Rate of Utilization, Relative Prices and Investment Behavior

By

M. I. Nadiri
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1. INTRODUCTION

The recent literature on investment behavior seems to follow two major
tables of development: (1) to explicitly formulate costs of adjustments
as integral parts of the theory of investment, and empirically estimate
such models,¹ and (2) to explore the empirical relevance and importance

¹ See, for example, Lucas [30], Schramm [43] and Treadway [46].

of relative prices in determining investment behavior.² Though the

² Most of the work on this problem has been mainly in-
spired by works of Jorgenson and associates. See [21], [22], [24]
and [25]. For an evaluation of the role of relative prices and output in determining investment behavior see [11].

former approach is now very popular and may lead to significant results,
there are some important issues in the context of the now traditional approach that require further analysis. Some of these issues are set out below.

(a) Unlike empirical models of employment, most of the existing investment models assume the rate of utilization of capital stock to be fixed and often equal to unity.\(^3\) Such an omission leads to an improper specification of the user cost of capital services and replacement policy.\(^4\) Also, the elasticity of output with respect to measured capital will be overestimated when its rate of utilization is ignored.\(^5\)

(b) Not much effort has been made to introduce factor prices other than rental on capital and output prices in the investment models. The separate response of investment behavior to changes in output and relative prices has not been sufficiently explored. Nor is much known about the short-run and long-run elasticities of investment with respect to output and relative prices.

\(^3\) This is true of all the empirical works of Jorgenson and associates and Eisner. See footnote 2, [8] and [10].

\(^4\) See Keynes [28], pp. 66-73 and Tobin [45].

\(^5\) See Feldstein [14] and Nerlove [38].
(c) We need to know the degree of return to scale, the role of technological change, and the long-run elasticity of substitution from the investment models.\textsuperscript{6}

\textsuperscript{6}It is of course possible to estimate these parameters by estimating production functions directly. However, approaching the problem from the input side has certain advantages described by Nerlove \textsuperscript{[37]}. These, relatively unexplored issues will be analyzed in this paper with the aid of an investment model that incorporates relative prices of labor and capital services, changes in output, technological change, and the rates of utilization of capital and labor. This model is cast in terms of demand for capital services and is analogous to models explaining behavior of man-hours. The model is sufficiently general to combine most important features of the existing econometric models. By constraining certain parameters of this model the distributed lag accelerator suggested by Eisner \textsuperscript{[8]}, the neoclassical model proposed by Jorgenson \textsuperscript{[25]}, and the accelerator-residual funds model developed by Meyer-Kuh \textsuperscript{[34]} and others,\textsuperscript{7} can be deduced as special cases.

\textsuperscript{7}See, for example, \textsuperscript{[2]}, \textsuperscript{[13]}, \textsuperscript{[35]} and \textsuperscript{[40]}

The model is estimated using quarterly data for the U.S. total manufacturing sector for the period 1949-I to 1960-IV; the stability of the estimated function is tested by fitting the regression to the
period of 1949-I to 1962-IV. In Section 2 the general features of the issues are related. Section 3 deals with the specification of the model. Essentially, we modify and extend Jorgenson's investment model. The choice of this model as the point of departure is due to the flexibility of its formulation and to its superior empirical performance among the available quarterly investment models.\(^8\) Section 4 explains the empirical results, examines the intertemporal stability of the model and its empirical performance against an autoregressive investment model. The nature of the data and the specification of the variables of the model are discussed in the statistical appendix.

2. SETTING OF THE PROBLEM

Output can be increased by augmenting the stocks of inputs, \(K\) and \(N\), or their respective rates of utilization, \(m\) and \(h\). In the very short run, for output to increase by increasing \(h\) may require only that the rate of utilization of the existing stock of capital, \(m\), increase (i.e., the machines must be used for longer hours). However, if the pressure of demand persists the firm may consider increasing its labor force, \(N\). Increasing \(N\) may require the firm (a) to increase \(m\) above its normal level, \(m^*\), provided no excess capacity exists, (b) to reactivate idle machinery with no change in the rate of utilization, \(m\), or (c) add to the firm's stock of capital by increasing outlays on investment. It is possible and most probable, that a combination of such policies will be
pursued by the firm. The critical factors which will affect a firm's policy will be the degree of permanency of expected output, the relative prices properly defined, the degree of substitution between capital and labor, the time lag between appropriating the outlays for investment and installing the new machinery, etc.

Increasing m and h induces additional costs: hiring new workers, paying over-time, depreciation costs due to intensive use of the equipment. The last cost reflects the loss of earnings due to rising maintenance bills. It may or may not be independent of depreciation due to exogeneous factors, especially in industries with a rapid rate of technical progress. In the short run, m and h will serve as adjustment

\[ \text{\footnotesize In such industries depreciation due to use will be rather small because the value of obsolete equipment is quite low. A higher rate of utilization also affects the timing of replacement by advancing the date for scrapping the equipment. Moreover, depreciation due to use may not be constant but rather, may vary with the rate of utilization see [42] and [44].} \]

factors and will return to their equilibrium levels, \( m^* \) and \( h^* \), when the firm has had time to adjust its capital stock and labor force. Thus it is possible to have no substitution between \( K \) and \( N \) in the short run, but some substitution when the inputs are defined in terms of services. Thus, variation in the rate of utilization of capital stock affects the marginal productivity of capital, the user cost, and the replacement policy of the firm.
The precise form of the relative prices that enter an investment function depends on the assumptions about the marginal productivity conditions for the inputs. If both marginal productivity conditions of capital and labor hold, the relevant measure of relative prices is the ratio of factor prices. However, if the marginal productivity condition for capital services holds, leaving the marginal productivity of labor to its own destiny, then the appropriate relative price is the rental on capital deflated by price of output.

3. SPECIFICATION OF THE MODEL

Assume a production \( Q = F(N, h, K, m, T) \) where \( Q \) is the rate of output, \( N \) is employment, \( h \) is the hours worked, \( K \) is stock of capital goods, \( m \) is the rate of utilization of \( K \), and \( T \) is the existing level of technology. Suppose the firm minimizes its expected total costs, defined by equation (1), subject to the constraints (2) and (3).

\[
C_t = wN h_t + q I_t 
\]

\( Q_t = F(N_t, h_t, K_t, m_t, T) \) \hspace{1cm} (2)

\[
I_t = \dot{K}_t + (\delta_0 + \delta_1 m_t) K_t \hspace{1cm} (3)
\]

\( w \) is average wage per man-hour; \( q \) is the price of capital goods; \( \dot{I} \) is the increment to capital services, \( \dot{K} \) is the net increase in capital services; \( \delta_0 \) is the rate of depreciation due to passage of time or technical change and \( \delta_1 \) is the rate of depreciation due to intensity of use of the capital stock.
The least cost combination for $L, K, h, \text{and } m$ can be stated by solving the calculus of the variation problem of minimizing (1) subject to constraints (2) and (3).

$$F_L = hw$$

$$F_K = q [\delta_0 + \delta_1 m + r - \frac{\partial}{\partial q}] = \overline{c}$$

$$F_h = Nw (1 + \varepsilon_h)$$

$$F_m = q (1 + \varepsilon_m) K$$

$$p = \lambda$$

where $F_L, F_K, F_m$ and $F_h$ are the marginal productivities of the variables, $\overline{c}$ is the user cost of capital services,

$$\varepsilon_h = [-\frac{dw}{dh} \cdot \frac{h}{w}] \text{ and } \varepsilon_m = [-\frac{dc}{dm} \cdot \frac{m}{c}].$$

To simplify, we rewrite the production function (2) as

$$Q = G (\overline{K}, \overline{L}, T)$$

where $\overline{K} = Km$ is capital services and $\overline{L} = Nh$ is labor services. Assuming that $m$ and $h$ are given, the demand for labor and capital services can be formulated in terms of relative prices and level of output.\(^\text{10}\)

\(^{10}\)This would be true for any homogeneous production function.

See Allen [1], Dhrymes [7] and . Rosen [41].

The relevant marginal productivity conditions are:

$$G_1 = w/p$$

$$G_2 = \overline{c}/p$$
where $G_1$ and $G_2$ are marginal products of labor and capital services.

For empirical estimation of changes in capital service, we need to specify the form of the production function and the assumptions made about the marginal productivity conditions, (6a) and (7a). Suppose the production function is a CES, i.e.,

$$Q = \gamma [\delta (K)^{-\rho} + (1 - \delta)(L)^{-\rho}]^{-\upsilon / \rho} e^{f(t)} \quad (2c)$$

where $\gamma$ is the efficiency parameter, $\delta$ is the intensity parameter, $\rho$ is the substitution parameter, $\upsilon$ is the degree of homogeneity of the production function. The last term in (2c) depicts a curvilinear dis-embodied technical change. The marginal productivity conditions for $\bar{K}$ and $\bar{L}$ are:

$$F_{\bar{K}} = \delta \upsilon \gamma^{-\rho / \upsilon} Q^{1+\rho / \upsilon} (K)^{-(\rho+1)} e^{-\rho / \upsilon} e^{f(t)} = \frac{c}{p} \quad (6b)$$

$$F_{\bar{L}} = (1-\delta) \upsilon \gamma^{-\rho / \upsilon} Q^{1+\rho / \upsilon} (L)^{-(\rho+1)} e^{-\rho / \upsilon} e^{f(t)} = \frac{w}{p} \quad (7b)$$

where $c$, $w$ and $p$ are respectively the rental on capital services, wage rate, and the price of output. We assume that the stochastic error term is associated only with the production function (2c) and the economic relations (6b) and (7b) hold without any error.\footnote{This assumption is necessary, otherwise a complex problem of identification will arise. See Nerlove [37], pp. 107-109.}

The specific determinants of desired capital services, $K^*$, and consequently of its rate of change, will depend on the production function parameters $\rho$ and $\upsilon$ and whether one or both equations, (6b) and (7b) hold. The following cases are of interest:
1. Assume the production function is Cobb-Douglas, \( Q = AK^\alpha L^\beta e^{f(t)} \).

1.a. If \( v = \alpha + \beta = 1 \) and the marginal productivity condition (6b) holds

\[
\frac{K^*}{K} = \alpha \frac{DQ}{c}
\]  

(9)

1.b. If \( v = \alpha + \beta \neq 1 \) and both marginal productivity conditions (6b) and (7b) hold

\[
\frac{K^*}{K} = A_0 Q^{1/v} (w/c)^{\delta/v} e^{1/uf(t)}
\]  

(9a)

where

\[
A_0 = \left[ A \left( \frac{\beta}{\alpha} \right) \right]^{-1/v}
\]

2. Assume the production function is CES.

2.a. If \( v = 1 \) and condition (6b) holds.

\[
\frac{K^*}{K} = A_1 Q (\frac{p}{c})^{\sigma} e^{(1-\sigma)f(t)}
\]  

(9b)

where

\[
A_1 = \left[ \delta \gamma^{-\rho} \right]^{\sigma} \text{ and } \sigma \text{ is the elasticity of substitution, } \sigma = \frac{1}{1+\rho}
\]

2.b. If \( v \neq 1 \) and condition (6b) holds

\[
\frac{K^*}{K} = A_3 (\frac{p}{c})^{\sigma} Q^{[\alpha+(1+\sigma)/v]} e^{[1-\sigma]/uf(t)}
\]

where

\[
A_3 = \left[ \delta \gamma^{-\rho/v} \right]^{\sigma}
\]

2.c. If \( v \neq 1 \) and both conditions (6b) and (7b) hold

\[
\frac{K^*}{K} = A_4 Q^{1/v} z^{1/\rho} e^{-1/uf(t)}
\]  

(9d)

where

\[
A_4 = \delta^{-1/v}
\]

and

\[
z = \left( \hat{\delta} + (1-\hat{\delta}) \left[ \hat{\sigma}/(1-\hat{\delta}) \right]^{\rho/\sigma} (\frac{\sigma}{\lambda})^{\rho/\sigma} \right)
\]

\( \hat{\delta} \) and \( \hat{\rho} \) are the estimated values of \( \delta \) and \( \rho \) in the production (2c).
They are obtained by estimating the expansion path of $\bar{K}$ and $\bar{L}$.\textsuperscript{12}

\textsuperscript{12} estimation

This is the two-step procedure used in [4] and [5]. For a useful discussion of this procedure see [37].

By estimating the appropriate investment functions, i.e., the first difference of equations (9) to (9d), the magnitude of the returns to scale and the elasticity of substitution of the underlying production function consistent with the data can be ascertained.

Generally, there are some delays in adjusting the desired capital services of the firm due to difficulties of procuring new capital, reactivating old machinery, costs of changing the rate of utilization of capital and overtime, imperfections of the markets, adjustments in other production inputs, etc. To take account of the adjustment of actual to desired capital services we postulate a Koyck adjustment mechanism, i.e.,

$$\frac{\bar{K}_t}{\bar{K}_{t-1}} = \left[ \frac{\bar{K}^*}{\bar{K}_{t-1}} \right] \lambda$$

where $\bar{K}_t$ and $\bar{K}^*$ are respectively the actual and desired capital services for and $\lambda$ is the coefficient of adjustment. Inserting $\bar{K}^*$ into equation (10) from any of the equations (9) to (9d), taking logs, and first differencing, will produce the estimating function

$$\tilde{dlnK}_t = a_0 + \sum_{i=1}^{k} \tilde{dlnQ}_{t-1} + \sum_{f=0}^{1} \sum_{j=1}^{\phi} \tilde{dlnK}_{t-j} = \sum_{i=0}^{1} \tilde{dln\phi}_{t-i} + \tilde{dlnK}_{t}$$

where $\phi$ is the relative price variable. Note the lag response of investment to changes in output and relative prices is not the same
in equation (11). There is no economic reason for assuming, as Jorgenson and others have done,\textsuperscript{13} that investment will respond in the same manner
to changes in output and relative prices. Output changes are generally frequent and contain large transitory components. The expected changes in demand can be approximated by several past changes in the level of actual output. Factor prices, on the other hand, do not change greatly and can be extrapolated into the future with some degree of confidence.\textsuperscript{14}

\textsuperscript{13} See, for example, [24] and [25].

\textsuperscript{14} An alternative explanation can be given in terms of "putty-clay" hypothesis. According to this hypothesis, changes in relative prices affect the equilibrium value of capital services but changes in level of output require additional capital at the prevailing prices. In the context of our model, the responses of changes in $K_t$ to changes in relative prices should be much smaller and less rapid than those to changes in level of output. The empirical results substantiate this statement (see p. \ref{p:empirical}, and Table \ref{table:empirical} equation \ref{eq:empirical}). The finding here reverses the speculation we made in [11]. The explanation for the divergence of the two results is mainly due to constraining the coefficients of $\phi$ before $t-1$ to zero and the introduction of the rate of utilization.
Note, also, that equation (11) implies disembodied technological change of quadratic or higher polynomial form in the production function.\textsuperscript{15} The time structure of changes in capital services is de-

\textsuperscript{15}The effect of nonneutral technical change cannot be tested directly from this equation. For an indirect test see p. 167

picted by $\sum_{j=1}^{\omega} I_{\omega}$, which includes the Pascal and Koyck distributed lage mechanisms as special cases.
3a. RELATION WITH EXISTING MODELS

The existing econometric investment models differ from each other mainly in the specification of the time structure of investment, cost of capital services, and the role of financial variables. Conceptually, they are not very different. They can be deduced from the marginal efficiency approach or the derived demand for capital services. The two approaches are, of course, the same.\(^{16}\) The conceptual framework of equation (11) is quite general and includes most features of the available econometric investment models. If the rate of utilization of capital, \(m\), is unity or constant and the elasticity of investment with respect to relative prices is zero, we get the essence of Eisner's investment model. Investment expenditure becomes a function of past changes in output.\(^{17}\) On the other hand, if the output and price elasticities of investment are equal to unity, the production function is Cobb-Douglas with constant returns to scale, and \(m = 1\), we obtain Jorgenson's

\(^{16}\) See Jorgenson [23].

\(^{17}\) With these assumptions equation (11) can be written as

\[
dlnK_t = b_0 + \sum_{i=1}^{n} b_i Q_{t-i} + \omega_1 dlnK_{t-1}
\]

See [8], [9] and [10].
investment function. \(^{18}\) Hickman's investment model is obtained if \(m = 1\)

\(^{18}\) One of the assumptions of Jorgenson's model is that the time structure of investment response to output changes and relative prices are the same. Equation (11) when empirically estimated should have equal number of lagged output and price variables to resemble Jorgenson's model. The results for such a function is reported in [11].

and the price elasticity of investment is low or zero. \(^{19}\)

\(^{19}\) Hickman has attempted in [20] to separate the effect of output changes and relative prices on investment behavior. He has used measures similar to our own for relative prices in his regressions but found little effect of prices on investment expenditures. See also [43].

The investment models developed by Meyer and Kuh [34] and their followers \(^{20}\) suggest that the determinants of investment and expenditure differ due to business cycle developments and that financial variables are important determinants of investment behavior. In the upswing the critical determinant of investment is the rate of utilization of capacity;

\(^{20}\) See footnote 7.
in the downswing the primary determinant of investment is the gross re-
tained earnings of the firm. The main elements of the accelerator-
residual funds theory can be explained in terms of relative factor cost
in our model. During the expansionary phase of the business cycle, rate
of utilization, m, approaches unity and both factor prices, w and c, and
the level of output all increase. It is the shift in the demand function
which is of critical importance during this phase of the business cycle.
The financial factors serve as mild restraints on investment and the
ratio of output to price elasticities of investment is high in this phase
of the cycle. Higher costs of factors require larger financial resources
which, if adequate, could lead to postponing investment expenditure.\textsuperscript{21}

\textsuperscript{21}It can also be argued that financial variables affect the co-
efficient of adjustment, \( \lambda \) of equation (11). Adequate financial re-
sources make it easier to buy capital goods by supplying funds for
immediate payments or by allowing higher prices to be paid to obtain
a faster flow of factor services. This interpretation is a version
of Eisner-Strotz' argument \cite{12}. Nonetheless, it is through relative
prices that the firm can influence the flow of capital services.

In the recessionary periods, given the demand for investment, the supply
of funds schedule must shift downward if the investment is to increase
according to the accelerator-residual funds theory. It is through this
reduction in factor costs, as Meyer and Glauber \cite{35} have argued, that
the supply of funds "pushes" investment in the recessions. Thus, proper
specification and measurement of the factor costs eliminates the need for including ad hoc financial variables in the investment functions.

4. THE EMPIRICAL RESULTS

Various versions of our model were fitted to the data for the total manufacturing sector for the period 1949-I to 1960-IV. The variables of the regression equations were selected on the basis of a priori correct signs and statistical significance of their coefficients. The significance criterion adopted was that the ratio of the coefficients to its standard error should be, at least, greater than one.²² This condition insures maximization of the adjusted coefficient of determination, $\bar{R}^2$.

The preliminary estimating equation chosen on the basis of these criteria with a Koyck lag adjustment mechanism was the following.²³

$$d\ln K_t = a_0 + \sum_{i=0}^{5} \eta_i d\ln Q_{t-1} + \psi_0 d\ln \phi_t + \tau_1 t + \omega_1 d\ln K_{t-1}$$  \hspace{1cm} (12)

²²The exact condition for this test is that the t value of each coefficient is greater than one/19]. However, most of the coefficients of the regression equations in Table I pass the conventional test of having t values greater than two.

²³Several lagged values of $\phi_t$ were used but their coefficients had either the wrong sign or were statistically insignificant. This was true irrespective of which variant of the relative price, i.e., p/c or w/c, is used as a measure of $\phi$. 
The results for the equation (12), when $\phi$ is defined as $w/c$ are indicated in Table I, column 1. Several features of these estimates are of interest: (a) the coefficients of changes in output trace a declining path and are all statistically significant, (b) the current relative price variable, $\phi_t$ seems to be a good proxy for future prices,\(^{24}\) (c) the time trend, $t$ is positive but statistically insignificant, (d) the adjustment coefficient implied by the regression equation is $\lambda = .06$, (e) the $R^2$ and standard error of the regression are very good according to conventional standards, and (f) judging from the value of D/W test statistics there is some serial correlation in the residuals.

The test for autocorrelation of errors based on the Durbin-Watson ratio is biased toward randomness for fitted regressions containing a lagged dependent variable.\(^{25}\) The conventional method of eliminating auto-

\(^{24}\)See footnote 23.

\(^{25}\)See Nerlove-Wallace [39].

\(^{26}\)A simple example can clarify this point. Suppose net investment, $N_t$, is determined by the following equation

\[ \text{dlnK}_t = a_0 + \sum_{t=0}^{6} \text{dlnQ}_{t-i} + \sum_{f=0}^{1} \text{dln}\phi_{t-f} + \sum_{j=1}^{2} \text{dlnK}_{t-j} \]
\[ N_t = \omega_0 \Delta S_t + \omega_1 \Delta P_t + N_{t-1} + \nu_t \quad (12a) \]

where \( \Delta S_t \) and \( \Delta P_t \) are respectively changes in the levels of sales and profit. Assume further that \( \nu_t \) is serially correlated, i.e.,

\[ \nu_t = \alpha \nu_{t-1} + \varepsilon \quad (12b) \]

\( \varepsilon \) is randomly distributed. If we subtract \( \alpha N_{t-1} \) from both sides of equation (12a) and rearrange the terms we get

\[ N_t = \omega_0 (\Delta S_t - \alpha \Delta S_{t-1}) + \omega_1 (\Delta P_t - \alpha \Delta P_{t-1}) + (\omega_2 + \alpha) N_{t-1} - \omega_2 \alpha N_{t-2} + \nu_t \quad (12c) \]

The Durbin-Watson test statistics is an appropriate measure of the serial correlation of equation (12c). See Griliches [15] and Malinvaud [31], pp. 468-472.

The statistical results of equation (13) are indicated in Table I, column 2.\(^{27}\) The coefficients of output changes are all positive and statistically

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\(^{27}\)Time was not included in the new regression. The coefficient of the time trend was generally insignificant. When a quadratic time variable (\( t \) and \( t^2 \)) was added to equation (13) the sign of the \( t \) was positive and \( t^2 \) had a negative coefficient, suggesting that technical change may have increased capital requirements during the sample period but at a decreasing rate. However, both coefficients were statistically insignificant.

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significant. However, they trace an almost V-shaped path. This pattern is due to the autoregressive transformation. Jorgenson and Eisner have
also found some fluctuation in the coefficients of output and other explanatory variables of their models. The price elasticity of investment increases when \( \phi_{t-1} \) is included in the regression equation; \( \phi_{t-1} \) has the correct positive sign but is statistically not significant. Finally, the coefficient of \( \text{dln}K_{t-2} \) is statistically significant, has the correct negative sign,\(^{28}\) and its inclusion increases the adjustment co-

\(^{28}\) See footnote 26.

efficient of the model to about 0.08. Also, the goodness of fit statistic of the model improves substantially due to this transformation; \( R^2 \) and \( S_y \) of the regression improve compared to those of equation (12) and the serial correlation in the residuals is eliminated.

To test hypotheses stated on page (8) six measures of \( \phi \) are used in estimating equation (13). Three measures refer to the ratio of output price to user costs, i.e., \( p/c \), \( p/c_1 \), and \( p/c \). The remaining three measures are ratios of factor prices \( w/c \), \( w/c_1 \), and \( w/c \).\(^{29}\) The overall

\(^{29}\) \( p \) and \( w \) are respectively the wholesale price index and wage rate. \( c \) is the user cost (defined according to the expression explained in the appendix) when \( m = 1 \) and \( \delta_1 = 0.0 \); \( c_1 \) is the user cost with \( m = 1 \) but \( \delta_1 = 0.02 \) and \( \bar{c} \) is the user cost with \( m \neq 1 \) and \( \delta_1 = 0.02 \). See the statistical appendix, p. A-4.
results of each equation are generally the same as those reported in column 2, Table I. For the sake of brevity, we present in columns 3 and 4 of Table I, the regression results with \( \phi = \frac{w}{c_1} \) and \( \phi = \frac{w}{c} \). Similar patterns are discernible when \( \phi \) is defined in terms of price of output and user cost. Table II presents the short-run and long-run elasticities of capital services with respect to output and different measures of relative prices for each of the five regression equations of Table I. Moreover, the elasticities for regression equations with relative prices defined as the ratio of output price to user cost are presented in the lower part of Table II. The average variance of adjustment lags are reported in columns 5 and 6 of this table.

The general conclusions drawn from experiments with various forms of our model are that (1) the price elasticity of investment is much below unity and about nine times smaller than its output elasticity and (2) diminishing return to scale, \( \nu < 1 \), prevails in the U.S. manufacturing sector. The results are consistent with either equation (9c) or (9d) of page 18 with \( 0 < \sigma < 1 \) and \( \nu < 1 \).

These general conclusions are substantiated by the two-step estimation procedure mentioned briefly on page 8. The following auxiliary equation was fitted to the same set of data in [36].

\[
\ln \left( \frac{X}{L} \right)_t = b_0 + b_1 \ln \left( \frac{w}{c} \right) + b_2 \ln \left( \frac{X}{L} \right)_{t-1} + b_3 t
\]

The estimates were \( \hat{\sigma} = .20 \) and \( \hat{\nu} = .98 \) and \( b_3 \approx 0 \). The coefficient of \( t \) suggests that technological change may have had a neutral effect on factor proportion.\(^{30}\)

\(^{30}\) See Nerlove [37] for proof.
Using $\hat{\sigma} = .20$ and $\hat{\delta} = .96$ to construct the relative price variable, $\phi$, according to expression (9d) on page (8), equation (13) was reestimated. The results are indicated in column 5 of Table I. Note that the results are exactly the same as those in column 2 where $\phi$ is measured as a ratio of $w/c$. The only difference is that the price elasticity of capital services is higher by about 30 per cent when the model is estimated by the two-step procedure.

Several specific results, mainly of directional character, should be noted:

1. The short-run and long-run output elasticities of investment exceed their price elasticities no matter what measure of relative prices is used. The long-run output and price elasticities, of course, exceed the short-run elasticities.

2. The output and price elasticities seem to increase irrespective of which variant of the relative price is used with $\delta_1$ and decrease when $m<1$. This implies that firms, changing their rate of utilization, may approach a constant return case by reducing excess capacity.

3. The price and output elasticities are generally higher in equations with $\phi$ defined as the ratio of output prices to user cost.

4. The coefficients of $\text{dln}K_{t-1}$ and $\text{dln}K_{t-2}$ have somewhat smaller values in equations with $\phi = p/c$ variant than when relative factor prices are used as a measure of $\phi$. The adjustment coefficients of the model are slightly sensitive to which concept of relative price is employed. Within each category of
relative prices, the coefficients of the lagged dependent variable, however, decrease with $\delta_1$ and increase with m<1.

(5) The $R^2$ and $S_y$ of the regression equations are quite similar. The average and variance of the adjustment lag are higher in equations with $\phi$ defined as a ratio of output prices to user cost than where it is measured in terms of factor prices.

4a. TIME PATH OF THE MODEL

The empirical results in Table I support a Pascal distributed lag adjustment mechanism. The coefficients of the lagged dependent variables are statistically significant at the 5 per cent level of confidence and are somewhat sensitive to relative prices and rate of utilization. Inclusion of $d\ln K_{t-2}$ improves the value of the D/W statistics from 1.5 to about 2.0 and the long-run output and price elasticities of investment are affected by the form of the lag structure (equation (1) and (2), Table I). With a Koyck adjustment lag mechanism, the lag is very flat and slow to converge, in contrast to the Pascal lag structure. The average lag adjustment implied by equation (12) is about four years with a standard deviation of over four years. While the same statistics for equation (13) are respectively about two years and one and a half quarters (equations (I-1) and (I-2), Table II).

Estimates of the coefficients of the distributed lag function, $\omega_i$, calculated from estimates of the coefficients of lagged dependent variables of equation (2), Table I, are:
Estimated $\omega_i$

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<td>$\omega_i$</td>
<td>.0567</td>
<td>.0982</td>
<td>.1014</td>
<td>.0934</td>
<td>.0830</td>
<td>.0728</td>
<td>.0635</td>
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<td>.0481</td>
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<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
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<tr>
<td>$\omega_i$</td>
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<td>.0365</td>
<td>.0318</td>
<td>.0277</td>
<td>.0241</td>
<td>.0210</td>
<td>.0183</td>
<td>.0160</td>
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</tbody>
</table>

This adjustment pattern suggests that the effects of changes in output or relative prices rise to a maximum in the third quarter before beginning to decline slowly. About 43 per cent of the total effect of these changes is accomplished during the first year, about 70 per cent by the end of the second year, and about 92 per cent by the end of the fourth year. These results are in sharp contrast to the findings of Jorgenson-Stephenson [26] who find, for the total manufacturing sector: (a) zero response of investment to changes in desired capital stock in the first year, and (b) a peak response in the seventh quarter.\textsuperscript{31}

\textsuperscript{31}See [11] for further evaluations of these results.

The average adjustment lag is about 7 quarters which is consistent with the results reported by Mayer [32] and Solow-Kareken [3]. However, it is smaller than Jorgenson's findings of 8.5 quarters and much greater than the 2.7 quarters estimate of the average lag reported by Griliches and Wallace [17]. In equations with the ratio of output price to user cost, the average lag is greater and less stable than equations with w/c variables. The variance of the adjustment lag is about two to three quarters and is mainly affected by relative prices. In equations with
the wage/user cost variable the variance of the adjustment lag is smaller and rather stable; but in equations with output price and user cost ratios the variance has larger values. The variance of the adjustment lag implied by equation (13) is lower than that reported by Solow-Kareken and slightly higher than the estimate obtained by Griliches and Wallace. On the whole, the average lag of our model is shortened when the rate of utilization and depreciation cost due to use are taken into account. The variance of the lag increases if \( m \) is assumed to be unity and \( \delta_1 \) is constrained to zero.

4b. TESTS OF PERFORMANCE

A stringent test of the performance of a quarterly econometric investment model is to contrast its results with those of an autoregressive model.\(^{32}\) The autoregressive model used as a standard of comparison has

\(^{32}\) A recent study [26] comparing the goodness of fit and predictive powers of a set of econometric investment models indicated that only one of the econometric investment models performed as well as the autoregressive model.

the following form:

\[
\ln K_t = .0011 + 1.4850 \ln K_{t-1} - .6499 \ln K_{t-2} + .1925 \ln K_{t-3} - .1892 \ln K_{t-4} \quad (15)
\]

\[
R^2 = .9276
\]

\[
S_y = .0014
\]

\[
D/W = 2.0599
\]
(15) includes the usual naive models as special cases. Note every form of equation (13) performs better in terms of fit than the autoregressive equation. Even if we cast equations (13) and (15) in terms of investment expenditure, our analytical model performs better than the autoregressive equation. This is a significant result considering that most investment functions have failed this test.

To test the stability of our model we fitted equations (I-2) to (I-4) of Table I to equivalent periods, 1949-I to 1962-IV and 1958-I to 1962-IV. The choice of these periods was completely arbitrary. The results for the first sample period were almost identical to those obtained for the period 1949-I to 1960-IV. There were no changes in $R^2$, D/W, and $S_Y$. The statistical significance of the output and relative prices variables in fact increased when the sample period was extended. Similar results were obtained for the same period, 1958-I to 1964-IV. There were noticeable differences, however: the coefficients of the output variable became less stable and those of the relative prices were slightly less significant. On the whole, the model satisfactorily passes the test of intertemporal stability.

In the previous discussion, we have implicitly assumed that the elasticity of capital stock, $K$, with respect to $m$ is unity. To test this assumption $\ln K_t$ was regressed on the same set of independent variables plus $\ln m_t$. The results are reported in column (6) of Table I. Several features of these results are of interest. First, we find that $0< \frac{\ln K_t}{\ln m_t} < 1$. The coefficients of $\ln m_t$ is positive, small, but statistically significant. Similar results have been reported recently
by Eisner [10a] for capital stock and by Feldstein [14] for labor input. In disequilibrium, $\ln m_t$ may represent a short-term adjustment phenomenon. When the demand for investment rises, the firm increases its rate of utilization above the normal rate $m^*$ because of the delay involved in increasing capacity. As new investment takes place, $m$ is allowed to return to $m^*$.

Second, inclusion of $\ln m_t$ reduces the short-run and increases the long-run output elasticity of investment expenditure substantially by reducing the coefficients of the lagged dependent variables. Similar changes occur in the short- and long-run price elasticities of investment. However, the value of the Durbin-Watson statistics increases somewhat and the average and variance of the adjustment lag structure change drastically. We cannot, at this stage, argue unequivocally that constraining the coefficients of capital and its rate of utilization to equality is erroneous (as we have in production function (2a). However, indications are that investment models should be estimated in a context of a more complete framework such as that sketched by equations (4) to (8) on page (6). Such an effort will certainly throw some light on the interrelationships between the stocks of the input and their rate of utilization.
CONCLUSIONS

Our objective in this paper was to develop a demand function for capital services and estimate it empirically. The following general conclusions emerge from the preceding analysis.

(1) A CES production function seems to be the underlying structure of the demand for capital services in the U.S. total manufacturing or diminishing sector. The elasticity of substitution is very small and constant/return to scale seems to prevail in this sector. The price elasticity of capital services, though very small, affects the timing and adjustment of capital services and cannot be dismissed as an unimportant variable affecting investment behavior. There is need for further exploration to improve the formulation and measurement of relative prices in investment functions.

(2) The rate of utilization of capital stock affects the returns to scale parameter and influences the elasticity of substitution through its effect on the user cost of capital services. This effect is not very strong on empirical grounds but the statistical results suggest the direction of the influence.

(3) Aside from the measurement problems, the precise form of relative prices in an investment model depends on the assumptions about the marginal productivity condition of capital and labor services. Empirically, however, it does not make much difference whether the ratio of output price to user cost or wage to user cost is used in the investment functions. The only difference is that elasticity of substitution is somewhat higher when the ratio of output price to user cost is used.
(4) The time trend does not seem to influence behavior of capital services in the manufacturing sector during the period 1949-I to 1962-IV. However, this does not mean that technological change is not an important determinant of investment behavior. A time trend is probably a poor proxy to capture the full effects of a complex phenomenon such as technology.

(5) The Pascal lag adjustment mechanism seems to be the relevant lag structure for a quarterly model of capital services in the U.S. total manufacturing sector. The mean and variance of capital services suggested by a Koyck adjustment mechanism are too large and thus unacceptable. The average and variance of the lag adjustment of our model seem to be broadly consistent with the findings of other investigators. The response of capital services to changes in output and relative prices is rather immediate and not zero for the first few quarters as reported by Jorgenson and others.

(6) The conceptual framework of the model developed here is general and most of the existing econometric investment models are special cases of it. The model is empirically very stable and performs better than a set of alternative models and autoregressive investment functions.