Coûts d'ajustement et productivité du travail

dans quatre pays de l'OCDE

Pierre MOHNEN
Département des sciences économiques
Université du Québec à Montréal

Ishaq NADIRI
New York University et
National Bureau of Economic Research

Mai 1985
RÉSUMÉ

L'étude vise à comparer deux modèles de coûts d'ajustement, et à quantifier l'impact du déséquilibre dans les facteurs de production sur la croissance et le ralentissement de la productivité du travail. Un modèle de production brute et un modèle de valeur ajoutée sont estimés pour les secteurs manufacturiers des États-Unis, du Japon, de l'Allemagne fédérale et de la France sur la période 1965-66/1977-78. Le stock de capital physique et le stock de capital de recherche et développement sont considérés comme des facteurs quasi-fixes tandis que le travail et les consommations intermédiaires (ou le travail et l'énergie suivant le modèle) sont traités comme des facteurs variables.

L'analyse dans le cadre de la production brute fait ressortir l'inertie du travail et l'évolution cyclique de la productivité du travail. L'estimation paramétrique des poids afférents aux facteurs de production dans le calcul de la croissance de la productivité du travail révèle que la prise en compte du déséquilibre élimine sensiblement le résidu, souvent interprété comme progrès technique, mais n'explique guère le ralentissement de la productivité du travail.
ABSTRACT

The focus of this study is to compare two adjustment cost models and to quantify the impact of disequilibrium in factors of production on the growth and slowdown of labour productivity. A gross output model and a value added model are estimated for the manufacturing sectors of the U.S., Japan, West Germany and France over the period 1965-66/1977-78. The physical capital stock and the stock of R&D knowledge are considered as quasi-fixed inputs, whereas labor and materials (or labor and energy depending on the model) are treated as variable inputs.

The analysis within the gross output framework reveals the sluggishness of labor and the cyclical behavior of labor productivity. The parametric estimation of the weights pertaining to the various inputs in the growth accounting of labor productivity reveals that the introduction of disequilibrium reduces the residual, often interpreted as technical progress, but explains very little of the labor productivity slowdown.
1. Introduction

L'analyse de la productivité émane de la théorie de la production. Les études empiriques sur différents aspects de la production peuvent être groupées en quatre catégories: les modèles statiques d'équilibre du producteur, où les facteurs de production sont toujours à leur niveau désiré de long terme; les modèles statiques de déséquilibre, qui ne décrivent que les situations de court terme, où certains inputs sont totalement fixes; les modèles dynamiques d'ajustement partiel sans fondement théorique qui supposent, comme leur nom l'indique, un ajustement partiel au nouvel optimum à court terme; et finalement, les modèles de coûts d'ajustement qui permettent de justifier les règles d'ajustement et qui peuvent être considérés comme des modèles d'équilibre dans une perspective dynamique et comme des modèles de déséquilibre dans une perspective statique. Notre étude utilise deux modèles de coûts d'ajustement pour analyser la productivité du travail dans le secteur manufacturier des Etats-Unis, du Japon, de la République Fédérale d'Allemagne et de la France de 1965-66 à 1977-78. La différence la plus importante entre les deux modèles réside dans la définition de l'output: dans un cas, il s'agit de la valeur ajoutée, dans l'autre, de la production brute. La sensibilité des estimations à ces deux spécifications fait l'objet du premier volet de cette étude.

Le premier volet de notre étude examine les effets qu'exercent les prix des facteurs de production et le montant à produire sur la productivité du travail en soulignant l'importance de la rigidité de certains facteurs dans ce contexte et la sensibilité des estimations aux deux spécifications de modèle.

Le deuxième volet a trait au débat sur le ralentissement de la productivité du travail. En effet, les années 70 ont été marquées dans les pays industrialisés par un ralentissement quasi général et parfois très marqué de la productivité du travail dans les différents secteurs de
l'économie (voir Boyer et Petit (1979), Guinchard (1984)). Pour expliquer ce phénomène, de nombreuses études ont été faites selon différentes optiques et ont abouti à des conclusions différentes (voir Kendrick (1984)). Les explications peuvent être groupées en deux catégories. Pour les uns, le ralentissement est lié à la crise du pétrole de 1973, à la récession et aux excédents de capacité de production qui s'ensuivent. Pour ces auteurs, le ralentissement de la productivité est un malaise accidentel, de nature temporaire, qui cependant n'est pas sans avoir des retombées à long terme: si l'énergie et le capital sont des inputs complémentaires, le renchérissement de l'énergie ralentira l'accumulation du capital et par là, la diffusion du progrès technique (Hudson et Jorgenson (1978)); par ailleurs, l'augmentation du prix de l'énergie peut retarder le progrès technique (Jorgenson et Fraumeni (1981)), et rendre le stock de capital partiellement désuet (Daily (1981)). Pour les autres, le ralentissement de la productivité est expliqué par des forces de long terme. Ces auteurs parlent d'une artériosclérose du système économique (Lindbeck (1983)), ils mentionnent la fin de l'ère de l'exode rural, des économies d'échelle et des rattrapages de retard technologique vis-à-vis des États-Unis (Maddison (1979)) et ils soulignent le ralentissement des dépenses en recherche et développement (R&D) créatrices de nouvelles idées (Kendrick (1981)). Notre étude tente de contribuer à la discussion de ce problème en examinant si le déséquilibre du producteur à court terme, dû aux coûts d'ajustement, explique le ralentissement de la productivité du travail et si les efforts de recherche et développement contribuent sensiblement à la croissance. L'attention est principalement centrée sur le secteur manufacturier français.

L'article est organisé de la manière suivante. Nous présentons d'abord, dans la deuxième section, la formulation théorique des deux modèles et ensuite, dans la troisième section, la méthode d'estimation économétrique employée. Nous décrivons les sources et la construction des données de base dans la quatrième section. Dans la cinquième section, nous comparons les estimations des deux modèles et déduisons de celles-ci l'importance et la direction des effets prix et output sur la productivité du travail. Dans la sixième section, nous mesurons
l'importance du déséquilibre dans la croissance et le ralentissement de
la productivité du travail en France et nous comparons les sources de
croissance dans les secteurs manufacturiers français, américains,
allemands et japonais. Enfin, dans la dernière section, nous proposons
des voies de recherche future.

2. Modèles de coûts d'ajustement

Les deux modèles de coûts d'ajustement que nous avons adoptés
sont des extensions des modèles de Morrison et Berndt (1981) et de
Epstein et Yatchew (1983). Soit une firme qui, au temps t, produit un
bien (Q) à l'aide de quatre facteurs de production. Deux d'entre eux,
les consommations intermédiaire (M) et le travail (L), sont parfaitement
variables. Pour les deux autres, le stock de capital (K) et le stock de
R-D (R), tout changement de niveau entraîne des coûts d'ajustement. Pour
le capital, ces coûts représentent l'installation des machines, la
réorganisation des tâches de production et l'accoutumance aux nouvelles
conditions de travail. Le stock de R-D\(^1\)) est un facteur de production
dans la mesure où il conduit à concevoir de nouveaux produits et/ou de
nouveaux procédés de production, générant ainsi de nouvelles ventes et/ou
des réductions de coûts de production. Les coûts d'ajustement de la R-D
ont trait aux difficultés de mettre sur pied et de mener à terme un
programme de recherche, et de réussir à trouver et à commercialiser de
nouveaux produits ou procédés de production (Mansfield (1968)). Nous
supposons que la firme a des anticipations ou attentes stationnaires
concernant les niveaux de production, les prix relatifs et le niveau
technologique. En outre, elle escompte les coûts futurs à un taux
d'actualisation constant, r. Son objectif est de minimiser la valeur
actuelle de ses coûts variables et de ses investissements bruts futurs:
(1) \( \min \tau \sum_0^{\infty} \left\{ (1-u_t) \left[ w_t L_{t+\tau} + M_{t+\tau} + q_{kt} \left( \Delta K_{t+\tau} + \delta_k K_{t+\tau} - 1 \right) \right] \\ + q_{rt} \left( \Delta R_{t+\tau} + \delta_r R_{t+\tau} - 1 \right) \right\} (1+r)^{-\tau} \)

étant donné sa technologie décrite par la fonction de production

(2) \( Q_t = F (M_t, L_t, K_{t-1}, R_{t-1}, \Delta K_t, \Delta R_t, T) \).

Cette minimisation conduit à des sentiers optimaux pour les quatre facteurs de production. Le salaire \( w_t \), les prix d'achat du capital \( q_{kt} \) et de la R-D \( q_{rt} \), de même que la fonction objectif sont divisés arbitrairement par le prix des consommations intermédiaires\(^2\). Les symboles \( \delta_k \) et \( \delta_r \) représentent respectivement les taux de dépréciation des stocks de capital et de R-D, \( u_t \) est le taux de taxation sur les bénéfices des sociétés. La fonction de production est censée être concave par rapport aux inputs et convexe par rapport aux coûts d'ajustement \( \Delta K_t \) et \( \Delta R_t \), où \( \Delta K_t = (K_t - K_{t-1}) \) et \( \Delta R_t = (R_t - R_{t-1}) \). \( T \) représente le progrès technologique exogène à la firme à l'encontre du progrès endogène généré par la R-D. Il faut noter également que les facteurs quasi-fixes affectent la production avec un délai d'une période.

Selon la théorie de la dualité, si une firme minimise ses coûts variables, la technologie peut être décrite par une fonction de coût variable normalisée

(3) \( G_t = G (w_t, K_{t-1}, R_{t-1}, \Delta K_t, \Delta R_t, Q_t, T) \)

concave par rapport aux prix des facteurs variables, et convexe par rapport aux facteurs fixes et aux coûts d'ajustement (voir Lau (1976)).
Comme représentation fonctionnelle de la fonction de coût variable normalisée, nous avons choisi la forme suivante:

\[ G = Q_t \left[ \alpha_o + \alpha_w w_t + \frac{1}{2} \alpha_{ww} w_t^2 + \alpha_{TT} T + \frac{1}{2} \alpha_{TTT} T^2 + \alpha_{wT} w_t T \right] \]

\[ + \alpha_{K_t} K_{t-1} + \alpha_{R_t} R_{t-1} + \frac{1}{2} \alpha_{KK} K_{t-1}^2 + \frac{1}{2} \alpha_{RR} R_{t-1}^2 \]

\[ + \alpha_{Kt} \frac{K_{t-1}}{u_t} \frac{R_{t-1}}{u_t} \frac{(\Delta K_t)^2}{u_t} + \frac{1}{2} \alpha_{RR} \frac{(\Delta R_t)^2}{u_t} \]

\[ + \alpha_{wK} w_t K_{t-1} + \alpha_{wR} w_t R_{t-1} \]

\[ + \alpha_{KT} K_{t-1}^T + \alpha_{RT} R_{t-1}^T. \]

Cette fonction est empruntée à Morrison et Berndt (1981) et peut être considérée comme une approximation quadratique d'une fonction générale de coût variable normalisée avec imposition de rendements d'échelle constants, de séparabilité des coûts d'ajustement et de nullité des coûts d'ajustement marginaux à l'état stationnaire.

De la fonction (4), nous pouvons déduire immédiatement les fonctions de demande de facteurs variables. En effet, selon le lemme de Shephard, \( \frac{\partial G_t}{\partial w_t} = L_t \). Dès lors, la demande travail est:

\[ L_t = \alpha_w + \alpha_{ww} w_t + \alpha_{wK} K_{t-1} + \alpha_{wR} R_{t-1} + \alpha_{wT} T. \]

\[ \frac{L_t}{u_t} = \frac{w_t L_t}{u_t} + M_t, \] la demande de consommations intermédiaires s'obtient de façon résiduelle: 3)
\[ M_t = \alpha_o - \frac{1}{2} \alpha_{WW} w_t^2 + \alpha_T T - \frac{1}{2} \alpha_{TT} T^2 + \alpha_{WR} w_t T \]

\[ + \frac{\alpha_K}{Q_t} K_t - 1 + \frac{\alpha_R}{Q_t} R_t - 1 + \frac{1}{2} \alpha_{KK} \left( \frac{K_t - 1}{Q_t} \right)^2 + \frac{1}{2} \alpha_{RR} \left( \frac{R_t - 1}{Q_t} \right)^2 \]

\[ + \frac{\alpha_{KR}}{Q_t} K_t - 1 R_t - 1 + \frac{1}{2} \alpha_{KK} \left( \frac{\Delta K_t}{Q_t} \right)^2 + \frac{1}{2} \alpha_{RR} \left( \frac{\Delta R_t}{Q_t} \right)^2 + \alpha_{KR} K_t - 1 T + \alpha_{RT} R_t - 1 T. \]

La forme de la fonction de coût variable normalisée conduit à des demandes dérivées de facteurs de production sous forme de rapports input-output.

Les trajectoires optimales de capital et de R-D sont données par les conditions de premier ordre du problème de minimisation intertemporelle, qui mènent aux équations de différence seconde suivantes \( t = 0, 1, \ldots, \infty \):

\[ -B x_{t+1} - C + [A + (2+r) B] x_{t+1} - (1+r) B x_{t+1} - 1 = a_t \]

où \( B = \left[ \begin{array}{c} \alpha_{KK} \\ 0 \end{array} \right], A = \left[ \begin{array}{cc} \alpha_{KK} & \alpha_{KR} \\ \alpha_{KR} & \alpha_{RR} \end{array} \right], x_{t+1} = \left[ \begin{array}{c} K_{t+1} \\ R_{t+1} \end{array} \right], \]

\[ a_t = \left[ \begin{array}{c} \alpha_K + \alpha_{WK} w_t + \alpha_{KT} T + c^K_t \\ \alpha_R + \alpha_{WR} w_t + \alpha_{RT} T + c^R_t \end{array} \right] Q_t \]

\[ c^K_t = q_t (r + \delta_K) / (1-u_t), \]

et \( c^R_t = q_t (r + \delta_R). \)
D'une part, ces conditions décrivent les quantités de capital et de R-D optimales à long terme, quant \( x_{t+\tau-1} = x_{t+\tau} = x_{t+\tau+1} \):

\[
(8) \quad x_t^* = A^{-1}a_t.
\]

D'autre part, pour \( \tau = 0 \), elles déterminent les investissements optimaux en capital et R-D à la période \( t \). Comme l'ont prouvé Prucha et Nadiri (1981), les investissements nets désirés au temps \( t \) prennent la forme de l'ajustement flexible généralisé:

\[
(9) \quad \Delta x_t = M(x_t^* - x_{t-1})
\]

où

\[
M = \begin{bmatrix}
m_{kk} & m_{kr} \\
m_{rk} & m_{rr}
\end{bmatrix}.
\]

3. Estimation économétrique

Dans le modèle I, on suppose que les trajectoires et les demandes de long terme de capital et de R-D sont indépendantes. Cette hypothèse implique \( \alpha_{kr} = 0 \), et elle simplifie considérablement les équations à estimer. Pour le capital, nous avons:

\[
(10) \quad \frac{K_t - K_{t-1}}{Q_t} = m_{kk} \frac{(K_t^* - K_{t-1})}{Q_t}
\]

où

\[
m_{kk} = -\frac{1}{2} \left\{ \frac{\alpha_{kk}}{\alpha_{kk}} - \left[ \left( \frac{\alpha_{kk}}{\alpha_{kk}} \right)^2 + \frac{4\alpha_{kk}}{\alpha_{kk}} \right]^{\frac{1}{2}} \right\}
\]
\[ K_L^* = \left( -\frac{1}{\alpha_{KK}} \right) \left[ \alpha_K + \alpha_{WK} \omega_t + \alpha_{KT} T + c_t \right] Q_t, \]

et, pour la R-D, nous avons une expression analogue. La matrice M est diagonale.\(^4\)

Le modèle II admet la possibilité d'elasticités croisées entre les deux facteurs quasi-fixes (sauf dans le cas de la France où l'estimation n'a pas été possible). En règle générale, il n'est plus possible, si \( \alpha_{KR} \) est non nul, d'exprimer les coefficients d'ajustement en fonction des paramètres structurels du modèle. En effet, si nous substituons (8) et (9) dans (7), nous obtenons un système d'équations quadratiques en M:

\[(11) \quad BM^2 + (A + rB)M - A = 0.\]

Il n'est, en général, pas possible de résoudre M en fonction de A et de B: mais il est facile de résoudre A en fonction de M et de B. Epstein et Yatchew (1983) ont eu ainsi l'idée de reparamétrer le modèle en replaçant A par sa solution dans (11):

\[(12) \quad A = BM(M + rI)(I - M)^{-1}.\]

Au lieu d'estimer les coefficients des matrices A, B et \( a_t \), on peut, suite à cette transformation, estimer le modèle de manière équivalente à partir des coefficients des matrices M, B et \( a_t \). Pour réduire encore davantage le nombre de coefficients à estimer, il est possible de tirer parti de la symétrie de la matrice C = -BM et d'exprimer les équations en fonction des matrices C, B et \( a_t \). (voir Mohlen, Nadiri, Prucha (1985)).
La technologie et le comportement du producteur sont estimés à partir de quatre équations décisionnelles: la demande de travail (5), la demande de consommations intermédiaires (6) et les investissements nets en capital et R-D (10). Un terme d'erreur est ajouté à chaque équation du modèle. À chaque période, le vecteur des termes d'erreur est supposé avoir une distribution normale. Dans les deux modèles, le système d'équations simultanées est estimé par la procédure itérative des "équations apparemment indépendantes", qui se trouve être équivalente à une estimation par maximum de vraisemblance, en raison de la récursivité du système (voir Morrison et Berndt (1981)). L'estimation est itérative tant sur la matrice de variance-covariance contemporaine que sur les paramètres, qui apparaissent dans les équations de manière non-linéaire. La méthode d'itération employée est celle de Gauss-Newton du logiciel TSP. Le niveau de convergence est fixé à 0,001. En présence d'autocorrélaison du terme d'erreur dans certaines équations, les variables de ces équations sont préalablement transformées sous l'hypothèse que le processus d'autocorrélaison est du premier ordre.

Les multiples restrictions d'égaleitée des paramètres à travers les équations du modèle sont imposées à priori. En outre, les paramètres estimés doivent satisfaire certaines restrictions d'inégalité. La fonction du coût variable $G(.)$ doit remplir les conditions de concavité et de convexité mentionnées plus haut. Par conséquent, nous devons avoir $\alpha_{vv} < 0$ et $A$ semi-définie positive. Finalement, il a été prouvé que $C$ doit être semi-définie négative (voir Prucha et Nadiri (1981)). Ces restrictions d'inégalité sont imposées au début de l'estimation dans la recherche de valeurs initiales pour les paramètres et relâchées par la suite. Il s'est avéré que, à cause de la nonlinéarité et des multiples restrictions d'égaleitée paramétriques, il est important de bien choisir le point de départ pour arriver à une solution qui soit convergente et qui corresponde à des estimations théoriquement acceptables.
4. Description des données


Consommations Intermédiaires: Les consommations intermédiaires (M), y compris l'énergie, proviennent des mêmes sources que la production brute. Pour les données tirées de Norsworthy et Malmquist (1983), ont été ainsi regroupées les séries concernant l'énergie et les consommations non-énergétiques. Les consommations intermédiaires prennent en compte les transactions entre les différents secteurs manufacturiers.7)

R-D: Le stock de R-D (R) est construit par la méthode chronologique à partir des dépenses réelles en recherche et développement faite et financée par le secteur manufacturier. Le taux de dépréciation est fixé à 10%. La valeur de départ est obtenue en divisant les dépenses en R-D de la première période par la somme du taux de dépréciation de la R-D et du taux de croissance de la valeur ajoutée réelle. Les dépenses nominales en R-D sont publiées par l'OCDE (1979) et (1982b). L'indice des prix implicite du PNB sert de déflateur pour les dépenses de R-D.


Coût d'usage du capital: Le coût d'usage du capital ($\tilde{c}^K$) est construit selon la formule $\tilde{c}^K = \tilde{q}^K (r + \delta^K)/(1-u)$ où $\tilde{q}^K$ est l'indice implicite des prix à l'investissement, $r$ le taux d'intérêt, $\delta^K$ le taux de dépréciation du stock de capital, $u$ le taux de taxation sur les bénéfices des sociétés. Les séries nominales et réelles de l'investissement utilisées pour calculer le prix implicite de l'investissement proviennent des mêmes sources que les stocks de capital. Pour le Japon, c'est le déflateur des investissements en outils et bâtiments publié par la Bank of Japan (1981) qui est utilisé pour $\tilde{q}^K$. Dans le modèle II, le taux d'intérêt réel est fixé à 4% tandis que, dans le modèle I, la série du taux d'intérêt nominal est celle du "government bond yield" publiée par le Fonds Monétaire International (1979). Le taux de dépréciation est dérivé implicitement de la formule de la méthode chronologique de construction du capital.

Le taux d'impôt sur les bénéfices des sociétés est tiré de Pechman (1983) pour les États-Unis, Pechman et Kaizuka (1976) pour le Japon, et des lois du 16 octobre 1934 et du 31 août 1976 pour l'Allemagne. Nous supposons que $u = 48\%$ pour le Japon depuis 1972, que $u$ est égal à une moyenne pondérée des taux d'impôt sur les bénéfices retenus et sur les dividendes à raison de 2/3 et 1/3 respectivement pour l'Allemagne fédérale, et finalement que pour la France $u = 50\%$ sur toute la période d'observation. La taxation sur les bénéfices des sociétés n'est pas prise en considération dans le modèle I.

Coût d'usage de la R-D: Le coût d'usage de la R-D ($\tilde{c}_R$) est construit comme $\tilde{c}_R = \tilde{q}_R (r + \delta_R)/2$ où $\delta_R = .10$ et $\tilde{q}_R$ est l'indice des prix du PNB.


**Prix des consommations intermédiaires:** Le prix des consommations intermédiaires ($P^M$) est le prix implicite déduit des données nominales et réelles. Pour l'Allemagne avant 1970, nous utilisons le "Index der Grundstoffpreise" publié dans Statistisches Bundesamt (1983).

**Conversion des données:** Afin de rendre les données internationales comparables tout en éliminant les variations dues au taux de change, les parités de pouvoir d'achat pour le revenu domestique brut en 1970, contenues dans Summers, Kravis et Heston (1980), sont utilisées pour convertir toutes les données nationales en dollars américains.

5. **Déterminants de la productivité du travail**

La productivité est un concept dont la signification dépend essentiellement de la mesure adoptée. La productivité du travail est une productivité partielle qui calcule l'efficacité dans l'utilisation des ressources par rapport à un seul facteur de production, le travail. Par contre, la productivité totale évalue l'efficience de la production par rapport à une moyenne pondérée des facteurs de production. Nous nous servons du concept de productivité du travail pour plusieurs raisons: cette mesure, la plus répandue dans la littérature, nous donne l'occasion de comparer nos résultats à ceux des autres approches; elle nous permet également d'analyser le rôle joué par les facteurs de production autres que le travail; et elle est intuitivement plus facile à comprendre et à relier à la compétitivité des entreprises manufacturières.
L'examen des déterminants de la productivité du travail requiert une certaine vue de l'exogénéité des facteurs causaux. Nous adoptons dans la présente étude une perspective d'équilibre partiel où le volume de production et les prix des facteurs de production sont exogènes tandis que les demandes de facteurs de production et les accumulations de capital physique et de connaissances dérivées de la recherche sont endogènes. Cette vue est certes critiquable à long terme mais peut très bien se défendre à court terme.

La particularité de notre étude est d'estimer la structure de la production à travers un modèle dynamique d'ajustement aux situations de déséquilibre à la place des modèles traditionnels d'équilibre du producteur. De nos résultats d'estimation nous pouvons dégager l'effet des variables exogènes sur la productivité du travail. Tel est l'objet de cette section. La prochaine section analysera le rôle des autres facteurs de production dans la croissance de la productivité du travail en faisant abstraction de l'effet causal.

Comme nous l'avons indiqué, il nous a paru utile d'estimer deux modèles assez différents de coûts d'ajustement et de comparer leurs résultats. Indépendamment de l'hypothèse de séparabilité des processus d'ajustement du capital et de la R-D, le modèle I diffère du modèle II principalement dans la définition de l'output : le modèle I explique la valeur ajoutée, y compris l'énergie, à l'aide du travail, de l'énergie, du stock du capital et du stock de R-D tandis que le modèle II explique la production brute à l'aide du travail, des consommations intermédiaires, et des deux facteurs quasi-fixes. Le modèle I fait donc l'hypothèse cruciale de la séparabilité entre les facteurs intermédiaires et les facteurs primaires dont l'énergie. Cette hypothèse explique en majeure partie les différences de résultats entre les deux modèles.

Le modèle I se distingue aussi du modèle II sur quelques autres points. Il contient, outre le progrès technique découlant des efforts de recherche et développement, une composante de progrès technique exogène représentée par un trend temporel tandis qu'on a été amené à supprimer ce
trend dans le modèle II. Le modèle I ne prend pas en compte la taxation des bénéfices des sociétés dans le calcul d'optimisation du producteur et utilise un taux d'intérêt nominal qui fait varier les coefficients d'ajustement. Dans le modèle I, les prix des facteurs de production sont normalisés par le salaire ce qui entraîne que la demande de travail devient l'équation résiduelle (6). Il faut donc noter que, dans le modèle I, M représente l'énergie et w le prix de l'énergie relatif au salaire horaire. Finalement, dans le modèle I, la France a fait en réalité l'objet d'une estimation sur la base d'un modèle commun pour les secteurs manufacturiers français et italien, l'utilisation de variables dichotomiques permettant à chaque pays d'avoir ses propres coefficients (voir Cardani-Mohnen (1984))\textsuperscript{8}).

Les estimations des différents paramètres des deux modèles sont présentées dans le tableau I\textsuperscript{9}). La plupart des coefficients sont significatifs et l'ajustement des équations aux variables dépendantes est bonne. L'équation de la demande de travail est la moins satisfaisante, les carrés des coefficients de corrélation entre la valeur observée et la valeur prédite par la forme réduite étant de .66 pour la France, .80 pour les États-Unis, .52 pour le Japon et .89 pour la RFA, tandis qu'ils dépassent 90% pour les demandes des autres facteurs de production. Le traitement du travail comme facteur variable dans la modélisation est peut être une hypothèse trop forte et laisse ainsi à désirer. Les estimations de l'élasticité de la demande du travail à la production viendront confirmer cette hypothèse.

Le tableau II donne les coefficients d'ajustement, qui expriment la vitesse à laquelle les facteurs quasi-fixes tendent vers leur optimum de long terme. Les deux modèles confirment notre attente qu'il faut plus de temps pour ajuster le stock de R-D que le stock de capital physique. Ceci peut s'expliquer par les délais additionnels associés à la création d'innovations, à leur application et à leur commercialisa-
tion. À l'exception des États-Unis, les coefficients d'ajustement croisés ne sont pas significatifs. Si on tient compte des ajustements croisés, les deux modèles conduisent à des vitesses d'ajustement à peu près égales aux États-Unis. Pour le Japon et la RFA, au contraire, le modèle de production brute produit des ajustements beaucoup plus lents que le modèle de valeur ajoutée. Une explication possible est la suivante: si le prix relatif des consommations intermédiaires augmente, ce qui fut le cas durant la période étudiée, comme celles-ci et les facteurs quasi-fixes sont des substituts dans le modèle II (voir Mohnen, Nadiri, Prucha (1985)), les stocks de capital et de R-D désirés à long terme y sont plus élevés que dans le modèle I, où consommations intermédiaires et facteurs quasi-fixes sont séparables; par conséquent, à investissements égaux, il faut que les vitesses d'ajustement soient plus faibles pour des écarts à combler plus élevés. Aux États-Unis, la différence entre les coefficients d'ajustement du capital est minime à cause du faible degré de substitution entre consommations intermédiaires et capital et le coefficient d'ajustement de la R-D y est même plus grand dans le modèle de production brute à cause de la complémentarité entre consommations intermédiaires et stock de R-D (voir Mohnen, Nadiri, Prucha (1985)). La rapidité avec laquelle le capital semble s'ajuster dans le modèle II pour la France est un peu surprenante. En fait, les difficultés d'estimation ont conduit à renoncer à spécifier des ajustements croisés pour la France.

Le tableau III présente les elasticités-prix de la demande de travail à long terme dans les deux modèles. Il convient, à la lecture du tableau III, de garder à l'esprit que $P_M$ représente le prix de l'énergie dans le modèle I et celui des consommations intermédiaires dans le modèle II. Les elasticités sont calculées en dérivant l'équation de la demande de travail par rapport aux prix des facteurs en tenant compte de l'endogénéité des stocks de capital et de R-D à long terme. Nous analysons les elasticités de long terme afin de pouvoir examiner l'ensemble des effets prix sur la productivité du travail, car, à court terme, les possibilités de substitution se limitent aux deux facteurs variables. Nous savons cependant que, à court terme, l'élasticité de la demande de
travail par rapport au salaire est moins importante qu'à long terme, selon le principe de Le Châtelier, et égale à l'inverse de l'élasticité de la demande de travail par rapport au prix des consommations intermédiaires, selon la propriété d'homogénéité de degré zéro de la demande des facteurs de production par rapport aux prix.

Pour un niveau de production donné, les élasticités de la productivité du travail par rapport aux prix des facteurs sont simplement les inverses des élasticités correspondantes de la demande de travail. Dès lors, les résultats du tableau III suggèrent les conclusions suivantes: les élasticités-prix de la productivité du travail sont inférieures à l'unité, donc les variations de la productivité du travail engendrées par les variations de prix des inputs sont proportionnellement moins grandes que ces dernières; la productivité du travail est peu sensible aux coûts d'usage des facteurs quasi-fixes; à variations de prix égales, l'effet du prix de l'énergie sur la productivité du travail est de loin inférieur à celui du prix des consommations intermédiaires; une hausse du salaire améliore la productivité du travail puisqu'elle encourage les firmes à utiliser d'autres facteurs au lieu du travail, tandis que une hausse du prix de l'énergie ou des consommations intermédiaires diminue la productivité du travail; les coûts d'usage du capital et du stock de R-D quant à eux produisent un effet négatif ou positif selon qu'il y a substitution ou complémentarité entre le travail et ces facteurs; puisque le modèle II permet une gamme de substituabilité plus large que le modèle I, il n'est pas étonnant qu'en règle générale l'évolution du salaire est plus favorable à la productivité du travail dans le modèle II; dans ce dernier modèle, la France et l'Allemagne fédérale semblent avoir moins de flexibilité que les États-Unis et le Japon dans la substituabilité entre le travail et les autres facteurs.

Les élasticités à court terme de la demande de travail par rapport à la production sont données dans le tableau IV. À long terme, comme les rendements d'échelle sont constants, ces élasticités sont unitaires. À court terme, cependant, les élasticités des facteurs quasi-fixes sont inférieures à l'unité en raison des coûts d'ajustement, tandis

6. Révision de la comptabilité de la croissance en situation de déséquilibre.

Cette section a pour but de réexaminer la contribution des autres facteurs de production à la croissance de la productivité du travail, en tenant compte du déséquilibre des facteurs de production à court terme, et d'examiner l'effet du déséquilibre sur le ralentissement de la productivité du travail.

Sous l'hypothèse de rendements d'échelle constants, il est facile d'établir un lien entre la croissance de la production et celle de la productivité du travail. En différenciant la fonction de production (2), nous obtenons en termes de croissance:
(13) \[
\dot{Q} = \partial \ln F \dot{L} + \partial \ln F \dot{M} + \partial \ln F \dot{K} + \partial \ln F \dot{R} + \frac{\partial \ln F \Delta K + \partial \ln F \Delta R + TFP}{\partial \ln \Delta K + \partial \ln \Delta R}
\]

où les grandeurs pointées \(\dot{x}\) représentent les taux de croissance de la production et des inputs, \(\partial \ln F/\partial \ln z\) les élasticités de la production par rapport aux inputs et TFP le taux de croissance de la productivité totale calculée de manière résiduelle. Comme les rendements d'échelle sont constants, la somme des élasticités est unitaire. Dès lors, si nous soustrayons \(\dot{L}\) des deux côtés de l'équation (13), nous obtenons la croissance de la productivité du travail en fonction des croissances des autres facteurs de production relatives à celle du travail et en fonction de la croissance de la productivité totale:

(14) \[
\dot{Q} - \dot{L} = \frac{\partial \ln F (\dot{M} - \dot{L}) + \partial \ln F (\dot{K} - \dot{L}) + \partial \ln F (\dot{R} - \dot{L})}{\partial \ln M + \partial \ln K + \partial \ln R}
\]

\[+ \frac{\partial \ln F (\Delta K - \dot{L}) + \partial \ln F (\Delta R - \dot{L}) + TFP}{\partial \ln \Delta K + \partial \ln \Delta R} \]

Notons que les rendements d'échelle se réfèrent aux six facteurs de production, les quatre facteurs traditionnels et les deux facteurs négatifs de coûts d'ajustement, que l'on pourrait également considérer comme outputs. Par rapport aux quatre facteurs traditionnels, les rendements d'échelle sont donc croissants.
Le problème crucial qui se pose est de mesurer les élasticités de la production, qui ne sont pas observables. Deux directions ont été suivies par les économistes. Les premiers se basent sur l'hypothèse d'équilibre du producteur leur permettant d'estimer les élasticités de la production par les parts respectives des inputs dans le coût total (par exemple, Christensen, Cummings et Jorgenson (1980)); les seconds recourent à l'estimation des élasticités du produit à partir d'une fonction de production sans faire d'hypothèse sur le comportement du producteur (par exemple, Mairesse et Cunéo (1984) et Cunéo (1984)). Nous adoptons ici une troisième approche, consistant elle aussi à estimer les élasticités de la production mais à partir d'un modèle bien précis de comportement du producteur, qui reconnaît le caractère non-optimal des inputs observés face aux prix et à la demande en vigueur.


Le tableau V présente ainsi l'analyse comptable de la croissance de la productivité du travail dans le secteur manufacturier français pour les périodes 1965-69, 1969-73, et 1973-77. La décomposition repose sur l'identité suivante:
où $x_1 = M, K, R$ et $s_i$ est la part respective de $M, K, R$ dans le coût total. Pour faire ressortir le rôle du déséquilibre nous isolons ainsi sous la rubrique "fausse pondération" la contribution à la croissance des inputs qui n'est pas prise en ligne de compte sous l'hypothèse d'équilibre du producteur.

Il apparaît au tableau V que l’erreur de pondération s’élève à 15% de la croissance de la productivité du travail dans le secteur manufacturier français. Le même ordre de grandeur a été obtenu pour les autres pays. La prise en compte du déséquilibre dans les facteurs de production par rapport à la demande et aux prix des facteurs en vigueur réduit donc d'un tiers à la moitié l'ordre de grandeur du progrès technique résiduel dans les analyses comptables de la croissance.

L'effet du déséquilibre n'a pas fortement varié au cours des trois périodes et ne peut donc être blâmé pour le ralentissement de la productivité du travail après 1973. Les coûts d'ajustement eux-mêmes ont plutôt joué un rôle de stabilisateur car les investissements tant en capital physique que en R-D, générateurs de coûts d'ajustement, se sont ralentis après 1973. Cependant la croissance des stocks de capital physique et de R-D ne s'est pas infiniment suffisamment par rapport à celle du nombre d'heures travaillées après 1973 pour expliquer le ralentissement de la productivité du travail. L'explication du ralentissement réside donc en fin de compte dans une moindre substitution des consommations intermédiaires au travail et surtout dans le ralentissement du progrès.
technique résiduel. À ce propos, il est intéressant de remarquer que le progrès technique résiduel a commencé à baisser avant la crise de 1973 selon les résultats du tableau V.

Étant donné les bouleversements créés par le choc pétrolier, il est un peu surprenant de constater que les déséquilibres pris en compte dans notre modèle n'expliquent pas le ralentissement de la productivité du travail. Cinq raisons peuvent être évoquées. Premièrement, les inputs les plus en déséquilibre n'ont qu'un faible poids dans la comptabilité de la croissance; deuxièmement, les déséquilibres peuvent être très accentués certaines années mais s'estomper sur une moyenne de plusieurs années; troisièmement, les déséquilibres des divers inputs se neutralisent mutuellement étant donné qu'un déséquilibre correspond à un déplacement le long d'un isoquant de la fonction de production; quatrièmement, les firmes étant sans cesse en-dessous de leurs stocks optimaux de capital et de R-D, une baisse de ceux-ci conduit en fait à une réduction du déséquilibre; et cinquièmement, notre modèle ne prend pas en compte les phénomènes de sous-utilisation et d'obsolescence du capital, souvent mentionnés pour expliquer le ralentissement de la productivité.

Le tableau VI présente la décomposition de la croissance de la productivité du travail dans les secteurs manufacturiers des quatre pays. Les contributions de chaque input y sont mesurées par les élasticités par rapport à la production estimées du modèle II. Avec cette pondération, la contribution de la R-D en France devient quatre fois plus importante et celle du capital physique 30% plus élevée que les contributions obtenues sous l'hypothèse d'équilibre du producteur (voir tableau V).

Le capital et la R-D ont joué en France dans la période 1965-1977, un rôle aussi important qu'au Japon. L'influence de l'accumulation est un peu plus faible aux États-Unis, mais l'est beaucoup plus en Alle-
magne fédérale. Pour les quatre pays, la substitution des consommations intermédiaires au travail explique la plus grande part de la croissance de la productivité du travail, surtout en Allemagne fédérale. Les coûts d'ajustement n'ont quant à eux qu'une influence directe très faible ralentissant de 2 à 4% la croissance de la productivité du travail. Au total, entre 15% et 25% de la croissance de productivité du travail reste affecté au progrès technique résiduel.

Il importe cependant de faire quelques réserves sur l'interprétation de nos résultats du point de vue de la croissance. D'abord nos estimations ne sont pas d'une grande précision, et servent plutôt à donner un ordre de grandeur relative. En outre, les évolutions du secteur manufacturier dans son ensemble peuvent dissimuler des évolutions sectorielles assez divergentes (voir Guinchard (1984)). Enfin, il ne faut pas perdre de vue que les facteurs de la croissance ne représentent que des déterminants immédiats et excluent les effets induits ou secondaires, tels que le ralentissement de la diffusion du progrès technique liée à l'accumulation du capital ou l'extension de la demande provoquée par la découverte de nouveaux produits.

Si nous comparons nos résultats à ceux obtenus par d'autres chercheurs, il convient d'ajouter deux remarques. Il faut d'abord souligner que la notion de progrès technique résiduel demande à être interprétée avec prudence et que son ampleur varie notamment avec la définition même de l'output. Ainsi, nos résultats sur la décomposition de la croissance de la productivité du travail sont similaires à ceux obtenus par Norsworthy et Malmquist (1983) pour les États-Unis et le Japon; en revanche, notre estimation du progrès technique résiduel est d'environ 50% inférieure à celle obtenue en moyenne par Kendrick (1981) après soustraction de l'effet dû à la recherche et développement. Il faut par ailleurs remarquer que la contribution du stock de R-D est limitée au rendement privé et aux dépenses financées intéièrement. L'effet de la
R-D serait plus important qu'il ne l'est déjà, compte tenu de son poids relativement minime dans le coût total de production, si les externalités qui lui sont liées étaient prises en compte. Ainsi Kendrick (1981) obtient sur ce point des résultats supérieurs aux nôtres parce qu'il retient un taux de rendement social de R-D de 50% (voir également Mairessse et Cunéo (1984) et Cunéo (1984)).13)

7. Conclusion

Les conclusions que cette étude dégage ouvrent de nouvelles perspectives de recherche. L'inertie du travail ressort nettement de nos estimations du modèle de production brute et suggère que non seulement les stocks de capital et de R-D mais aussi le travail soient traités comme facteurs quasi-fixes dans des modèles de production. La prise en compte du déséquilibre dans la détention des facteurs de production à court terme affecte les estimations de la technologie et des sources de la croissance mais ne semble pas jouer un rôle important dans le ralentissement de la productivité. Peut-être faudrait-il envisager d'autres sources de déséquilibre que les coûts d'ajustement pour expliquer le ralentissement de la productivité, en particulier les variations dans l'utilisation du capital. Nos résultats illustrent également le rôle non négligeable des connaissances accumulées de recherche et développement dans la croissance des économies industrielles. Cependant, pour encore mieux cerner l'importance sociale de la recherche, il conviendrait de mesurer ses externalités d'un secteur à l'autre.
NOTES

1) Plus exactement le stock non déprécié des connaissances accumulées à l'aide de dépenses en recherche et développement.

2) Le choix du numéraire est arbitraire.

3) La fonction quadratique présente l'inconvénient de traiter de manière asymétrique la demande des deux facteurs variables. Le choix arbitraire du numéraire fixe ainsi lequel de ces deux facteurs est dérivé de manière résiduelle. Dans le modèle I, le salaire est en fait choisi comme numéraire, et la demande de travail est l'équation résiduelle.

4) La formule des coefficients d'ajustement est légèrement différente de celle de Morrison et Berndt (1981) parce que notre modèle est formulé en temps discret.

5) Etant donné la faible taille de l'échantillon, aucune correction supplémentaire n'est apportée aux cas où le processus d'autocorrélation des erreurs semble être d'un ordre supérieur à un.

6) Par exemple, si α_{ww} > 0, la fonction G(.) n'est pas concave et par conséquent la fonction du coût variable normalisée n'est plus une représentation duale de la technologie (voir Lau (1976)).

7) Dans le modèle I, l'énergie, le travail et le capital ont pu être corrigés de leur composante en R-D, de façon à éviter une double compatibilité. Dans le modèle II, on a été amené à déduire le total des dépenses de R-D des consommateurs intermédiaires.

8) Dans l'étude de Cardani-Mohnen (1984), une variable dichotomique particulièrement à un des deux pays est introduite en interaction avec les régressseurs de chaque équation afin que les paramètres estimés diffèrent entre les secteurs manufacturiers français et italien tout en respectant les égalités paramétriques du modèle. Seuls deux coefficients sont impo-
sés à être égaux dans les deux pays \((\alpha_{K} et \alpha_{WW})\) pour des raisons de régularité de la fonction de coût variable.


10) Les vitesses d'ajustement en présence d'ajustements croisés sont calculées selon la formule \(v_{KT} = m_{KK} (R^*_t - R_{t-1}) / (K^*_t - K_{t-1})\). Pour les États-Unis, nous obtenons, avec le modèle II, comme vitesses d'ajustement, .26 et .17 pour le capital et la R-D respectivement.

11) La présence d'ajustements croisés \((c_{KR} non nul)\) entraîne des difficultés de convergence et des valeurs incroyablement petites pour les coefficients d'ajustement en France. Les estimations des coefficients d'ajustement français sont donc sujettes à caution. Il convient cependant de rappeler que les coefficients d'ajustement croisés ne sont significatifs que pour les États-Unis.

12) C'est la complémentarité entre le travail et les facteurs quasi-fixes pour des niveaux donnés des autres inputs, et non la complémentarité mesurée par les elasticités-prix croisées, qui importe dans ce contexte (voir Morrison et Berndt (1981)).

Références bibliographiques


### Tableau I: Estimations du maximum de vraisemblance des modèles du coûts d'ajustement dans les secteurs manufacturiers des E.U., du Japon, de la RFA et de la France, 1965-1977*

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>MODELE I</th>
<th></th>
<th>MODELE II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E.U.</td>
<td>JAPON</td>
<td>RFA</td>
<td>FRANCE</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>1.79 (9.76)</td>
<td>5.01 (61.71)</td>
<td>3.31 (8.64)</td>
<td>2.73 (20.53)</td>
</tr>
<tr>
<td>$\alpha_K$</td>
<td>-0.93 (-2.84)</td>
<td>-2.41 (-18.11)</td>
<td>-2.45 (-3.60)</td>
<td>-0.95 (-4.73)</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>-0.32 (-4.32)</td>
<td>-0.14 (-34.67)</td>
<td>-0.19 (-7.85)</td>
<td>-0.14 (-1.79)</td>
</tr>
<tr>
<td>$\alpha_{K\bar{K}}$</td>
<td>0.75 (2.18)</td>
<td>0.90 (6.19)</td>
<td>1.79 (2.91)</td>
<td>0.42 (2.41)</td>
</tr>
<tr>
<td>$\alpha_{\bar{R}R}$</td>
<td>0.22 (3.02)</td>
<td>0.05 (10.82)</td>
<td>0.16 (5.80)</td>
<td>0.03 (0.67)</td>
</tr>
<tr>
<td>$C_{K\bar{K}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$C_{\bar{R}R}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$C_{K\bar{R}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\alpha}_{K\bar{K}}$</td>
<td>4.22 (3.47)</td>
<td>1.67 (4.05)</td>
<td>2.54 (4.08)</td>
<td>4.81 (2.69)</td>
</tr>
<tr>
<td>$\bar{\alpha}_{\bar{R}R}$</td>
<td>5.42 (8.07)</td>
<td>0.42 (14.69)</td>
<td>1.42 (3.51)</td>
<td>3.97 (0.58)</td>
</tr>
<tr>
<td>Model II</td>
<td>Model I</td>
<td>E.U.</td>
<td>Japan</td>
<td>FRA</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>------</td>
<td>-------</td>
<td>-----</td>
</tr>
<tr>
<td>(0.96)</td>
<td>(1.20)</td>
<td>0.75</td>
<td>0.98</td>
<td>(0.52)</td>
</tr>
<tr>
<td>(0.99)</td>
<td>(1.17)</td>
<td>0.95</td>
<td>0.80</td>
<td>(0.73)</td>
</tr>
<tr>
<td>(0.64)</td>
<td>(0.38)</td>
<td>0.41</td>
<td>0.97</td>
<td>(0.15)</td>
</tr>
<tr>
<td>(0.83)</td>
<td>(0.86)</td>
<td>0.60</td>
<td>0.06</td>
<td>(0.20)</td>
</tr>
<tr>
<td>(0.86)</td>
<td>(0.63)</td>
<td>0.17</td>
<td>0.05</td>
<td>(0.14)</td>
</tr>
<tr>
<td>(0.68)</td>
<td>(0.80)</td>
<td>0.14</td>
<td>0.85</td>
<td>(0.13)</td>
</tr>
<tr>
<td>(0.66)</td>
<td>(0.70)</td>
<td>0.16</td>
<td>0.77</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.69)</td>
<td>(0.72)</td>
<td>0.40</td>
<td>0.78</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.72)</td>
<td>(0.76)</td>
<td>0.16</td>
<td>0.75</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.75)</td>
<td>(0.79)</td>
<td>0.15</td>
<td>0.73</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.78)</td>
<td>(0.82)</td>
<td>0.14</td>
<td>0.71</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.81)</td>
<td>(0.86)</td>
<td>0.13</td>
<td>0.69</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.84)</td>
<td>(0.90)</td>
<td>0.12</td>
<td>0.67</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.87)</td>
<td>(0.94)</td>
<td>0.11</td>
<td>0.65</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.90)</td>
<td>(0.98)</td>
<td>0.10</td>
<td>0.63</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.93)</td>
<td>(1.02)</td>
<td>0.09</td>
<td>0.61</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.96)</td>
<td>(1.06)</td>
<td>0.08</td>
<td>0.59</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(0.99)</td>
<td>(1.10)</td>
<td>0.07</td>
<td>0.57</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.02)</td>
<td>(1.14)</td>
<td>0.06</td>
<td>0.55</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.05)</td>
<td>(1.18)</td>
<td>0.05</td>
<td>0.53</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.08)</td>
<td>(1.22)</td>
<td>0.04</td>
<td>0.51</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.11)</td>
<td>(1.26)</td>
<td>0.03</td>
<td>0.49</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.14)</td>
<td>(1.30)</td>
<td>0.02</td>
<td>0.47</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.17)</td>
<td>(1.34)</td>
<td>0.01</td>
<td>0.45</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(1.20)</td>
<td>(1.38)</td>
<td>0.00</td>
<td>0.43</td>
<td>(0.12)</td>
</tr>
</tbody>
</table>

*See chi-squares and parentesses next to their respective lines.*
<table>
<thead>
<tr>
<th>*</th>
<th>0.02</th>
<th>0.08</th>
<th>0.11</th>
<th>0.23</th>
<th>0.07</th>
<th>0.26</th>
<th>0.26</th>
<th>0.15</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>-0.01</td>
<td>-0.10</td>
<td>-0.26</td>
<td>0.49</td>
<td>0.92</td>
<td>0.29</td>
<td>0.53</td>
<td>0.22</td>
<td>RK</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>0.01</td>
<td>0.01</td>
<td>0.24</td>
<td>0.27</td>
<td>0.38</td>
<td>0.35</td>
<td>0.32</td>
<td>KR</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.24</td>
<td>0.27</td>
<td>0.38</td>
<td>0.35</td>
<td>0.32</td>
<td>0.35</td>
<td>0.32</td>
<td>KK</td>
</tr>
<tr>
<td>FRANCE</td>
<td>RF</td>
<td>JAPAN</td>
<td>EU</td>
<td>RF</td>
<td>JAPAN</td>
<td>EU</td>
<td>RF</td>
<td>JAPAN</td>
<td>d'États</td>
</tr>
<tr>
<td>MODÈLE II</td>
<td></td>
<td></td>
<td></td>
<td>MODÈLE I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Coefficients d'estimation</td>
</tr>
</tbody>
</table>

* Les valeurs non-significatives sont marquées d'un astérisque.
Tableau III: Elasticités-prix de long terme de la demande de travail dans les secteurs manufacturiers des É.U., du Japon, de la RFA et de la France, en 1970*

<table>
<thead>
<tr>
<th>Elasticités</th>
<th>MODELE I</th>
<th>MODELE II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E.U.</td>
<td>JAPON</td>
</tr>
<tr>
<td>$\varepsilon_{LPM}$</td>
<td>.03</td>
<td>.01</td>
</tr>
<tr>
<td>$\varepsilon_{LPL}$</td>
<td>-.12</td>
<td>-1.02</td>
</tr>
<tr>
<td>$\varepsilon_{LPK}$</td>
<td>.05</td>
<td>.92</td>
</tr>
<tr>
<td>$\varepsilon_{LPR}$</td>
<td>.04</td>
<td>.09</td>
</tr>
</tbody>
</table>

* Symboles: $\varepsilon_{LPi}$ = Elasticité à long terme de la demande de travail par rapport au prix du facteur $i$, où $i$ = M (énergie dans le modèle I, consommations intermédiaires dans le modèle II), L (travail), K (capital), R (R-D).
Tableau IV: Elasticités à court terme de la demande de travail à l'output dans les secteurs manufacturiers des E.U., du Japon, de la RFA et de la France, en 1970

<table>
<thead>
<tr>
<th></th>
<th>E.U.</th>
<th>JAPON</th>
<th>RFA</th>
<th>FRANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODELE I</td>
<td>1.43</td>
<td>1.85</td>
<td>1.65</td>
<td>1.49</td>
</tr>
<tr>
<td>MODELE II</td>
<td>.83</td>
<td>.97</td>
<td>.51</td>
<td>.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Périodes</th>
<th>Q/L</th>
<th>S_M/M/L</th>
<th>S_K/K/L</th>
<th>S_R/R/L</th>
<th>Fausse Proportion</th>
<th>Coûts d'ajustement</th>
<th>Progrès technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-1969</td>
<td>5.81</td>
<td>49%</td>
<td>14%</td>
<td>1%</td>
<td>14%</td>
<td>-6%</td>
<td>29%</td>
</tr>
<tr>
<td>1969-1973</td>
<td>5.89</td>
<td>56%</td>
<td>12%</td>
<td>1%</td>
<td>12%</td>
<td>-1%</td>
<td>20%</td>
</tr>
<tr>
<td>1973-1977</td>
<td>4.65</td>
<td>53%</td>
<td>17%</td>
<td>2%</td>
<td>14%</td>
<td>-0.2%</td>
<td>15%</td>
</tr>
</tbody>
</table>

* Décomposition basée sur l'équation (15)

Notation: Q= output brut

M= consommations intermédiaires

L= heures-travail

K= stock de capital

R= stock de R-D
<table>
<thead>
<tr>
<th>Pays</th>
<th>Productivité du travail</th>
<th>Facteurs de production</th>
<th>Coûts d'ajustement</th>
<th>Progrès technique résiduel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>K</td>
<td>R</td>
</tr>
<tr>
<td>Etats-Unis</td>
<td>2.26</td>
<td>64%</td>
<td>18%</td>
<td>3%</td>
</tr>
<tr>
<td>Japon</td>
<td>8.02</td>
<td>63%</td>
<td>24%</td>
<td>3%</td>
</tr>
<tr>
<td>Allemagne</td>
<td>5.99</td>
<td>75%</td>
<td>10%</td>
<td>2%</td>
</tr>
<tr>
<td>France</td>
<td>5.26</td>
<td>52%</td>
<td>23%</td>
<td>4%</td>
</tr>
</tbody>
</table>

* Décomposition basée sur l'équation (13)
Comparison and Analysis of Productivity Growth and R&D Investment in the Electrical Machinery Industries of the United States and Japan

M. Ishaq Nadiri and Ingmar R. Prucha

4.1 Introduction

During the 1970s the growth rates of labor productivity in the Japanese manufacturing sector dramatically exceeded those of the United States, particularly in such key industries as primary metals, chemicals, electrical machinery, and transportation equipment. This enabled the Japanese to reach and eventually surpass levels of U.S. labor productivity in these industries (Grossman 1985). Although each of these Japanese industries is a key competitor to the U.S. high-technology industries in both the domestic and in the world market, the electrical machinery industry stands out in certain respects. It has experienced very rapid growth in output and productivity and high rates of capital formation both in the United States and Japan. Also, a substantial amount of research and development (R&D) resources—over 20% of total R&D expenditures in total manufacturing—is concentrated in this industry in both countries. Furthermore, Japan has increased its share of free world exports in electrical machinery from 22% in 1971 to 48% in 1981 and has also dramatically increased its share of U.S. imports of electrical machinery products over the same period (Grossman 1985).

M. Ishaq Nadiri is the Jay Gould Professor of Economics at New York University and a research associate of the National Bureau of Economic Research. Ingmar R. Prucha is an associate professor of economics at the University of Maryland and a research associate of the National Bureau of Economic Research.

The authors would like to thank Jack Triplett, Ernie Zampelli, and the participants of the NBER Conference on Research in Income and Wealth for valuable comments. Yuzu Kumasaka and Yoichi Nakamura were extremely helpful in the preparation of the Japanese data. The authors would further like to thank Elliot Grossman for providing them with his data set and Jennifer Bond, Mike Peltzman, and Ken Rogers for their help with the U.S. data. Nancy Lemrow provided very able research assistance. The research was supported in part by NSF grant PRA-8108635 and by the C. V. Starr Center's Focus Program for Capital Formation, Technological Change, Financial Structure, and Tax Policy. The authors would also like to acknowledge the support with computer time from the Computer Science Center of the University of Maryland.
Because of these characteristics, we have chosen to examine the productivity performance of this industry in the United States and Japan. The analysis is based on a dynamic factor demand model. The model links intertemporal production decisions by explicitly recognizing that the level of certain factors of production cannot be changed without incurring some costs. These costs are often referred to as "adjustment costs" and are defined here in terms of forgone output from current production. Not all inputs are subject to adjustment costs; some inputs, like materials, which can be adjusted very easily, are called variable factors while others, like capital and R&D, which are subject to adjustment cost (and only adjust partially in the first period), are referred to as quasi-fixed inputs. Since output growth has been fairly high in the electrical machinery industry both in the United States and Japan, we have not imposed a priori constant returns to scale. Rather, returns to scale are estimated from the data. Since the rate of R&D investment in the electrical machinery industry has been very rapid, we have also incorporated R&D explicitly as one of the inputs. The stocks of physical capital and R&D are considered to be quasi-fixed inputs, while labor (hours worked) and materials are considered to be variable factors in the production process. Using the structural parameter estimates, we analyze the sources of growth in output, labor productivity, and total factor productivity.

The paper is organized as follows. In section 4.2 we provide a brief description of the behavior of productivity growth as well as input and output growth in the electrical machinery industries of the United States and Japan. Section 4.3 describes the basic features of the analytical model. In section 4.4 we describe the results obtained by estimating the model using annual data. We report output and price elasticities of the variable and quasi-fixed factors of production in the short run, the intermediate run, and the long run, and we calculate the speeds of adjustment of the quasi-fixed factors—physical and R&D capital. Section 4.5 is devoted to examining the sources of output and factor productivity growth rates. Summary and conclusions are offered in section 4.6. Mathematical details of the analytic model are given in appendix A. Appendix B contains the data description. Explicit formulas for expressions used in the decomposition of total factor productivity growth are given in appendix C.

4.2 Some Descriptive Characteristics

In this section, we provide a brief description of total and partial factor productivity growth and the growth of gross output, labor, materials, capital, and R&D in the electrical machinery industry for the periods 1968–73 and 1974–79. We refer to these periods as the pre-OPEC and the post-OPEC periods, respectively.

Average growth rates for gross output and factor inputs for the two periods are given in table 4.1. For the pre-Opec period, the growth rates were extremely high for Japan in comparison to the United States. However, in the
<table>
<thead>
<tr>
<th>Period</th>
<th>Average annual rates (%)</th>
<th>Input shares in total</th>
<th>RAED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968–72</td>
<td>4.2</td>
<td>16.9</td>
<td>6.4</td>
</tr>
<tr>
<td>1974–79</td>
<td>4.0</td>
<td>6.4</td>
<td>6.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Japan</th>
<th>United States</th>
<th>Japan</th>
<th>United States</th>
<th>Japan</th>
<th>United States</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>4.2</td>
<td>16.9</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Japan</th>
<th>United States</th>
<th>Japan</th>
<th>United States</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average annual rates</td>
<td>4.2</td>
<td>16.9</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Input shares in total</td>
<td>4.0</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Table 4.1 Growth of Output and Inputs and Input Shares in the U.S. and Japanese Electrical Machinery Industries, 1968–72 and 1974–79.
post-OPEC period, the Japanese electrical machinery industry experienced a substantial drop in rates of growth of output and of most inputs. For example, the average output growth rate declined from 16.9% to 6.4% for Japan while increasing from 4.2% to 4.9% in the United States. Still, the level of output growth rates for the Japanese industry remained high compared to the U.S. industry. The average growth rate of capital over the period 1968–73 was twice as high in Japan as in the United States even though the U.S. industry experienced a healthy 5.4% per annum growth rate over this period. However, Japan's rate of growth in capital formation decelerated by more than 40% after 1973. Materials inputs grew much faster in Japan than in the United States in the pre-OPEC period, but again Japan experienced a dramatic slowdown in the growth rate of this input during the second period.

As indicated in table 4.1 the R&D stock grew at a much more rapid rate in Japan than in the United States in both periods, reflecting the very high rate of growth in R&D investment in Japan. In both the U.S. and Japanese electrical machinery industries the growth in the stock of R&D slowed down in the 1974–79 period. The input shares in total cost shown in the lower panel of table 4.1 indicate, for Japan, a tendency toward increase in the labor share and a decline in the share of materials in the two periods. The cost shares in the United States are generally very stable in this industry over the two periods.

The growth rate of labor measured in hours worked shows a dramatically different pattern in the two countries. It increased from −0.5% in 1968–73 to 1.4% in 1974–79 in the United States, while in Japan the growth in this input declined from 4.3% to an actual reduction of −2.5%. This phenomenon is consistent with the general pattern of employment in the two countries: Japan experienced declines in employment in several industries while the United States experienced increases in employment in most industries (Griliches and Mairesse, in this volume; Norsworthy and Malmquist 1983).

As demonstrated by table 4.2, an important characteristic of the electrical machinery industry in both countries is the high ratio of R&D investment in output. While the ratio of capital investment in value added or gross output in this industry is generally lower than in total manufacturing, the opposite is true for R&D investment. The R&D ratios in the electrical machinery industry are two to three times as large as those in total manufacturing. It is also important to note that in the U.S. electrical machinery industry the R&D investment ratios are considerably higher than the capital investment ratios, while the opposite is true in Japan.

Total and partial productivity growth rates based on a gross output measurement framework are shown in table 4.3. Both total and labor-productivity growth rates were much higher in the Japanese electrical machinery industry than in the United States. This was particularly true in the pre-OPEC period. Unlike the aggregate manufacturing sector (Norsworthy and Malmquist 1983), total factor productivity growth was rising in this industry in the two countries over the two periods. The differences in the growth of labor productivity in the industries of the two countries are substantial. In the United
States, labor productivity grew about 4.7% in 1968–73 and declined to 3.6% in 1974–79; in Japan, the corresponding growth rates are 12.6% and 8.9%, respectively. Substantial improvements in materials productivity in this industry in both countries in the post-OPEC period are also noted.

Thus, the elements of the Japanese productivity "miracle" can also be observed in the electrical machinery industry: high rates of labor-productivity growth accompanied by rapid growth rates of output and other inputs such as materials, capital, and R&D before 1973 and diminishing but still very high rates of labor-productivity growth after 1973 accompanied by a substantial falloff in the growth rates of output and other inputs. To explore the reasons for these productivity patterns, we proceed to estimate the production structure of the electrical machinery industry of the two countries.

### 4.3 Model Specification

The model specified below generates a set of factor demand equations for both variable inputs (materials and labor) and the quasi-fixed inputs (capital and R&D). Each demand equation allows for the effect of changes in output,
changes in relative prices, and technological change. Also, the model allows for the interaction (i.e., nonseparability) of the quasi-fixed inputs, capital and R&D, during the adjustment process. From the structural parameters various underlying features of the technology, such as the degree of economies of scale and the output and price elasticities of the inputs in the current and subsequent periods, can be measured. Finally, these parameters can be used to decompose the factors that affect total and labor-productivity growth rates in the Japanese and U.S. electrical machinery industries.

Consider a firm that employs two variable inputs and two quasi-fixed inputs in producing a single output from a technology with internal adjustment costs. Specifically, assume the firm’s production function takes the form:

\[ Y_t = F(V_t, X_{t-1}, \Delta X_t, T_t), \]

where \( Y_t \) denotes gross output, \( V_t = [V_{t1}, V_{t2}]' \) is the vector of variable inputs, \( X_t = [X_{t1}, X_{t2}]' \) is the vector of end-of-period stocks of the quasi-fixed inputs, and \( T_t \) is an exogenous technology index. The vector \( \Delta X_t = X_t - X_{t-1} \) represents the internal adjustment costs in terms of foregone output.

The firm’s input markets are assumed to be perfectly competitive. It proves convenient to describe the firm’s technology in terms of the normalized restricted cost function defined as \( G(W_t, X_{t-1}, \Delta X_t, Y_t, T_t) = \tilde{V}_{t1} + W_t\tilde{V}_{t2} \). Here \( \tilde{V}_{t1} \) and \( \tilde{V}_{t2} \) represent the cost-minimizing amounts of variable inputs needed to produce the output \( Y_t \) conditional on \( X_{t-1} \) and \( \Delta X_t \), and \( W_t \) denotes the price of \( V_{t2} \) normalized by the price of \( V_{t1} \). We assume that the normalized restricted cost function satisfies standard properties. In particular \( G(\cdot) \) is assumed to be convex in \( X_{t-1} \) and \( \Delta X_t \) and concave in \( W_t \); compare, for example, Lau (1976).\(^2\)

Given the presence of large firms in the electrical machinery industries of both the United States and Japan, we do not impose a priori constant returns to scale. Rather, we allow the technology to be homogeneous of (constant) degree and determine the returns to scale parameter \( \rho \) from the data.\(^3\) Given that \( F(\cdot) \) is homogeneous of degree \( \rho \), the corresponding normalized restricted cost function is of the following general form:\(^4\)

\[ G(W_t, X_{t-1}, \Delta X_t, T_t) = G(W_t, X_{t-1}/Y_t^{1/\rho}, \Delta X_t/Y_t^{1/\rho}, T_t)Y_t^{1/\rho}. \]

In the empirical analysis we take materials, \( M \), and labor (hours worked), \( L \), as the variable factors and the stocks of capital, \( K \), and research and development, \( R \), as the quasi-fixed factors. We adopt the convention \( V_t = M \), \( V_{t1} = L \), \( X_t = K \), \( X_{t2} = R \), \( W \) is the real wage rate; the price of materials is the numeraire. In the empirical analysis, we further take \( T_t = t \), that is, technical change, other than that reflected by the stock of R&D, is represented by a simple time trend. We specify the following functional form for the normalized restricted cost function:
\[ G(W_t, X_{t-1}, \Delta X_t, Y_t, T_t) = (\alpha_0 + \alpha_w W_t + \alpha_{ww} W_t^2 + \alpha_{ww} W_t^2/2)Y_t^{1/\phi} \]
\[ + a'X_{t-1} + b'X_{t-1}W_t + c'X_{t-1}T_t \]
\[ + X_{t-1}AX_t/(2Y_t^{1/\phi}) + \Delta X_t/B \Delta X_t/(2Y_t^{1/\phi}) \]

where

\[ a = \begin{bmatrix} \alpha_k \\ \alpha_s \end{bmatrix}, \quad b = \begin{bmatrix} \alpha_{kw} \\ \alpha_{sw} \end{bmatrix}, \quad c = \begin{bmatrix} \alpha_{kk} \\ \alpha_{sk} \end{bmatrix}, \quad A = \begin{bmatrix} \alpha_{kk} & \alpha_{ks} \\ \alpha_{sk} & \alpha_{ss} \end{bmatrix}, \quad B = \begin{bmatrix} \alpha_{kk} & 0 \\ 0 & \alpha_{ss} \end{bmatrix} \]

In light of the above discussion, we can view (3) as a second-order approximation to a generalized normalized restricted cost function that corresponds to a homogeneous technology of degree \( \phi \). Expression (3) is a generalization of the normalized restricted cost function introduced by Denny, Fuss, and Waverman (1981) and Morrison and Berndt (1981) for linear homogeneous technologies. As in these references we impose parameter restrictions such that the marginal adjustment costs at \( \Delta X_t = 0 \) are zero. The convexity of \( G(\cdot) \) in \( X_{t-1} \) and \( \Delta X_t \), and concavity in \( W_t \), implies the following inequality parameter restrictions: \( \alpha_{kk} > 0, \alpha_{ss} > 0, \alpha_{kk} \alpha_{ss} - \alpha_{kk} \phi > 0, \alpha_{sk} > 0, \alpha_{kk} > 0, \alpha_{sw} < 0 \).

We assume that in each period \( t \) (for given initial stocks \( X_{t-1} \) and static expectations on relative factor prices, output, and the technology) the firm derives an optimal plan for inputs in period \( t, t + 1, \ldots \) such that the present value of the future cost stream is minimized, and that the firm chooses its inputs in period \( t \) accordingly. In each period, the firm revises its expectations and the optimal plan for its inputs, based on new information.

A mathematical formulation and analysis of the firm's optimization problem is given in appendix A. It is shown there that the implied demand equations for the quasi-fixed factors, capital and R&D, are in the form of an accelerator model. We denote the accelerator matrix with \( M = (m_{ij})_{i,j = k,k} \). The firm's demand equations for the variable factors, labor and materials, can be derived from the restricted cost function via Shephard's lemma. Instead of estimating the parameter matrices \( A \) and \( B \), it proves advantageous to estimate the matrices \( C = (c_{ij})_{i = k, j = k,k} = -BM \) and \( B \) (and to express \( A \) as a function of \( C \) and \( B \)). The matrix \( C \) is found to be symmetric and negative definite. Explicit expressions for the resulting demand equations for labor, materials, capital, and R&D are given in equations (A4) and (A5) of appendix A.

### 4.4 Empirical Results

In this section, we report the structural parameter estimates for the U.S. and Japanese electrical machinery industries as well as implied estimates for short-run, intermediate-run, and long-run price and output elasticities.

A detailed description of the data sources and the variables of the model is given in the appendix B. The data on gross output, materials, labor, capital
and R&D are in constant 1972 dollars and yen and have been normalized by their respective sample means. Prices were constructed conformably. The model parameters were estimated by full-information maximum likelihood from the demand equations (A4) and (A5); for further details see appendix A.

### 4.4.1 Parameter Estimates

The structural parameter estimates are given in table 4.4. As indicated by the squared correlation coefficients between actual and fitted data, the estimated factor demand equations seem to fit the data quite well. (Fitted values are calculated from the reduced form). The parameter estimates are, in general, statistically significant. For both the United States and Japan, the parameter estimates satisfy the theoretical restrictions. In particular, the estimates for $c_{xx}$, $c_{rr}$, and $\alpha_{pp}$ are negative, and those for $\alpha_{xx}$, and $\alpha_{rr}$, and $(c_{xx}c_{rr} - c_{xx}^2)$ are positive. The variables underlying the estimates for the U.S. and Japanese electrical machinery industries are, as explained above, measured in

<table>
<thead>
<tr>
<th>Parameters</th>
<th>United States</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_x$</td>
<td>1.83</td>
<td>1.45</td>
</tr>
<tr>
<td>$p$</td>
<td>1.21</td>
<td>1.39</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>-0.95</td>
<td>-0.47</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>-0.65</td>
<td>-0.67</td>
</tr>
<tr>
<td>$\alpha_{rr}$</td>
<td>-0.19</td>
<td>-0.05</td>
</tr>
<tr>
<td>$\alpha_{rr}$</td>
<td>0.22</td>
<td>0.02</td>
</tr>
<tr>
<td>$c_{xx}$</td>
<td>-2.05</td>
<td>-0.58</td>
</tr>
<tr>
<td>$c_{xx}$</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>$c_{xx}$</td>
<td>8.70</td>
<td>2.57</td>
</tr>
<tr>
<td>$c_{rr}$</td>
<td>13.80</td>
<td>1.11</td>
</tr>
<tr>
<td>$c_{rr}$</td>
<td>1.91</td>
<td>1.33</td>
</tr>
<tr>
<td>$\alpha_{pp}$</td>
<td>-0.48</td>
<td>-0.81</td>
</tr>
<tr>
<td>$\alpha_{pp}$</td>
<td>0.29</td>
<td>0.39</td>
</tr>
<tr>
<td>$\alpha_{pp}$</td>
<td>-0.52</td>
<td>0.02</td>
</tr>
<tr>
<td>$\alpha_{pp}$</td>
<td>-0.28</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

**Table 4.4** Full Information Maximum-Likelihood Estimates of the Parameters of the Dynamic Factor Demand Model for the U.S. and Japanese Electrical Machinery Industries

- **Note:** Absolute values of the asymptotic t-ratios are given in parentheses. The $R^2$ values correspond to the squared correlation coefficients between the actual $M$, $L$, $K$, $R$ variables and their fitted values calculated from the reduced form.
different units. Hence, a direct comparison of individual parameter estimates is difficult. However, we do calculate various unit-free characteristics that allow a meaningful comparison.

In general the adjustment cost coefficients $\alpha_{kk}$ and $\alpha_{kr}$ are significantly different from zero. They are crucial in determining the investment patterns of the quasi-fixed factors via the accelerator coefficients. Omitting those terms would not only have resulted in a misspecification of the investment patterns but also (in general) in inconsistent estimates of the other technology parameters.

Table 4.5 shows the estimates for the accelerator coefficients $m_{kk}$, $m_{kr}$, $m_{rk}$, and $m_{rr}$. For both the U.S. and Japanese electrical machinery industries we find that the cross-adjustment coefficients $m_{kr}$ and $m_{rk}$ (as well as $c_{rr}$) are very small in absolute magnitude and are not significantly different from zero at the 95% level. In describing the adjustment speed, we can therefore concentrate on the own-adjustment coefficients $m_{kk}$ and $m_{rr}$. As a first observation, we note that the obtained estimates are quite similar across countries. For both the United States and Japan, capital adjusts faster than R&D. While capital closes approximately one-fourth of the gap between the initial and the desired stock in the first period, R&D only closes approximately one-seventh of its gap.

As remarked earlier, our specification does not impose a priori constant returns to scale. Rather, we estimate the scale elasticity (represented by $\rho$) from the data. For both countries, we find substantial and significant scale effects in the industry. For the United States, our estimate for the scale elasticity is 1.21; for Japan we obtained a considerably higher estimate of 1.39. As we explain in more detail in section 4.5, this difference in scale elasticities will translate into substantial differences in productivity growth. It is also interesting to note that, contrary to our finding of increasing returns to scale at the industry level, Griliches and Mairesse (in this volume) find decreasing returns to scale in the U.S. and Japanese total manufacturing sectors at the firm level.
4.4.2 Price and Output Elasticities

The own- and cross-price elasticities of labor, materials, capital, and R&D for 1976 are reported in table 4.6. The elasticities are calculated for the short run (SR), intermediate run (IR), and long run (LR) for each input for the electrical machinery industry in both the United States and Japan. All of the own-price elasticities have the expected negative sign. The magnitudes of the elasticities are fairly similar between the two countries. In the United States, the own-price elasticity of labor is the largest among the inputs followed by materials, R&D stock, and capital stock. In Japan, with minor exceptions, the same pattern holds; the quasi-fixed inputs, capital and R&D, seem to have a higher own-price elasticity in the Japanese than in the U.S. electrical machinery industry. These results are similar to those reported for the total manufacturing sectors of the United States and Japan in Mohnen, Nadiri, and Prucha (1986).

Although the cross-price elasticities are generally small in comparison to own-price elasticities, some of the elasticities are sizable. The elasticities of materials and R&D with respect to the wage rate, and the elasticities of labor, R&D, and capital inputs with respect to the price of materials, are quite large.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>United States</th>
<th></th>
<th></th>
<th>Japan</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SR</td>
<td>IR</td>
<td>LR</td>
<td>SR</td>
<td>IR</td>
<td>LR</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;W&lt;/sup&gt;</td>
<td>−.32</td>
<td>−.40</td>
<td>−.64</td>
<td>−.04</td>
<td>−.18</td>
<td>−.64</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;M&lt;/sup&gt;</td>
<td>.36</td>
<td>.41</td>
<td>.65</td>
<td>.09</td>
<td>.15</td>
<td>.36</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;C&lt;/sup&gt;</td>
<td>−.01</td>
<td>.02</td>
<td>.09</td>
<td>−.02</td>
<td>.04</td>
<td>.20</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;R&lt;/sup&gt;</td>
<td>−.01</td>
<td>−.02</td>
<td>−.08</td>
<td>−.03</td>
<td>−.01</td>
<td>.09</td>
</tr>
<tr>
<td>ε&lt;sub&gt;C&lt;/sub&gt;&lt;sup&gt;M&lt;/sup&gt;</td>
<td>.47</td>
<td>.55</td>
<td>.90</td>
<td>.37</td>
<td>.51</td>
<td>.85</td>
</tr>
<tr>
<td>ε&lt;sub&gt;C&lt;/sub&gt;&lt;sup&gt;L&lt;/sup&gt;</td>
<td>−.48</td>
<td>−.58</td>
<td>−1.12</td>
<td>−.38</td>
<td>−.44</td>
<td>−.57</td>
</tr>
<tr>
<td>ε&lt;sub&gt;C&lt;/sub&gt;&lt;sup&gt;R&lt;/sup&gt;</td>
<td>−.02</td>
<td>−.06</td>
<td>−.06</td>
<td>−.06</td>
<td>−.23</td>
<td></td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;L&lt;/sup&gt;</td>
<td>.04</td>
<td>.27</td>
<td>−.01</td>
<td>−.01</td>
<td>−.05</td>
<td></td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;M&lt;/sup&gt;</td>
<td>.10</td>
<td>.17</td>
<td>.38</td>
<td>.27</td>
<td>.46</td>
<td>.99</td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;C&lt;/sup&gt;</td>
<td>−.05</td>
<td>−.09</td>
<td>−.17</td>
<td>−.13</td>
<td>−.23</td>
<td>−.48</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;L&lt;/sup&gt;</td>
<td>−.04</td>
<td>−.08</td>
<td>−.18</td>
<td>−.14</td>
<td>−.24</td>
<td>−.49</td>
</tr>
<tr>
<td>ε&lt;sub&gt;L&lt;/sub&gt;&lt;sup&gt;R&lt;/sup&gt;</td>
<td>−.01</td>
<td>−.01</td>
<td>−.04</td>
<td>−.01</td>
<td>−.01</td>
<td>−.02</td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;L&lt;/sup&gt;</td>
<td>−.05</td>
<td>−.09</td>
<td>−.27</td>
<td>.19</td>
<td>.33</td>
<td>.91</td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;M&lt;/sup&gt;</td>
<td>.11</td>
<td>.20</td>
<td>.65</td>
<td>−.05</td>
<td>−.08</td>
<td>−.23</td>
</tr>
<tr>
<td>ε&lt;sub&gt;R&lt;/sub&gt;&lt;sup&gt;C&lt;/sup&gt;</td>
<td>−.01</td>
<td>−.01</td>
<td>−.03</td>
<td>−.01</td>
<td>−.01</td>
<td>−.04</td>
</tr>
<tr>
<td>ε&lt;sub&gt;C&lt;/sub&gt;&lt;sup&gt;C&lt;/sup&gt;</td>
<td>−.06</td>
<td>−.10</td>
<td>−.34</td>
<td>−.14</td>
<td>−.24</td>
<td>−.65</td>
</tr>
</tbody>
</table>

*Note:* ε<sub>z</sub> is the elasticity of factor Z = materials (M), labor (L), capital (K), and R&D (R) with respect to s = price of materials (w<sup>M</sup>), labor (w<sup>L</sup>), capital (c<sup>K</sup>), and R&D (c<sup>R</sup>).
in both countries. Materials are substitutes for other inputs, except for R&D in the United States. Labor and R&D are substitutes in the United States and weak complements in the Japanese electrical machinery industry. Labor and capital and R&D and capital are complements in both countries.

The output elasticities of the inputs for 1976 are shown in Table 4.7. The long-run elasticities of the inputs are .8 and .7, respectively, for the United States and Japan, reflecting fairly sizable economies of scale. The results are consistent with Fuss and Waverman (in this volume), Nadiri and Prucha (1982, 1987) and Nadiri and Shankerman (1981). The patterns of the output elasticities, particularly in the United States, indicate that the variable factors of production, labor and materials, respond strongly in the short run to changes in output. This is because both labor and materials in the United States and materials in Japan overshoot their long-run equilibrium values in the short-run to compensate for the sluggish adjustment of the quasi-fixed factors. They slowly adjust toward their long-run equilibrium values as capital and R&D adjust. The output elasticities of capital and R&D are small in the short-run but increase over time and are quite similar. At least in the short-run and intermediate-run, the output elasticities of both the variable and quasi-fixed factors substantially exceed their own-price elasticities. It is surprising that, except for the labor input, the patterns of input responses are similar in both countries.

Thus, the production structure of the electrical machinery industry in the two countries, characterized by the patterns of factor input substitution and complementarity as well as the degree of scale, is qualitatively similar. Quantitatively, there are some differences in scale and in the responses of inputs to changes in prices and output in the two industries. Both industries are characterized by increasing returns to scale. However, the Japanese industry has a higher scale, which substantially influences its productivity growth and is a major source of divergence between the productivity growth rates in this industry in the two countries.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>United States</th>
<th></th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SR</td>
<td>IR</td>
<td>LR</td>
</tr>
<tr>
<td>$e_{m}$</td>
<td>1.19</td>
<td>1.07</td>
<td>.82</td>
</tr>
<tr>
<td>$e_{l}$</td>
<td>1.07</td>
<td>1.06</td>
<td>.82</td>
</tr>
<tr>
<td>$e_{k}$</td>
<td>.20</td>
<td>.34</td>
<td>.82</td>
</tr>
<tr>
<td>$e_{r}$</td>
<td>.14</td>
<td>.24</td>
<td>.82</td>
</tr>
</tbody>
</table>

*Note: $e_{p}$ is the elasticity of factor $Z = \text{materials (M), labor (L), capital (K), and R&D (R)}$ with respect to output ($Y$).
4.5 Productivity Analysis

Using the estimates of the production structure, we can quantitatively examine the sources of output and productivity growth. The contributions of the factor inputs, technical change, and adjustment costs to output growth are shown in table 4.8. This decomposition is based on the approximation:

\[
\Delta \ln Y_t = \frac{1}{2} \sum_{i=1}^{k} [\varepsilon_{x_i}(t) + \varepsilon_{x_i}(t-1)] \Delta \ln Z_{x_i} + \frac{1}{2} \lambda_r(t) + \lambda_r(t-1),
\]

with \( Z_1 = L, Z_2 = M, Z_3 = K, Z_4 = R, Z_5 = \Delta K, \) and \( Z_6 = \Delta R. \) The \( \varepsilon_{x_i} \)'s denote respective output elasticities and \( \lambda_r(t) = (1/Y_t) \left( \partial Y_t/\partial t \right) \) denotes technical change.9

The average growth of gross output was very rapid in Japan in the period 1968–73, but growth decelerated substantially in the period 1974–79. For the United States, output growth rates were similar in the two periods. The contributions of various inputs to the growth of output differ considerably between the two periods and the two industries. The most significant source of gross output growth is materials growth, particularly in Japan. The contribution of capital is larger in Japan than in the United States, but falls in both countries over the post-OPEC period. The R&D stock contributes significantly to the growth of output in both industries. In the post-OPEC period its contribution falls in the United States but remains the same for Japan. The large contribution of R&D to the output growth may come as a surprise but can be explained by two factors. First, the share of R&D investment in gross output, as noted earlier, is very high in the electrical machinery industries of both countries; second, the marginal product of R&D, because of the relatively large adjustment costs and the considerable degree of scale, is fairly large in the two industries. The direct contributions of the adjustment costs are fairly small, as one would expect. The contribution of technical change is clearly important in explaining the growth of output in both industries. Its contribution is twice as large in Japan as in the United States.

In table 4.9 we provide a decomposition of labor-productivity growth. This decomposition is based on the approximation:

\[
\Delta \ln (Y_t/L_t) = \frac{1}{2} \sum_{i=2}^{k} [\varepsilon_{x_i}(t) + \varepsilon_{x_i}(t-1)] \Delta \ln (Z_{x_i}/L_t)
\]

\[
+ \frac{1}{2} \lambda_r(t) + \lambda_r(t-1) + (p-1) \Delta \ln L_r,
\]

where \( p \) is the scale elasticity.10 The most significant contribution again stems from the growth of materials, particularly in Japan, although the contribution of physical capital is also important. In comparison to the results reported by Norsworthy and Malmquist (1983) for the total manufacturing sector, the contribution of physical capital is somewhat larger for the United States but
Table 4.8  
Sources of Output Growth for the U.S. and Japanese Electrical Machinery Industries: Average Annual Rates of Growth (in %)

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Gross Output</th>
<th>Labor Effect&lt;sup&gt;*&lt;/sup&gt;</th>
<th>Materials Effect&lt;sup&gt;*&lt;/sup&gt;</th>
<th>Capital Effect&lt;sup&gt;*&lt;/sup&gt;</th>
<th>R&amp;D Effect&lt;sup&gt;*&lt;/sup&gt;</th>
<th>Adjustment Cost</th>
<th>Technical Change</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Capital</td>
<td>R&amp;D</td>
<td></td>
</tr>
<tr>
<td>United States:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.06</td>
<td>.12</td>
<td>.73</td>
</tr>
<tr>
<td>1968–73</td>
<td></td>
<td>4.2</td>
<td>-.24</td>
<td>1.83</td>
<td>.87</td>
<td>1.18</td>
<td>-.09</td>
<td>.04</td>
<td>.86</td>
</tr>
<tr>
<td>1974–79</td>
<td></td>
<td>4.9</td>
<td>.39</td>
<td>1.06</td>
<td>.69</td>
<td>.31</td>
<td>-.26</td>
<td>-.34</td>
<td>1.55</td>
</tr>
<tr>
<td>Japan:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.09</td>
<td>-.12</td>
<td>2.55</td>
</tr>
<tr>
<td>1968–73</td>
<td></td>
<td>16.9</td>
<td>.94</td>
<td>14.32</td>
<td>2.12</td>
<td>.7</td>
<td>-.26</td>
<td>-.34</td>
<td>1.55</td>
</tr>
<tr>
<td>1974–79</td>
<td></td>
<td>6.4</td>
<td>-.66</td>
<td>2.08</td>
<td>1.10</td>
<td>.72</td>
<td>.09</td>
<td>-.12</td>
<td>2.55</td>
</tr>
</tbody>
</table>

<sup>*</sup>Growth rate of input weighted by average output elasticity.
Table 4.9  Decomposition of Labor Productivity Growth in the U.S. and Japanese Electrical Machinery Industries. Average Annual Rates of Growth (in %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>United States:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1968-73</td>
<td>4.68</td>
<td>-.04</td>
<td>2.07</td>
<td>.91</td>
<td>1.28</td>
<td>.06</td>
<td>.12</td>
<td>.73</td>
<td>-.44</td>
</tr>
<tr>
<td>1974-79</td>
<td>3.56</td>
<td>.15</td>
<td>.43</td>
<td>.37</td>
<td>.12</td>
<td>-.07</td>
<td>.04</td>
<td>.86</td>
<td>1.66</td>
</tr>
<tr>
<td>Japan:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1968-73</td>
<td>12.63</td>
<td>.81</td>
<td>10.24</td>
<td>1.33</td>
<td>.56</td>
<td>-.13</td>
<td>-.26</td>
<td>1.55</td>
<td>-1.48</td>
</tr>
<tr>
<td>1974-79</td>
<td>8.95</td>
<td>-.47</td>
<td>4.48</td>
<td>1.54</td>
<td>.86</td>
<td>.05</td>
<td>-.16</td>
<td>2.55</td>
<td>.10</td>
</tr>
</tbody>
</table>

$^*$Growth rate of input per unit of labor, weighted by average output elasticity.
smaller for Japan. The contribution of R&D is somewhat smaller and rising for Japan. For the United States, the contribution of R&D is very substantial in the pre-OPEC period but only marginal in the post-OPEC period. The direct contribution of adjustment costs is again small. The contribution of technical change is very substantial (particularly in Japan) and rising in both countries.

The labor effect (given by the last term on the right-hand side of (5)) follows from the fact that scale is not equal to one. The contribution of this term to labor-productivity growth is shown in the second column of table 4.9. Its effect is positive in Japan in the pre-OPEC period and negative in the post-OPEC period. The opposite is the case for the United States. This reflects the growth pattern of the labor input in the two industries over the two periods.

Denny, Fuss, and Waverman (1981) have shown that if all factors are variable, then the traditional measure of total factor productivity (using cost shares) can be decomposed into two components, one attributable to scale and one to technical change. Nadiri and Prucha (1983, 1990) extend this decomposition to technologies with adjustment costs. More specifically, consider the Törnqvist approximation of the growth rate of total factor productivity, \( \Delta TFP_p \), defined implicitly by:

\[
\Delta \ln Y_r = \frac{1}{2} \sum_{i=1}^4 [s_{x_i}(t) + s_{x_i}(t-1)] \Delta \ln Z_{x_i} + \Delta TFP_p
\]

with \( Z_1 = L, Z_2 = M, Z_3 = K, Z_4 = R \), and where the \( s_{x_i} \)’s denote respective long-run cost shares. Given increasing returns to scale and adjustment costs we find that the output elasticities \( e_{x_i} \) exceed the cost shares \( s_{x_i} \). As a consequence, as is evident from a comparison of equations (4) and (6), total factor productivity will not equal technical change. Prucha and Nadiri (1983, 1990) shows that total factor productivity growth can be decomposed as follows:

\[
\Delta TFP_p = (1 - \rho^{-1}) \Delta \ln Y_r + \phi_v + \phi_a + \frac{1}{2} [\lambda_x(t) + \lambda_x(t-1)].
\]

where \( \lambda_x = (1/\rho) \lambda_y \). The first term on the right-hand side of (7) represents the scale effect and the last term the pure effect of technical change on the growth of total factor productivity. The terms \( \phi_v \) is attributable to the fact that, in short-run temporary equilibrium, the rate of technical substitution between the quasi-fixed and variable factors differs from the long-run price ratios. We will refer to \( \phi_v \) as the temporary equilibrium effect. The terms \( \phi_a \) reflects the direct adjustment-cost effect in terms of forgone output due to the presence of \( \Delta K \) and \( \Delta R \) in the production function. We will refer to \( \phi_a \) as the direct adjustment-cost effect. Explicit expressions for the terms \( \phi_v \) and \( \phi_a \) (and a further discussion of those terms) are given in appendix C.

Table 4.10 presents the decomposition of total factor productivity based on
Table 4.10  
Decomposition of Total Factor Productivity Growth in the U.S. and Japanese Electrical Machinery Industries for Respective Sample Periods (in %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale effect +</td>
<td>1.04</td>
<td>3.38</td>
</tr>
<tr>
<td>Temporary equilibrium effect +</td>
<td>.28</td>
<td>.16</td>
</tr>
<tr>
<td>Direct adjustment-cost effect +</td>
<td>.03</td>
<td>-.04</td>
</tr>
<tr>
<td>Technical change +</td>
<td>.60</td>
<td>1.49</td>
</tr>
<tr>
<td>Unexplained residual =</td>
<td>.10</td>
<td>-.24</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>2.04</td>
<td>4.74</td>
</tr>
</tbody>
</table>

(7) for the sample periods used in estimating the production technology of the U.S. and Japanese electrical machinery industries. The scale effect is, by far, the most important contributor to total factor productivity growth. This is particularly the case in the Japanese industry where the output growth was very rapid and the estimated degree of scale larger than in the U.S. industry. The temporary equilibrium effect, $\phi_1$, is fairly large in the United States and about twice as big as in the Japanese electrical machinery industry. The direct effect of the adjustment costs, $\phi_2$, is negligible. The combined effect of $\phi_1$ and $\phi_2$ is 15% and 4% of the measured total factor productivity growth for the United States and Japan, respectively, and hence not negligible, particularly for the United States. Consequently, if zero adjustment costs would have been imposed, a nonnegligible portion of measured total factor productivity growth would have been misclassified. In addition, inconsistency of the estimates of the underlying technology parameters would have distorted the decomposition of total factor productivity growth. The contribution of technical change to the growth of total factor productivity is second only to the scale effect. For each of the sample periods, the unexplained residual is small.

4.6 Conclusion and Summary

In this paper, we have modeled the production structure and the behavior of factor inputs and have analyzed the determinants of productivity growth in the U.S. and Japanese electrical machinery industries. These industries have experienced a very high rate of output growth, are technologically very progressive (measured by the rate of expenditures on R&D), and are highly competitive in the domestic U.S. and world markets. Our model allows for scale effects and the quasi fixity of some of the input factors. It also incorporated R&D to capture the high-technology feature of the industry. Other inputs considered are labor, materials, and physical capital. We have also allowed for exogenous technical change using a time trend. The model was estimated using annual data from 1960–80 and 1968–80 for the United States and Japan, respectively.
The main results of this paper can be summarized as follows:

(i) The production structure of the electrical machinery industry in both countries is characterized by increasing returns to scale; the Japanese electrical machinery industry exhibits higher returns to the scale than the US industry. The responses of the factors of production to changes in factor prices and output in the short run, intermediate run and long run are similar in the two industries. Materials are generally found to be substitutes for other inputs. Other inputs are generally complements except for labor and R&D in the U.S. industry. Capital and R&D are found to be quasi fixed, and their adjustment speeds are found to be similar across countries. The stock of capital adjusts much faster than the stock of R&D.

(ii) The elements of the so-called Japanese productivity miracle noted by others are, to a large extent, present in the electrical machinery industry: high rates of labor-productivity growth accompanied by rapid output growth and input growth before 1973 and diminishing but still high rates of labor productivity growth after 1973 accompanied by a substantial slowdown in the growth rates of outputs and factor inputs.

(iii) Based on the structural estimates of our model, we identify the following sources of growth of output and labor productivity: (a) The most important source of output and labor-productivity growth is the growth of materials for both pre- and post-OPEC periods in both countries. Technical change and capital were found to be the next most important factors. For the United States, capital's contribution exceeds that found at the total manufacturing level; the reverse is true for Japan. (b) Consistent with the high ratio of R&D expenditures to gross output in the electrical machinery industry, we find significant contributions of R&D to both output and labor-productivity growth. However, the R&D contribution to both has significantly declined in the United States from the pre-OPEC to the post-OPEC period.

(iv) The most important source of growth in total factor productivity for both countries is the scale effect. This is particularly true in Japan due to the higher scale elasticity and higher rate of growth of output. A significant portion of the differential of total factor productivity in the electrical machinery industry in the United States and Japan is due to the greater contribution of economies of scale to the growth of total factor productivity in Japan. Technical change is the second most important contributor. In the context of our dynamic model the rate of technical substitution for the quasi-fixed factors deviates in the short run from the long-run relative price ratios. This source also explains part of the traditional measure of total factor productivity growth.

Our model provides a richer framework for the analysis of productivity growth than some of the conventional approaches by incorporating dynamic aspects, nonconstant returns to scale, and R&D. The omission of dynamic
aspects will typically result in inconsistent estimates of the technology parameters and a misallocation in the decomposition of measured total factor productivity growth. However, a number of issues remain unresolved.

(i) Given the rapid expansion of the electrical machinery industries in the United States and Japan, it seems important to explore the effect of nonstatic expectations on the input behavior and its implications for the productivity growth analysis.

(ii) It may also be of interest to explore a more general lag structure for the quasi-fixed factors and to adopt a more general formulation of the model that allows for scale to vary over the sample period.

(iii) A further area of research is the decomposition of labor into white- and blue-collar workers and the modeling of white-collar workers as potentially quasi-fixed. The quasi-fixity of labor may be particularly important in Japan where employment is considered fairly long term.

(iv) Finally, an important extension of the model would be to incorporate explicitly the role of demand and thereby analyze the role of the utilization rate on productivity growth.

Appendix A

Estimated System of Factor Demand Equations

Given the assumptions of section 4.3, the firm’s optimum problem in period \( t \) can be written as

\[
\begin{align*}
\min_{\{K_{t+1}, R_{t+1}, \gamma, \alpha, \bar{\gamma}, \bar{\alpha}\}} \quad & \text{PVC}_t = \sum_{t=0}^{\infty} \left[ [G_{t+\tau} + \hat{Q}_t^b(\Delta R_{t+\tau} + \delta_R K_{t+\tau})] (1 - u_t) \\
& + \hat{Q}_t^c(\Delta K_{t+\tau} + \delta_K K_{t+\tau})] (1 + r)^{-\tau},
\end{align*}
\]

where the restricted cost function \( G_{t+\tau} = G(\bar{W}_t, K_{t+\tau}, R_{t+\tau}, \Delta K_{t+\tau}, \Delta R_{t+\tau}, \bar{\gamma}, \bar{\alpha}) \) is defined by (3). With \( \hat{Q}_t^c \) and \( \hat{Q}_t^b \) we denote the acquisition price of capital and R&D normalized by the price of materials, respectively, \( \delta_K \) and \( \delta_R \) denote the depreciation rates of capital and R&D, respectively, \( u_t \) is the corporate tax rate, and \( r \) is the constant (real) discount rate. Expectations are characterized with a caret ('). We maintain \( \bar{W}_t = W_t, \hat{Q}_t^c = Q_t^c \), and \( \hat{Q}_t^b = Q_t^b \). R&D expenditures are assumed to be expended immediately. The minimization problem (A1) represents a standard optimal control problem. Its solution is well known and implies the following system of quasi-fixed factor demand equations in accelerator form:¹³
\[ \Delta K_i = m_{kk}(K_i^* - K_{i-1}) + m_{kr}(R_i^* - R_{i-1}), \]
\[ \Delta R_i = m_{rk}(K_i^* - K_{i-1}) + m_{rr}(R_i^* - R_{i-1}), \]
where
\[
\begin{bmatrix}
K_i^* \\
R_i^*
\end{bmatrix} = - \begin{bmatrix}
\alpha_{kk} & \alpha_{kr} \\
\alpha_{rk} & \alpha_{rr}
\end{bmatrix}^{-1} \begin{bmatrix}
\alpha_k + \alpha_{kw} W_i + \alpha_{kr} T_i + C_i^k \\
\alpha_r + \alpha_{rw} W_i + \alpha_{rr} T_i + C_i^r
\end{bmatrix} \hat{Y}_i^{1/p},
\]
with \( C_i^k = Q_i^k (r + \delta_r)/(1 - u) \) and \( C_i^r = Q_i^r (r + \delta_r) \). The matrix of accelerator coefficients \( M = (m_{ij})_{i,j,K,R} \) has to satisfy the following matrix equation:
\[ BM^2 + (A + rB)M - A = 0; \]
(A3)

Furthermore, the matrix \( C = (c_{ij})_{i,j,K,R} = -BM \) is symmetric and negative definite. Unless we impose separability in the quasi-fixed factors, that is, \( \alpha_{kr} = 0 \), which implies \( m_{kr} = 0 \), (A3) cannot generally be solved for \( M \) in terms of \( A \) and \( B \). We can, however, solve (A3) for \( A \) in terms of \( M \) and \( B \): \( A = BM(M + rI) (I - M)^{-1} \). Since the real discount rate \( r \) was assumed to be constant, \( M \) is constant over the sample. Hence, instead of estimating the elements of \( A \) and \( B \), we may estimate those of \( M \) and \( B \). To impose the symmetry of \( C \) we can also estimate \( B \) and \( C \) instead of \( A \) and \( B \). Let \( D = (d_{ij})_{i,j,K,R} = -MA^{-1} \), and we observe that \( A = C - (1 + r)(B - B(C + B)^{-1}B) \) and that \( D = B^{-1} + (1 + r)(C - rB)^{-1} \) are symmetric. It is then readily seen that we can write (A2) equivalently as:
\[ \Delta K_i = d_{kk}[\alpha_k + \alpha_{kw} W_i + \alpha_{kr} T_i + C_i^k] \hat{Y}_i^{1/p} \]
\[ + d_{kr}[\alpha_k + \alpha_{kw} W_i + \alpha_{kr} T_i + C_i^k] \hat{Y}_i^{1/p} \]
\[ + [c_{kk}/\alpha_{kk}] K_{i-1} + [c_{kk}/\alpha_{kk}] R_{i-1}; \]
(A4)
\[ \Delta R_i = d_{kk}[\alpha_k + \alpha_{rw} W_i + \alpha_{rr} T_i + C_i^r] \hat{Y}_i^{1/p} \]
\[ + d_{rr}[\alpha_r + \alpha_{rw} W_i + \alpha_{rr} T_i + C_i^r] \hat{Y}_i^{1/p} \]
\[ + [c_{rr}/\alpha_{rr}] K_{i-1} + [c_{rr}/\alpha_{rr}] R_{i-1}; \]
where
\[ d_{kk} = 1/\alpha_{kk} + (1 + r) [c_{kk} - r\alpha_{kk}] / e, \]
\[ d_{kr} = 1/\alpha_{kk} + (1 + r) [c_{kk} - r\alpha_{kk}] / e, \]
\[ d_{rr} = -(1 + r)c_{rr}/e, \]
and
\[ e = (c_{kk} - r\alpha_{kk})(c_{kk} - r\alpha_{kk}) - c_{kk}^2. \]
The firm’s demand equations for the variable factors can be derived from the
normalized restricted cost function via Shephard’s lemma, as \( L_i = \partial G_{i,0}/\partial W_i \)
and \( M_i = G_{i,0} - W_i L_i \):

\[
L_i = \left[ (\alpha_u + \alpha_{uw}W_i + \alpha_{uw}T_i) \hat{Y}_i^{\gamma} + \alpha_{wx}K_{i-1} + \alpha_{wx}R_{i-1} \right],
\]

\[
M_i = \left[ (\alpha_o - 1/2\alpha_{ow}W_i) \hat{Y}_i^{\gamma} + \alpha_{ox}K_{i-1} + \alpha_{ox}R_{i-1} + \alpha_{ox}T_i \right] + \alpha_{ox}R_{i-1}T_i + \left[ \frac{1}{2}\alpha_{xx}K_{i-1}^2 + \alpha_{xx}K_{i-1}R_{i-1} + \frac{1}{2}\alpha_{xx}R_{i-1}^2 \right] / \hat{Y}_i^{\gamma},
\]

where

\[
\alpha_{xx} = c_{xx} - (1 + r)(\alpha_{xx} - (\alpha_{xx})^2(\alpha_{xx} + c_{xx})/f),
\]

\[
\alpha_{xx} = c_{xx} - (1 + r)(\alpha_{xx} - (\alpha_{xx})^2(\alpha_{xx} + c_{xx})/f),
\]

\[
\alpha_{xx} = c_{xx} - (1 + r)(\alpha_{xx} + c_{xx})/f.
\]

and

\[
f = (\alpha_{xx} + c_{xx})(\alpha_{xx} + c_{xx}) - c_{xx}^2.
\]

The complete system of factor demand equations consists of (A4) for the
quasi-fixed factors and (A5) for the variable factors. This system is nonlinear
in parameters and variables. For the empirical estimation, we have added sto-
chastic disturbance terms to each of the factor demand equations. When neces-
sary, we have corrected for first-order autocorrelation of the disturbances.
Expectations on gross output were calculated as follows. We first estimated a
first-order autoregressive model for output that was then used to predict \( Y \),
rationally.

Appendix B

Data Sources and Construction of Variables

U.S. Electrical Machinery Industry

Gross Output: Data on gross output in current and constant 1972 dollars
were obtained from the U.S. Department of Commerce, Office of Business
Analysis (OBA) data base and correspond to the gross output series of the
U.S. Department of Commerce, Bureau of Industrial Economics (BIE). Gross
output is defined as total shipments plus the net change in work in process
inventories and finished goods inventories.
Labor: Total hours worked were derived as the sum of hours worked by production workers and nonproduction workers. Hours worked by production workers were obtained directly from the OBA data base. Hours worked by nonproduction workers were calculated as the number of nonproduction workers times hours worked per week times 52. The number of nonproduction workers was obtained from the OBA data base. Weekly hours worked by nonproduction workers were taken to be 39.7. A series of total compensation in current dollars was calculated by multiplying the total payroll series from the OBA data base with the ratio of compensation of employees to wages and salaries from the U.S. Department of Commerce, Bureau of Economic Analysis (1981, 1984).

Materials: Materials in current dollars were obtained from the OBA data base. Materials in constant 1972 dollars were calculated using deflators provided by the U.S. Department of Commerce, Bureau of Economic Analysis.

Value Added: Value added in current and constant 1972 dollars was calculated by subtracting materials from gross output.

Capital: The net capital stock series in 1972 dollars and the current and constant 1972 dollar gross investment series were taken from the OBA data base. The method by which the capital stock series is constructed is described in the U.S. Department of Labor, Bureau of Labor Statistics (1979). The user cost of capital was constructed as $c_k = q^x(r + \delta_k)/(1 - \omega)$, where $q^x =$ investment deflator, $\delta_k =$ depreciation rate of the capital stock, $\omega =$ corporate tax rate, and $r = 0.05$.

R&D: The stock of total R&D is constructed by the perpetual inventory method with a depreciation rate $\delta_k = 0.1$. The benchmark in 1958 is obtained by dividing total R&D expenditures by the depreciation rate and the growth rate in real value added. The nominal R&D expenditures are taken from the National Science Foundation (1984) and earlier issues. To avoid double counting we have subtracted the labor and material components of R&D from the labor and material inputs. The gross domestic product (GDP) deflator for total manufacturing is used as a deflator for R&D.

All constant dollar variables were normalized by respective sample means. Prices were constructed conformably.

Japanese Electrical Machinery Industry

Gross Output: For the period 1970–80, the data series on gross output in current and constant 1975 yen were obtained from Economic Planning Agency (1984). The data for the period before 1970 were constructed by connecting these series with the corresponding series reported in Economic Planning Agency (1980) via identical growth rates.

Labor: Total hours worked were calculated as total numbers of employees times monthly hours worked times 12. For the period 1970–80, the number
of employees was taken from Economic Planning Agency (1984). For the period before 1970 the number of employees was calculated by connecting this series with the employment index provided by the Economic Planning Agency (EPA). Monthly hours worked for the period 1977–80 were obtained from the Statistics Bureau (1985). For previous years, monthly hours worked were calculated by using the monthly hours work index provided by the EPA. For the period 1970–80, total compensation is reported in the EPA (1984).

For the period before 1970, total compensation was calculated by connecting this series with an index on cash earnings provided by EPA.

Value Added: For the period 1970–80, data on value added in current and constant 1975 yen were obtained from the EPA (1984). The data for the period before 1970 were obtained by connecting these series with the corresponding series reported in the EPA (1975) via identical growth rates.

Materials: Materials in current and constant 1975 yen were calculated as the differences between gross output and value added.

Capital Stock: Data for the stock of capital and gross investment in 1975 yen were taken from the EPA (1985). A series for current dollar gross investment was obtained from the Japanese Ministry of Finance. This series was adjusted in such a way that it coincided with the constant yen EPA series in 1975. The user cost of capital was constructed analogously to that for the United States.

R&D: Current yen R&D expenditures are taken from Organization for Economic Cooperation and Development (1983 and earlier issues). To avoid double counting we have subtracted the labor and material component of R&D from the labor and material inputs. The GDP deflator for total manufacturing is used as the deflator for R&D. The stock of R&D is constructed analogously to that for the United States with 1965 as the benchmark year.

All constant yen variables were transformed to a 1972 base and then normalized by respective sample means. Prices were constructed conformably.

Appendix C

Expressions in TFP Growth Decomposition

In the following we give explicit expressions for the temporary equilibrium effect and the direct adjustment-cost effect in the decomposition (7) of total factor productivity growth. We make use of the cost-share weighted index of aggregate inputs $F$ defined as

$$
\Delta \ln F_t = \frac{1}{2}[\Delta \ln F^t + \Delta \ln F^{t-1}],
$$

$$
\Delta \ln F^t = s_N(\tau)\Delta \ln M^t + s_L(\tau)\Delta \ln L^t + s_K, (\tau)\Delta \ln K^t - 1
$$

$$
+ s_N, (\tau)\Delta \ln R^t - 1,
$$
where $\tau = t, t - 1$. The cost shares are defined as $s_\tau(t) = M_\tau/TC_\tau$, $s_{\tau-1}(t) = W_{t-1}/TC_\tau$, $s_{\tau-1}(t) = C_\tau^oK_{\tau-1}/TC_\tau$, $s_{\tau-1}(t) = C_\tau^oR_{\tau-1}/TC_\tau$, with $TC_\tau = M_\tau + W_{t-1} + C_\tau^oK_{\tau-1} + C_\tau^oR_{\tau-1}$. Here $C_\tau^o$ and $C_\tau^o$ denote, respectively, the rental price of capital and R&D normalized by the price of materials; compare appendix A. The following expressions for the temporary equilibrium effect $\phi_1$ and the direct adjustment cost effect $\phi_2$ are taken from Nadiri and Prucha (1983, 1990) and can be derived by comparing equations (4) and (6):

$$\phi_1 = \frac{1}{2p} \sum_{\tau=\tau}^{\tau-1} [(-\frac{\partial G_{\tau}/\partial K_{\tau-1} - C_\tau^o)}{G_{\tau}/\partial Y_{\tau}}) [\Delta ln K_{\tau-1} - \Delta ln F_{\tau}]]$$

$$+ \frac{1}{2p} \sum_{\tau=\tau}^{\tau-1} [(-\frac{\partial G_{\tau}/\partial R_{\tau-1} - C_\tau^o)}{G_{\tau}/\partial Y_{\tau}}) [\Delta ln R_{\tau-1} - \Delta ln F_{\tau}]],$$

$$\phi_2 = \frac{1}{2p} \sum_{\tau=\tau}^{\tau-1} [(-\frac{\partial G_{\tau}/\partial K_{\tau}}{G_{\tau}/\partial Y_{\tau}}) [\Delta ln K_{\tau} - \Delta ln F_{\tau}]]$$

$$+ \frac{1}{2p} \sum_{\tau=\tau}^{\tau-1} [(-\frac{\partial G_{\tau}/\partial R_{\tau}}{G_{\tau}/\partial Y_{\tau}}) [\Delta ln R_{\tau} - \Delta ln F_{\tau}]],$$

where $(\tau = t, t - 1)$. In long-run equilibrium both the temporary equilibrium effect and the direct adjustment-cost effect are zero since $\partial G/\partial K + C^o = \partial G/\partial R + C^o = \partial G/\partial K = \partial G/\partial R = 0$. Furthermore both effects are zero if all factors (and hence the aggregate input index) grow at the same rate.

Notes

1. The total factor productivity growth rates are calculated from the Törnqvist approximation formula (using long-run cost shares). The divergence in total factor productivity growth rates is much more pronounced in a value-added measurement framework. However, Norsworthy and Malmquist (1983) found that such a framework is inappropriate—at least at the total manufacturing level.

2. The restricted cost function $G(\cdot)$ is furthermore assumed to satisfy $G_{x_i} < 0, G_{x_j} > 0, G_x < 0, G_x > 0$.

3. Clearly the scale elasticity depends for general $F(\cdot)$ on the various factor inputs. However, to keep the model specification reasonably parsimonious, we have assumed that $F(\cdot)$ is homogeneous of constant degree $\rho$.


6. These coefficients have been calculated from the estimates in table 4 observing that $M = -B^{-1}C$.

7. We note that those adjustment speeds are consistent with earlier results obtained by Mohren, Nadiri, and Prucha (1986) for the total manufacturing sectors of the two countries.
8. Let \((X_{t}, V_{t})_{t=0}^{\infty}\) denote the optimal plan values of the inputs in periods \(t, t + 1, \ldots\), corresponding to the firm's optimization problem in period \(t\). Short-run, intermediate-run, and long-run elasticities then refer to the elasticities of \(X_{t}\), and \(V_{t}\), in periods \(\tau = 0, 1, \ldots, \infty\), respectively (\(X_{t} = X_{t}^\tau\)).

9. The contribution of each of the variables is calculated by multiplying the respective (average) elasticities with the growth rate of the corresponding variable. The output elasticities are computed from the estimated restricted cost function using standard duality theory. For both variable and quasi-fixed factors, those output elasticities exceed long-run cost shares because of increasing returns to scale. For the quasi-fixed factors the output elasticities also differ from long-run cost shares because of adjustment costs.

10. This approximation is readily obtained from the decomposition of output growth by noting that the sum of the output elasticities must equal scale.

11. For an excellent discussion of problems in measuring technical change see Griliches (1988).

12. Note that in this table technical change corresponds to \(\lambda_t = \lambda_t p\) while in tables 4.8 and 4.9 technical change corresponds to \(\lambda_t\). Furthermore, note that the decompositions of output growth and labor productivity growth in tables 8 and 9 only gives the direct effect of adjustment costs. The "indirect" temporary equilibrium effect in those decompositions is accounted for by using (estimated) output elasticities rather than cost shares as weights.


14. Such a reparametrization was first suggested by Epstein and Yatchew (1985) for a somewhat different model with a similar algebra. Mohlen, Nadiri, and Prucha (1986) used such a reparametrization within the context of a constant returns to scale model. Recently Madan and Prucha (1989) generalized this approach to the case where \(B\) may be nonsymmetric.

References


Grossman, E. S. 1985. Productivity and international competition: United States and


Productivity Growth in Japan and the United States

Edited by Charles R. Hulten

Prefatory Note

Introduction
Charles R. Hulten

Dale W. Jorgenson and Masahiro Kuroda

Comment: Robert M. Schwab

Dale W. Jorgenson, Hikaru Sakuramoto, Kanji Yoshioka, and Masahiro Kuroda

Melvin Fuss and Leonard Waverman

4. Comparison and Analysis of Productivity Growth and R&D Investment in the Electrical Machinery Industries of the United States and Japan
M. Ishaq Nadiri and Ingmar R. Prucha

Catherine Morrison

Comment: Ingmar R. Prucha

Ernst R. Berndt, Shun'neke Mori, Takamitsu Sawa, and David O. Wood

Comment: Kanji Yoshioka

7. Productivity Growth and Changes in the Terms of Trade in Japan and the United States
Catherine Morrison and W. Erwin Diewert

Edwin Dean, Masako Darrough, and Arthur Neef

Comment: Masahiro Kuroda

9. The Taxation of Income from Capital in Japan: Historical Perspectives and Policy Simulations
Tatsuya Kitazaki and Toshiaki Tachibanaki

10. Taxes and Corporate Investment in Japanese Manufacturing
Fumio Hayashi

Zvi Griliches and Jacques Maresh

Comment: Edwin Mansfield

Reply: Zvi Griliches and Jacques Maresh

12. Compositional Change of Heterogeneous Labor Input and Economic Growth in Japan
Hajime Imamura

Comment: Walter Y. Oi

13. Technical Change and Human Capital Acquisition in the U.S. and Japanese Labor Markets
Hong W. Tan

Comment: Romesh Dhan

14. Labor Disputes and Productivity in Japan and the United States
Alice C. Lam, J. R. Norworth, and Craig A. Zabala

Comment: Mary Jean Bowman

List of Contributors

Author Index

Subject Index
Order Form
The University of Chicago Press

Order Department
The University of Chicago Press
11030 South Langley Avenue
Chicago, Illinois 60628

Please send me the following books. I understand that if for any reason I am not completely satisfied, they may be returned within ten days for a full refund or cancellation of charges.

<table>
<thead>
<tr>
<th>ISBN PREFIX: 0-226-36059-8</th>
<th>AUTHOR/TITLE</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C. Hulten, ed./PRODUCTIVITY GROWTH</td>
<td>$60.00</td>
</tr>
</tbody>
</table>

Check, money order, or credit card number must accompany orders from individuals.

☐ Check or money order enclosed. Publisher pays postage.

(Add 8% sales tax for orders sent to Chicago addresses; 7% for other Illinois addresses)

Charge to my ☐ MasterCharge ☐ Visa

Credit card number

Bank ID (MasterCharge only)

Credit card expires

Signature

Name

Address

City ___________________________ State ___________ Zip ___________

AD 0002
On the specification of accelerator coefficients in dynamic factor demand models

Ingmar R. Prucha
University of Maryland, College Park, MD 20742, USA
National Bureau of Economic Research, Cambridge, MA 02138, USA

M. Ishaq Nadiri
New York University, New York, NY 10003, USA
National Bureau of Economic Research, Cambridge, MA 02138, USA

Received 30 April 1990
Accepted 29 May 1990

Recent empirical application of dynamic factor demand models employed different formulae for the accelerator coefficient. In this note we compare and analyze the respective formulae used for the accelerator coefficient.

1. Introduction

Empirical applications of dynamic factor demand models are often based on a linear-quadratic specification of the firm's (restricted) profit or cost function. Typically the derived demand equations for the quasi-fixed factors can then be formulated in form of an accelerator model. Different empirical specifications of the accelerator model corresponding to different formulae for the accelerator coefficient have been considered in the literature. Differences in the formulae for the accelerator coefficients result from differences in the assumptions maintained with respect to the firm's planning horizon. Differences also result since in some empirical applications the firm's control problem is directly formulated in discrete time while in others it is first formulated in continuous time. In the latter case the continuous time solution is then approximated by a discrete time analog for empirical implementation. The discrete time analog of the continuous time solution typically employed in the empirical literature differs from the solution obtained if the firm's control problem is directly formulated in discrete time.

* We would like to thank Ernst R. Berndt for helpful comments. We retain, however, full responsibility for any shortcomings.

Section 2 compares the accelerator coefficients corresponding to a discrete time infinite horizon model with those corresponding to the discrete time finite horizon model introduced in Prucha and Nadiir (1982). This model assumes that the firm calculates the present value of the profit or cost stream beyond the actual planning horizon under the simplifying assumption of static expectations and a constant firm size. Some numerical results reported in Prucha and Nadiir (1986) and Nadiir and Prucha (1989b) suggest that the finite horizon model approximates the infinite horizon model well even for moderate sizes of the planning horizon. In this paper we utilize results by Lewis (1982) on explicit forms on the inverse of tridiagonal matrices to present an explicit analytic expression for the optimal control solution and, in particular, for the accelerator coefficient corresponding to the finite horizon model. We utilize those analytic expressions to compare the accelerator coefficient corresponding to the infinite and finite horizon model numerically for a wide range of parameter values. The explicit analytic expressions also provide theoretical insight as to how the solution corresponding to the finite horizon model converges to the solution corresponding to the infinite horizon model as the planning horizon is extended towards infinity.

In section 3 we provide a numerical comparison of the accelerator coefficient derived from the discrete time infinite horizon model with the accelerator coefficient corresponding to a continuous time infinite horizon model, and discuss potential problems if the latter is used in empirical applications.

Since explicit analytic expressions for the accelerator coefficients corresponding to the infinite horizon model are generally not available in case of several non-separable quasi-fixed factors we restrict the following discussion to the case of a single quasi-fixed factor. For definiteness of the exposition we consider a profit maximizing firm. Since, in essence, only the mathematical structure of the first-order conditions is of importance, the subsequent analysis also applies readily to the case of a cost minimizing firm. Variable factors can be readily incorporated by working with a restricted profit or cost function.

2. Accelerator coefficients: Infinite vs. finite horizon specification

Consider a firm that produces a single output good \( y_t \) from a single quasi-fixed factor \( x_t \). More specifically, assume that the firm's technology is described by the following (generalized) linear-quadratic production function:

\[
y_t = F(x_{t-1}, \Delta x_t) = \alpha_0 + \alpha_k x_{t-1} + \left(1/2\right)\alpha_{xx} x_{t-1}^2 + \left(1/2\right)\alpha_{xx} \Delta x_t^2,
\]

where \( \Delta x_t = x_t - x_{t-1} \) appears in the production function to model internal adjustment costs in terms of forgone output due to changes in the quasi-fixed factor. To ensure that the production function satisfies the usual concavity conditions we maintain \( \alpha_{XX} > 0 \) and \( \alpha_{xx} > 0 \). The firm is assumed to maximize the present value of current and future profits. The firm's profits in period \( t \) are given by:

\[
\Pi(x_t, x_{t-1}, q_t) = F(x_{t-1}, \Delta x_t) - q_t(x_t - (1 - \delta)x_{t-1}),
\]

where \( q_t \) denotes the after tax acquisition price of the quasi-fixed factor normalized by the output price and \( \delta \) denotes the depreciation rate.

---

3 Of course, the subsequent analysis also applies trivially to the case of several separable quasi-fixed factors, since separability essentially allows us to solve a separate control problem for each quasi-fixed factor.
First consider the case of an infinite planning horizon and assume that the firm's objective is to maximize the expected present value of current and future profits. The firm's profit function is linear-quadratic. Therefore the well-known certainty equivalence principle implies that we can find the firm's closed loop feedback optimal control solution more conveniently if in each period \( t \) we find and select the quasi-fixed input \( x_t \) such that it maximizes the following certainty equivalence analog of the firm's objective function:

\[
\lim_{t \to \infty} \sum_{\tau=0}^{\infty} \Pi(x_{t+\tau}, x_{t+\tau-1}, E_t q_{t+\tau})(1+r)^{-\tau}.
\]

(3)

Here \( E_t \) denotes the expectations operator conditional on information available at time \( t \) and \( r > 0 \) denotes the real discount rate. The above optimal control problem is standard [cf. Hansen and Sargent (1980, 1981), Kollintzas (1985, 1986) and Madan and Prucha (1989)].* The input sequence optimizing (3) must satisfy the following set of Euler equations:

\[
-x_{t+\tau+1} + \left[ \phi + 2 + r \right] x_{t+\tau} - (1+r) x_{t+\tau-1} = a_{t+\tau}, \quad \tau = 0, \ldots, \infty,
\]

(4)

where \( \phi = \alpha_{XX}/\alpha_{X}\Gamma \) and

\[
a_{t+\tau} = \left( 1/\alpha_{XX} \right) \left[ -\alpha_{X} + E_t q_{t+\tau} (1+r) - E_t q_{t+\tau+1} (1-\delta) \right].
\]

The roots of the characteristic polynomial corresponding to (4) are given by

\[
\lambda_{1,2} = (1/2) \left\{ \phi + 2 + r \mp \left[ (\phi + r)^2 + 4\phi \right]^{1/2} \right\}.
\]

(5)

Clearly \( \lambda_1, \lambda_2 = 1 + r \) with \( \lambda_1 \) less than unity and \( \lambda_2 \) greater than unity. The optimal control solution for \( x_t \) is uniquely determined by the following accelerator model [cf., e.g., Madan and Prucha (1989)]:

\[
x_t = mx_t^* + (1-m) x_{t-1},
\]

(6)

\[
m = 1 - \lambda_1 = -\left( 1/2 \right) \left\{ \phi + r - \left[ (\phi + r)^2 + 4\phi \right]^{1/2} \right\},
\]

\[
x_t^* = m^{-1} \sum_{\tau=0}^{\infty} \gamma_{t-\tau}, \quad \gamma_{t-\tau} = \left[ \lambda_1/(1+r) \right]^{t-\tau+1}.
\]

Next consider the case of a finite but shifting planning horizon. Following Prucha and Nadiri (1982, 1986) and Nadiri and Prucha (1989b) we assume that the firm chooses its inputs under the simplifying conditions of static expectations and a constant firm size beyond the actual planning horizon. That is, we assume that the firm maximizes (3) in each period \( t \) subject to the constraints \( x_{t+\tau} = x_{t-\tau} \) and \( E_t q_{t+\tau} = E_t q_{t-\tau} \) for all \( \tau \geq T \). The objective function of the firm is then given by:

\[
\sum_{\tau=0}^{T} \Pi(x_{t+\tau}, x_{t+\tau-1}, E_t q_{t+\tau})(1+r)^{-\tau} + \Pi(x_{t-T}, x_{t-T}, E_t q_{t-T})(1+r)^{-T}.
\]

(7)

* As usual in the literature, we restrict the solution space to the class of processes \( x_t \) that are of mean exponential order less than \( (1+r)^{1/2} \). This ensures, together with the assumption that \( q_t \) is of mean exponential order less than \( (1+r)^{1/2} \), the finiteness of the objective function for all processes in the solution space.
The term

\[ \Pi(x_{t+\tau}, x_{t+\tau}, E_t q_{t+\tau})/[r(1+r)^\tau] = \sum_{\tau=T+1}^{\infty} \Pi(x_{t+\tau}, x_{t+\tau}, E_t q_{t+\tau})(1+r)^{-\tau} \]

represents the present value of the stream of profits the firm obtains by maintaining its operation beyond the actual planning horizon at the same level as at the end of the actual planning horizon. Clearly, the input sequence optimizing (7) must satisfy:

\[ -x_{t+\tau} + [\phi + 2 + r]x_{t+\tau} - (1+r)x_{t+\tau} = a_{t+\tau}, \quad \tau = 0, \ldots, T-1, \]

\[ [\phi + r]x_{t+\tau} - r x_{t+\tau} = a_{t+\tau}, \]

with \( a_{t+\tau} \) for \( \tau = 0, \ldots, T-1 \) as defined above and \( a_{t+\tau} = (1/\alpha \gamma X - x + E_t q_{t+\tau}(t + \delta)) \). The optimal quasi-fixed input in period \( t \) defined by (8) is given by the following accelerator model (proof on request from author)

\[ x_t = m_t x_t^* + (1 - m_t)x_{t-1}, \]

\[ m_t = \left[ \sigma_1(\lambda_1/\lambda_2)^{T+1} - \sigma_2 / \left[ \sigma_1(\lambda_1/\lambda_2)^{T+1} - \sigma_2 \right] \right], \]

\[ x_t^* = m_t^{-1} \sum_{t=0}^{T} \gamma_{t,T} a_{t+\tau}, \]

\[ \Theta_{t,T} = \frac{\sigma_1(\lambda_1/\lambda_2)^{T+1} - \sigma_2 / \left[ \sigma_1(\lambda_1/\lambda_2)^{T+1} - \sigma_2 \right] \gamma_{t,T} a_{t+\tau}}, \]

where \( \sigma_1 = (1+r)(\lambda_1 - 1)(\lambda_2 - 1), \sigma_2 = (\lambda_1 - (1+r))^2, \sigma_3 = (1+r)(\lambda_1 - 1)^2 \) and \( \gamma_{t,T} = \gamma_t \) for \( \tau = 0, \ldots, T-1 \) and \( \gamma_{t,T} = [(1+r)/r]^{T-t} \). Since \( \lambda_1/\lambda_2 < 1 \) we see immediately upon inspection of the expressions in (9) that \( m_t \to m \) and \( \Theta_{t,T} \to 1 \) as \( T \to \infty \) (for fixed \( t = 0, \ldots, 1 \ldots \)). The speed of convergence depends only in the magnitude of the ratio \( \lambda_1/\lambda_2 = \lambda_1^2/(1+r) \); the smaller the

---

Table 1

Comparison of the accelerator coefficients \( m \) and \( m_T \) for five, ten and twenty period planning horizons (i.e., \( T = 4, 9, 19 \)).

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \phi )</th>
<th>( m )</th>
<th>( m_T )</th>
<th>( m )</th>
<th>( m_T )</th>
<th>( m )</th>
<th>( m_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>( T = 4 )</td>
<td>( T = 9 )</td>
<td>( T = 19 )</td>
<td>( T = 4 )</td>
<td>( T = 9 )</td>
<td>( T = 19 )</td>
<td>( T = 4 )</td>
</tr>
<tr>
<td>0.02</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.03</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.04</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.05</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.06</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.07</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.08</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 2
Comparison of the accelerator coefficients $m$ and $m^*$.

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$r = 0.02$</th>
<th>$r = 0.05$</th>
<th>$r = 0.08$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m$</td>
<td>$m^*$</td>
<td>$m$</td>
</tr>
<tr>
<td>1000.000</td>
<td>0.999</td>
<td>99.989</td>
<td>0.999</td>
</tr>
<tr>
<td>1000.000</td>
<td>0.998</td>
<td>31.612</td>
<td>0.998</td>
</tr>
<tr>
<td>100.000</td>
<td>0.990</td>
<td>9.990</td>
<td>0.989</td>
</tr>
<tr>
<td>10.000</td>
<td>0.914</td>
<td>5.152</td>
<td>0.912</td>
</tr>
<tr>
<td>1.000</td>
<td>0.612</td>
<td>0.990</td>
<td>0.604</td>
</tr>
<tr>
<td>0.500</td>
<td>0.493</td>
<td>0.697</td>
<td>0.483</td>
</tr>
<tr>
<td>0.100</td>
<td>0.261</td>
<td>0.306</td>
<td>0.250</td>
</tr>
<tr>
<td>0.050</td>
<td>0.191</td>
<td>0.213</td>
<td>0.179</td>
</tr>
<tr>
<td>0.010</td>
<td>0.086</td>
<td>0.090</td>
<td>0.074</td>
</tr>
<tr>
<td>0.001</td>
<td>0.023</td>
<td>0.023</td>
<td>0.015</td>
</tr>
</tbody>
</table>

magnitude of this ratio the higher the speed of convergence. Under static expectations it is readily seen that $x_{n+1}^* = x_n^*$.

We note that $\lambda_1$ and $\lambda_2$ and the accelerator coefficients $m$ and $m_T$ only depend on $\Phi$ and $r$. In table 1 we compare those accelerator coefficients for different values of $\Phi$ and $r$ and different sizes of the planning horizon. Since $\phi = \alpha_{xx}/\alpha_{xx}$, large values of $\phi$ correspond to small adjustment costs and vice versa (for a given value of $\alpha_{xx}$). In case of zero adjustment costs, i.e. $\alpha_{xx} = 0$, we have $\phi = \infty$. The results in table 1 show that the finite horizon model approximates the infinite horizon model very closely already for moderate sizes of the planning horizon, even for small values of $\phi$ and $r$; for practical purposes a planning horizon of 10 period seems generally sufficient. This demonstrates that previous results reported in Prucha and Nadiri (1986) and Nadiri and Prucha (1989b) for, essentially, specific values of $\phi$ and $r$ hold more generally.

3. Accelerator coefficients: Continuous time vs. discrete time solution

In deriving dynamic factor demand equation for empirical estimation we can, in principle, first solve a continuous time optimal control problem and then specify the estimating equations as discrete time approximations to the continuous time optimal control solution. This approach was, e.g., taken by Berndt, Fuss, and Waverman (1977, 1979, 1980), Berndt, Morrison and Watkins (1981), Denny, Fuss and Waverman (1981), Morrison (1986), and Morrison and Berndt (1981). The discrete approximations of the accelerator equations (obtained from a continuous time model) that were utilized in this important literature were generally also of the form (6) but with the accelerator coefficient $m$ replaced by

$$m^* = -(1/2) \left( r - \sqrt{r^2 + 4\phi} \right)^{1/2}.$$  

In the following we analyze the properties of this formula in more detail. Clearly, in the case of zero adjustment costs, i.e. $\phi = \infty$, the (discrete time) accelerator coefficient should equal unity to allow for instantaneous adjustment, i.e. $x_n = x_{n+1}^*$. It is not difficult to see that $m \to 1$ and $m_T \to 1$ as

---

5 We emphasize that the solution of the finite horizon model considered here is such that the contributions of the unstable root tend to zero as the planning horizon extends to infinity.
\( \phi \rightarrow \infty \). However, as was pointed out briefly in Mohnen, Nadiri and Prucha (1983), \( m^* \rightarrow \infty \) as \( \phi \rightarrow \infty \). Consequently, specifications of empirical accelerator models that utilize (10) as the formula for the accelerator coefficient violate basic theoretical properties. Also, while accelerator models defined by (6) or (9) allow for proper tests of the hypothesis of zero adjustment costs, this is not the case for specifications based on (10).

In table 2 we compare \( m^* \) with \( m \) for different values of \( r \) and \( \phi \). As expected, the table shows a dramatic divergence of \( m^* \) and \( m \) for small adjustment costs. However, on the positive side, the table also reveals that the values for \( m^* \) and \( m \) are similar for the magnitudes of accelerator coefficients often reported in the empirical literature on capital investment. We have furthermore compared full information maximum likelihood estimates for \( m^* \) and \( m \) from several empirical examples and found that the divergence for those estimates is similar but (especially for high values of the accelerator coefficients) less pronounced as compared to that for the actual accelerator coefficients reported in table 2.

References


THE NEW PALGRAVE
A DICTIONARY OF ECONOMICS

EDITED BY
JOHN EATWELL
MURRAY MILGATE
PETER NEWMAN

Volume 2
E to J

M
Joint production. The network of cost relationships in a multi-product firm is much more complicated than in a single product one and the nature of these relationships has important implications for the structure and size of the firm, the organization and regulation of the industry, and the pattern and intensity of resource employment. The general neoclassical multi-product/multi-input decision framework presupposes that the firm is producing more than one output, but the central question is why do firms diversify into multiple outputs. The answer may lie in the characteristics of the cost functions and the nature of the demand facing the firm. We shall therefore briefly discuss: (1) the main causes of joint production, (2) the econometric techniques proposed for testing the presence of joint production and (3) the implication of joint production for the organization of the industry.

CAUSES OF JOINT PRODUCTION.

Joint production includes two cases: (1) when there are multiple products, each produced under separate production processes - i.e. the production function is non-joint; and (2) when several outputs are produced from a single production process. In the first case 'joint production' is a problem of aggregation while in the second case it is a technological phenomenon of 'intrinsic jointness'. Thus, writing a production function or cost function with several inputs is by itself not an evidence of joint production, it is the absence of non-jointness which is a crucial test.

Recent literature has identified a variety of reasons for joint production but three cases stand out: economizing of some shareable inputs or economies of scope; jointness due to output interactions; and uncertainty on the demand side.

(1) Economies of scope. Suppose that a vector of outputs \( y = (y_1, \ldots, y_s) \) and a vector of primary inputs \( x = (x_1, \ldots, x_n) \) are technically related by the production structure characterized by its dual, the joint cost function \( C = g(w, y) \) where \( w = (w_1, \ldots, w_k) \) is the vector of input prices. Further assume that the cost function is non-additive with respect to all partitions of the commodity set. Economy of scope is defined for the partition of commodity set, \( h \), as

\[
\sum_{j} C(x_j, w) = \sum_{j} [C(x_j, w)],
\]

where \( \sum C(x_j, w) \) is the total cost when each commodity is produced separately (Lloyd, 1983). Economies (diseconomies) of scope will exist by this definition for a given partition of commodities.

Economies of scope may arise from fixed inputs such as physical and human capital that are shared or utilized without complete congestion. Some fixed inputs may be imperfectly divisible and could not easily be shifted from one production to another so that the production of a subset of commodities may leave excess capacity in some stage of production. Another possibility is that some of the inputs may have a quasi-public characteristic which when purchased for use in one production process can be at least partially used in the production of other commodities.

(2) Economies of scope may also be due to interrelationships among products: two or more commodities may be produced jointly, at lower cost than if they were produced separately even in the absence of excess capacity and shareable inputs in the production process. An example will be a production process where \( y_1 = f(x) \) but \( y_2 = f(x', y_1) \) which characterizes many industrial and agricultural production processes. Another example is the case where it is not possible to produce zero quantities of the commodities produced jointly, i.e. the multiple output-input function \( F(y, x) = 0 \) is restricted to the combination \( (x, y) \) that precludes all of the output vector \( y \) to be zero. Examples of such production can be found in agriculture (wool and mutton) and some chemical processes.

(3) Demand conditions are also important for the product structure of the firms; firms may avoid declines in revenue because of market saturation by producing new products, thereby substituting economies of scope for the economies of scale that the firm cannot achieve given the market conditions it faces. Another reason for joint output is attributed to uncertainty and risk aversion (Lloyd, 1983). Firms choose commodity diversification as a strategy to reduce risk in an environment of uncertainty though no jointness exists in their production process. Suppose a firm's profits from each commodity is random because the output and input prices are random variables; the firm maximizes the expected utility of aggregate profit given the joint probability distribution of the random variables. Under these sets of assumptions the firm will produce multiple outputs even though there is no technological reason for doing so. The presence of uncertainty plays the same role as shareability of input or intrinsic jointness of output in generating economies of scope.

ECONOMETRICS OF JOINT PRODUCTION. A major problem has been the difficulty of specifying a sensible and estimable functional form for the multi-product technology. The flexible functional form developed by Christensen et al. (1973), Diewert (1971), and Lau (1978) has made it possible to use the flexible production or cost functions, and particularly the translog cost or profit functions, to approximate multiple output technology. Other more suitable joint cost functions can be formulated but since the translog cost function is often used in empirical studies, we employ it for illustrative purposes. Consider the cost function

\[
\ln C = \ln x_0 + \sum_j \ln x_j + \sum_i \beta_i \ln y_i + \frac{1}{2} \sum_{i,j} \gamma_{ij} \ln x_i \ln x_j + \frac{1}{2} \sum_i \delta_i \ln y_i + \ln w \]

(1)

which is a quadratic approximation to an arbitrary multiple output cost function. The nature of the cost relationships can be tested by imposing the necessary parameter restrictions. For example, if it turns out that \( \gamma_{ij} = \delta_i = \beta_j \), then the cost function is separable, i.e. the ratio of any two marginal costs is independent of factor prices or factor intensities; then the cost function can be written as \( C(y, w) = H(y)k(w) \). Another important feature of the production structure is non-jointness which is that total cost of producing all outputs be the same as the sum of the cost of producing each output separately, i.e.,

\[
C(y, w) = \sum_i C_i(y_i, w_i) = \sum_i [C_i(y_i, w_i)],
\]
$C(y, w) - \Sigma_{i} g_{i}(y, w)$. This implies that the marginal cost of each output is independent of the level of any output. In terms of (1) the condition of non-jointness is $\delta_{i} = -\beta_{i}$, for $k \neq l$. Hall (1972) has shown that no multiple output technology with constant return to scale can be both separable and nonjoint; in fact all nontrivial separable technologies are inherently joint and cannot be used empirically to test hypotheses about jointness.

Ordinary translog cost function (1) and (cost function with logarithmic output variables) is inappropriate to measure economies of scope. By definition, if any of the outputs is zero the multiple product firm's cost will be zero, which suggests that if a firm specializes completely in one of the outputs it must incur no costs whatsoever. To overcome this problem it is necessary to modify (1) by performing a Box-Cox transformation on output variables, i.e. substitute $x_{i}^{*} = (x_{i} - 1)/\lambda$, where $\lambda$ is a parameter to be estimated. Another possibility is to formulate alternative cost functions such as the linear generalized Leontief joint cost function proposed by Hall (1972) or the CES multiple output cost function stated in the next section. Both are well defined for zero output levels.

Note two other issues: when allocable fixed inputs are the sole cause of jointness the dual production models (multi-product costs or profit functions) are not very useful because they can recover the production function in the sum of the inputs and not in terms of individual allocations. This arises because all of the input allocations have the same market price, which enters the cost and profit functions. Appropriately specified primal model would permit identification of such allocations (Shumway et al., 1984). Also, a Giffen effect may arise in the case of multiproduct production. As the direct substitution effect of a change in price may not be negative for a factor in a single product line, although it will be over all lines. Moreover, the cross-substitution effect may not be equal in an individual product line (Hughes, 1981).

The measures of economies of scale and scope are, respectively,

$$S = \frac{\lambda}{\log C^{-1}} \cdot \bigg( \frac{\text{log } C^{-1}}{\text{log } x_{i}^{*}} \bigg)$$

and

$$S_{x} = \frac{C(y_{x}^{*}, 0) + \cdots + C(0, y_{x}^{*}) - C(y_{x}^{*}, \ldots, y_{x}^{*})}{C(y_{x}^{*}, \ldots, y_{x}^{*})}$$

and the relationship between them is shown to be

$$\sum_{i=1}^{n} \beta_{i} S_{i} = \frac{1}{1 - S_{x}}$$

where $S_{i}$ are measures of product specific economies of scale and $\beta_{i}$ are roughly equal to the share of the variable cost of producing each output. If there is a sufficiently large economy of scope, it could result in economies of scale on the entire product set even if there is a constant return or some degree of diseconomies of scale in the separate products.

A number of econometric studies summarized by Bailey and Friedlaender (1982) and those by Denny and Pinto (1978), Brown et al. (1979), Griffen (1977), Vincent et al. (1980), Just et al. (1983), and others have shown that at industry level (particularly in agriculture) multiple output production technologies prevail with differing degrees of jointness and economies of scope. However, further studies are required. Particularly, the role of technological progress in changing the intertemporal structure of the cost relations by unbundling some joint costs and giving rise to new ones requires considerable attention.

INDUSTRY STRUCTURE. Multi-product technology has important implications for the organization and regulation of industry. The characteristics of the underlying cost relationships could determine the optimal number of firms that may populate an industry; the industry may be dominated entirely by a single firm producing all of the output or may be characterized by duopolistic oligopolistic or competitive forms. Baumol (1977) among others has formulated conditions for natural monopoly to prevail. When the cost function is subadditive the efficient supply condition is a single firm that can produce industry output at lower costs than two or more firms. The degree of contestability in many multiproduct industries depends to a great extent on the nature of the multiple output cost function. For example, consider the cost function

$$C(y_{1}, \ldots, y_{n}) = \frac{F + \left( \sum_{i=1}^{n} \gamma_{i} y_{i} \right)^{-1}}{\beta}$$

where $F > 0$ is the fixed cost. Depending on the parameter values of (2) four market structure possibilities can be identified: (1) if $(F > 0, \beta > 1, \gamma > 0)$, the industry is a natural monopoly; (2) if $(F = 0, x < 1, \beta > 0)$, the industry is competitive; (3) if $(F = 0, \beta > 0, \gamma > 0, \beta > 1)$, $n$ specialized firms each producing the industry output of the specialized good will constitute the industry; (4) finally, if $(F > 0, \beta > 0, \gamma > 0, \beta > 1)$, at small levels of output, either a single firm ($\beta > 1$) or a number of specialized firms ($\beta < 1$) will populate the industry while at large levels of output, several smaller specialized firms will constitute the industry. Similar experiments can be carried out with the modified translog cost function (1).

The degree of contestability deduced from the characteristics of the cost relations (given sustainable prices) has policy implications for the entry of new firms, the degree of concentration in a market, and antitrust laws. In contestable markets, mergers may not be anticompetitive; the theory of joint production is also important for considering the boundary issue between regulated and unregulated portions of an industry and the related problem of cross-subsidization. Another policy concern is the potential pathological substitution effects in multiple-production processes that, at least in the short run, may lead to possible bottlenecks in factor utilization and may to some extent negate the effect of particular policies.

M. ISHAQ NAZIRI

See also COST CURVES, COST FUNCTIONS, DUALITY, VON NEUMANN TECHNOLOGY.

BIBLIOGRAPHY


1029
THE NEW
PALGRAVE
A DICTIONARY OF ECONOMICS

EDITED BY
JOHN EATWELL
MURRAY MILGATE
PETER NEWMAN

Volume 3
K to P

M
sufficient, it is maintained that output can be expanded more than proportionately with the labour employed in manufacture (increasing returns to scale).

Mars used these two examples to draw a distinction between the 'heterogeneous' manufacture (exemplified by Petty's watch-making activity) in which the final output is obtained by simple assemblage of 'partial and independent products', and the more sophisticated 'organic' manufacture (exemplified by Smith's pin factory) in which a series of successive operations gradually transforms the original raw material into the finished product.

Smith referred to three arguments in favour of the technical superiority of an ever increasing division of labour:

1. first, to the increase of dexterity in every particular workman;
2. secondly, to the saving of the time which is commonly lost in passing from one species of work to another;
3. lastly, to the invention of a great number of machines which facilitate and abridge labour, and enable one man to do the work of many (Smith, 1776, p. 17).

It has been observed that these arguments are not truly convincing. The importance attributed to increased dexterity conflicts with the relatively low level of skills required in contemporary factories (witness the common use of child labour). Time saving does not imply specialization by individuals: in principle, it could equally be attained by a suitable reorganization of the activity of a single artisan. And the introduction of machines does not seem to exhibit any necessary relation to the increasing division of tasks.

In fact, the new organization of labour associated with the factoy system did go along with the process of technical change associated with the industrial revolution. But its original role was primarily to discipline the manner in which the work was performed and to give the capitalist the power of controlling the production process in every single detail.

The introduction of machinery came after labour specialization and reinforced the need for a thorough organization of production. The effects of the introduction of the steam-engine and other complex machines were eventually studied by two scholars who possessed the necessary technical background, Charles Babbage (1832) and William Ure (1835), whose treatises were very popular at the time and were widely used by the economists (e.g. by John Stuart Mill and Marx). They conceived the control and management of a factory as that of a single complex machine, under the full control of the capitalist and with manual work brought to a minimum.

It is worth noticing that these speculations about the rational management of a highly mechanized factory were easily extended to society as a whole. At the turn of the century, Mikhail Tugan-Baranovsky (1905) dreamed of an economy in which machines were automatically produced by machines, and where the labour force was paradoxically reduced to one worker alone. In a similar vein, especially in Germany after World War I, we find many suggestions for a 'rational' organization of the economy as if it were a giant Konzern (as an extreme example, see the 'natural economy' proposed by Otto Neurath (1921) for the ephemeral Bavarian republic).

Giorgio Gilibert

See also INCREASING RETURNS

BIBLIOGRAPHY


production, modes of. See MODES OF PRODUCTION.

production: neoclassical theories. The economic theory of production is concerned with the characterization of the input demand and output supply functions based on a theory of profit maximization subject to a production function. Two sets of issues are involved: one is the technical constraint that describes the range of production processes available to the firm, and the other is the make-up of the markets where the firm's transactions take place. There is a substantial literature on the latter, which cannot be addressed here: we adopt the admittedly unrealistic assumption of 'perfect competition' in both commodity and factor markets. Our purpose is to discuss the properties of the production technology in the context of the neoclassical theory of the multiple-product and multiple-input firm, identify the specific forms of the production function which are proposed in the literature, and discuss the duality principles as well as some of the new dynamic factor demand model analyses.

NEOCLASSICAL THEORY OF PRODUCTION. Consider a firm that produces $m$ products and employs $n$ inputs; its objective is to maximize profits given as:

$$\Pi = \sum_{i=1}^{m} p_i y_i + \sum_{i=1}^{m} p_i q_i = \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i = \sum_{i=1}^{m} x_i,$$

where $y_i (i = 1, \ldots, m)$ are the outputs and $q_i(s = n + 1, \ldots, m)$ are output prices; $x_i = -x_i (i = 1, \ldots, m)$ are the inputs, and $p_i (i = 1, \ldots, m)$ are the input prices. Profit, $\Pi$, is maximized subject to the production function

$$f(x_1, x_2, \ldots, x_n, y_1, \ldots, y_m) = 0.$$  

$y_1, y_2, \ldots, y_m$ are often called net outputs; they have positive signs for outputs and negative signs for inputs.
Assuming that the production function \( f(x) \) is \( 1 \) twice differentiable, i.e.,
\[
\frac{\partial f(x)}{\partial y} = f_y \quad \text{and} \quad \frac{\partial f(x,y)}{\partial y} = f_{xy}, \quad (i,j = 1, \ldots, m+n),
\]
exist; (2) increasing in the \( n \) new inputs, i.e. the derivatives, \( f_i \), are always positive; and (3) convex (subject to the condition \( f(x) = 0 \), the function is \textit{strictly} convex); the optimal production plan of the firm can be stated using the familiar Lagrangian function:
\[
L(y_1, y_2, \ldots, y_m, \lambda) = \Pi + \lambda f(y_1, y_2, \ldots, y_m),
\]
\[
= \sum_{i=1}^{m+n} p_i y_i + \lambda f(y_1, y_2, \ldots, y_m),
\]
where \( \lambda \) is a Lagrange multiplier associated with the constraint \( f(x) = 0 \). There are \( m+n \) first-order conditions that can be interpreted as equality between the marginal profitability of each net output and its revenue or cost. The Lagrange multiplier is the change in profit made by the firm with respect to a change in its production plan. Manipulating these equalities, we obtain familiar expressions such as the marginal transformation among commodities and inputs, the marginal rate of technical substitution among inputs and the expansion path of inputs. It follows that the profit-maximizing output and input levels, \( y_1(1), \ldots, y_m(1), \) are the net supply functions. For outputs, the equations \( f \), are the usual supply functions; for inputs, the negative of the demand functions. Thus net supply functions exist provided that the marginal profitability conditions are satisfied and that the production function has the appropriate properties.

### Properties and Form of the Production Functions

The characterization of the input demand and output supply functions depends on the specific properties of the production function. A number of studies have tried to specify these properties and discover more flexible functional forms to accommodate various economic effects often embedded in the production process. Some economic concepts of interest are listed below. Though the concepts shown in Table 1 are defined in terms of a single-output production function, they can easily be extended to multiple-output production functions. Given the production function \( y = f(x, \lambda) \), where \( x \) is a vector of inputs and \( \lambda \) the index of technological change, it is possible to deduce expressions shown in Table 1 for returns to scale, shares of factors of production, price elasticity and elasticity of substitution, as well as various indices of disembedded technological change. Other effects such as indices of \textit{embodied} technical change can also be derived. By imposing specific restrictions on these effects, different functional forms of the production function can be obtained. Of this array of economic effects, those associated with returns to scale, degree of substitution among inputs and the type and nature of technological change, have received prominent attention in the literature.

Table 1. A partial list of economic effects related to the production function

| Output level | \( y = f(x, \lambda) \)
| Returns to scale | \( \mu = \sum_{i=1}^{m} x_i f_i / f \)
| Distributive share | \( s_i = x_i f_i / \sum_{i=1}^{m} x_i f_i \)
| Own 'price' elasticity | \( e_i = f_i f_{xy} / (f f_{xy}) \)
| Elasticity of substitution | \( e_q = f f_{xy} / (f f_{xy} - f f_{xy}) \)
| (1) Rate of technical change | \( T = f_i f / f \)
| (2) Acceleration of technical change | \( T = (f_i f) - (f f f) / f^2 \)
| (3) Rate of change of marginal products | \( \mu_i / \mu = f_i f / f \)

Disembedded technological change:
- \( e_q \) is either homogeneous of some degree, or logarithmic, the additivity condition holds.

Most of the theoretical formulations of the production functions described in the literature implicitly assume that separability conditions prevail. The \( f(x) \) is weakly separable with respect to partition \( R \) when the marginal rate of substitution (MRS) between any two inputs \( x_i \) and \( x_j \) from any subset \( N_i, i = 1, \ldots, s \), is independent of the quantities outside \( N \). (Leontief, 1947; Green, 1964; Berndt and Christensen, 1973) or \( \delta f(x,y)/\delta x_i \) = 0. \textit{Strong separability}, on the other hand, exists when MRS between any two inputs inside \( N \) does not depend on the quantities outside \( N \) or \( N_i \) of \( f_{xy} - f_{yx} = 0 \).

Functional separability plays an important role in aggregating heterogeneous inputs and outputs, deriving value-added functions and estimating production functions. It also opens up the possibility of consistent multi-stage estimation, which may be the only feasible procedure when large numbers of inputs and outputs are involved in the production activity of complex organizations.

### Production: Neoclassical Theories

The existence and magnitude of these effects. These properties of the production function—homogeneity, additivity and separability—have played an important role in the derivation of input demand and output supply functions. A homogeneous production function of degree \( k \) is defined as:
\[
f(\lambda x_1, \ldots, \lambda x_n) = \lambda^k f(x_1, \ldots, x_n); \quad \lambda > 0
\]
and a monotonic transformation of a homogeneous production function yields a homothetic production function in
\[
y = f(x_1, \ldots, x_n).
\]
This family of production functions is characterized by straight-line expansion paths through the origin. Additivity may take the form:
\[
f(\lambda x_1) + \cdots + f(\lambda x_n) = f(x_1, \ldots, f(x_n, \ldots, f(x_n, \ldots, f(x_n, x_n)))) = 0 \quad \text{for any} \quad \lambda > 0.
\]
These economic effects arise from the inherent nature of the underlying production process, and the specific form of the production function is therefore critical in determining...
substitution functions (VES) where \( \sigma \) is dependent on economic variables such as input mix (Liu and Hildebrand, 1965; Kudryk, 1972). Efforts to relax the homogeneity property have led to the development of a number of homothetic production functions that make the returns to scale depend on output and/or input mix (Zellner and Revankar, 1969; Färe, Jansson and Knox Lovell, 1978). A major advance has been the formulation of non-homothetic functions by Christensen, Jorgensen and Lau (1973), who formulated the translog production function, which does not use a priori impose restrictive constraints such as homotheticity, constancy of \( \sigma \), additivity, and so on.

**TECHNICAL PROGRESS.** Technical progress deals with the process and consequences of shifts in the production function due to the adoption of new techniques which either have a neutral effect on the producing process or change the input-output relationships. Neutrality of technical change can be measured by its effect on certain economic variables such as capital-output, output-labour and capital-labour ratios, which should remain invariant under technical change. Several definitions of technical progress have been proposed, such as (1) product-augmenting, (2) labour- or capital-augmenting, and (3) input-decreasing and factor-augmenting, amongst others (Beckmann, Sato and Schupack, 1972). However, the most familiar definitions are the Hicks, Harrod, and Solow forms of technical progress.

Part of technical change can be endogenous and would be determined by the firm to maximize its long-run profit. Technical knowledge is expensive to produce but, once produced, its transmission cost is almost zero, giving rise to the 'spillover' and 'appropriability' characteristics of inventions. Attempts have been made to incorporate R&D as an input in the neoclassical production function and cost functions, to estimate its contribution to the firm's productivity growth and cost behaviour, and to measure its spillover effects on other firms or industries (Nordhaus, 1969; Griliches, 1979). The results indicate substantial private and social rates of return to R&D (Mansfield, 1969). Changes in relative prices and output not only affect endogenous technical change but also the rate of factor productivity and the bias of technical change, which will in turn alter the structure of the production process (Jorgenson and Fraumeni, 1981).

**DUALITY.** A major advance in the economic theory of production has been the dual formulation of production theory (Shephard, 1953; Dice, 1974; Fuss and McFadden, 1978). The main features of this approach is to recover through indirect functions—that is, by means of a dual representation such as profit or cost functions—the properties of the underlying production function. The dual approach not only contributes important insights of its own but also offers more immediate empirical applications. A mapping of the characteristics of the transformation function and its dual cost function is indicated in Table 2.

The cost formulation is used extensively in econometric studies. This approach has two main advantages: (1) demand and supply functions can be derived as explicit functions of relative prices and output without imposing arbitrary constraints on production patterns required in the traditional methodology; (2) cost and profit functions are computationally simple and permit testing of a wider class of hypotheses by utilizing economic variables (Nadiri, 1982).

**DYNAMIC FACTOR DEMAND MODELS.** These types of production functions emphasize the intertemporal aspect of the production process by focusing on the movement from one equilibrium state to another. The models incorporate costs of adjustment that are incurred in order to change the level of quasi-fixed inputs, costs which can take two forms. The first type is internal: as the firm adjusts its quasi-fixed factors it must face either a higher purchase price or some other external factor (Lucas, 1967; Gould, 1968) or a higher financing cost for the accumulation of these inputs (Steigum, 1983). The second type is internal and reflects the fact that firms must make the trade-offs associated with producing current output and diverting some of the resources from current production to accumulate capital for future production (Treadway, 1974).

Suppose the firm maximizes its present value:

\[
V = \int_0^{\infty} \left( Py - WL - rK - GK \right) e^{-rt} dt
\]

subject to the production function \( f(y, L, K, K) = 0 \) and the initial condition \( K(0) = K_0 \). \( P \) is the price of output, \( y \) is the level of output, \( W \) is the nominal wage, \( r \) is the user cost of capital, \( G \) is the purchase price of investment, \( K \) is a vector of capital inputs, \( L \) is labour, and \( K \) is net investment. \( K \) is introduced in production on the assumption that firms produce essentially two types of outputs: \( y \), to sell, and \( K \), the internally accumulated capital which will be used in future production. \( K \) is assumed to be neither perfectly fixed nor perfectly variable. Suppose, in addition, that the production function is characterized by the relation \( y + C(K) - g(K, L) = 0 \), where \( C \) and \( g \) are continuous and the marginal products of \( f_L \) and \( f_K \) are positive and diminishing.

From the necessary conditions, it follows that for perfectly variable inputs its marginal product must equal its price, while for the quasi-fixed inputs the discounted sum of future net values of its marginal product must equal the sum of the purchase price of investment and the marginal value of real product foregone as a consequence of expansion at the rate \( K \).

### Table 2. Comparison of the properties on the transformation function and its dual cost function

<table>
<thead>
<tr>
<th>Property A on the transformation function ( F(y, x) )</th>
<th>Property B on the cost function ( C(y, p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Non-increasing in ( y )</td>
<td>Non-decreasing in ( y )</td>
</tr>
<tr>
<td>2 Uniformly decreasing in ( y )</td>
<td>Uniformly increasing in ( y )</td>
</tr>
<tr>
<td>3 Strongly upper semi-continuous in ( (y, x) )</td>
<td>Strongly lower semi-continuous in ( (y, p) )</td>
</tr>
<tr>
<td>4 Strongly lower semi-continuous in ( (y, x) )</td>
<td>Strongly upper semi-continuous in ( (y, p) )</td>
</tr>
<tr>
<td>5 Strongly continuous in ( (y, x) )</td>
<td>Strongly continuous in ( (y, p) )</td>
</tr>
<tr>
<td>6 Strictly quasi-concave from below in ( x )</td>
<td>Continuously differentiable in ( p )</td>
</tr>
<tr>
<td>7 Continuously differentiable in positive ( x )</td>
<td>Strictly quasi-concave from below in ( p )</td>
</tr>
<tr>
<td>8 Twice continuously differentiable strictly</td>
<td>Twice continuously differentiable and strictly</td>
</tr>
<tr>
<td>differentiably quasi-concave from below in ( x )</td>
<td>quasi-concave from below in ( p )</td>
</tr>
</tbody>
</table>

994
The basic problem in this type of model is to deal with expectations about future prices of inputs and outputs. A simple and often-used approach to the problem is to assume static expectations, but that begs the question. Uncertainty about future prices are handled in two ways, either by approximating optimization under uncertainty with certainty equivalence, which requires a quadratic objective function and linear constraint (Hansen and Sargent, 1981) or by making adjustment costs a function of the level of the quasi-fixed inputs, which exploits the expectations of future prices that are contained at the quasi-fixed input levels.

There is a fairly large and growing theoretical and empirical literature using the dynamic production function or its dual, dynamic profit or cost functions. The main result of these models is that, because of the existence of adjustment costs, substitution possibilities and technological biases may be limited in the short run, and the effects of prices and tax changes on factor demands may be quite different from their effects in the long run.

ECONOMIES OF SCALE AND SCOPE. An important extension of the theory of the firm has been the production and pricing behaviour of a multi-product firm when economies of scale prevail. To derive the net supply functions, the necessary conditions for equilibrium noted earlier break down when increasing returns or declining long-run average costs prevail. In such cases, monopolistic organization of an industry may offer cost advantages over production by a multiplicity of firms. An interesting and important question is what are the necessary and sufficient conditions for a multi-product firm to be a monopolist and for it to be sustainable against entry (Baumol, 1977). The condition for natural monopoly is that a cost function be strictly and globally subadditive in the set of commodities $C(y^1 + \ldots + y^n) < C(y^1) + \ldots + C(y^n)$, which means that it is always cheaper to have a single firm produce whatever combinations of outputs is supplied to the market. If the output vectors are restricted to be orthogonal, then the production function exhibits economies of scope. The natural monopoly is a sustainable set of products set at prices that do not attract rivals into the industry (Baumol, Bailey and Willig, 1977). Even if rivals are attracted, the monopoly may be able to protect itself from entry by changing its prices. But, by definition, only a sustainable vector of prices can prevent entry and yet remain stationary. The conditions necessary for sustainability are (1) the products are weak gross substitutes; (2) the cost function exhibits strictly decreasing ray average costs; and (3) the cost function is also transversal convex. Ramsey prices often ensure sustainability under specified circumstances.

M. Ishaq Nadiri

See also CORN-DOUGLAS FUNCTIONS: COST AND SUPPLY CURVES; COST FUNCTIONS; HUMPHREY PRODUCTION FUNCTION; JOINT PRODUCTION; SUPPLY FUNCTIONS.

BIBLIOGRAPHY


production, prices of. See PRICES OF PRODUCTION.

production and cost functions. The traditional starting point of production theory is a set of physical technological possibilities, often represented by a production or transformation function. The development of the theory parallels the firm's objective (cost minimization or profit maximization) and leads to input demands (and output supplies in the case of profit maximization) constructed from an explicit consideration of the underlying technology (i.e. derived directly from the production function).
Introduction

Over the past few decades, tax policy has been an important instrument of the government to influence both fluctuations and growth in the economy. Initially, economists emphasized the link between taxes and demand as the channel through which tax policy affected the cyclical variability and secular trend of output and thereby employment and capital accumulation. Now, however, we have come to recognize that there are also significant direct influences of tax policy on production. Tax rates, credits and allowances affect the costs incurred by firms in hiring labour, investing in equipment and structure, using energy, and undertaking research and development. The purpose of this paper is to analyze the models used to evaluate the effects of tax policy on the structure of production and to discuss the substantive results arising from the empirical applications of these models.

The predominant focus in studies of production has been related to the degree to which factors of production are substitutable and to the biases associated with technological change. Substitution possibilities characterize the manner in which firms alter their input demands in the face of changing relative factor prices. As a consequence, since tax policy operates through factor prices, the knowledge of the substitutability or complementarity of inputs permits us to determine how factor proportions change as tax policy initiatives are introduced.

Technological change generates new structures of production. In general, technological change is biased toward some factors of production and away from others, in the sense that the new production processes tilt relative input demands. However, technological change does not occur in a vacuum, but rather it is influenced by the same determinants as factor demands, namely product and factor prices. Hence, tax policy can affect technological change. For example, assume a credit is introduced on investment which lowers the effective factor price of capital. If technological change is of the variety which is biased towards capital, then the cost of undertaking technological change falls with the investment credit, causing the rate of technological progress to advance.

The ability of tax policy to influence factor demands and the rate of technological change may be severely hampered in the short-run because substantial costs of adjustment may have to be incurred in order to change factor demands. The quasi-fixedness of certain inputs (for example, skilled labour, equipment and structures, research and development) limits substitution possibilities and technological biases in the short-run. This implies that adjustment in the quasi-fixed factors occurs gradually and is not immediate.

In the short run, the effects of changes in tax policy on factor demands may be quite different from the effects occurring in the long-run. For example, a decrease in the corporate income tax rate can lower the effective cost of production, therefore causing output to expand. If there are some factors which are quasi-fixed, then increases in short-run output production occur more intensely using the variable factors of production. However, as the adjustment costs are incurred (the lower tax rate could also help in this regard) and investment takes place, the quasi-fixed factors are substituted for the variable factors in the long-run. The existence of adjustment costs changes the manner in which taxes affect production and the effectiveness of the policy.

---

1 The authors would like to express their thanks to Erwin Diewert, Mel Fuss, John Heilwell, Robert Lawrence, Jack Mintz, Jim Poterba, Nick Rowe and Larry Summers for their comments and suggestions on this paper.

Session III
Adjustment costs fix the level of the quasi-fixed factors in the short-run and cause the gradual adjustment to their long-run magnitudes. However, the rate at which the quasi-fixed factors are utilized in the short-run may also vary. If utilization of the quasi-fixed factors is not costless (for example, due to overtime and shift wage premiums or greater depreciation costs), then firms may find it desirable to leave idle portions of the quasi-fixed factors in order to meet future production requirements. The rates of utilization of the quasi-fixed factors are determined in the short-run in conjunction with the demands for the variable factors of production. Hence, utilization rates depend on product and factor prices and therefore on tax policy. For example, a decrease in the corporate income tax rate which causes output to expand can generate increases in the demand for variable factors and increases in the rates of quasi-fixed factor utilization. Since the latter is now costly, resources will be redirected from investment and the future expansion of the quasi-fixed factors towards the greater utilization of the current stocks.

Corporate tax rates, credits and allowances influence the long-run substitution of all factors of production, the short-run utilization and the dynamic adjustment of the quasi-fixed factors. This survey is based upon these themes. We develop a general intertemporal model of production, emphasizing the role of present and expected future corporate income taxes, credits and allowances along with costly adjustment and utilization of the quasi-fixed factors. We then proceed to specialize the general model in order to highlight each of the three themes and their interaction with tax policy. We also discuss the various ways in which empirical implementation of the theoretical models has been undertaken, along with the relevant results from the empirical investigations. The empirical studies are restricted to those which include the array of corporate tax, credit and allowance rates, have emphasized the role of tax policy on production structure, and are explicitly based on an optimization model of production decisions. The latter criterion enables us to establish a clear link between the theoretical and empirical models, and to see the problems in empirical implementation. The survey is organized along the following lines: In the next section we develop the general theoretical model. The following section focuses on the issues of tax policy, long-run factor substitution and the rate of technological change. In the fourth section we specifically discuss the issues of taxes and quasi-fixed factor adjustment costs, while in the next section the topic centers on the short-run utilization of the quasi-fixed factors. Lastly, we discuss some policy implications which emerge from the analysis and empirical results.

A model of production, investment and taxation

There are two objectives of this section. First a model is developed in which the effect of taxes on production and investment can be analyzed within the general themes of factor substitution, adjustment and utilization, output expansion and technological change. The second objective is to provide a framework in which to organize and evaluate the empirical research on this topic.

We begin by characterizing production and investment decisions. We assume that a firm produces $i$ outputs using $n$ non-capital inputs and $m$ capital inputs. The technology is represented by

$$\mathbf{T}(y_i, v, K_i^N, K_i^O, I_i, A_i) = 0$$

(1)

where $T$ is the transformation function, $y_i$ is an $i$ dimensional vector of output quantities, $K_i^N$ is an $m$ dimensional vector of 'new' capital (or beginning of period capital) input quantities, $K_i^O$ is a $m$ dimensional vector of 'old' capital (or end of period capital) input quantities.
quantities, and \( A \) represents an indicator of autonomous technological change.\(^2\) (The subscript \( t \) represents the time period.) The transformation function is twice continuously differentiable, increasing in \( y_t, K^0_t, l_t \) and decreasing in \( v_t, K^N_t \). Generally the transformation function is decreasing in \( A \) (in other words, technological progress). The transformation function is also concave in \( y_t, K^0_t, K^N_t, v_t \) and \( l_t \).

The specification of the technology is flexible enough to include the costs associated with installation and utilization of capital. The costs associated with capital utilization are introduced in a manner similar to the general approach developed by Hicks (1946), Malinvaud (1953), Bliss (1975), and Dievert (1980). Each time, the firm combines the beginning of period capital inputs \( (K^0) \) with the non-capital inputs to produce the outputs and the capital inputs to be used for future production \( (K^N) \). Thus, the firm produces two kinds of output: one type for current sale \( (y_t) \) and one type for future production \( (K^N) \). Utilization is captured through the selection of capital for future production. The choice of the end of period capital reflects decisions on the utilization of the capital inputs which are available at the beginning of the period. The specific processes of capital utilization are embedded or internal to the production process and are captured by the transformation function.

Capital adjustment or installation is costly. This is reflected by the vector of investment flows \( (l_t) \) in the transformation function. The installation costs are internal to the production process since the specific processes of capital installation are captured by the transformation function. Resources devoted to installing capital must be directed away from producing current output and repairing existing capital. The cost of installing additional capital is the opportunity cost of foregone current output and foregone capital repairs. Thus, equation (2) becomes the usual formula of depreciation by evaporation. We can see this by noting that depreciation is \( (K^N_t - K^0_t) = \delta_t K^N_t \) where \( \delta_t \) is defined as an \( m \) dimensional diagonal matrix of depreciation rates. Thus, equation (2) can be re-written as \( K^N_{t+1} = l_t - (I_m - \delta_t)K^N_t \), where \( I_m \) is the \( m \) dimensional identity matrix. Clearly, if \( \delta_t \) is time invariant and exogenous, then equation (2) becomes the usual formula of depreciation by evaporation.

The distinction between stock and flow decisions can be noted from equations (1) and (2). At any time \( t \), the beginning of period capital stocks are predetermined. Thus there exists a given bundle of capital services embedded in each stock of capital available to the firm. The firm selects the flow of services from each of the given capital stocks or the rates of utilization to combine with the non-capital (or variable) inputs to produce outputs or to install additional capital stocks. The choice on the rates of utilization are captured through the decisions on the end of period capital stocks. These

---

\(^2\) All variables in equation (1) are measured as flows of services. The term 'capital inputs' is meant to suggest factors of production obtained from stocks which can be accumulated. These stocks can represent the traditional equipment, structures and land and also pertain to research and development and various types of skilled labour.

Session III

181
end of period stocks along with the newly installed capital represent the capital stocks available to the firm at the beginning of the period $t+1$. 3

A firm generates revenue, hires variable inputs, utilizes its capital stocks, invests and finances its operations such that the flow of funds is

$$p_i v_t - w_t v_t - q_i l_t - \Delta B_t - p_{n+1} \lambda N_{n+1} - r_{BP} B_t - T_{ct} - D_t = 0. \quad (3)$$

The vector of output prices is $p_i$, $w_t$ is the vector of variable input prices, $q_i$ is the vector of capital purchase prices, $\Delta B_t$ is the nominal value of bond issues (net of retirements), $p_{n+1}$ is the price of new shares, $\lambda N_{n+1}$ is the number of new shares, $r_{BP}$ is the interest rate on the corporate bond, $T_{ct}$ and $D_t$ are corporate income taxes and dividends. 4 (The superscript T stands for vector transposition.) We assume that prices are given in all markets.

The flow of funds can be further decomposed by considering the nature of the corporate income taxes. These taxes are defined by a tax rate of $0 < u_{ct} < 1$, based on revenues net of variable input costs, interest payments, capital cost allowances, investment tax credits and allowances. Revenues net of variable input costs and interest payments are straightforward items. Next consider the capital cost allowances. In general, a firm is permitted depreciation deductions equal to $D_t$, on one dollar of the original cost of the $i$th capital of age $t$. Since capital must be fully depreciated, it must be the case that

$$\sum_{i=0}^{\infty} D_t = 1, \quad i = 1, \ldots, m.$$ 

The depreciation deductions at time $t$ for a particular type of capital installed at different times is

$$\sum_{i=0}^{\infty} q_{it} l_{it} D_t, \quad i = 1, \ldots, m.$$ 

Governments generally offer incentives to undertake investment. These incentives are often in the form of tax credits such that at time $t$ with a credit rate of $0 < u_{it} < 1$, $i = 1, \ldots, m$, the investment tax credit is

$$ITC_{it} = u_{it} q_{it} l_{it}, \quad i = 1, \ldots, m. \quad (4)$$

Moreover, the investment tax credit can reduce the depreciation base for tax purposes of the capital stocks. This means that the depreciation deductions for tax purposes or the capital cost allowances are reduced by the investment tax credit. Hence the capital cost allowance at time $t$ is

$$3 \text{ In this paper, vintages of capital stocks are not distinguished. The reasons are first that the empirical work in this area has focused on the putty-clay type of vintage model (see Bischoff (1971), King (1972), S nombreux (1974), and Macomber (1982)). In other words, once installed factor proportions are fixed. This implies that changes in tax rates and incentives cannot affect the rate of factor substitution of installed capital. Second, these studies assume that the service life of capital is constant, which means that tax policy does not affect the rate of capital utilization. The model in the text could be modified to allow for alternative vintages of capital. The transformation function in this case would depend on the vector of all past investment flows for all types of investment rather than on the vectors of beginning- and end-period capital. (See Dewan (1955b)).}

$$4 \text{ The focus is not on the financial decisions of the firm and so it is assumed that the firm issues one kind of bond and one kind of share.}$$

182 Corporate Taxes and Incentives and the Structure of Production
CCA_t = \sum_{\tau=0}^{\infty} q_{i-t-1} (1 - \phi_{i-t}) D_{i,t}, \quad i = 1, \ldots, m. \tag{5}

where \( \phi_{i-t} \) is the proportion of the investment tax credit which reduces the depreciation base for tax purposes. In Canada, \( \phi_{i-t} = 1 \) and the U.S. \( \phi_{i-t} = 0.5 \).

Besides the capital cost allowance and investment tax credit, a third type of investment incentive relates to additions to the rate of investment. For example, incentives of this nature have been introduced to stimulate R&D expenditures. In Canada, from 1978 to 1984, there was a tax allowance of 50 per cent on current R&D expenditures in excess of the average of the previous three years. In the U.S., a tax credit of 25 per cent exists since 1981, on current R&D expenditures in excess over the average expenditures undertaken during the previous three years. An allowance at time \( t \) based on incremental investment can be defined as

\[ \text{IIA}_{i,t} = \gamma_{i} \sum_{\tau=0}^{\infty} \mu_{i-t} q_{i-t-1} b_{i-t-1}, \quad i = 1, \ldots, m. \tag{6} \]

where

\[ \mu_{0} = 1, \quad \mu_{i-t} - \mu_{i-t-1} < 0, \quad \gamma_{i}, \quad \delta > 0 \]

and

\[ \gamma_{i} = \begin{cases} a_{i} > 0 & \text{if } \sum_{\tau=0}^{\infty} \mu_{i-t} q_{i-t-1} b_{i-t-1} > 0, \\ 0 & \text{otherwise} \end{cases} \]

where \( a_{i-t} \) is the allowance rate on the \( i \)th capital stock in period \( t \). To see the magnitude of the incremental allowance, suppose that in order to obtain the allowance, current investment expenditure must exceed the average of the past three years. Thus, \( \mu_{0} = 1, \mu_{t} = -0.3, \delta = 0.2, 3 \). In addition, suppose current expenditures are $1.00, while expenditures for the previous three years are $0.75, $0.50 and $0.25, respectively. Thus, the incremental expenditures upon which the allowance is based is $1.00 - $0.50 - $0.25. If the allowance rate is 0.5, then the firm obtains an allowance of $0.25.

Combining equations (4), (5), and (6) yields corporate income taxes at time \( t \) to be

\[ T_{i,t} = u_{i,t} \left[ \rho_{i} y_{i} - w_{i} v_{i} - r_{i} B_{i} - \mathbb{1}_{m} (\text{CCA}_{i} + \text{IIA}_{i}) \right] - \mathbb{1}_{i} \rho_{i} \text{ITC}_{t} \tag{7} \]

where \( \mathbb{1}_{m} \) is the \( m \) dimensional identity vector, \( \text{CCA}_{i}, \text{IIA}_{i} \) and \( \text{ITC}_{i} \) are \( m \) dimensional vectors of the capital cost allowances, incremental investment allowances, and the investment tax credits respectively.

Substituting equation (7) into the flow of funds which is given by equation (3), we can write

\[
F_{i} = \left[ \frac{D_{i}}{\rho_{i} N_{i}} + \frac{\Delta D_{i}}{\rho_{i} N_{i}} \right] \rho_{i} N_{i} - r_{i} (1 - u_{i}) B_{i} \\
- \Delta (\rho_{i} N_{i}) - \Delta B_{i}, \tag{8}
\]

\[ \]
where
\[
F_t = [D_t Y_t - W_t Y_s (1 - u_{ct}) - q_t^T l_t + \int^T u_{ct} (CCA_t + II_A_t) + ITC_t]
\]
which is the flow of funds to the shareholders and bondholders.

Share market equilibrium requires that
\[
r_{st} = D_t / (p_{st} N_{st}) + \Delta p_{st} / p_{st},
\]
where \(r_{st}\) is the rate of return on equity, and defining
\[
V_t = p_{st} N_{st} + B_t
\]
so that \(\Delta V_t = \Delta (p_{st} N_{st}) + \Delta B_t\), then equation (8) can be rewritten as
\[
F_t = \left[r_{st} (1/(1 + \delta)) + r_{bt} (1 - u_{ct}) \delta_t / (1 + \delta)\right] V_t - \Delta V_t
\]
where \(\delta_t = B_t / V_t\). The rate of return on financial capital can be defined as
\[
r_t = r_{st} / (1 + \delta_t) + r_{bt} (1 - u_{ct}) \delta_t / (1 + \delta_t).
\]
Thus, equation (9) implies that the flow of funds to the shareholders and bondholders plus any capital gains equals the return on financial capital.

The objective of the firm is to operate in the interest of its shareholders by maximizing the expected present value of the flow of funds to the shareholders. In the present context, because the rates of return on bonds and shares are exogenous to the firm, and therefore cannot be influenced by shareholder behavior, the objective is equivalent to maximizing the expected present value of financial capital (or in other words, the expected present value of the flow of funds to shareholders and bondholders). The objective function which can be obtained from equation (9) by solving for the present value of financial capital and applying expectations, can be written as
\[
J_t = E_t \sum_{s=1}^\infty a(t, s) \left[p_t^s Y_s - W_t Y_s (1 - u_{cs}) - Q_t^s l_s + \int^T u_{cs} M_s\right].
\]
where \(E_t\) is the expectation operator conditional on information known at time \(t\), the discount rate is \(a(t, s) = 1, a(t, t+1) = 1/(1 + \delta),\). \(Q_s\) is an \(m\) dimensional vector of capital purchase prices net of taxes such that
\[
Q_s = q_s \left(1 - v_{is} - \sum_{r=0}^\infty a(t, s + r) a(t, s)^{-1} u_{cs+r} \left[(1 - \phi_{is+r} \gamma_{is+r}) D_r + \gamma_{is+r} \mu_r\right]\right).
\]
\(M_s\) is an \(m\) dimensional vector such that the \(i\)th element is \(\sum_{r=0}^\infty q_{is+r} l_{is+r} \left[(1 - \phi_{is+r} \gamma_{is+r}) D_r + \gamma_{is+r} \mu_r\right].\)

\(M_s\) represents the tax reduction due to the capital cost allowances and the increment in investment allowances arising from past investment expenditures. In deriving equation (10), we have made use of the fact that capital purchase prices are modified by the investment tax credit, the capital cost allowance (which may be reduced in part by the credit) and the incremental investment allowance. In addition, we have separated,
type of investment expenditure into the portion at any time t which relates to the present
and the portion which is a legacy of the past (given by the vector \( M_k \)). Clearly, at any
time t the latter does not figure into the maximizing program because, from the vantage
point of the present, it is predetermined.

The post-tax capital purchase prices contain the allowance on incremental invest-
ment. To see how the latter affects the post-tax purchase prices and reduces taxes, as-
sume that the corporate income tax rate is fixed and equal to \( \gamma \), the allowance rate is
fixed and equal to \( \gamma \), and the discount rate is constant and equal to \( \rho \). In addition, as-
sume that the allowance is based on current investment expenditures in excess of the
average of the past three years. Suppose there is one type of capital and a firm incurs
an investment expenditure in year 1 of $1 (q \_1 = 1). This expenditure will add $1 to the
incremental allowance in year 1. Thus, the tax reduction from the allowance is \( u \_c \_1 \gamma \). $1.
In year 2, however, the $1 expenditure will decrease taxes through the allowance by
one-third of \( u \_c \_2 \gamma \). $1. Discounting the latter magnitude back to year 1 yields \( u \_c \_1 \gamma \frac{1}{(1 + \rho)^3} \).
In years 3 and 4 the discounted tax reductions from the allowance are \( u \_c \_3 \gamma \frac{1}{(1 + \rho)^3} \) and \( u \_c \_4 \gamma \frac{1}{(1 + \rho)^3} \) respectively. The $1 expenditure increases
the incremental allowance in the year the expenditure was increased and then reduces
the allowance over the next three years. Thus the present value of the tax reduction due
to the incremental allowance is

\[
\frac{1}{(1 + \rho)} \sum_{i=1}^{3} \frac{1}{(1 + \rho)^i} \cdot U \_c \_i \gamma \frac{1}{(1 + \rho)^3}
\]

if \( u \_c = 0.46 \), \( \gamma = 0.5 \) and \( \rho = 0.15 \), then $1 \( u \_c \_2 \gamma 25 = $0.06 \), which is the present value of the
tax reduction from the investment allowance generated by the $1 expenditure.

The firm maximizes the right side of equation (10) by selecting the vector of outputs, variables inputs, levels of investment and used (or end of period) capital stocks, subject to:

1. To see how the latter affects the generation of new (or beginning of period) cap-
ital stocks (equation (2)). This program can be undertaken in two stages. First, condi-
tional on the beginning of period capital stocks and the technology, the firm determines
its output supplies, variable factor demands and end of period capital stocks. This is the
set of short-run decisions. With this solution, the firm proceeds to the intertemporal
problem in order to determine the beginning of period capital stocks.

The short-run problem is defined by

\[
\max_{v_t, y_t, K_t} \left( \frac{1}{1 - u_c} \cdot w_t \cdot v_t \right) + E_t Q_t \cdot K_t^2
\]

\[
\text{s.t.} \quad T (y_t, v_t, K_t^N, K_t^G, K_t^{N+1}, K_t^G, A_t) = 0.
\]

The first order necessary conditions for any time period are (including the constraint in
(11)):

\[
\mu_t (1 - u_c) - \lambda \nabla T_y = 0 \quad \text{(12.1)}
\]

\[
-w_t (1 - u_c) - \lambda \nabla T_v = 0 \quad \text{(12.2)}
\]

\[
E_t Q_t - \lambda \nabla (T_C - T_d) = 0 \quad \text{(12.3)}
\]

where \( \lambda \) is the Lagrangian multiplier and \( \nabla T_i \) represent the first order partial derivatives of outputs (i=y), variable inputs (i=v), end of period capital stocks (i=0) and investment levels (i=i). Equation sets (12.1) and (12.2) are standard. They imply that relative product prices equal the respective rates of product transformation and relative variable factor prices equal the respective rates of factor substitution. Equation (12.3)
implies that relative net of tax capital stock expected purchase prices equal the respective relative marginal values of capital utilization \( (T_0) \) net of the marginal costs of capital installation \( (T_1) \).

It is clear from equation set (12.1) and (12.2) that tax policy influences output supplies and variable factor demands through its effect on the quasi-fixed factors. There are two reasons for this result. First, the corporate income rate does not affect output supplies and variable factor demands directly because it is based on revenues net of variable input costs or variable profits. The corporate income tax is a variable profits tax and as such it is based on a residual of the firm's income stream, given capital utilization, installation and accumulation. The second reason is that all allowances and credits are actually based on the quasi-fixed factors. As a consequence, output supplies and variable factor demands are affected by tax policy through their link with the intertemporal decisions governing the quasi-fixed factors.

In this model there are three ways in which quasi-fixed factor decisions interact with output supplies and variable factor demands. First, there is the traditional route through factor substitution and output expansion. This is the link between \( \gamma \) and \( v_t \) on the one hand and \( K^R \) on the other. Second, there is the interrelationship through capital installation which is the link between decisions on \( v_t \) and \( v_t \) and decisions on \( T_1 \). Third, there is the interaction between capital utilization, \( K^R \), and output supplies, \( \gamma \), and variable factor demands, \( \gamma \), To see the role of each of these interrelationships, let us assume for the moment that the costs of capital utilization and installation are separable from the production technology. In other words, \( T_{10} = T_{01} = T_{00} = T_{0} = 0 \). This means that changes in the corporate income tax, credit and allowance rates only affect output supplies and variable factor demands through changes in the beginning of period quasi-fixed factors. The channel is as follows. A change in tax policy in period \( t \) elicits a change in capital utilization and installation in period \( t \). This causes the quasi-fixed factors at the beginning of period \( t+1 \) to change which in turn generates changes in period \( t+1 \) output supplies and variable factor demands. There is no direct link between capital utilization or installation and variable input demands and output supplies.

The other two channels arise from capital utilization and installation. If utilization and installation decisions are not separable from production decisions then from equation set (12) a change in tax policy generates contemporaneous effects on output supplies and variable factor demands. In addition, the effects on utilization and installation alter the quasi-fixed factors available for production in the succeeding period which in turn affects output supplies and factor demands in this later period.

The solution to the short-run program given by equation set (12) can be substituted into (11) to define the post tax variable profit function (see Diewert (1973)):

\[
\tau_t = \Pi_t (P_t, W_t, E_t, K^N_t, K^N_{t+1}, A_t)
\]

(13.1)

where \( \tau_t \) is a twice continuously differentiable function which is increasing in \( P_t = P_t (1 - u_{00}) \), and \( E_t \), increasing in \( K^N_t \) and decreasing in \( W_t = W_t (1 - u_{01}) \) and \( K^N_{t+1} \), convex and homogeneous of degree 1 in the prices \( P_t, E_t, \) and \( W_t \); concave in \( K^N_t \) and \( K^N_{t+1} \). The post-tax variable profit function is defined such that differentiating it with respect to the post tax prices \( (P_t, W_t, \) and \( E_t) \) yields,

\[
\nabla \Pi_P = y_t
\]

(13.2)

If tax credits or allowances are defined on the variable factors of production then these instruments of tax policy would directly affect output supplies and variable factor demands.
\[ \nabla \Pi_w = v_t \]
\[ - \nabla \Pi_Q = K_Q^t. \]

This result, known as Hotelling's Lemma, implies that the short-run equilibrium can be either characterized by equation set (12) and the transformation function (defined by the constraint in (11) or by equation set (13). The attractive feature of the latter approach is that reduced form output supply, variable factor demand, and quasi-fixed factor utilization functions are readily obtainable from the variable profit function.

The second stage of the program involves the intertemporal determination of the beginning of period quasi-fixed factor demands. This can be obtained by substituting the post tax variable profit function into the expected present value of financial capital (which is the right side of equation (16)). Thus the firm desires to

\[ \max_{K_{Q+1}^t} E_t \sum_{s=t}^{\infty} \beta (t,s) \left[ \pi_2 \left( P_t, W_t, E_t Q_s, K_s^N, K_{s+1}^N, A_s \right) - Q_t^s K_s^{N+1} \right]. \]  

(14)

The first order necessary conditions for any time period are

\[ E_t \left[ \frac{\partial \Pi_t}{\partial K_{N+1}^t} - \alpha (t, t + 1) \frac{\partial \Pi_{t+1}}{\partial K_{N+1}^t} \right] = 0 \]  

(15)

Equation set (15) implies that the present value of marginal variable profit from a quasi-fixed factor available for production must be balanced against the marginal cost of obtaining this output.\(^6\) The marginal cost contains the post tax purchase price of additional capital and the decline in variable profits due to installing and maintaining the quasi-fixed factor for future production. This is the classic trade-off between greater future post-tax profits due to larger capital stocks versus smaller present post-tax profits in order to obtain the larger capital stocks.

There are some interesting features contained in equation set (15). First, not only contemporaneous but all future tax, credit and allowance rates enter each equation through the post-tax purchase prices of the capital stocks. Second, embedded in the post-tax variable profit function is the manner in which the quasi-fixed factors interact with each other and with the variable input demands in determining output supplies. Third, utilization of the quasi-fixed factors is endogenous and governed by the post tax variable profit function. In other words, the specification of the post tax variable profit function implies a specification of quasi-fixed factor utilization.

The complete model consists of equation sets (13) and (15). We can see that empirical models which do not consider the potentially important influences of changes in present and future tax credit and allowance rates on output expansion, factor substitution, and quasi-fixed factor utilization and installation may be assuming away significant effects of tax policy on the structure of production.

### Taxes, factor substitution and productivity growth

The theoretical model previously developed is complex in that it involves the analysis of corporate taxes and the structure of production in a dynamic context. The empirical literature on taxation and the structure of production has, in recent times, moved towards the implementation of a general model of production in order to address the is-

\[ \text{We also assume that } \lim_{s \to \infty} a (t,s) E_t Q_{s+t} K_s^N = 0 \quad t = 1, \ldots, m. \]
sues related to the influence of taxes on factor substitution, adjustment, and utilization as well as output expansion and technological change. The purpose of this and the following sections of this paper is to analyse, within the context of the general theoretical model, the empirical work on the interaction between taxes and production decisions. We undertake this task by discussing the substantive empirical findings along with the nature of the models used to obtain these results.

The first issue we discuss pertains to the effects of taxes on factor substitution. We can address this issue by assuming that utilization and installation are costless and current prices and tax policy are always expected to persist. Thus the determination of production decisions can be simplified to the following two-stage procedure. First, the problem defined by (11) is simplified to

\[
\begin{align*}
\max_{(y^*, v^*)} & \quad (p^T y^* - w^T v^*) (1 - u_c) \\
\text{s.t.} & \quad T(y^*, v^*, K^N, A^i) = 0. \\
\end{align*}
\]

(16)

This leads to equations similar to (12.1) and (12.2). In addition, a post-tax variable profit function can be defined in a similar fashion to equation (13.1) with the derived conditions similar to equations (13.2) and (13.3). In this simpler context, the variable profit and derived conditions (with respect to the post-tax prices) are

\[
\begin{align*}
n^*_t &= \Pi(P, W, K^N, A^i) \\
\nabla \Pi_p &= y_t \\
\nabla \Pi_w &= v_t.
\end{align*}
\]

(17.1) \hspace{1cm} (17.2) \hspace{1cm} (17.3)

Although the properties of the post-tax variable profit function are similar to those for the function given by the right side of (13.1), there are some differences. First, in the case defined by equation (17.1), post-tax variable profits are defined as revenue minus variable input costs. The value of the unutilized quasi-fixed factors does not have to be added to revenue, as in the general model, because utilization is costless and thereby exogenous. Moreover, this implies that the post-tax purchase prices of the quasi-fixed factors do not enter the variable profit function. Second, because there are no installation costs, future quasi-fixed factors are not part of the domain of the variable profit function.

The second stage of the production problem is to

\[
\max_{(K^N_{s+1})} \sum_{s=1}^{\infty} (1 + \rho)^{-s+1} \left( \Pi(P, W, K^N_s, A^i) - Q^T (K^N_{s+1} - \langle m - \delta \rangle K^N_s) \right).
\]

(18)

The fact that quasi-fixed factor utilization is costless means that these factors are fully utilized and any depreciation can simply be defined to be exogenous and constant over time. The m dimensional diagonal matrix of constant depreciation rates is \( \delta \) and capital accumulates by \( K^N_{s+1} = I_s + (m - \delta) K^N_s \). The first-order necessary conditions for this program are

\[
\nabla \Pi / \partial K^N_{s+1} - W_s = 0
\]

(19)

where \( W_s \) is vector of post-tax rental rates such that

\[
W_s = Q_s (p + \delta) = q_s (p + \delta) (1 - v_i - d_i - d_a) \quad i = 1, \ldots, m.
\]

We have defined the present value of capital cost allowances as \( d_c \) and the present value of incremental investment allowances as \( d_a \). Clearly equation (19) is just a spe-
cial case of equation (15). The equilibrium consists of equation set (17) and equation (19).

In this model, the emphasis is on the influence of tax policy on output supplies and factor demands. This can be described geometrically by assuming there is a single output \((z = 1)\) and two inputs \((n = m = 1)\); there is no distinction here between variable and quasi-fixed factor. The analysis of an increase in the investment tax credit or capital cost allowance is straightforward. An increase in either of these policy instruments lowers the relative factor price of capital. The firm chooses a new cost minimizing mix of inputs for the given output. This mix is relatively more capital intensive. In addition, at the given output level, the marginal cost of production declines and therefore output supply expands.

The analysis is somewhat different when the incremental investment allowance increases. The reason is that the firm can only take advantage of the incremental allowance if current investment expenditure exceeds an average of past expenditures. In the following Figure, the firm produces on the given isoquant.

**FIGURE:** Incremental investment allowance and factor demands

The minimum cost equilibrium in the absence of any taxes or tax incentives is denoted by \(E^1\), with relative factor prices reflected by the isocost line AB. Suppose an incremental allowance on capital is introduced. This has the effect of lowering the rental rate such that the new isocost line CD reflects the relative factor prices inclusive of the allowance. Thus the isocost line CD is steeper than AB. In addition, CD has been drawn so that it is tangent to the isoquant at \(E^2\). The point \(E^2\) represents the minimum cost equilibrium inclusive of the incremental allowance. Next, let us assume that the capital
stock upon which the incremental allowance is based is $K_0$, where by construction the
isocost lines AB and CD intersect. In this situation, with capital stock levels greater than
$K_0$, the relevant isocost line is AB. Thus the effective isocost curve is $CB^1B$. Moreover,
this isocost curve represents the same production costs as those given by the isocost line
AB (measuring cost in labour units). Hence, the firm is indifferent between the equilibria
given by $E^1$ and $E^2$, as each represents the identical minimum cost to produce the same
output. The firm produces the output with the same cost with or without the incremental
allowance. Suppose now that the base for the allowance declines to $K_0$. In this case,
the effective isocost curve is $CB_2F$, which represents lower production costs relative to
$CB^1B$. Thus, the firm produces at a minimum cost by using the incremental allowance.
The equilibrium point is $E^2$. With a base of $K_0$, the effective isocost curve is $CB_2G$. The
firm produces at a minimum cost given by the isocost line AB and so the equilibrium
point is $E^1$. The firm does not use the incremental allowance. Notice that if the base
quantities of capital are always less than the undistorted cost minimizing level, then the
firm will always utilize the incremental allowance.

The empirical implementation of the model defined by equations (17) and (19)
necessitates a functional form for the post-tax variable profit function. Moreover, because
the post-tax rental rates of capital are time invariant, we can combine the two
stages of production decisions and correspondingly define a post-tax profit function. In
addition, empirical implementation is often mainly concerned with factor substitution,
and so it is generally assumed that output levels are predetermined. In this instance, only
the cost function needs to be specified. Many different functional forms for the cost
function have been introduced over the years (see Berndt and Khaled [1979]). Probably
the one most often used in this context is the translog (see Christensen, Jorgenson and
Lau [1973], Fuss and McFadden [1978] and Dievert [1980]).

$$\ln c = a_0 - \sum_{i=1}^{n} a_i \ln y_i + \sum_{j=1}^{n} a_j \ln w_j + a_{11} t + .5 \sum_{i=1}^{n} a_{ii} \ln y_i \ln y_i$$

$$+ .5 \sum_{j=1}^{n} a_{jj} \ln w_j \ln w_j + .5 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln y_i \ln w_j$$

$$+ .5 a_{tt} t^2 + \sum_{i=1}^{n} a_{it} \ln y_i t + \sum_{j=1}^{n} a_{jt} \ln w_j t + u_c,$$

where $c = \sum_{j=1}^{n} w_j v_j$

and $u_c$ is the stochastic disturbance of the cost function. By symmetry, the parameters
satisfy $a_{it} = a_{ti}$, $i, t = 1, \ldots, r$, $a_{ij} = a_{ji}$, $j = 1, \ldots, n$, $a_{ij} = a_{ji}$, $i, j = 1, \ldots, r$, and by homogeneity of degree 1 in the factor prices

$$\Sigma a_{ij} = 1, \quad \Sigma a_{ij} = 0, \quad s = 1, \ldots, n, \quad \Sigma a_{ij} = 0$$

In addition, the cost function is concave and nondecreasing in the factor prices and
nondecreasing in output. Applying the equivalent of Hotelling's Lemma, known as
Shepherd's Lemma, to the cost function, the conditional factor demands (conditional

---

9 Since there are no adjustment costs, all inputs are variable. Also, time (t) designates the rate of autonom-
ous technological change. The time subscript is deleted from each of the variables.

190 Corporate Taxes and Incentives and the Structure of Production
since outputs are exogenous) are derived by differentiating the cost function with respect to the factor prices. Thus,

\[ s_j = \alpha_j + \sum_{s=1}^{\alpha} \alpha_{ij} \sum_{n=1}^{W_j} n \sum_{i=1}^{n} Y_i + \alpha_{P} 1 - u_j \quad j = 1, \ldots, n \]  

(21)

where \( s_j = W_j Y_j / c \) is the jth input cost share and \( u_j \) is the stochastic disturbance of the jth input cost share. Stochastic disturbances have been appended to equations (20) and (21). These disturbances reflect errors of optimization and errors in the data. The disturbance in the cost function can also reflect stochastic shocks (for example, productivity shocks) to the technology.\(^{10}\)

The model consists of equations (20) and (21). However, in estimating the unknown parameters, only \( n \) of the \( n+1 \) equations are used because one of the errors can always be written as a linear combination of the others and therefore one of the equations adds no new information. The easiest way to see this is to use equation set (21). The cost function is \( C(y,W) \) and \( \alpha_n C(y,W) / \alpha n W_j \) are the terms on the right side of (21), not including the stochastic error. Thus, from (21),

\[ \sum_{i} s_i = \sum_{i} W_i \partial C(y,W) / \partial W_i C(y,W) + \sum_{i} u_i \quad j = 1, \ldots, n \]

Since the cost shares sum to unity

\[ \sum_{i} s_i = 1 \]

and since the cost function is homogeneous of degree 1 in the factor prices,

\[ \sum_{i} W_i \partial C(y,W) / \partial W_i = C(y,W) \]

then it must be true that

\[ \sum_{i} u_i = 0. \]

There has been a great deal of empirical work over the years estimating the cost structure for firms and industries. To various degrees, tax rates, credits, and allowances have been included in the factor prices. However, few studies have explicitly investigated the effects of changes in tax policy on variable factor demands. An exception is the paper by Keeselman, Williamson and Berndt (1977). In this study, a single output, three-factor translog cost function was estimated in the absence of technological change and for a technology which exhibits constant returns to scale. Thus, in terms of the cost function given by the right side of equation (20), \( \ell = 1 \) (single output and so let \( \alpha_0 = \alpha_y \), \( \alpha_w = 0 = \alpha_m = \alpha_{ym} \), \( j = 1, \ldots, n - 1 \) by constant returns to scale, \( \alpha_0 = \alpha_n \) by the absence of technological change, and \( n = 3 \) (three inputs). The inputs are blue-collar workers, white-collar workers and capital. The

---

\(^{10}\) The disturbances in the share equations could also reflect technology shocks. However, in this case, the disturbance in the cost equation must be contemporaneously correlated with each of the factor prices in order for technology shocks to appear in each of the share equations. This does not pose any theoretical difficulties but adds to the estimation problems.

Session III

191
effects of an investment tax credit along with two types of employment tax credits on the factor demands were simulated. One employment tax incentive was an employment tax credit and the other was a marginal (or incremental) employment tax credit.

The results from the elimination of the investment tax credit for the period 1962 to 1971 for U.S. manufacturing were that total labour demand would have been around 0.7 percent higher over the period. Employment of blue-collar workers would have been about 1.1 percent higher, while employment of white-collar workers would have fallen about 0.3 percent. These results reflect the findings that white and blue-collar workers are mildly substitutable, capital and blue-collar workers are substitutes and capital and white-collar workers are complements. Also, average costs and thereby product price would have been about 0.8 percent higher.

Next, Kesselman, Williamson and Berndt considered the effects of the imposition of an employment tax credit. First, the imposition was on a per man-hour basis and second on the wage bill. In each simulation the cost of the employment tax credit was set equal to the revenue gain from eliminating the investment tax credit. In both cases, the effects were quite small and the tax credit on a per man-hour basis was relatively more favourable to blue-collar workers compared to white-collar workers. The converse was true for the credit based on the wage bill. The greatest influence of the employment tax incentives arose from the incremental tax credit. A base of 0.5 of the previous year's wage bill doubled the impact on factor demands relative to the effects of an employment tax credit based on a percentage of the wage bill. This result occurred because the incremental employment tax credit channelled subsidies to the firm for additional employment beyond a base magnitude. Hence, the same policy cost can generate a larger percentage change in the price of subsidized units of labour through an incremental credit. Provided, of course, that the firm utilizes the incremental tax credit.

The previous empirical analysis focused on the effects of tax incentives on factor demands. However, in the long-run equilibrium framework (defined by equations (20) and (21)), which admit multiple outputs, non-constant returns to scale and non-neutral technological change, it is also possible to investigate the effects of tax policy on scale economies, scope economies, and the rate of productivity growth. There has been an empirical analysis of the effects of tax policy on scale and scope economies, but Fraumeni and Jorgenson (1980) and Jorgenson (1981) have studied the dependency of productivity growth on tax rates and incentives.

To see how productivity growth can be affected by tax policy, refer to equation (20). Since the rate of productivity growth is defined as the proportional decline in production costs over time, this rate can be obtained by differentiating equation (20) with respect to t,

$$-\frac{\partial \ln c}{\partial t} = - \left( a_{t} + a_{tt}t + \sum_{i=1}^{L} a_{t} \ln y_{t} + \sum_{j=1}^{n} a_{j} \ln w_{j} \right).$$  

(22)

We can observe then that the rate of productivity growth is a function of the government's tax policy. Tax policy operates through the factor prices which, in turn, influence the rate of productivity growth.

The coefficients, in equation (22), relating to the factor prices characterize how the rate of productivity growth responds to changes in the tax, credit and allowance rates. For example, suppose a credit is offered to the jth input which causes its factor price to decline by 1 percent. The effect on the rate of productivity growth is found by differentiating (22) with respect to \( \ln w_{j} \). Thus, in this case \( a_{j} \) characterizes the manner in which the rate of technological change is influenced by an increase in the tax credit on
the \( j \)th factor of production. If \( \alpha_{jI} > 0 \), then the rate of productivity growth increases as the tax credit increases, while if \( \alpha_{jI} < 0 \), the converse arises.

The \( \alpha_{jI} \) coefficients show the biases of technological change. They indicate the effect of changes in technology on the input cost shares. For example, technological change for the \( j \)th input gives the change in the cost share of the \( j \)th input in response to changes in technology represented by time. This can be seen from equation set (21).

If we differentiate the \( j \)th share by time, the effect is determined by \( \alpha_{jI} \). Hence the factor biases of technological change characterize how the rate of productivity growth is influenced by tax policy.

Generally, we define technological change as factor-using if the bias of technological change for the factor is positive (that is for the \( j \)th input \( \alpha_{jI} > 0 \)). In other words, if changes in technology result in an increase in the cost share of the \( j \)th input, then technological change is \( j \)th factor-using. Conversely, if changes in the technology result in a decrease in the cost share of the \( j \)th input, then technological change is \( j \)th factor-reducing (or saving).

The biases of technological change express the dependence of factor cost shares on the technology and also characterize the dependence of the rate of productivity growth on the input prices and thereby on tax policy. For example, technological change, which is \( j \)th factor-using, means that an increase in the factor price of the \( j \)th input decreases the rate of productivity growth. Similarly, technological change which is \( j \)th factor-reducing means that an increase in the factor price of the \( j \)th input increases the rate of productivity growth. The lesson to be learned from this analysis is that it is not sufficient for the government to provide tax incentives in order to improve productivity performance. The factor biases associated with technological change must be determined in order to characterize how the rate of productivity is influenced by the factor prices.

Fraumeni and Jorgenson [1980] have estimated the biases of technological change for 35 industries in the U.S. for the period 1952-1979. They assumed that the technology exhibits constant returns to scale, and so \( \alpha_{iI} = 0 \), \( i = 1, \ldots, \ell \) in equation (22); also, there was a single output and four inputs, which were capital, labour, energy and materials. The pattern of technological change that occurred most frequently was capital-using, labour-using, energy-using and materials-reducing. This pattern arose for 19 of 35 industries. This implies that increases in the factor price of capital, labour and energy decreased the rate of productivity growth.

The overall conclusion of Jorgenson and Fraumeni was that effective tax rates on corporate income were inversely correlated with the rates of productivity growth. This result arose from the fact that tax policy has reduced the rental rate on capital which increased the rate of productivity growth because the latter was capital-using. They found that effective tax rates declined sharply between 1960 and 1965 while the rate of productivity growth attained the postwar peak of 2.11 percent during this period. From 1965-1969, effective tax rates rose substantially while the rate of productivity growth declined to 0.05 percent. Effective tax rates declined from 1969 to 1972 and have remained relatively constant since that time but productivity growth increased slightly from 1969 to 1972 and fell dramatically from 1973. They attributed the latter decline to the energy price increases. In light of this conclusion, which has been the subject of much debate (see Nadiri and Schankerman [1981], Baily [1981] and Clark [1982]), they recommended that tax policy should be introduced to decrease the factor prices of capital and labour.
In Canada, little work has been done on investigating the effects of tax, credit and allowance rates on factor substitution and productivity growth. In general, much more empirical work needs to be done, even in the context of long-run equilibrium. First, little is known about the effects of tax policy on scale and scope economies. In order to capture these effects, it is necessary to estimate cost (or profit) functions which do not incorporate the maintained hypotheses of constant returns to scale (or for that matter homotheticity) and of a single output. Second, the treatment of technological change is quite simplistic. Technological development does not usually occur autonomously; it is also part of production and investment decisions. Indeed, the demand for research and development capital, which is an important element of technological change, is itself a function of the array of factor prices and the quantities of outputs. Thus, as is the case of the other factors of production, the demand for R&D capital depends on the various taxes, credits and allowances.

Taxes and factor adjustment

In the previous section we considered the effects of taxes on factor substitution and productivity growth in the context of our general model by assuming that factors of production could be costlessly adjusted and utilized. Suppose now it is assumed that a subset of factors of production can be costlessly adjusted while for the remaining inputs installation costs must be incurred, and so the latter are quasi-fixed factors.

Generally, two types of models have been developed which relate to factor adjustment. The first type emphasizes the trade-off between future increases in the quasi-fixed factors (and thereby future increases in output levels) and higher present costs associated with increasing adjustments speeds. The higher costs appear either as higher purchase prices of the quasi-fixed factors or as higher interest rates associated with financing the accumulation of these factors. The former costs have been considered by Lucas [1967], Gould [1968] and Mussa [1977], while the latter costs have been considered by Steigum [1983]. These models are able to capture the positive correlation between capital costs and investment and determine the magnitude of adjustment speeds associated with the quasi-fixed factors. The determination of investment in the quasi-fixed factors is the mechanism by which the adjustment process of these inputs are determined. There is, however, no relationship between the variable factor demands and investment in the quasi-fixed factors. Thus the costs of faster adjustment are not reflected in lower current production levels.

The second type of factor adjustment model recognizes that changes in quasi-fixed factor demands alter variable factor demands and thereby current output supplies. In this context, the costs of adjustment are reflected in lower current output levels. Thus, in adjusting quasi-fixed factors, the benefits of increased future output supplies are balanced by the costs of decreased present output supplies. This type of model emphasizes internal costs of adjustment through the technology and the cost is represented by foregone current output. The first model type emphasizes external adjustment costs, represented either by rising quasi-fixed factor purchase prices or by rising interest costs.

---

11 Recently Rao and Preston (1983) have investigated the effects of factor prices on factor demands and the rate of productivity growth for 9 Canadian manufacturing industries and 8 non-manufacturing Canadian industries for the period 1957-1979. They did not investigate the effects of tax policy on the structure of production. Surprisingly, their results were quite different than those obtained by Fraumeni and Jorgenson. In particular, technological change generally appeared to be capital-reducing.

12 An excellent survey on the role of R&D capital in production activities is by Griliches (1979).
rates. The models incorporating internal adjustment costs have been developed by Treadway [1971, 1974], Mortensen [1973] and Epstein [1981].

The model developed in this paper incorporates internal adjustment costs. With costly quasi-fixed factor adjustment but costless utilization, the first stage of the production decisions is given by (16) except the transformation function is now defined as

\[ T(y_t, v_t, K^{N}_t, K^{N}_{t+1} - (l_m - \delta) K^{N}_t, A_t) = 0 \]

and prices are not time invariant. Quasi-fixed factor utilization is costless and consequently depreciation is exogenous and constant over time, so that investment demands are

\[ I_t = K^{N}_{t+1} - (l_m - \delta) K^{N}_t. \]

The first order necessary conditions are similar to equation set (13) such that the variable profit function and derived conditions (with respect to the post-tax prices) are

\[ \pi^v_t = \Pi_t(P_t, W_t, K^{V}_t, K^{V}_{t+1}, A_t) \]  
(23.1)

\[ \nabla \Pi_t = y_t \]  
(23.2)

\[ -\nabla \Pi_t = v_t. \]  
(23.3)

In this case, after tax variable profits are defined as revenue minus variable input costs and future capital services enter the domain of the post-tax variable profit function because quasi-fixed factor adjustments are costly to undertake.

The second stage of the production problem is to

\[ \max_{K^{N}_{t+1}} \sum_{s=1}^{S} \alpha(s) \left( \prod_{t} P_t, W_t, K^{N}_s, K^{N}_{s+1}, A_s \right) - Q_s^T \left( K^{N}_{s+1} - (l_m - \delta) K^{N}_s \right). \]  
(24)

The first order necessary conditions for any time period are

\[ E_t \left[ \nabla \frac{\partial \Pi_t}{\partial K^{N}_{t+1}} - Q_t + \alpha (t + 1) \left( \nabla \frac{\partial \Pi_{t+1}}{\partial K^{N}_{t+1}} + (l_m - \delta) Q_{t+1} \right) \right] = 0. \]  
(25)

The equilibrium consists of equation sets (23) and (25). The empirical implementation of the model is generally quite complex and a number of procedures have been introduced in the literature. The complexity of the model relates to equation set (25) and the first procedure confronts this difficulty by placing enough structure on the technology and expectations in order for equation set (25) to have a closed form solution. We shall deem this procedure the direct approach. The direct approach restricts the technology represented by the variable profit function (or variable cost function if output is exogenous) to a quadratic specification and adjustment costs depend only on the first order changes in the quasi-fixed levels. In addition, the expectations process must be specified in the model. Berndt, Fuss and Waverman [1979], Denny, Fuss and Waverman [1981] and Berndt and Morrison [1981] impose static expectations. Sargent [1978], Meese [1981] and Hansen and Sargent [1980] have imposed rational expectations. Static expectations are to be understood in the context of continuous plan revision and the belief that current prices, tax credit and allowance rates persist. Current period plans are the only ones that are actually carried out. Rational expectations are to be understood in the context of generating forecasts of prices, taxes, credit and allowance rates which are the ones that best fit the actual time series. In this case, restrictions are im-

---

13 Hansen and Sargent [1981] have developed a model where adjustment costs do not have to depend on first order differences in the quasi-fixed factors. Their procedure has not as yet been implemented.
posed on the model (in other words, cross-equation restrictions on parameters) which reflect the maintained expectations processes.\textsuperscript{14}

The direct approach can be presented in the following context. Assume that there is a single output ($t = 1$) and so the technology can be represented by a production function which is assumed to be

\[ y_1 = \alpha^T V_1 + 0.5V_1^T AV_1 + 0.5(V_{t+1} - V_1)^TB(V_{t+1} - V_1) + H(t) \]  

(25)

where $V_t$ is the $n \times m$ vector of inputs which may be variable or quasi-fixed, $\alpha$ is an $n \times m$ vector, $A$ and $B$ are symmetric and negative definite matrices, and $H$ captures autonomous technological change as a function of time. The matrix $B$ is diagonal and represents the costs of adjustment in terms of foregone output.\textsuperscript{15} If a factor is variable then the relevant diagonal in $B$ is zero, while if the factor is quasi-fixed then the relevant diagonal is positive. In this manner, variable and quasi-fixed factors are distinguished.

Prices evolve according to the following process:

\[ S_t - \psi + \sum_{i=1}^{n} \theta_i S_{t-i} = G(t) + \xi_t \]  

(27)

where $S_t$ is the $n \times m$ vector of the post-tax factor prices normalized by the post-tax price of output ($p_i(1 - u_i)$). Indeed, $S_t$ is a vector of both variable and quasi-fixed post-tax prices (in other words, it contains both $W_i$ and $Q_i$). Also $\psi$ is a $m \times n$ vector, $\theta_i$ is a $m \times n$ dimensional matrix and $\xi_t$ is a $m \times n$ vector of white noise processes, and $G$ reflects the time trend.

The objective is to

\[ \max_{(V_{s+1}, \Phi V_{s})} \mathbb{E}_t \sum_{s=0}^{\infty} (1 + \rho)^{-s-1} \left[ y_s - S_b^T (\Gamma(V_{s+1} - (1 + n - 1) V_s) + \phi V_s) \right] \]  

(28)

with $\Gamma$ an $m \times n$ dimensional diagonal matrix with a 1 in the diagonal if the factor is quasi-fixed and a 0 if the factor is variable. $\Phi$ is a diagonal matrix defined conversely to $\Gamma$. $\delta$ is the diagonal matrix of constant depreciation rates, and the diagonal is zero for a variable factor. If the $i$th factor is quasi-fixed then $V_{is}$ is given. In addition, it must be assumed that the discount rate is known with certainty. This assumption is unavoidable if closed form solutions are to be obtained for multiple quasi-fixed factor production programs. The firm maximizes (28) by selecting the relevant factor demands subject to the technology (26) and price expectations (27).\textsuperscript{16}

The solution to this problem (see Kushner [1971] or Astrom [1970]) is the set of flexible accelerator factor demand equations,

\textsuperscript{14} In a recent paper, Epstein and Yatchew (1985) develop and estimate a model which assumes that the technology is quadratic with adjustment costs based on first order differences and expectations are based on autoregressive processes. Because they estimate the quadratic production function and autoregressive expectation equations along with the derived factor demand equations, they could test all of the cross equation restrictions implied by the production plan and expectations processes.

\textsuperscript{15} The specification of adjustment costs, which depend on net rather than gross changes in the quasi-fixed factors is not an important difference, provided that depreciation is exogenous because quasi-fixed factor utilization is costless.

\textsuperscript{16} Here both stages of the production decisions are combined into a single stage. In addition, a production function is specified because there is only a single output. We could just as easily have tackled this special case of the general model in two stages.

196 Corporate Taxes and Incentives and the Structure of Production
\[ V_{t+1} - V_t = M (V_t - V_{t+1}^*) \]  
\[ (29) \]

where
\[ V_{t+1}^* = A^{-1} ( \omega_t - \alpha ), \]
\[ \omega_t = C \sum_{s=t}^{\infty} (l_{m+n} + C)^{-s+1} \left[ E_t S_s - \Gamma \left( l_{m+n} - \delta \right) E_t S_{s+1} \right], \]
\[ C = AB^{-1} (1 + \rho) + R + M^T. \]

\( R \) is the \( m+n \) diagonal matrix with \( \rho \) in the diagonal and \( M \) is the stable adjustment matrix which solves the quadratic \( M^2 - (1 - \rho) B^{-1} AM - \rho M = B^{-1} A (1 + \rho) = 0. \)

The model which can be estimated consists of equations (26), (27) and (29) with stochastic error terms appended to equations (26) and (29). The disturbance terms in these latter two equations can reflect optimization or measurement errors. In addition, the disturbance in the production function, (26), can also reflect shocks to the technology.\(^{17}\) Berndt, Fuss and Waverman (1977) developed a special case of the above model which incorporated the corporate income tax credit, the physical investment tax credit and the physical capital cost allowance. They assumed that there was a single quasifixed factor and static price expectations. Under the assumption of exogenous output, the first of the two stages relating to the production decisions can be determined by the specification of a variable cost function (as opposed to a variable profit function when output supplies are endogenous). Assuming a quadratic variable cost function which is normalized by the first variable factor

\[ c' / W_i = \alpha_0 + \alpha_y y + \sum_{j=2}^{n} \alpha_j W_j + \alpha_k K + \alpha_t t + .5 \alpha_{yy} y^2 \]
\[ + .5 \sum_{j=2}^{n} \sum_{k=2}^{n} \alpha_{jk} W_j W_k + .5 \alpha_{kk} K^2 + .5 \alpha_{tt} t^2 \]
\[ + \sum_{j=2}^{n} \alpha_{yt} W_j t + \alpha_{kt} K^2 t + \alpha_{tt} t K^2 + u_0 \]  
\[ (30) \]

where
\[ c' / W_i = v_i + \sum_{j=2}^{n} W_j v_i \]

\( W_i \) is the normalized after-tax variable factor price, \( c' / W_i \) is the normalized after-tax variable cost, \( \Delta K = K_{it} - K_{it} \), and the parameters satisfy \( \alpha_{jk} = \alpha_{kj}, j = 2, \ldots, n \) by symmetry. Normalizing the variable cost function has the effect of imposing homogeneity of the first degree in the factor prices. The normalized variable cost function must also be nondecreasing and concave in the factor prices, nonincreasing and convex.

\(^{17}\) The disturbances in the factor demand equations (29) can also reflect technology shocks. However, by a similar argument to that presented in footnote 10, estimation problems arise because the error in the production function, from which the factor demands are derived, must be contemporaneously correlated with each of the factors in order for technology shocks to appear in each of the factor demand functions.

Session III

197
in the quasi-fixed factor, nondecreasing in output and nondecreasing and convex in net investment. Applying Shepherd's Lemma to the normalized variable cost function yields the conditional variable factor demand functions

\[ v_j = a_j + \sum_{s=2}^{n} a_{js} W_s + a_{jy} y - a_{jk} K_{k}^{N} + a_{kj} - u_j, \quad j = 2, \ldots, n. \]  

(31)

The stochastic disturbances \( u_0 \) and \( u_j, \quad j = 2, \ldots, n \) have been added to the variable cost and conditional variable factor demand functions. The error terms reflect the same kind of phenomena as described for the errors of equations (20) and (21). Equations (30) and (31) represent the first stage of the production decisions or the short-run equilibrium.

The determination of investment is governed by a flexible accelerator because this model is a special case of (28). The investment equation is

\[ K_{k+1} - K_k = M (K_k - K_{k-1}) + u_k. \]

(32)

where \( M \) is the stable adjustment matrix which solves the quadratic \( M^2 + (a_{kk}/a_{kj} + \rho) M - a_{kk}/a_{kj} = 0 \), and

\[ K_{k+1}^{Ne} = \left( -1/a_{kk} \right) \left[ a_k + a_{ky} y + \sum_{j=2}^{n} a_{jk} W_j + a_{kt} - W_k \right]. \]

is the long-run equilibrium demand for the quasi-fixed factor. \( W_k = Q_k(p + \delta) \) is the after-tax rental rate on this factor, and a stochastic disturbance has been added to the investment equation.\(^{16}\)

The model consists of equations (30), (31) and (32). Moreover, because the variable cost function is normalized, the errors in equations (30), (31) and (32) are linearly independent. The first variable factor conditional demand function has already been eliminated. Thus equations (30), (31) and (32) can be used to estimate the unknown parameters.

Berndt, Fuss and Waverman estimated this model for U.S. manufacturing for the period 1947-1974 and incorporated the corporate income tax rate, investment tax credit and capital cost allowance into the post-tax rental rate on capital. From our point of view, the most significant result of this paper was that the long-run price elasticities on the conditional factor demands (both variable and quasi-fixed) were considerably smaller than their counterparts obtained in models with no adjustment costs. This means that, for U.S. manufacturing, the influence of tax policy on long-run factor demands was significantly smaller than previous empirical evidence showed. The misspecification, by assuming all factors can be costlessly adjusted, caused an upward bias in the influences of factor prices, and thereby tax policy on input demands.

The second approach to the empirical implementation of the intertemporal production model is the dual approach developed by Rockafeller (1970), Benveniste and Schienkman (1979), McLaren and Cooper (1980) and Epstein (1981). The focus of this approach is not the variable profit function defined by (23.1) (or the variable cost function) but rather the value function defined by equation (10). Unlike the direct approach, dynamic duality can handle much more general specifications of the technology, in-

\(^{16}\) \( W_k \) is also defined in the discussion after equation (19). It is an outcome of the static price and tax expectations assumption. The disturbance in the investment equation represents optimizing or measurement errors. If the disturbance reflects technology shocks, then the error in the normalized variable cost function must be contemporaneously correlated with the quasi-fixed factor.
cluding the quasi-fixed factor adjustment mechanisms. However, the treatment of expectations formation processes is much more limited using the dual approach.

The dual approach can be presented in the following context. Assume that there is a single output \( t = 1 \) and the technology is represented by the general production function

\[
y_t = F(v_t, \mathbf{K}_t, \mathbf{K}_{t+1} - (\ell - \delta)\mathbf{K}_t, A_t).
\]

(33)

In addition, assume that there are static expectations on the prices, tax, credit and allowance rates and the firm's discount rate is constant.

The objective of the firm is to

\[
\max_{\{v_t, \mathbf{K}_{t+1}\}} \sum_{t=1}^{\infty} (1 + \rho)^{t-1} F(v_t, \mathbf{K}_t, \mathbf{K}_{t+1} - (\ell - \delta)\mathbf{K}_t, A_t) - W^r v_t
\]

\[
- Q^T (\mathbf{K}_{t+1} - (\ell - \delta)\mathbf{K}_t).
\]

(34)

with \( \mathbf{K}_t \) given, and the post-tax prices of the variable factors \( W \) and of the quasi-fixed factors \( Q \) are normalized by the post-tax price of output. This problem is a special case (combined into a single stage) of the one defined by equations (23) and (24). Rather than proceeding directly, we can use the Hamilton-Jacobi equation (see Arrow and Kurz (1970) and Dreyfus (1965)). Define the maximized value of (34) as \( J(\mathbf{K}_t, W, Q) \) and thus

\[
\rho J(\mathbf{K}_t, W, Q) = F(v_t, \mathbf{K}_t, \mathbf{K}_{t+1} - (\ell - \delta)\mathbf{K}_t, A_t)
\]

\[
- W^r v_t - Q^T (\mathbf{K}_{t+1} - (\ell - \delta)\mathbf{K}_t) + J(\mathbf{K}_{t+1}, - \mathbf{K}_t)
\]

(35)

where \( \rho = \ln (1 + \rho) \) and the factor demands are evaluated at the solution to the problem defined by (34). The solution to the problem (in other words, the factor demands) are found by differentiating both sides of (35) by the post-tax factor prices. Thus

\[
\mathbf{K}_{t+1} = \mathbf{K}_t + \frac{1}{J_{K}^T} [\rho J_Q + \mathbf{K}_t] - \mathbf{K}_t
\]

(36.1)

\[
v_t = - \alpha J_W + J_{KW}(\mathbf{K}_{t+1} - \mathbf{K}_t)
\]

(36.2)

\[
y_t = \alpha \left[ J_K (\mathbf{K}_t, W, Q) - J_{KW} W - J_{Q} Q \right]
\]

\[
- \left[ J_{K} - W^T J_{KW} - Q^T J_{KD} \right] (\mathbf{K}_{t+1} - \mathbf{K}_t).
\]

(36.3)

Equation (36.3), which is the output supply function, is derived by substituting equations (36.1) and (36.2) into (35).

By appending error terms to equation set (36) and postulating a functional form for the value function, \( J(\mathbf{K}_t, W, Q) \), the model can be implemented empirically. Epstein and Denny (1983) have investigated investment behaviour for U.S. manufacturing, Bernstein and Nadir (1985) have estimated the spillovers that are associated with R&D investment for U.S. firms, and Bernstein (1986) has estimated the benefits of physical and R&D investment tax incentives for Canadian firms using dynamic duality. In all cases, an intertemporal cost minimizing approach was used, because the stream of output was assumed to be exogenous.

In his model of tax incentives and the structure of production, Bernstein (1986) assumed that labour was the sole variable factor, while physical and R&D capital were
quasi-fixed factors. The discount rate was treated as a constant and there were static expectations on the prices.\(^9\) The value function was assumed to be of the form

\[
J(K^N, W, Q, y) = -0.5 \left[ Q^T W T \right] \begin{bmatrix} B_{QQ} & B_{QW} \\ B_{WQ} & B_{WW} \end{bmatrix} \begin{bmatrix} Q \\ W \end{bmatrix} y + \left[ Q^T A^{-1} + a^{-1} \right] K^N + \left[ Q^T A^{-1} h + h_0 \right] a^{-1}
\]

(37)

where the matrices \( B_{QQ}, B_{QW}, B_{WQ}, B_{WW} \) and \( A \), the vectors \( a \) and \( h \) and the scalar \( h_0 \) represent the unknown parameters. The matrices \( B_{QQ} \) and \( B_{WW} \) are symmetric and negative definite. \( B_{QQ} \) is an \( m \)-dimensional matrix (since there are \( m \) quasi-fixed factors) and \( B_{WW} \) is an \( n \)-dimensional matrix (since there are \( n \) variable factors). The stable adjustment matrix is given by \( [I_m - A] \), where \( A \) is an \( m \)-dimensional matrix and \( I_m \) is the \( m \)-dimensional identity matrix. This functional form for the value function is linear in output and the quasi-fixed factors and quadratic in the post-tax factor prices.\(^10\)

The results from the empirical work based on a sample of about 30 firms over the period 1975-1980 were that physical and R&D capital were complements both in the short and long-runs, while each type of capital was a substitute for labour. Both types of capital responded to changes in their own post-tax purchase prices. However, the demands for capital were quite price inelastic. Even in the long-run, the own price elasticities of the capital inputs were less than .4. Labour demand was relatively more price responsive in both the short and long-runs. The adjustment process for physical capital was shorter than for R&D capital. The latter took about six years to adjust while the former took about four years. Moreover, the capital stocks were complementary to each other along the adjustment path. In other words, increases in the stock of physical capital shorten the adjustment period of R&D capital.

Changes in the three types of tax incentives were considered in this study. First, a 1 percent increase in the physical investment tax credit generated increases in the demand for physical capital of .022 percent in the short-run and .055 percent in the long run. Similarly, the demand for R&D capital increased by .010 percent in the short-run and .029 percent in the long-run. Moreover, when the output effects of the physical investment tax credit increases were considered, the demands for all the inputs increased.

Second, an increase in the R&D investment tax credit also affected the structure of production. However, these effects were smaller relative to an equivalent increase in the physical investment tax credit. The third incentive was the R&D incremental investment allowance. An increase in this allowance affected the structure of production, but generated the smallest effects of all three incentives.

The fact that the empirical results were based on a dynamic model permitted the investigation of short- and long-run effects on factor demands from tax policy initiatives. In addition, the speed of the adjustment process was estimated. Bernstein determined the annual adjustment from the short to the long-run effect of any tax policy initiative. In the study, this type of analysis was conducted for R&D expenditures, because of its focus on policies influencing R&D investment.

Changes in tax credit and allowance rates decreased post-tax factor prices and thereby decreased production and adjustment costs. Using an intertemporal application

---

\(^9\) Epstein and Denny (1983) estimated models with both static expectations and expectations generated by first order autoregressive processes. However, in the latter case the processes were estimated independently of the production decisions.

\(^10\) In addition, this functional form is consistent with aggregation conditions guaranteeing the existence of a representative firm (see Dievert (1980), Epstein and Denny (1983) and Blackorby and Schwarm (1983)).

---

Corporate Taxes and Incentives and the Structure of Production
of Shepherd's Lemma based on the value function, permitted the determination of the
cost to the government, in terms of foregone tax revenues, of increases in the tax credit
and allowance rates. However, this analysis does not necessarily capture changes in
efficiency associated with changes in tax policy (see Diewert (1985(a)), and Jorgenson
and Stoker (1985)). Bernstein investigated the relative effectiveness of alternative tax
policies on the structure of production when the cost to the government across tax policy
changes was equalized. In addition, a calculation was made of the actual cost to the
government of alternative tax policy initiatives. The calculations showed that changes
in tax credit and allowance rates directed towards R&D investment generated about $3.82
of R&D expenditure per dollar of lost tax revenue at the existing level of output. More-
over, an increase in the physical investment tax credit generated around $0.66 of R&D
expenditure per dollar of lost tax revenue. This figure increased to around $1.51 when
output effects were considered. Hence, there may be important cross effects arising from
government tax policy changes directed towards a particular factor of production or type
of investment. Excluding these cross effects biases the cost estimates and the influence
of tax policy on production and investment.

The third approach to the implementation of the model given by equations (23) and
(25) is to treat the first order conditions for the quasi-fixed factors as implicit equations
and not obtain closed form solutions. This is the approach developed and implemented
by Kennan (1979), Hansen and Singleton (1982), Pindyck and Rotemberg (1983), and
Bernstein and Nadiri (1986). This approach which may be referred to as the implicit ap-
proach, involves the specification of a functional form for the variable profit (or variable
cost function) which is jointly estimated with the reduced form variable factor demand
equations and the implicit equations for the quasi-fixed factors. This approach permits
a great deal of flexibility in the specifications of the technology and the expectations
generating processes because the first order conditions for the quasi-fixed factors to not
have to be solved.

There are two difficulties with the implicit approach. First, because closed form sol-
solutions are not obtained for the quasi-fixed factors, there are no conditions in the model
guaranteeing the optimality (existence and uniqueness) of the factor demands for any
set of price trajectories. In other words, the terminal or transversality conditions are ig-
nored, as only the first order conditions are used. In terms of the estimation of the model,
since the estimator ignores the information contained in the transversality conditions, it
must not be asymptotically efficient. However, the direct and dual approaches require
the choice of particular expectation generating processes (as well as the choice of a
technology). This necessitates that these processes be incorporated into the restrictions
imposed in the estimation. An incorrect choice leads to inconsistent, as well as
asymptotically inefficient estimates (see Gourieroux, Laffont and Monfort (1979)).

The second difficulty with the implicit approach is that because the quasi-fixed factor
demands are not determined, we cannot characterize the properties of these demand
functions through time. We can only investigate the long-run properties of the quasi-
fixed factor demands. Wickens (1982) has suggested a solution to this difficulty. Replace
all expected values of future variables with their realizations to produce an observable
but incomplete system of equations. The system is then completed by adding equations
characterizing the determinants of future values of the variables in terms of any variables
known in the current period. Estimation of the complete system will be consistent but
not asymptotically efficient. Moreover, through this augmented system of equations we
can determine the short as well as the long-run properties of the quasi-fixed factor de-
mands. This method has not as yet been used to estimate models of production struc-
ture and to determine the effects of tax policy on this structure.