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sufficient, it is maintained that output can be expanded more than proportionately with the labour employed in manufacture (increasing returns to scale).

Smith used these two examples to draw a distinction between the 'heterogeneous' manufacture (exemplified by Petty's watch-making activity) in which the final output is obtained by a series or successive operations of fairly simple assemblage or 'partial and independent products', and the more sophisticated 'organic' manufacture (exemplified by Smith's pin factory) in which a series of successive operations gradually transforms the original raw material into the finished product.

It has been observed that these arguments are not truly convincing. The importance attributed to increased dexterity conflicts with the relatively low level of skills required in contemporary factories (witness the common use of child labour). Time saving does not imply specialization by individuals in principle, it could equally be attained by a suitable reorganization of the activity of a single artisan. And the introduction of machines does not seem to exhibit any necessary relation to the increasing division of tasks.

In fact the new organization of labour associated with the factory system did go along with the process of technical change associated with the industrial revolution. But its original role was primarily to discipline the manner in which the work was performed and to give the capitalist the power of controlling the production process in every single detail.

The introduction of machinery came after labour specialization and reinforced the need for a thorough organization of production. The effects of the introduction of the steam-engine and other complex machines were eventually studied by two scholars who possessed the necessary technical background, Charles Babbage (1832) and William Ure (1835); their tracts and other complex machines were eventually studied by two scholars who possessed the necessary technical background, Charles Babbage (1832) and William Ure (1835); their tracts and other complex machines were eventually studied by two scholars who possessed the necessary technical background, Charles Babbage (1832) and William Ure (1835); their tracts.


Assuming that the production function $f(\cdot)$ is \(1\) twice differentiable, i.e.
\[
\frac{\partial f}{\partial y_i} = f_i \quad \text{and} \quad \frac{\partial^2 f}{\partial y_i \partial y_j} = f_{ij} \quad (i, j = 1, \ldots, m+n),
\]
exist; \(2\) increasing in the net outputs, i.e. the derivatives, $f_i$ are always positive; and \(3\) convex (subject to the condition $f(\cdot) = 0$, the function is strictly convex); the optimal production plan of the firm can be stated using the familiar Lagrangian function:
\[
L(y_1, y_2, \ldots, y_{m+n}, \lambda) = \Pi + \lambda f(y_1, y_2, \ldots, y_{m+n})
\]
where $\lambda$ is a Lagrange multiplier associated with the constraint $f(\cdot) = 0$.

There are $m+n$ first-order conditions that can be interpreted as equality between the marginal profitability of each net output and its revenue or cost. The Lagrangian multiplier is the change in profit made by the firm with respect to a change in its production plan. Manipulating these equalities, we obtain familiar expressions such as the marginal transformation of commodities and inputs, the marginal rate of technical substitution among inputs and the expansion path of inputs. It follows that the profit-maximizing output and input levels, $\bar{y}_i(1, \ldots, m+n)$, and the Lagrange multiplier, $\lambda$, are functions of the prices $p_i(1, \ldots, m+n)$. That is:
\[
\bar{y}_i = z(p_1, \ldots, p_{m+n})
\]
and
\[
\lambda = \lambda(p_1, \ldots, p_{m+n}). \quad (i = 1, \ldots, m+n).
\]

\(\lambda\) is homogeneous of degree one, while $z(\cdot)$ is homogeneous of degree zero. $\bar{y}_i$ are the net supply functions. For outputs, the equations $\bar{y}_i$ are the usual supply functions; for inputs, they are the negative of the demand functions. Thus net supply functions exist provided that the marginal profitability conditions are satisfied and that the production function has the appropriate properties.

## PROPERTIES AND FORM OF THE PRODUCTION FUNCTIONS.

The characterization of the input demand and output supply functions depends on the specific properties of the production function. A number of studies have tried to specify these properties and discover more flexible functional forms to accommodate various economic effects often imbedded in the production process. Some economic concepts of interest are listed below. Though the concepts shown in Table 1 are defined in terms of a single-output production function, they can easily be extended to multiple-output production functions. Given the production function $y = f(\cdot, t)$, where $x$ is a vector of inputs and $t$ the index of technological change, it is possible to deduce expressions shown in Table 1 for returns to scale, shares of factors of production, price elasticity and elasticity of substitution, as well as various indices of disembodied technical change. Other effects such as indices of embodied technical change can also be derived. By imposing specific restrictions across these effects, different functional forms of the production function can be obtained. Of this array of economic effects, those associated with returns to scale, degree of substitution among inputs and the type and nature of technological change, have received prominent attention in the literature.

These economic effects arise from the inherent nature of the underlying production process, and the specific form of the production function is therefore critical in determining the existence and magnitude of these effects. These properties of the production function—homogeneity, additivity, and separability—have played an important role in the derivation of input demand and output supply functions. A homogeneous production function of degree $k$ is defined as:
\[
f(\lambda x_1, \ldots, \lambda x_n) = \lambda^k f(x_1, \ldots, x_n); \quad \lambda > 0
\]
and a monotonic transformation of a homogeneous production function yields a homothetic production function in $y = g(f(\lambda x_1, \ldots, x_n))$. This family of production functions is characterized by straight-line expansion paths through the origin. Additivity may take the form:
\[
f'(\lambda x_1) + \cdots + f'(\lambda x_n) = f'(x_1) + \cdots + f'(x_n) = 0 \quad \text{for any } \lambda > 0
\]
where $y_i$ represents net output of $i$th commodity, some of which are inputs to the production process. If the function $f'$ is either homogeneous of some degree, or logarithmic, the additivity condition holds.

Most of the theoretical formulations of the production functions described in the literature implicitly assume that separability conditions prevail. The $f(x)$ is weakly separable with respect to partition $R$ when the marginal rate of substitution (MRS) between any two inputs $x_i$ and $x_j$ from any subset $N_s, s = 1, \ldots, r$, is independent of the quantities outside $N_s$ (Leontief, 1947; Green, 1964; Berndt and Christensen, 1973) or $\frac{\partial f(x_i)}{\partial x_i} = 0$. Strong separability, on the other hand, exists when $\text{MRS}$ between any two inputs inside $N_s$ and $N_t$ does not depend on the quantities outside $N_s$ and $N_t$ or $\frac{\partial f}{\partial x_i} = 0$.

Functional separability plays an important role in aggregating heterogeneous inputs and outputs, deriving value-added functions and estimating production functions. It also opens up the possibility of consistent multi-stage estimation, which may be the only feasible procedure when large numbers of inputs and outputs are involved in the production activities of highly complex organizations.

A major preoccupation in the literature for empirical estimation of production functions has been to find flexible functional forms. Well-known functions (e.g. the Leontief and Cobb–Douglas production functions) impose restrictions of zero and one, respectively, on the elasticity of substitution, $\sigma$, while for CES production functions, $\sigma$ is an arbitrary constant to be estimated. Attempts to relax this stringent requirement have led to the development of the variable elasticity of...
substitution functions (VES) where \( \sigma \) is dependent on economic variables such as input mix (Liu and Hildebrand, 1965; Kadiyala, 1972). Efforts to relax the homogeneity property have led to the development of a number of non-homothetic production functions that make the returns to scale dependent on output and/or input mix (Zellner and Revankar, 1969; Färe, Jansson and Knox Lovell, 1978). A major advance has been the formulation of non-homothetic production functions by Christensen, Jorgenson and Lau (1973), who formulated the translog production function, which does not a priori impose restrictive constraints such as homotheticity, constancy of \( \sigma \), additivity, and so on.

**TECHNICAL PROGRESS.** Technical progress deals with the process and consequences of shifts in the production function due to the adoption of new techniques which either have a neutral effect on the production process or change the input–output relationships. Neutrality of technical progress can be measured by its effect on certain economic variables such as capital–output, capital–labour and capital–labour ratios, which should remain invariant under technical change. Several definitions of technical progress have been proposed, such as (1) product-augmenting, (2) labour- or capital-augmenting, and (3) input-decreasing and factor-augmenting, amongst others (Beckmann, Sato and Schupack, 1972). However, the most familiar definitions are the Hicks, Harrod, and Solow forms of technical progress.

Part of technical change can be endogenous and would be determined by the firm to maximize its long-run profit. Technical knowledge is expensive to produce but, once produced, its transmission cost is almost zero, giving rise to the 'indivisibility' and 'inappropriability' characteristics of inventions. Attempts have been made to incorporate R&D as an input in the neoclassical production and cost functions, to estimate its contributions to the firm's productivity growth and cost behaviour, and to measure its spillover effects on other firms or industries (Nordhaus, 1969; Griliches, 1979.) The results indicate substantial private and social rates of return to R&D (Mansfield, 1969). Changes in relative prices and output not only affect endogenous technical change but also the rate of factor productivity and the bias of technical change, which will turn alter the structure of the production process (Jorgenson and Fraumeni, 1981).

**DUALITY.** A major advance in the economic theory of production has been the dual formulation of production theory (Shephard, 1953; Diewert, 1974; Fuss and McFadden, 1978). The main features of this approach is to recover through indirect functions—that is, by means of a dual representation such as profit or cost functions—the properties of the underlying production function. The dual approach not only contributes important insights of its own but also offers more immediate empirical applications. A mapping of the characteristics of the transformation function and its dual cost function is indicated in Table 2. The cost formulation is used extensively in econometric studies. This approach has two main advantages: (1) demand and supply functions can be derived as explicit functions of relative price and output without imposing arbitrary constraints on production patterns required in the traditional methodology; (2) cost and profit functions are computationally simple and permit testing of a wider class of hypotheses by utilizing economic variables (Nadiri, 1982).

**DYNAMIC FACTOR DEMAND MODELS.** These types of production functions emphasize the intertemporal aspect of the production process by focusing on the movement from one equilibrium state to another. The models incorporate costs of adjustment that are incurred in order to change the level of quasi-fixed inputs, costs which can take two forms. The first type is internal: as the firm adjusts its quasi-fixed factors it must face either a higher purchase price for these factors (Lucas, 1967; Gould, 1968) or a higher financing cost for the accumulation of these inputs (Steigum, 1983). The second type is internal and reflects the fact that firms must make the trade-off between producing current output and diverting some of the resources from current production to accumulate capital for future production (Treadway, 1974).

Suppose the firm maximizes its present value:

\[
V = \int_0^\infty \{Py - WL - rK - gK\} e^{-r} dt
\]

subject to the production function \( f(y, L, K) = 0 \) and the initial condition \( K(0) = K_0. \) \( P \) is the price of output, \( y \) is the level of output, \( W \) is the nominal wage, \( r \) is the user cost of capital, \( G \) is the purchase price of investment, \( K \) is a vector of capital inputs, \( L \) is labour, and \( K \) is net investment. \( K \) is introduced in production on the assumption that firms produce essentially two types of outputs: \( y, \) to sell, and \( K, \) the internally accumulated capital which will be used in future production. \( K \) is assumed to be neither perfectly fixed nor perfectly variable. Suppose, in addition, that the production function is characterized by the relation \( y + C(K) - g(K, L) = 0, \) where \( C \) and \( g \) are continuous and the marginal cost functions of \( f_L \) and \( f_K \) are positive and diminishing.

From the necessary conditions, it follows that for perfectly variable inputs its marginal product must equal its price, while for the quasi-fixed inputs the discounted sum of future net values of its marginal product must equal the sum of the purchase price of investment and the marginal value of real product foregone as a consequence of expansion at the rate \( \dot{K}. \)

### Table 2. Comparison of the properties on the transformation function and its dual cost function

<table>
<thead>
<tr>
<th>Property A on the transformation function ( F(y, x) )</th>
<th>Property B on the cost function ( C(y, p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Non-increasing in ( y )</td>
<td>Non-decreasing in ( y )</td>
</tr>
<tr>
<td>2 Uniformly decreasing in ( y )</td>
<td>Uniformly increasing in ( y )</td>
</tr>
<tr>
<td>3 Strongly upper semi-continuous in ( (y, x) )</td>
<td>Strongly lower semi-continuous in ( (y, p) )</td>
</tr>
<tr>
<td>4 Strongly lower semi-continuous in ( (y, x) )</td>
<td>Strongly upper semi-continuous in ( (y, p) )</td>
</tr>
<tr>
<td>5 Strongly continuous in ( (y, x) )</td>
<td>Strongly continuous in ( (y, p) )</td>
</tr>
<tr>
<td>6 Strictly quasi-concave from below in ( x )</td>
<td>Continuously differentiable in positive ( p )</td>
</tr>
<tr>
<td>7 Continuously differentiable in positive ( x )</td>
<td>Strictly quasi-concave from below in ( p )</td>
</tr>
<tr>
<td>8 Twice continuously differentiable strictly continuously quasi-concave from below in ( x )</td>
<td>Twice continuously differentiable and strictly differentiable quasi-concave from below in ( p )</td>
</tr>
</tbody>
</table>
ECONOMIES OF SCALE AND SCOPE. An important extension of the dynamic profit or cost functions. The main result of these expectations about future prices of inputs and outputs. A important question is what are the behaviour of a multi-product firm when economies of scale prevail. To derive the net supply functions, the necessary conditions for equilibrium noted earlier break down when increasing returns or declining long-run average costs prevail. In such cases, monopolistic organization of an industry may offer cost advantages over production by a multiplicity of firms. An interesting and important question is what are the necessary and sufficient conditions for a multi-product firm to be a natural monopoly and for it to be sustainable against entry (Baumol, 1977). The condition for natural monopoly is that a cost function be strictly and globally substitutive in the set of commodities \( C(y^1 + \cdots + y^n) < C(y^1) + \cdots + C(y^n) \), which means that as only one firm produce whatever combinations of output is supplied to the market. If the output vectors are restricted to be orthogonal, then the production function exhibits economies of scope. The natural monopoly is a sustainable set of products set at prices that do not attract rivals into the industry (Baumol, Bailey and Willig, 1977). Even if rivals are attracted, the monopoly may be able to protect itself from entry by changing its prices. But, by definition, only a sustainable set of prices can prevent entry and yet remain stationary. The conditions necessary for sustainability are (1) the products are weak gross substitutes; (2) the cost function exhibits strictly decreasing ray average costs; and (3) the cost function is also transversely convex. Ramsey prices often ensure sustainability under specified circumstances.  

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See also CORB-DOUGLAS FUNCTIONS; COST AND SUPPLY CURVES; COST FUNCTIONS; HUMBLAG PRODUCTION FUNCTION; JOINT PRODUCTION; SUPPLY FUNCTIONS.

BIBLIOGRAPHY


