there is a time lag between actual and desired real cash balances of the firm. The lag could be due to uncertainty about the demand conditions, incomplete information about financial markets, etc. It may also reflect the disequilibrium in other assets of the firm. That is, adequate cash balances are needed to facilitate and lower the adjustment costs of other assets. We postulate a simple Koyck distributed lag mechanism to depict the adjustment process of actual to desired real cash balances, i.e.,
\[
m_t^* = m_{t-1}^* - \gamma (m_{t-1}^* - m_{t-2}^*),
\]
where \( \gamma \) is the adjustment coefficient. Combining equations (6a) or (6b) with (8) and rearranging we get the following estimating equations:

a. restricted model
\[
\ln m_t = a_0 + a_2 \ln(c/\bar{w})_t + a_3 \ln X_t + a_4 \ln m_{t-1} + \xi_1
\]

b. unrestricted model
\[
\ln m_t = b_0 + b_1 \ln(c/\bar{w})_t + b_2 \ln X_t + b_3 \ln m_{t-1} + \xi_2
\]
where \( a_4 = (1-\gamma_1) \) and \( b_2 = (1-\gamma_2); \gamma_1 \) and \( \gamma_2 \) are the adjustment coefficients in the two equations. The a priori hypothesis (7) suggests that
\[
\begin{align*}
& a_1 < 0, a_2 > 0, a_3 > 0, \\
& b_1 < 0, b_2 > 0, b_3 < 0, b_4 > 0, b_5 > 0.
\end{align*}
\]

Variable \( X \) depicts the scale of operation of the firm and can be measured by total assets, total output, or the level of sales of the firm. Our model suggests using the flow variables such as output or sales as the scale variable. Setting the problem of measurement of \( X \) aside for the moment, we assume that the firm maintains a desired relation between its real cash balances and its expected level of output, \( X. \) It forecasts expected output by a simple schema such as
\[
\hat{X} = \frac{X_{t-1} - C_0}{X_{t-1}} (C_0 - C_1),
\]
where \( C_0 \) and \( C_1 \) are discount rates. From equation (10), \( X \) can be interpreted as a trend extrapolation \( X_{t-1} \) with allowance for deviations from trend, \( X_{t-1} \). Substituting for \( X \) in equations (9a) and (9b) from equation (10) and rearranging we get the final forms of the estimating equations of our model:

a. restricted model
\[
\ln m_t = a_0 + a_2 \ln(c/\bar{w})_t + a_3 \ln X_t + a_4 \ln m_{t-1} + \xi_1
\]

b. unrestricted model
\[
\ln m_t = b_0 + b_1 \ln(c/\bar{w})_t + b_2 \ln X_t + b_3 \ln m_{t-1} + \xi_2
\]
b. unrestricted model

\[ \ln x_t = b_0 + b_1 \ln r_t + b_2 \ln \pi_t + b_3 dln r_t + b_4 \ln \left( \frac{c}{w} \right)_t + b_5 \ln x_{t-1} + b_6 \ln x_{t-2} + \varepsilon_{2t} \]

(11b)

where \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) are random errors from the regression coefficients of equations (11a) and (11b). \( c_0 \) and \( c_1 \) can be interpreted as the elasticities of real cash balances with respect to "permanent" and "transitory" output. They can easily be calculated from the regression coefficients of either equation. For example, the values of these parameters in equation (11b)

\[ c_0 = b_5 \quad \text{and} \quad c_1 = b_6 \]

In the long run, \( c_1 \) will tend to be zero and \( c_0 \) may take values greater, equal, or smaller than unity. The magnitude of this coefficient depends on the mechanics of pooling small transactions together, such as using calculating machines, computers, etc., which allow for economies of scale in managing cash balances. In such a case, the income elasticity of real cash balances will be less than one.

II. Empirical Results

Both versions of the model were fitted to the quarterly data for seventeen subindustries of the manufacturing sector for the period 1948-I to 1960-IV. The results for total manufacturing have been reported elsewhere [24] and will not be shown here. The industry classification used in this study is stated in Table 1. This classification is based on the OBE-SEC Investment Survey. The main empirical results of the model for various industries are indicated in Tables 2 and 3. In Table 2 the results for the unrestricted form of the model are displayed while Table 3 gives the results for the restricted model. The long-run output interest cross elasticities of real cash balances are indicated in Table 4. The average and variance of the lag adjustment of real cash balances in various industries are also indicated in this table. In Table 5 the results of a set of conceptual experiments designed to test the intratemporal stability of the model and existence of aggregation errors are presented. The empirical results of the restricted and unrestricted version of the model are similar in their general features. The specific-point estimates of the coefficients of similar variables in both models, of course, differ. However, for the sake of brevity, we will restrict the discussion to the unrestricted version of the model, which is richer in economic content.

The overall results of the model for each industry in terms of the familiar statistical criteria--high R², low standard error of estimate (S), and absence of strong serial correlation--are encouraging. The fit of the model, however, is not quite satisfactory in industries 14, 15, and 16. The model on the whole seems to perform much better in terms of correct signs and goodness of fit in aggregate industries, such as total durables and total nondurables than in their subindustries. The signs of the variables are generally consistent with the a priori hypotheses stated on page 463. The major exception seems to be the lagged output variable, \( x_{t-1} \), which has a positive coefficient in a few industries. The sign of the coefficient of relative factor prices is positive except in two industries, 07 and 08. A negative sign for \( c/w \) is inconsistent with the hypothesis that real cash balances are substitutes with capital and labor.

The variation in the coefficients of the price and output variables across industries is probably a reflection of the prevailing industrial structure of these industries. To interpret our results properly, it is necessary to relate these variations to the specific developments of the industries. For example, the finding of economies of scale with respect to real cash balances in an industry, could be due to entry of new firms or expansion of the existing firms in that industry. Specifically, the price elasticities are affected whether the structure of the industry is dominated by small or large firms. Moreover, the rate of technical progress, the supply conditions, and the demand factors differ among various industries. Unfortunately, detailed considerations of these matters are not feasible in the context of this study and therefore are not discussed.

A. The Price Elasticities of Real Cash Balances

The price variables relevant for decision-making are the expected factor prices. However, they are not known and have to be approximated from past data according to some assumed expectations hypotheses. We have used very simple expectations hypotheses, i.e., the current values of \( v, r, \pi, \) and \( \pi \) and the two period lagged \( c/w \) are good proxies of the expected values of these variables in our model. These hypotheses, though theoretically inadequate and empirically too simple, seem to be good first approximations. However, this aspect of the model requires further study which must await further research.

The role of the interest rate as a determinant of cash balances has been widely discussed and is a controversial issue in the literature. The controversy seems to center on the magnitude of the interest elasticity of cash balances and whether short- or long-term rates are the appropriate variable in the demand function for money. Friedman [13] considers interest elasticity of money to be very low while writers like Brunner-Meltzer [4] think cash balances are approximately unitary elastic. Laidler [21] has argued that short-term interest is the theoretically and empirically relevant variable in a demand function for money because money is a close substitute for short-term securities, and short-term rates are empirically better proxies for the "interest rate" variable. However, some other studies indicate that long-term interest rate empirically performs better.

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<table>
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<th>Industry Code</th>
<th>OBE-SIC Industry</th>
<th>SIC Industry</th>
</tr>
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<tr>
<td>01</td>
<td>Total durables</td>
<td>19, 24, 25</td>
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<tr>
<td></td>
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<td>32-39</td>
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<td>02</td>
<td>Primary iron and steel</td>
<td>331-2</td>
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<tr>
<td>03</td>
<td>Primary nonferrous metal</td>
<td>333-9</td>
</tr>
<tr>
<td>04</td>
<td>Electrical machinery and equipment</td>
<td>36</td>
</tr>
<tr>
<td>05</td>
<td>Machinery, except electrical</td>
<td>35</td>
</tr>
<tr>
<td>06</td>
<td>Motor vehicles and equipment</td>
<td>371</td>
</tr>
<tr>
<td>07</td>
<td>Transportation equipment, excluding motor vehicles</td>
<td>372-9</td>
</tr>
<tr>
<td>08</td>
<td>Stone, clay, and glass</td>
<td>32</td>
</tr>
<tr>
<td>09</td>
<td>Other durables</td>
<td>19, 24, 25</td>
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<td></td>
<td></td>
<td>34, 38, 39</td>
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<tr>
<td>10</td>
<td>Total nondurables</td>
<td>20-23, 26-31</td>
</tr>
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<td>11</td>
<td>Food and beverages</td>
<td>20</td>
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<tr>
<td>12</td>
<td>Textile mill products</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>Paper and allied products</td>
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<td>14</td>
<td>Chemical and allied products</td>
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<td>15</td>
<td>Petroleum and coal products</td>
<td>29</td>
</tr>
<tr>
<td>16</td>
<td>Rubber products</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>Other nondurables</td>
<td>21, 23, 27, 31</td>
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</table>
Whether short- or long-term interest rate is the appropriate variable in demand for corporate real cash balances is difficult to decide on a priori grounds. Empirically, the long-term rate did perform better than the short-term interest rate in a majority of the industry regressions. This result can be rationalized in two different ways. Movements of the long-term interest rate may represent the systematic part of fluctuations of the short-term interest rates and thus the long-term interest rate is a good proxy for the interest rate variable in the demand function for money. Moreover, in the context of our cost minimization model, the opportunity cost of holding money is the highest return foregone. Switching between cash and short-term securities does not mean that the short-rate is the most lucrative opportunity foregone by holding cash. Holding short-term securities may involve some additional opportunity costs as well.

The results in Table 4 show that the short- and long-run interest elasticity of real cash balances are generally lower than unity and vary from industry to industry. The results on interest elasticity of real cash balances seem to be in contrast to both the Friedman [13] and Brunner-Meltzer [4] findings mentioned above. There is a negative coefficient for lnr in each industry regression and it is statistically significant in 11 of the 17 industries. The short-run interest elasticities of real cash balances suggested by the value of the coefficient of this variable ranges from -.0399 in industry 04 to a high of -0.4686 in industry 07. The long-run interest elasticity of real cash balances of various industries is indicated in column 4 of Table 4 and is calculated as \( b_1/1-b_7 \), where \( b_1 \) and \( b_7 \) are the coefficients of \( \text{ln}r_t \) and \( \text{ln}m_{t-1} \). The short- and long-run interest elasticity of real cash balances is generally higher in the nondurable industries; however, the highest interest elasticity of real cash balances occurs in a durable industry, 07. The higher interest elasticity for the nondurable industries may indicate the greater dependence of these industries on the money market. Note that the absolute value of the long-run interest elasticity is generally greater than the magnitudes of the cross elasticity or real cash balances with respect to \( \text{c}/\text{w} \). That is, the response of real cash balances to the long-term interest rate is greater than their response to movements of factor prices. This result supports Tobin’s [31] contention that substitution between liquid assets is much greater than between money and physical inputs, capital or labor.

The capital gains variable, \( \text{dlnr}_t \), has the correct positive coefficient in all of the regression equations, suggesting that when the firm expects capital losses it will adjust its portfolio by shifting from holding securities to cash balances. Unfortunately, the coefficient of \( \text{dlnr}_t \) is not statistically significant in most industry regressions. If capital gains is considered a short-term phenomenon, in the long run we expect the coefficient of \( \text{dlnr}_t \) to equal zero. Ignoring the large standard error of this variable, we observe that the magnitude of its coefficient is generally similar to that of \( \text{lnr}_t \). However, in industries 04, 07, 15, and 17, the two coefficients differ substantially.

Expectations about changes in general price level influence real cash balances through changes in the purchasing power of money. There have been studies attempting to examine empirically the role of price changes on real cash balances. However, most of them are confined to hyper-inflationary periods and little is known about the effect of price changes on money holdings during periods of slowly rising prices. The main difficulty has been the formulation and empirical approximation of an adequate price expectation hypothesis. In this study an extraordinarily simple expectation model is employed, i.e., current actual change in price level is a good proxy of expected changes in price level. It is possible to use more complicated hypotheses but such an attempt is not made here. Nonetheless, our simple price expectation performs adequately; \( \text{dlnp}_t \) has a negative sign in all the regressions and its coefficient is statistically significant in more than half of the regressions. When \( \text{dlnp}_{t-1} \) was used instead of \( \text{dlnp}_t \) in the regressions, the coefficients of the price variables did not change very much; using \( \text{dlnp}_{t-1} \) in addition to \( \text{dlnp}_t \) did not improve the results either. However, a better price expectations hypothesis is needed to improve our estimates of the influence of changes in the price level on real cash balances.

Ignoring the high standard error of \( \text{dlnp}_t \) in several industries we can see that the price elasticity of real cash balances varies greatly among various industries. To compute the elasticities of \( m_t \) with respect to \( p \) we need to convert \( p \) from an index number by multiplying the coefficient of \( c/w \) by the mean value of \( p_t/p_{t-1} \) for the sample period (about .023). The short-run price elasticity of real cash balances, not shown in Table 4, ranges from -.0016 in industry 06 to -.0964 in industry 07 while the long-run elasticity varies from -.0009 to -.2941. Of course, the high standard error of \( \text{dlnp}_t \) makes these estimates suspect.

The interesting result of this study is the performance of the relative factor price variable \( c/w \) in various industries. The choice of a two-period lag for this variable is based on the findings for total manufacturing industry, reported in [24]. When rental on capital and wage rates were separately introduced in the regression equation for a few arbitrarily chosen

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### TABLE 2

The Determinants of Real Cash Balances in the U.S. Manufacturing Industries\textsuperscript{a,b}

Unrestricted Model

1948-I to 1960-IV

<table>
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<tr>
<th></th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
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<tr>
<td>c</td>
<td>0.0684</td>
<td>-0.1978</td>
<td>-0.8000</td>
<td>-0.3255</td>
<td>-0.4730</td>
<td>-1.0077</td>
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<td>ln(r_t)</td>
<td>-0.0838</td>
<td>-0.0659</td>
<td>-0.2316</td>
<td>-0.0399</td>
<td>-0.1369</td>
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<td>dln(r_t)</td>
<td>0.0499</td>
<td>0.0520</td>
<td>0.0903</td>
<td>0.1012</td>
<td>0.0740</td>
<td>0.0695</td>
<td>0.1110</td>
<td>0.0902</td>
<td>0.0679</td>
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<tr>
<td>dln(p_t)</td>
<td>0.0772</td>
<td>0.0545</td>
<td>0.2774</td>
<td>0.0906</td>
<td>0.1163</td>
<td>0.0891</td>
<td>0.1042</td>
<td>0.1204</td>
<td>0.0811</td>
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<tr>
<td>ln((\widetilde{c}/\widetilde{w}))(_{t-2})</td>
<td>-1.0901</td>
<td>-0.5815</td>
<td>-3.0948</td>
<td>-1.3333</td>
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<td>-0.6087</td>
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<td>ln(x_t)</td>
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<td>0.1603</td>
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<td>ln(x_{t-1})</td>
<td>0.2235</td>
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<td>ln(m_{t-1})</td>
<td>-0.1234</td>
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<td>-0.2732</td>
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<td>0.7323</td>
<td>0.6742</td>
<td>0.5465</td>
<td>0.6936</td>
<td>0.6557</td>
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<tr>
<td>(\hat{R}_t)</td>
<td>0.8989</td>
<td>0.7546</td>
<td>0.8564</td>
<td>0.8158</td>
<td>0.8870</td>
<td>0.8662</td>
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<tr>
<td>(S_y)</td>
<td>0.0283</td>
<td>0.0449</td>
<td>0.0627</td>
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<td>0.0480</td>
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<td>0.0578</td>
<td>0.0538</td>
<td>0.0422</td>
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<tr>
<td>D(\widetilde{W})</td>
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<td>2.1199</td>
<td>1.5188</td>
<td>1.9865</td>
<td>2.1218</td>
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<td>2.4235</td>
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(continued)

### TABLE 2 (continued)

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<td>-0.5345</td>
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<td>-0.5883</td>
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<td>-0.2559</td>
<td>-0.0375</td>
<td>-0.1111</td>
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<td>dln(r_t)</td>
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<td>-0.4954</td>
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<td>-0.1972</td>
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<td>0.0385</td>
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<td>0.0632</td>
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<tr>
<td>ln(x_{t-1})</td>
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\textsuperscript{a} The figures in parentheses are standard errors of the coefficients.

\textsuperscript{b} The coefficients of the dummy variables of each industry regression are not reported for the sake of brevity.
### TABLE 3

The Determinants of Real Cash Balances in the U.S. Manufacturing Industries<sup>a,b</sup>

#### Restricted Model,

1948-I to 1960-IV

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<th>Durables</th>
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<th>02</th>
<th>03</th>
<th>04</th>
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<th>09</th>
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<td>-.0740</td>
<td>-.0248</td>
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<tr>
<td>ln(Y&lt;sub&gt;t&lt;/sub&gt;)/Y&lt;sub&gt;t&lt;/sub&gt;-2</td>
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<td>(.0266)</td>
<td>(.0398)</td>
<td>(.0364)</td>
<td>(.0314)</td>
<td>(.0277)</td>
<td>(.0431)</td>
<td>(.0362)</td>
<td>(.0277)</td>
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<td>.7565</td>
<td>.7807</td>
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<td>(.1070)</td>
<td>(.0818)</td>
<td>(.0838)</td>
<td>(.1171)</td>
<td>(.0930)</td>
</tr>
</tbody>
</table>

R<sup>2</sup> | .8969 | .7510 | .8486 | .8194 | .8844 | .8600 | .8407 | .8873 | .8772 |

S<sub>y</sub> | .0279 | .0442 | .0628 | .0585 | .0474 | .0479 | .0576 | .0572 | .0417 |

D/W | 1.8995 | 2.1909 | 1.5384 | 1.9961 | 2.0938 | 2.0206 | 2.4284 | 2.2142 | 1.9703 |

(continued)

### TABLE 3 (concluded)

#### Nondurables

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<th>17</th>
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<td>(.1381)</td>
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<td>(.2084)</td>
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<td>(.1385)</td>
<td>(.1736)</td>
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<td>-.1510</td>
<td>.0553</td>
<td>-.4305</td>
<td>-.2099</td>
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<td>-.4500</td>
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<td>(.1438)</td>
<td>(.1619)</td>
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<td>(.0816)</td>
<td>(.1051)</td>
<td>(.1312)</td>
<td>(.1139)</td>
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</table>

R<sup>2</sup> | .8586 | .9988 | .9523 | .8856 | .7230 | .4593 | .7199 | .8158 |

S<sub>y</sub> | .0307 | .0317 | .0614 | .0542 | .0444 | .0744 | .0736 | .0424 |

D/W | 2.2399 | 2.0967 | 2.3882 | 1.8395 | 2.5590 | 1.7877 | 2.3639 | 1.9589 |

---

<sup>a</sup> The figures in parentheses are standard errors of the coefficients.

<sup>b</sup> The coefficients of the dummy variables of each industry regression are not reported for the sake of brevity.

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industries, the indication was that most of the explanatory power of \( \frac{c}{w} \) is due to variation in \( \frac{c}{p_1} \), where \( p_1 \) is the wholesale price index.

The coefficient of \( \frac{c}{w} \) is positive in all except two industries, 07 and 06. The coefficient of this variable is in general statistically significant primarily in the durable industries. The results seem to support the notion that real cash balances are gross substitutes for capital and labor. If we can assume that the capital/labor ratio responds to changes in the relative price variable, \( \frac{c}{w} \), an increase in the stock of real cash balances of the firm will influence the capital intensity of the firm. This is true especially in the durable industries. There seems to be some support of the theoretical arguments recently advanced by Sidrauskis [28] and Tobin [31], i.e., diversion of funds to cash balances reduces investable funds for capital expansion and may lead to a lower capital intensity.

B. Income Elasticity of Real Cash Balances

Two issues which have received considerable attention in the literature are whether income or wealth is the proper constraint on real cash balances of the economic unit and the proper measurement of these constraints.\(^{13}\) In the context of our model the theoretically relevant variable is a measure of output. In estimating equations (9a) and (9b) we used several measures for wealth and output as proxies of the scale parameter in these equations. Two measures of deflated total assets, excluding real cash balances, were used as measures of the wealth variable. The first measure of the wealth variable was defined as the total asset of the industry deflated by the GNP price deflator, and the second measure variable, \( T \), was a synthetic measure of total assets allowing for variation changes of specific assets on the consolidated balance sheet of the various industries.\(^{14}\) Three measures of output were used in the preliminary stages of the investigation. They were deflated sales \( (X) \), deflated actual output \( (X_0) \), and deflated capacity output \( (X^C) \), which was derived from the relation

\[ \frac{X^C - X_0}{X^C} = 1 - s, \text{ where } s \text{ is the rate of utilization.} \]

These measures of wealth and output variables and their lagged values were substituted for \( X_t \) and \( X_{t-1} \) in equations (11a) and (11b).

There was very little difference between the coefficients of the wealth variables, \( T, T^*, X, \) and \( X_0 \). Other coefficients remained stable except that the coefficients of \( \text{dlnp}_t \) and \( \text{lnm}_t \) were somewhat larger in equations with \( T \) and \( T^* \). The coefficients of \( X \) and \( X_0 \) were practically similar with the former being slightly more stable.

The coefficients of the scale variables when measured by \( X^C \) were generally smaller, suggesting that an increase in the rate of utilization, \( s \), would tend to decrease the desired stock of real cash balances of the firm; that is, the closer the path of actual output to that of capacity output, the smaller the need for financing inventories and precautionary balances. To put it differently, when the level of output of the firm falls, its cash flow declines, while cutting back the stock of its inputs requires some time. In the disequilibrium period the adjustment will be met by reducing the stock of real cash balances of the firm. Other coefficients of equations (11a) and (11b) remained quite stable in most industry regressions when \( X_C \) was used, except for the price variable, \( \text{dlnp}_t \), in equation (11b). The smaller magnitude and lack of statistical significance of the coefficient of \( \text{dlnp}_t \) can be explained by the unavoidable collinearity between prices and rate of utilization in the expansionary phase of the business cycle.

It was decided to use deflated sales, \( X \), as a proxy for output variables in the individual industry regressions. The long-run income elasticity of real cash balances is

\[ c_0 = \frac{b_5 + b_6}{(1-b_7)}, \]

where \( b_5 \), \( b_6 \), and \( b_7 \) are respectively, the coefficients of current and lagged sales and \( \text{lnm}_{t-1} \). The value of \( c_0 \) varies greatly among industries ranging from .0556 in industry 06 to 1.313 in industry 12. The elasticity of real cash balances with respect to the transitory component of actual income, calculated as \( c_1 = \frac{b_5}{b_7} \), is generally positive and smaller than the corresponding \( c_0 \) in each industry, as can be seen from columns 1 and 2 of Table 4. However, in industries 03, 05, 07, 08, and 12 \( c_1 \) is negative, which suggests that the real cash balances of these industries decline when the current income deviates from its past trend. This is a possible outcome but it is contrary to the arguments presented on page 463. Note, however, the short-run income elasticity of real cash balances \( (b_5 + b_6) \), as distinct from their elasticity with respect to the "transitory" income, is always positive but smaller than with respect to both \( c_0 \) and \( c_1 \) in each industry.

The important point to note is that, no matter which measure of the scale factor was used, the evidence suggested economies of scale of various magnitudes in every industry. The calculated value for \( \phi \), the scale parameter for various industries, is the reciprocal of figures shown in column 1 of Table 4, i.e., \( \phi = 1/c_0 \). These estimates suggest that most of the manufacturing industries economize cash balance when their scale of transactions increases. The only exceptions are industry 12.
TABLE 4

Elasticities of Real Cash Balances with Respect to Income, Interest
and Change in General Price Level; the Average and Variance
of the Adjustment in Various Industries

<table>
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<td>6.2558</td>
</tr>
</tbody>
</table>

$^a/$

$c_0$ is the long-run income elasticity of real cash balances measured as $\frac{b_0 + b_6}{1 - b_7}$ where $b_5$, $b_6$, $b_7$ are the coefficients of $lnX_t$ and $lnX_{t-1}$, and $lnm_{t-1}$ in equations (11a), (11b). The scale parameter $\rho = 1/c_0$.

c_1$ is the "transitory" income elasticity of real cash balances and is measured as $\frac{b_6}{b_7}$.

c_2$, $c_3$, and $c_4$ are the elasticities of real cash balances with respect to the rate of interest, change in general price level, and factor prices. They are calculated by dividing the coefficients of these variables in Table 2 by (1$-b_7$). The coefficients of the price variable in Table 2 is multiplied by .023, the mean of $\frac{P_t - P_{t-1}}{P_{t-1}}$ for the sample period.

$E_0$ and $V_0$ are the mean and variance of the adjustment lag defined as

$E_0 = \frac{b_7}{1 - b_7}$ and $V_0 = \frac{b_7}{(1 - b_7)^2}$.
where decreasing return (\(n=1\)) prevails and industries 03 and 13 where \(n=1\). This finding supports the theoretical propositions of Baumol [2], Miller and Orr [23], and Tobin [30], and contradicts Friedman's [13] high income elasticity and Brunner-Meltzer's [5] unitary income elasticity hypotheses of real cash balances with respect to the scale variable. The interesting point is that despite aggregation problems the income elasticity of real cash balances is consistently less than unity. However, the question of why our results are so at variance with the findings of these writers and the evidence from the cross-section data is left open at present. The nature of the data and the specification of our model could be responsible for the difference in the results.

C. The Adjustment Process

The short-term adjustment path of actual to desired real cash balances depicted by the coefficient of variable \(\ln M_{t-1}\) varies among different industries. These coefficients range from .69 to .80 but there are exceptional cases such as industries 07, 15, and 16. The empirical performance of the regressions, however, does not depend on the presence of the lagged dependent variable. When \(\ln M_{t-1}\) was dropped from the regressions the independent variables of the model explained on the average more than 70 per cent of the variance of real cash balances in various industries. The signs of the explanatory variables also remained quite stable.

The adjustment path for each industry is generally rapid. About 90 per cent of the gap between the actual and desired level of real cash balances in different industries is achieved within two years. This finding contradicts de Leeuw's [8] long adjustment hypothesis of cash balances. The average and variance of the adjustment lag, shown in columns 6 and 7 of Table 4, are similar in different industries except for industries 12 and 13. However, even with these two unsatisfactory cases our results are in contrast to the extraordinary values for these parameters implied by de Leeuw's findings. Our results on the lag adjustment of actual to desired real cash balances is closer to the one year speed of adjustment reported recently by Pige [12].

III. Some Conceptual Experiments

To evaluate the results of our model, several conceptual experiments were performed. These were designed to test the temporal stability of our model, the aggregation errors and give a comparison with a set of naive models.

1. Temporal stability of the model

To examine the intertemporal stability of the model, a test for structural change was carried out by fitting the model to the whole period 1948-I to 1964-IV and the subperiods 1948-I to 1960-IV and 1961-I to 1964-IV. The appropriate test for structural change is

\[
F = \frac{(S_{1} - S_{2})/k}{S_{2}/(n+z-2k)},
\]

where \(S_{1}\) is the sum square residuals of the regression for the whole period; \(S_{2}\) is the combined sum square residuals for the subperiods 1948-I to 1960-IV and 1961-I to 1964-IV; \(k\) is the number of parameters; \(n\) is the number of observations for the same period (1948-I to 1960-IV); and \(z\) is the number of observations outside the sample (1961-I to 1964-IV). This test is most powerful among invariant tests with the same level of Type I error. The calculated and critical values of this test for each industry are shown in Table 5. If the calculated value of \(F\) exceeds its critical value the null hypothesis of no structural change is rejected. On the whole the model passes this test successfully. The null hypothesis is rejected definitely in four out of seventeen industries, i.e., structural change has occurred in aggregate industries 01 and 10 and in subindustries 02, and possibly 05.

2. Test of aggregation

To test for aggregation errors the following test was developed.\(^{18}\) It consists of comparing the sum square residuals for the sample period of the aggregate industries, of total durables and total nondurables, with those of their individual subindustry regressions. The appropriate test statistics is defined by

\[
F[k(q-1), q(n-k)] = \frac{S_{M} - S_{I}}{k(q-1)q(n-k)}
\]

where \(S_{M}\) is the sum square residuals from a master regression using the model

\[
\begin{array}{ccc}
1 & 1 & \\
\vdots & \vdots & \vdots \\
q & q & \\
\end{array}
\begin{array}{ccc}
X_{1} & \varepsilon_{1} \\
X_{2} & \varepsilon_{2} \\
\vdots & \vdots & \beta_{+} \\
X_{q} & \varepsilon_{q} \\
\end{array}
\]

where \(y_{j}\) (j=1...q) is the vector of observations on real cash balances of the \(j^{th}\) subindustry, \(X_{j}\) is the matrix of observations on the independent variables, \(\beta_{j}\) is the vector of the coefficients, and \(\varepsilon_{j}\) is the vector of errors in the equation; \(q\) is the number of subindustries.
TABLE 5
Test of Structural Change and Aggregation Error in Manufacturing Industries

<table>
<thead>
<tr>
<th>Durable Industries</th>
<th>Calculated F Values</th>
<th>Critical F Values</th>
<th>Nondurable Industries</th>
<th>Calculated F Values</th>
<th>Critical F Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F (10, 48)</td>
<td>Fc (10, 48)</td>
<td></td>
<td>F (10, 48)</td>
<td>Fc (10, 48)</td>
</tr>
<tr>
<td>01</td>
<td>5.714</td>
<td>2.71</td>
<td>10</td>
<td>3.444</td>
<td>2.71</td>
</tr>
<tr>
<td>02</td>
<td>2.285</td>
<td>&quot;</td>
<td>11</td>
<td>1.875</td>
<td>&quot;</td>
</tr>
<tr>
<td>03</td>
<td>1.666</td>
<td>&quot;</td>
<td>12</td>
<td>0.787</td>
<td>&quot;</td>
</tr>
<tr>
<td>04</td>
<td>0.311</td>
<td>&quot;</td>
<td>13</td>
<td>1.888</td>
<td>&quot;</td>
</tr>
<tr>
<td>05</td>
<td>2.710</td>
<td>&quot;</td>
<td>14</td>
<td>1.894</td>
<td>&quot;</td>
</tr>
<tr>
<td>06</td>
<td>3.680</td>
<td>&quot;</td>
<td>15</td>
<td>1.892</td>
<td>&quot;</td>
</tr>
<tr>
<td>07</td>
<td>0.181</td>
<td>&quot;</td>
<td>16</td>
<td>1.018</td>
<td>&quot;</td>
</tr>
<tr>
<td>08</td>
<td>1.152</td>
<td>&quot;</td>
<td>17</td>
<td>1.866</td>
<td>&quot;</td>
</tr>
<tr>
<td>09</td>
<td>1.250</td>
<td>&quot;</td>
<td>&quot;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. Structural Change: \( F = \frac{(S_1-S_2)/k}{S_x/n+e-2k} \)

B. Aggregation Error: \( F = \frac{S_n-S_I}{k(g-1)/q(n-k)} \)

(1) Eight subindustries of total durables
\[ F = \frac{(1.9706-0.9109)/70}{0.9109/336} = 5.584 \quad F_c = 1.35 \]

(2) Seven subindustries of total nondurables
\[ F = \frac{(1.9333-0.9445)/60}{0.9445/294} = 5.13 \quad F_c = 1.38 \]

(3) Total durables and total nondurables
\[ F = \frac{(1.1140-0.0704)/10}{0.0704/84} = 4.202 \quad F_c = 1.95 \]

\( a \)

In these computations the seasonal dummy variables are included in calculation of k, the values of \( F_c \) are at .10 level of probability and those of \( F_c \) are at .05 level of confidence.
\[ S_i \text{ is the combined sum square residuals from the model} \]

\[
\begin{array}{c|cccccc}
    y_1 & X_1 & 0 & \ldots & 0 & 0 & \varepsilon_1 \\
    y_2 & 0 & X_2 & \ldots & 0 & 0 & \varepsilon_2 \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots + \vdots \\
    y_g & 0 & 0 & \ldots & X_g & 0 & \varepsilon_g \\
\end{array}
\]

where \( n \) is the number of observations in the sample period and \( k \) is the number of parameters.

A three way test of the aggregation errors is presented in Table 5: (a) combining the sum square residuals of industries 01 and 10 to calculate \( S_i \) and using the sum square residuals of the master regression for these two industries; (b) combining the sum square residuals for industries 02 to 09 to measure \( S_i \) and using the sum square residuals of the master regression for these industries; and (c) carrying out step (b) for industries 11 to 17. The calculated and the corresponding critical values of \( F \) with degrees of freedom \( k(q-1) \) and \( q(n-k) \) for each of these cases are(207,465),(980,517) given in Table 5. Generally the null hypothesis of no aggregation errors in the cases considered cannot be rejected. Thus, aggregate industry groupings such as total manufacturing, total durables, and total nondurables are not only subject to structural change but they cannot be used without aggregation error.

D. Comparison with Naive Models

The naive models employed for comparison purposes were the familiar equations

(a) \( \ln m_t = a_0 + a_1 \ln m_{t-1} \)

and

(b) \( \ln m_t = b_0 + b_1 \ln m_{t-1} \)

Our model performed better than both of these naive models in each industry. However, our results were similar to those of autoregressive scheme of the form

\[
\ln m_t = a_0 + a_1 \ln m_{t-1} + a_2 \ln m_{t-2} + a_3 \ln m_{t-3} + a_4 \ln m_{t-4}
\]

Comparison with the fourth order autoregressive model (16) is a very stringent test of a quarterly analytical model and it is not surprising that like most quarterly economic investment functions our model succeeds in this test only with a passing grade.19

Footnotes

1 [1], p. 59, [14].

2 For a discussion of interrelationship of real cash balance and capital intensity of a firm, see [28] and [32].

3 The appropriate measure of user cost of labor services is to define the total wage bill as \( W + s(r+q)^L \), where \( w \) is a wage rate, \( h(r+q) \) is the user cost of labor services, \( L \) is total man-hours, \( H \) is hours/man, \( h \) is the hiring cost, and \( q \) is the separation cost of labor. Holding \( H \) fixed, marginal labor cost is

\[
\tilde{w} = \frac{w + h(r+q)}{H} \text{ for } H \leq H_0
\]

and

\[
\tilde{w} = \frac{w(1+q) + h(r+q)}{H} \text{ for } H > H_0
\]

where \( \sigma = \frac{dw}{dH} \cdot \frac{H}{w} \) and \( H_0 \) is standard hours; neglecting the second relation and assuming \( h = \beta w \), the marginal cost of labor reduces to

\[
\frac{w(1 + \beta (r+q))}{H}. \text{ See Rosen [27] for further comments on the user cost of labor services.}
\]

4 [17], the recent modifications of the user cost of capital services proposed by Hall and Jorgenson [15] and Coen [7] are not included in our measure of \( \tilde{c} \).

5 The model can be specified in an alternative way, though the estimating equations may remain the same. Assume that there are four factors of production \( m, B, L \) and \( K; B \) is the net stock of bonds held by the firm, \( m, B, \) and \( K \) are part of the asset structure of the firm and each is subject to depreciation. Assume that the cost of capital to the firm is \( d. \) The firm produces output and financial services by combining its physical inputs \( K \) and \( L \) and by combining its financial assets. The problem is to minimize

\[
C = \tilde{w}L + \tilde{c}K + vm + \tilde{b}B
\]

subject to

\[
\tilde{x} = f_1([L,K]; \theta; [v,m]).
\]

Assuming that the production function has desirable properties we can deduce the implicit input prices as
\[ v_m = \frac{1}{1 + \mu} \left( d + \frac{e^s}{p} \right) \]
\[ v_B = \frac{1}{1 + \mu} \left( d - r + \frac{e^s}{p} \right) \]
\[ v_k = \frac{1}{1 + \mu} \left[ (d + \delta) + \delta_1 s - \frac{\delta_2}{q} \right] \]

and

\[ v_L = \bar{w} \text{ (see footnote (3)).} \]

Thus, when \( d \) increases labor services are substituted for \( K \), \( B \) and \( m \) while the substitutions between different assets depend on \( r \), capital gains, and the depreciation rates, \( \delta \) and \( \delta_1 \).

6
[3].

7
This interpretation critically depends on the assumption about the residuals of equations (11a) and (11b). \( \ln m_t \) and \( \xi_0 \) or \( \xi_2 \) will generally be correlated in these equations if \( \xi_1 \) or \( \xi_2 \) in equations (8) and (9b) are serially correlated. Then the least squares estimates of equations (11a) and (11b) will be biased. For further discussion see [29].

6
[4].

10
When this variable was omitted from the regressions the coefficients of the remaining variables became unstable. The statistical insignificance of the coefficient of \( \ln r_t \) in the industry regressions may imply that the industries keep only minimum balances to meet this transaction need.

11
[6].

12
The rental on capital, \( \bar{c} \), is assumed, due to a lack of data, to be the same for each industry but the cost of labor services, \( \bar{w} \), measured by gross hourly wage rate is considered to vary among industries. However, \( \bar{c}/\bar{w} \) moves in a similar pattern in every industry due to the close relationship of the movements of the wage rates of different industries.

13
[4], [5], [14].

14
See [24] for further discussion; for measurement of \( T^* \) see footnote 20.

15
[8], [16].

16
See [34] for a discussion of this issue.

17
See [18] for further discussion of this test.

18
[17].

19
[17].

20
Assume that there are \( N \) assets on the balance sheet of the firm; they are labeled as follows:

\( G_i \) = assets which are liquid in character but subject to price fluctuations such as inventories. Suppose there are \( n \) such assets on the firm's balance sheet;

\( H_i \) = assets which are of long term nature, e.g., plant and equipment. These assets are subject to price changes, depreciation, etc. Let the number of this type of assets be \( m \);

\( I_i \) = assets which are highly liquid and could easily be exchanged to cash, e.g., the short-term securities and receivables. The value of these assets is relatively fixed in nominal terms, thus, there is no need to adjust them for their "own" price changes. Assume there are \( I \) assets of this kind;

then, we may write:

\[ T^* = \left[ \sum_{i=1}^{n} \left( 1 + \frac{p_i^{s}}{p} \right) G_i + \sum_{i=1}^{m} \left( 1 + \frac{p_i^{k}}{p} \right) H_i + \sum_{i=1}^{I} I_i \right] / p, \]

where \( p_i^{s} \) = the change in wholesale price index, \( p_i^{k} \) = change in index of capital stock deflator, \( p_i \) = the index of price capital, \( P \) = the GNP price deflator (1958 = 100).
Statistical Appendix

The Nature of the Data and Specification of the Variables

The liquidity preference function specified by equations (1la) and (1lb) were fitted to quarterly data for each two-digit manufacturing industry for the period 1948-I to 1964-IV. The main sources of data were the various issues of the FTC-SEC Quarterly Financial Report [11] and the Economic Report of the President [36]. The QFR data were adjusted for sample changes. The data are not seasonally adjusted, but dummy variables were introduced in the regression equations to account for the effects of seasonal variation. The variables used in the regression equations are measured as follows:

- $m_t$ is the quarterly cash holdings of each industry deflated by quarterly GNP deflator, published in the August 1965 issue of SCB [34].

- $X_t$ is the scale variable; it is measured by deflated total assets ($T$), output ($X_o$), capacity output ($X_c$), or sales ($X$), mentioned below.

- $T_t$ is quarterly total assets deflated by GNP deflator.

- $T^*_t$ is quarterly adjusted wealth variable. 20

- $X_{0t}$ is quarterly deflated output defined as the sum of sales plus changes in inventories.

- $X_{ct}$ is the quarterly deflated capacity output defined as $X_{0t}/s$, where $s$ is the rate of utilization.

- $X_t$ is quarterly sales deflated by the quarterly GNP price deflator.

- $r_t$ is the quarterly long-term rate of interest on government bonds from various issues of the Federal Reserve Bulletin [10].

- $v$ is the opportunity cost of real cash balance ($\pi r - \pi^p/\pi^p$).

- $p$ is the GNP deflator from [36], $\dot{p} = dp/dt$.

- $\sigma$ is the user cost or rental of capital $c = \frac{a}{1-\mu} ([1-\nu]d + (1-ur)d + \delta_0 s)$. The data for constructing $\sigma$ are chiefly from the SCB [34] and Statistics of Income [33]. The income tax rate, $\nu$, is the ratio of corporate tax payments to gross corporate profits. The proportion of current replacement cost allowable for tax purposes, $\nu$, is the ratio of tax depreciation to replacement in constant price. A better measure of $v$ has been recently constructed by Coen [7], and Hall and Jorgenson [15], which takes into account changes in depreciation rules. We unfortunately had no access to this data and consequently our measure of $v$ is somewhat biased. $d$ is the cost of capital, computed by using the earnings price ratio and the rate of interest on government bonds. $t$, the proportion of the total cost of capital allowable for tax purposes, is the ratio of net monetary interest to total cost of capital. $s$ is the rate of utilization described below and $\delta_0$ is the rate of depreciation due to use and set to a value of .02 based on findings [24].

- $\frac{\dot{p}}{p} t$ is the BLS quarterly wage rate deflated by the index of GNP price deflator.

- $q_t$ is a four-quarter moving average of capacity utilization index constructed by dividing the Federal Reserve Industrial Production series by McGraw-Hill quarterly capacity index based on the question "how much did capacity increase"? The McGraw-Hill capacity index was interpolated using quarterly investment series as weights. 21 This capacity index is quite similar to the FRB capacity utilization rate.

- $Q_1$, $Q_2$, and $Q_3$ are dummy variables centered in the fourth quarter, to simplify presentation of the results the coefficients of these variables are not reported in the tables.

References


[34] ________, Survey of Current Business, various issues