Infrastructure Capital and Productivity Analysis
Cost- and Profit-Function Approaches

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I. Introduction

Since 1973 there has been a major concern in the United States and other advanced industrial countries about the slowdown of productivity growth. There have been many attempts to identify the sources of this slowdown in the US. Many explanations have been offered, such as (i) changes in the composition of the labor force, mainly due to the entrance of women and minorities with lower levels of skill; (ii) slowdown in growth of private capital stock, misallocation of its services, and underutilization of capacity; (iii) increased energy prices; (iv) slower investment in R&D capital; (v) the shift to a service-oriented economy; (vi) mismeasurement of output, particularly in service industries; and (vii) decline in investment in public infrastructure. This list can easily be extended to include many other explanations. Recently there has been a very lively discussion regarding the inadequacy of infrastructure capital as a cause of slowdown of productivity growth at the aggregate and industry levels.

Numerous studies have been undertaken to clarify the relationship between productivity growth and public infrastructure capital. These studies can broadly be classified as those which estimate a neoclassical production function augmented to include the publicly financed infrastructure capital stock as an input of production, and those which utilize the dual approach by estimating a cost or profit function. The second approach utilizes market data about the prices of private inputs and output, and treats public infrastructure capital as an unpaid factor of production. The nature of the data which has been used to estimate the production or cost functions is very diverse. Some studies use highly aggregate national and international data, others use regional and state level data. Some studies have used cross-section time series of metropolitan SMSA’s, while others have employed industry-level data. The studies in each of the two categories often differ in their coverage of industries, geographic region, methodology and use of econometric estimation techniques, and the reported results often reflect these differences.

What is clear, however, is that no consensus has yet emerged to explain the causes of the slowdown of productivity growth, including the role played by infrastructure capital. The proponents of each cause often cast the argument as though there is a single explanation of the productivity slowdown. There is in fact no one explanation of the productivity slowdown phenomenon, rather a convolving of several factors, including those listed and others not mentioned which may have been responsible for it. What has
often been missing is a general framework to identify the probable effects of demand and supply forces that have contributed to the productivity growth slowdown. The relative contributions of public infrastructure capital, or that of any other factor can best be evaluated within a general framework. The basic proposition of our approach is that a fully specified structural model is needed to evaluate the contribution of the various types of public sector capital to productivity growth. Most of the criticisms of the various studies reported in the literature could be accommodated within such an approach. Furthermore, since we will utilize data disaggregated in several dimensions, by industry, by functional use of the infrastructure, and by level of government, i.e. states, we will be able to analyze the influence of growth of regional (state) income, population, relative factor prices, technology and various types of public capital on regional and state growth rates of output and productivity by industry and states.

In this paper we attempt to:
(i) Review the findings reported in the literature on the effect of infrastructure capital productivity using the cost function framework;
(ii) Contrast these findings with those reported using the production function framework;
(iii) Propose a general methodology to decompose sources of total factor productivity growth, including the contribution of infrastructure capital; and
(iv) Outline an econometric model that could be used to analyze the disaggregated data of industry, state, and type of infrastructure being gathered by Apogee.

The rest of this paper is organized as follows: In section II we briefly summarize the main results based on production function estimation about the effect of public infrastructure capital on productivity growth. Our review is necessarily brief, but the interested reader may consult the comprehensive surveys by David A. Aschauer (1993) and Federal Highway Administrations Discussion Paper (1992). Section III is devoted to a survey of the literature based on cost and profit function estimates. We first answer why this alternative approach is often adopted to estimate the contribution of public infrastructure capital to growth of output and productivity; next we discuss briefly the results reported in the literature based on cost or profit functions. Our aim is not to provide an exhaustive survey, but to emphasize the major findings of a number of studies.
We also attempt to contrast these findings, wherever possible, with those based on the production function methodology.

In section IV we provide an analytical framework for decomposing total factor productivity (TFP) into several components reflecting both demand and supply factors that affect growth of output and total factor productivity. It is in the presence of these factors that we evaluate the relative contribution of public infrastructure capital to productivity growth. This methodology allows us to put enough structure on the data in order to delineate the effect of a number of forces which both theory and logic suggest may affect growth of output and productivity. According to our proposed methodology, we can trace the effect of aggregate demand, population growth, rise in real factor prices, technical change, and types of infrastructure capital on TFP growth. Further, we can evaluate both the direct and indirect effects of various types of public infrastructure capital on productivity growth in the presence of the structural determinants just mentioned.

To decompose TFP into its components using the methodology developed in section IV, we need to estimate the parameters of the demand and cost functions. We sketch the outline of the econometric model in section V. We hope to estimate this model using the disaggregated data being assembled presently by Apogee. The data requirements for estimating the model are briefly described in this section. They consist of the cross-section time series data on prices and quantities of output and inputs of industries by state classification. The econometric model we have outlined is very general, accounting for industries and regional or state differences; it can be tailored to study one or several industries across all 48 states or specialized to deal with regional and state classification alone.

The potential findings from the proposed research strategy and some concluding comments are presented in section VI.

II. What has been learned from the production function approach?

There is an extensive literature on the effect of public infrastructure capital on growth of output and productivity using the production function framework. The analysis falls into two categories: (a) aggregate production studies, and (b) regional or state level
production function analysis. Table 1 presents some of the characteristic features of a number of studies that are based on production function estimation. This literature began with a paper by David Aschauer (1989a) which stimulated an extensive discussion of the kind and magnitude of the impact of infrastructure capital on output and productivity growth.¹ He used an aggregate production function to argue that infrastructure capital financed by the public sector increased the capacity of the private sector to be more productive, and that public infrastructure investment stimulates private sector investment by enhancing the rate of return to private sector investment. Munnell (1990a) extended this line of argument, and her results generally support the proposition that there is a strong and significant effect of public capital on productivity.

Aschauer and Munnell employ aggregate time-series data on the United States to estimate the relationship between private output and the stock of nonmilitary public capital, which consists of the following equipment and structures: highways, streets, educational buildings, hospital buildings, sewer and water facilities, conservation and development facilities, gas, electric, and transit facilities, and other miscellaneous but nonmilitary structures and equipment. Aschauer finds that the elasticity of output with respect to public capital is 0.39. Munnell finds an elasticity of 0.33 for output per hour with respect to public capital. She uses the estimated coefficients of the aggregate production function to calculate annual percentage changes in multifactor productivity and concludes that the "drop in labor productivity has not been due to a decline in the growth rate of multifactor productivity or technical progress. Rather it has been due to a decline in the growth of public infrastructure."

Several criticisms of these results have been raised. Some have argued that the estimated elasticities and the implied marginal productivity of the public capital are extremely high. The marginal productivity of public infrastructure capital based on Aschauer's estimates will exceed that of the private capital stock by several times, which seems highly implausible (Aaron (1990)). Others have argued that the aggregate time

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<tr>
<td>ASCHAUER (1989)</td>
<td>Cobb-Douglas prod. func. and TFP regres. on t, g, cu</td>
<td>time series 1949-1985 private business economy</td>
<td>0.39- 0.36</td>
<td>CRS in all inputs, including public capital input</td>
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<td>0.37-0.41</td>
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<td>Significant</td>
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<tr>
<td>MUNNELL (1990a)</td>
<td>Cobb-Douglas prod. func. reproduces Aschauer</td>
<td>time series 1948-1987 private non-farm sector</td>
<td>0.34-0.41</td>
<td>CRS in all inputs; also priv. and publi. cap. coeff. equal</td>
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<td>MUNNELL (1990b)</td>
<td>Cobb-Douglas prod. func.?</td>
<td>cross-sect. time series 48 states 1970-1986</td>
<td>0.15</td>
<td>see Munnell 1991 and other references</td>
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<td>0.11</td>
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<td>0.22</td>
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<td>1.04</td>
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<td>GARCIA-MILA AND McGuire (1988)</td>
<td>Cobb-Douglas prod. func.</td>
<td>cross-sect. time series 14 annual obs. of 48 states gross state prod. labor, capital expenditures on education and highways</td>
<td>Highways: 0.045-0.044 Education: 0.16-0.072</td>
<td>Returns to Scale 1.04</td>
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<tr>
<td>HULTEN AND SCHWAB (1991a)</td>
<td>Cobb-Douglas prod. func.</td>
<td>time series 1949-1985 same as Aschauer</td>
<td>0.42</td>
<td>(-) coeff. for labor?</td>
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<td>First differ.</td>
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<td>0.028</td>
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<td>Insignificant</td>
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<td>First differ.</td>
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<td>Insignificant</td>
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<td>MERA (1972)</td>
<td>Cobb-Douglas prod. func.</td>
<td>Japan pooled data of regions and time 3 sectors primary secondary tertiary 4 classifications of social overhead capital</td>
<td>0.22</td>
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<td>0.20 (.50)</td>
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<td>0.12-0.18</td>
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<td>TFP regressions</td>
<td>USA and 11 OECD countries time series and country cross-section</td>
<td>Half of countries significant effect after 1960 Mixed support of Aschauer results</td>
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<td>FORD AND PORET (1991)</td>
<td>TFP regressions</td>
<td>cross section time series regional study of Snow-Sun Belt 1970-1986 Gross output value added</td>
<td>public capital insignif. in all regressions private capital insignif. in gross output regres. signif. in value added implying scale .88</td>
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*Coefficient of infrastructure capital*
series correlation may not reflect a causal relation, but may reflect a spurious correlation, i.e., both labor productivity and public infrastructure spending have declined over the same period due to other forces (Aaron (1990) and Tatom (1991)). A third issue is that a reverse causation may exist between public infrastructure capital and productivity growth; it is argued that the positive coefficient estimated in various studies may result from the effect of productivity growth on infrastructure capital rather than the reverse. Also there is some evidence of a lack of robustness when more recent data are used to estimate the aggregate production function of Aschauer and Munnell.

There are a number of production-function studies that utilize data at the state level. The data consist of time-series cross-section of data on 48 states for several years. This richer body of data has certain advantages which mitigate the possibility of spurious correlation over time. As a whole, the studies based on state-level data support a small but positive relationship between public infrastructure and productivity.

Munnell’s (1990b) elasticity coefficients: public capital 0.15; private capital, 0.31; and labor, 0.59; show that although public capital has a positive effect on output productivity, it is only half the size of the effect of private capital. For example, a 1 percent increase in public capital causes a 0.15 percent increase in output productivity, whereas a 1 percent increase in private capital causes a 0.31 percent increase in output productivity. However, calculating the marginal products shows that one more unit of public capital will increase output by the same amount as an additional unit of private capital. The results remain plausible when public capital is split into its three components—highways, water and sewer systems, and other. The first two, which are the largest part of core infrastructure, have much larger effects than does the "other" category. The coefficients are labor, 0.55; private capital, 0.31; highway stock, 0.06; water and sewer facilities, 0.12; and other public capital stock, 0.01.

Using Munnell’s data, Eisner (1991) reports that: for all functions considered, the significance of public capital holds up when the data are arranged to reflect cross-sectional variation, but disappears when the data are arranged to allow for time-series variation. The data tell us that states with more public capital per capita have more output per capita, but that a state that increases its public capital in some year does not as a result get more
output in that year. Therefore, Eisner regards the direction of causation between output and public capital as undecided and postulates that a lag structure will be needed to get a true time-series relationship between output and public capital.

Calculating manufacturing productivity growth rates for the years 1951 to 1978 for major regions of the United States, Hulten and Schwab (1984) test whether different rates of public capital growth correspond to different rates of productivity growth. They find that the differences in output growth are not due to differences of public infrastructures, but instead to variation in the rates of growth of capital and labor. In a later paper, Hulten and Schwab (1991) expand the analysis to include the years 1978 to 1986, but their conclusion remains the same: public infrastructure has had little impact on regional economic growth.

These disparate results are likely due to whether the unobserved state-specific characteristics are controlled in the estimation process. Holtz-Eakin (1992) has tested the hypothesis that the positive and strong effect of infrastructure will diminish or disappear if the state-specific effects are accounted for. An interesting recent study which provides a feel of the range of estimates is performed by McGuire (1992). McGuire estimates four specifications of a state-level production function with public capital as an input: Cobb-Douglas with no control for State effects; Cobb-Douglas controlling for State fixed and random effects; and translog with no control for State effects. The drawback of a Cobb-Douglas production function is that it restricts the elasticity of substitution between each pair of input variables to equal 1. The four specifications of the model yield broadly similar results, with public capital having a positive and statistically significant effect on GSP. The elasticity ranges from 0.035 to 0.394 in McGuire's new estimates.

When public capital is split into its three component parts (highways, water and sewers, and other) highways has the strongest impact, with elasticities ranging from 0.121 to 0.370. Water and sewers has a much smaller but usually significant effect, and other public capital is not statistically significant or has a negative effect on private output. Indeed, some economists hypothesize that state-level data may systematically underestimate the productivity value of public capital, because such data cannot capture the aggregate effects of public capital as a system.
Similar findings have been reported by a number of production function studies which utilize even more disaggregated data. Studies by Eberts (1988) Eberts and Fogarty (1987) and Duffy-Deno and Eberts (1989) use data at the metropolitan level. They test the direction of causation between infrastructure capital and output and estimate the magnitude of the elasticity of output with respect to infrastructure capital. Their findings suggest that the causation runs mostly from infrastructure capital to output growth and there is a positive but considerably smaller elasticity of output with respect to public capital than those based on aggregate production functions, relationship between infrastructure and growth of output and productivity.

From a reading of the evidence so far, based on production function studies it is possible to draw the following conclusions: (1) The early estimates based on aggregate production function analyses considerably overstated the magnitude of the effects of public infrastructure capital on growth of output and productivity. (2) The estimates on state level data indicate a much smaller contribution of infrastructure and that the composition of infrastructure capital matters; some types of infrastructure capital’s contribution are larger than others. (3) There are serious estimation problems in both aggregate national time series studies and state and regional level studies that lead to highly disparate results. (4) On the whole, it seems that the evidence points to a positive but small elasticity of output with respect to public infrastructure capital of about 0.10 to 0.20 at the national level and possibly a lower range at the regional level.

One reason for the wide range of estimates of the elasticity of output with respect to infrastructure capital based on production function estimates may be due to minimal modelling structure imposed on the data. If enough structure is not imposed on the data, provided that the underlying data are not subject to serious or major measurement problems, the parameter estimates of the underlying production structure are likely to be biased and the estimates are not likely to be robust. In estimating production functions, whether using national aggregates or state level data, the production function is treated as a purely technological relationship between output and inputs, and firms optimization decisions with respect to how much output to produce and what mix of inputs to use in the production process is not considered specifically. In reality, inputs and output are
simultaneously determined when firms optimize (minimize) their profit (costs). When firms’ optimization is explicitly considered, the marginal productivity conditions for the inputs should be estimated jointly with the production function. If the marginal conditions are not explicitly considered, the estimated production function parameters are likely to be seriously mismeasured.

III. Why the Cost Function Approach?

The production function approach has been used extensively to estimate the relationship between public infrastructure capital and productivity. Production estimates are easy to interpret, and production function analysis is familiar to many researchers. It is also claimed that the data requirements for estimating profit or cost functions are much greater than for estimating production functions. Some of these assertions, however, arise from certain misunderstandings:

First, the duality theory assures that we can derive the underlying production function parameters from the cost or profit functions. Public capital stock as an unpaid input could enter the production function or cost function in a similar way based on the same theoretical rationale. Measurement of total factor productivity and labor productivity can be estimated using either of these two approaches. In general, in order to estimate the underlying technology of a production process, one can examine either the production function or the associated cost (or profit) function. Under certain regularity conditions, there is a unique correspondence between the production and cost functions, and all information about the underlying technology is contained in both functions (Shephard (1970), Diewert (1974), and Chambers (1988)). However, the dual approach is statistically convenient, as well as better for analytical purposes.

Second, while the data requirement for estimating cost functions may appear to be somewhat greater than those required for estimating production functions, in reality it poses no significant problems. Most of the data used in production analyses are often in nominal terms which have to be converted to real values using appropriate price deflators. In estimating cost functions, aside from these deflators, it may be necessary to gather data on wages and the interest rate, which are usually quite easily available.
Third, the reason that the production function estimation does not require exactly the same data set, as we noted earlier, is that production function is treated as a purely technological relationship. But when firms optimize their production decisions they must equalize the marginal product of inputs to their prices. The estimating model would then include, besides the production function, the marginal productivity conditions for the inputs which involve prices of inputs and output. Thus, the data requirement for estimating the production function and cost (profit) functions are the same whenever optimizing behavior is explicitly considered.

There are, however, certain advantages to the dual approach, i.e., estimating profit or cost functions: First, note that the production function expresses the output in terms of inputs. Thus it is explicitly assumed that output is an endogenous variable, while the inputs are exogenous variables. However, it is not clear why inputs should be considered exogenous. According to economic theory, given the price of inputs, firms will choose their inputs and output to maximize their profit, or, for a given output, to minimize their cost. 

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Let $P$ be the vector price of private inputs and assume that firms choose quantities of private inputs so as to minimize the private costs of producing output $Y$. Then, under certain regularity conditions (see for instance, Dievert (1974)), there exists a cost function given by

$$C = C (Y, P, S, T),$$

where $Y$ is the gross output (value added), $X$ is a vector of private inputs, $S$ a vector of publicly-financed capital services, and $T$ an index of exogenous technological (or other) stocks. Applying Shephard's lemma in (1) we get the conditional input demands

$$X_i (Y, P, S, T) = \frac{\partial C}{\partial P_i}, \; \forall \; i.$$

Thus, it is clear that the inputs are endogenous variables and will depend on prices and other exogenous variables, including publicly-financed capital, $S$. Therefore, the estimation of a production function like $Y = F (X, S, T)$ suffers from simultaneous equation bias, and the OLS estimates will be biased. If it is assumed that some private inputs are fixed in the short-run, then the short run cost function can be defined by

$$VC = V C (Y, P, Z, S, T),$$

where $Z$ is the vector of fixed inputs. For the relationship between variable cost and long run total cost see, for instance, Varian (1984).
The cost- or profit-function approach, on the other hand, takes explicit account of the optimizing behavior of firms with respect to inputs and prices are the only exogenous variables. In addition, in most studies on infrastructure production is assumed to be of a Cobb-Douglas specification, which a priori imposes restrictive conditions of the substitutability of inputs. There is a need for more flexible functional forms.

The second reason for using the cost function is that it yields direct estimates of the various Allen-Uzawa elasticities of substitution. These parameters are the key to describing the pattern and degree of substitutability and complementarity among the factors of production. In the production function context, estimation of the elasticities of substitution requires that the matrix of production coefficients be inverted. This thus exaggerates the estimation errors and reduces the statistical precision of the computed elasticities of substitution (Nadiri and Schankerman (1981)). Furthermore, the effect of public capital on the demand for inputs can be directly estimated.\(^3\) If the effect is positive, the public capital and the private inputs are complements; if it is negative, the public capital and private inputs are substitutes.

The third advantage of the cost function approach is that joint estimation of the cost function and the derived demand for the inputs increases the degrees of freedom and enhances the statistical precision of the estimates. On the other hand, in principle one can of course estimate a production function together with the first-order conditions for profit maximization. However, one has to rely on the stronger hypothesis of profit maximization which might not be true (for instance, in regulated industries) instead of the weaker cost minimization hypothesis.

Finally, we can easily derive the marginal benefit of infrastructure capital from a cost function; the first derivative of cost with respect to public capital gives the marginal

\(^3\) This is given by the second partial derivatives of cost function:

$$\frac{\partial x_i}{\partial s_k} = \frac{\partial^2 c_i}{\partial p_i \partial s_k} \forall i, j$$

This condition shows the effect of public capital stock on the demand of private inputs.
benefit, in terms of cost reductions, of public capital services. The sign of the cost derivative with respect to infrastructure capital will indicate the direction of the effect. If it is negative, an additional unit of public capital stock will make the firm better off; if it is positive, worse off. Under general conditions of convexity of the technology of the firms (see Diewert (1986)), the equation indicated in the footnote can be considered the demand for publicly-financed capital. Furthermore, we can test the notion whether the amount of infrastructure is optimal. Summing the marginal benefits of infrastructure over all firms, and equating the sum of marginal benefits with the marginal cost of infrastructure $k$, we can derive the optimal amount of infrastructure services. By comparing the optimal amount of infrastructure capital services determined by the model and its actual level, it is possible to conclude whether there is an undersupply of infrastructure capital services in a particular geographical unit.

In general, the dual approach provides a richer framework to explicitly address such questions as: What is the effect of public capital on inputs of production? Does public capital have a crowding-out or crowding-in effect on private capital? What is the marginal benefit or willingness of the private sector to pay for an additional increase in public infrastructure? Is the level of publicly-provided capital optimal from the perspective of the private production sector?

IV. Estimates Based on the Cost (Profit) Function Approach

The number of studies using a cost function to analyze contribution of infrastructure capital and other types of publicly financed capital are small compared to the number of studies based on the production function. Some of the important features of these studies are presented in table 2. The dual approach has been applied in a set of diverse studies, using different types of data at the national and international level, state level data, and industry data, using different assumptions about the optimizing behavior of firms, and specifying different functional forms with special preference to the translog and generalized Leontief. In addition, different authors use different notions of public

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4 That is, $B_k (Y, P, S, T) = - \partial C / \partial S_k$ for $k$. 
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<thead>
<tr>
<th>Author</th>
<th>Specification</th>
<th>Cost</th>
<th>Direct Effect</th>
<th>Indirect Effect</th>
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<td>DENQ (1988)</td>
<td>USA 36 SMSA Manufacturing Industries 1970-78</td>
<td>Cost Savings</td>
<td>Profit Truncated Translog</td>
<td>Elasticity = 0.08 to 0.1</td>
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<td>CONRAD AND SETZ (1992)</td>
<td>West Germany Manufacturing, Construction, Transport 1960-1988 Time-Series</td>
<td>Cost Translog and MR = MC</td>
<td>Cost Savings</td>
<td>Elasticity = 0.45 to 0.49</td>
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<td>KEELER AND RING (1988)</td>
<td>USA Trucking Industry 1980-1988 Regional Pooled</td>
<td>Cost Translog</td>
<td>Cost Savings</td>
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<td>LYNDE AND RICHMOND (1992)</td>
<td>USA Nonfinancial Corporate Sector 1958-1989 Time-Series</td>
<td>Cost Translog</td>
<td>Cost Savings</td>
<td>Elasticity = 0.90</td>
</tr>
<tr>
<td>LYNDE AND RICHMOND (1993)</td>
<td>U.K. Manufacturing Sector 1966-71 to Added Value</td>
<td>Cost Translog</td>
<td>Cost Savings</td>
<td>Elasticity = 0.90</td>
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Table 2: Cost or Profit Function Estimates
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<th>Study</th>
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<th>Methodology</th>
<th>Core Description</th>
<th>Cost Savings Elasticity</th>
<th>Substitutes Elasticity</th>
<th>Complements Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>MORRISON AND SCHWARTZ (1991)</td>
<td>USA</td>
<td>Variable Cost Generalized Leontief P=MC</td>
<td>Core</td>
<td>Cost Savings Elasticity = -.10 to -.27</td>
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<tr>
<td>NADIRI AND MAMUNEAS (1991)</td>
<td>USA Manufacturing 12 2-digit industries 1955-1986 Pooled Industry Specific Effects</td>
<td>Cost Translog CRTS for Private Inputs</td>
<td>Total public Stock Adjusted with Capacity Utilization Rate</td>
<td>Cost Savings Elasticity = 0 to -.21</td>
<td>Substitutes Elasticity = 0 to -1.4</td>
<td>Complements Elasticity = .12 to .76</td>
</tr>
<tr>
<td>SHAH (1992)</td>
<td>Mexican Manufacturing Sector 28 3-digit Industries Pooled</td>
<td>Variable Cost Translog</td>
<td>Total Adjusted with industries' output proportion</td>
<td>Cost Savings</td>
<td>Complements Elasticity = -.006</td>
<td>Complements Elasticity = -.002</td>
</tr>
</tbody>
</table>
infrastructure. Some use the core infrastructures, others the total stock, and others adjust these stocks to reflect the collective nature of public capital investment.

For the United States, the cost function approach has been applied by Keeler and Ying (1988) to the trucking industry, Lynde and Richmond (1992) to the corporate business sector, and Nadiri and Mamuneas (1991) to disaggregate two-digit manufacturing industries. Also, Morrison and Schwartz (1991) estimate a variable cost function for the manufacturing sector by state, while Deno (1988) estimates a profit function for 36 metropolitan areas. For outside the US, Shah (1992) estimates a variable cost function for the Mexican manufacturing sector, Conrad and Seitz (1992) and Seitz (1992a, 1992b) estimate a cost function for West Germany’s manufacturing, construction and trade sector, and finally Bernd and Hansson (1991) estimate a variable cost function for the private sector of Sweden. Even though a single estimate cannot be provided for the effect of public infrastructure on the cost and consequently on its contribution to the productivity, all studies reach the conclusion that publicly financed capital contributes positively to productivity in terms of cost savings.

(a) Cost Elasticities

At the aggregate US level, Lynde and Richmond (1992) estimate a translog cost function using aggregate US nonfinancial corporate business sector data for the period 1958 to 1989. By imposing constant returns to scale in all inputs, public capital included, and by assuming that firms behave competitively, they estimate two cost-share equations: one for labor, and one for public capital. Due to the constant returns to scale assumption, they are able to define the cost share of public capital as one minus the output cost share. Their findings suggest that publicly financed infrastructures reduces the cost of the nonfinancial corporate business sector.

Nadiri and Mamuneas (1991) estimate a translog cost function for 12 industries of the manufacturing sector at the two-digit level for the period 1955 to 1986. They pool the data across the industries, but allow for industry-specific effects and estimate the cost function together with the share equations for labor and capital inputs. Their findings indicate that an increase of public infrastructures as well as publicly financed R&D reduces
the cost of the industries in their sample. However, Nadiri and Mamuneas had to adjust
the public capital with the capacity utilization rate in order to capture the collective nature
of public investment. In a revised version of their paper (1993), they find that this
adjustment does not bias the direction of the effect, even though the magnitude is slightly
affected. Nevertheless, Nadiri and Mamuneas estimate that the cost elasticity of public
infrastructure varies from industry to industry within the range 0 to -.21.

Morrison and Schwartz (1991) estimate a variable cost function using US state
level data for the total manufacturing sector over the period 1971 to 1987. They specify a
generalized Leontief cost function, treating private and infrastructure capital as exogenous.
They estimate a system of input-output equations for production labor, non-production
labor and energy, and a short-run output price equation (\( p = mc \)) to incorporate profit
maximization. The estimation is carried out for the regions-- Northeast, North Central,
South and West-- with pooling parameters for each state added as intercept terms on the
estimating equations. Their result suggest that the an increase of 1% of public capital
reduces the cost from .15% in the Northeast to .25% in the West. In addition, the authors
calculate the contribution of infrastructures to productivity growth for each region and
state.

Deno (1988) estimates a translog profit function at the regional level of 36
standard metropolitan statistical areas for the manufacturing industries from 1970 to 1978.
He estimates the effects of highway, sewer and water capital on output supply, and on
unconditional demands of capital and labor. In order to take into account the collective
nature of public capital, he multiplies the public capital stocks by the percentage of the
metropolitan population that is employed in the manufacturing sector. His findings suggest
that all types of public capital contribute positively to output growth, but that highway and
sewer capital contribute the most to output growth, capital formation and employment. He
finds that output supply responds strongly to total public capital with an elasticity of 0.69.
The corresponding elasticities for specific types of capital are 0.31 for highway capital,
0.30 for sewer capital, and 0.07 for water capital.

In particular, for the US road freight transport industry Keeler and Ying (1988)
estimate a translog cost function of regional truck firms for the period 1950 to 1973.
They find that the highway infrastructure has a significant effect on the productivity growth of the trucking industry, with substantial benefits of this investment, justifying about half of the cost of the Federal aid highway system. Dalenberg (1987) estimates cost functions across metropolitan areas and finds public capital and private capital to be complementary inputs. He finds that public capital lowers costs; however, he also finds, based on his estimates of the shadow price of public capital, that many SMSA’s have overinvested in public capital from the manufacturing sector’s point of view.

At the international level, Berndt and Hansson (1991) estimate a short-run (variable) cost function by specifying a labor requirement function and using aggregate data from the Swedish private sector, assuming that private capital as well as public capital are fixed in the short run. They find that public infrastructure and labor input are complements for the 1960’s and 1980’s, while they were substitutes for the 1970’s. The authors conclude that the increase of public infrastructures reduces private costs. In addition the authors estimate the ratio of the optimal amount of infrastructure capital to the existing capital stock by equating the marginal benefits of private and public capital with their corresponding rental prices and solving simultaneously for the optimal amounts of private and public capital. They find that for the period 1970 to 1988 there were excess amount of infrastructures for the private production sector of the Swedish economy.

Lynde and Richmond (1993) estimate a translog cost for U.K. manufacturing for the period 1966-1990 using quarterly data. They control for nonstationary effects in the time-series and classify changes in productivity according to four parts: (i) changes in the public capital to labor ratio; (ii) changes in economies of scale; (iii) changes in prices of intermediate inputs, including energy; and (iv) changes in technology. They find an average elasticity of output with respect to public capital of 0.20 and they attribute approximately 40 percent of the productivity slowdown to the decline in the public capital to labor ratio. Their estimates indicate that there is a significant role for public capital in the production of value-added output of the manufacturing sector.

Shah (1992) estimates a translog variable cost function treating labor and materials as variable inputs and private capital and public capital as fixed inputs. Shah uses data from 1970 to 1987 for twenty-six Mexican three-digit manufacturing industries
and takes into account, as do Nadiri and Mamuneas (1991) and Deno (1988), the usage of public infrastructures. Thus, he constructs the industry usage of public infrastructures to be proportional to public capital where the degree of proportionality is defined as the ratio of the industry's output to the output of all industries. He finds that the short run effect of infrastructures is to reduce variable cost implying that there is underinvestment in public capital.

Conrad and Seitz (1992) estimate a translog cost function together with marginal revenue equal to marginal cost condition for the manufacturing, construction and trade and transport sector of West Germany for the period 1960 to 1988. They find that the estimate of the shadow price of infrastructures is .06, 0.03 and .06 respectively, implying that there is substantial reduction of cost. Similar results are reported by Seitz (1992a,b) for the effect of core and total public capital on the cost of 31 two-digit industries of the West German Manufacturing sector for the period 1970 to 1987. These results are generated by estimating a generalized Leontief cost function.

(b) Effects of Public Capital on Employment and Private Capital

The public capital hypothesis asserts that the public capital has both a direct effect and an indirect effect on the productivity of private sector (see Tatom (1991b)). The direct effect arises under the assumption that marginal product of public capital is positive, i.e., an increase of public capital services increase the private sector output. The indirect effect arises under the assumption that the private and public capital are complements in production, i.e., the partial derivative of marginal product of private capital with respect to public capital is positive. If private and public capital are complements this hypothesis asserts that an increase of public capital raises the marginal productivity of private capital, and given the rental price of capital, private capital formation increases, further raising private sector output.

In the cost function framework the direct effect of infrastructure capital is measured by the magnitude of cost reduction, due to an increase of public capital. Its indirect effect is given by the magnitude of its effect on demand for public sectors factors of production. This sign of this effect will determine whether infrastructure capital is
biased toward one or another of the private inputs. To see the linkage between the direct
and indirect effects note that at the optimum, for a given output, $Y$, the cost function is:

$$ C (Y, P, S, T) = \sum P_i X_i^* $$

where $X_i^*$ is given by $\frac{\partial C}{\partial P_i}$. Differentiating the cost function with respect to $S$ yields

$$ \frac{\partial C}{\partial S_k} = \sum P_i \frac{\partial X_i^*}{\partial S_k} $$

which decomposes the cost change associated with an increase of public capital services
into adjustment effects of private inputs. If now all private inputs are substitutes with
public capital then an increase of public capital is always cost saving. The inverse of
course is not true. As has been shown so far, the literature of the cost function
framework reviewed supports the hypothesis that cost savings are associated with an
increase of public capital. Hence, if one of the private inputs is a complement of public
capital then cost savings can arise only if the substitution effects of the other private
inputs outweigh the complementary effect (see also Seitz (1992b)).

It is clear that a priori no sign can be assigned to the indirect effect of public
capital on the inputs of production. The sign and magnitude of the effect is an empirical
question. It seems that the cost function literature supports the hypothesis that labor and
public capital are substitutes. Lynde and Richmond (1992) find that the public capital
elasticity of labor is about .45, Nadiri and Mamuneas (1991) estimate labor elasticities
from 0 to 1.4, Seitz (1992a) an elasticity of .0004 for public roads, Seitz (1992b)
estimates .15 for public capital and finally the same substitutability relationship is found
by Conrad and Seitz (1992). Exceptions are Berndt and Hansson (1991) who find a short-
run weak complementarity between labor and public capital and Shah (1992) who finds
labor and public capital to be short and long-run weak complements. For the relationship
between public capital and private capital there is no clear cut evidence. The studies are divided between Conrad and Seitz (1992), Seitz (1992a,b) and Lynde and Richmond (1992) who find that public capital and private capital are complements and Shah (1992) and Nadiri and Mamuneas (1991) who find that there are substitutes with elasticities from 0.005, and 0.02 to 1.4 respectively. Also, Nadiri and Mamuneas (1991) have estimated that public capital and intermediate inputs are complements for the US manufacturing sector, while Conrad and Seitz (1992) report that they are substitutes for West Germany. Finally, Deno (1988) has estimated that labor and capital are both gross complements of public capital.\(^5\)

(c) **Optimal Provision of Public Capital and Its Rate of Return**

One question which has been raised in the literature and has important public policy implications is whether or not public capital is at its optimal level. In other words, is public capital under-supplied? Public capital can be considered as not only a public good, used as an intermediate input for the production of private goods, but also as providing services directly to the consumer. The optimal provision of public capital services in such cases will be given by the well-known Samuelson condition, as modified by Kaizuka (1965). This condition requires that public capital be provided at the point where the sum of marginal benefits of producers and consumers is equal to the marginal cost of providing one additional unit of public capital. However, the literature has so far emphasized only the producer benefits arising from infrastructures.

Ignoring the consumption sector, an alternative means of determining whether public capital is provided optimally is to consider the rate of return of public capital and compare it with the rate of return of public capital for the whole economy. The optimal provision of public capital requires that the rates of publicly provided and private capital be

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5. Note that substitutability and gross substitutability are different notions. Gross substitutability allows for the adjustment of output (see Chambers (1988)). One input can be a substitute for another input, as well as a gross complement, as long as both are non-regressive, and the induced output effect overcomes the substitution effect.
equalized. Thus, if the rate of return of public capital is higher than that of private capital, public capital is under-supplied and an increase of public investment is necessary. Nadiri and Mamuneas (1993a) find that the rate of return of public infrastructures implied by the industries of the manufacturing sector is about 7%, while the rate of return of private capital is about 9%. If, however, one considers that the industries of their sample is a small fraction of the economy's production sector, then the implied rate of return of infrastructure will exceed the rate of return of private capital.

Morrison and Schwartz (1991) take another approach. They compare the shadow price of public capital with the "user cost" of public capital, and find that the Tobin's q ratio of public investment exceeds one, suggesting that infrastructure investment has been too low for social optimization for the manufacturing sector of all regions in their sample. Similarly Shah (1992) estimates a Tobin's q equal to 1.04 for the Mexican manufacturing sector, and concludes that there is indication of underinvestment of public capital.

Finally, Berndt and Hansson (1992), by equating the marginal benefit of public infrastructures with its ex-ante rental price, solve for the optimal capital stock and then calculate the ratio of the optimal capital stock to the actual public capital. They find that this ratio is above one for the period 1960 to 1970, below one for the period 1970-1990, suggesting overinvestment, although the ratio is rising for the late 1980s. Thus they conclude that the "roads and highways were not as well maintained as had been the case in the 1970s and early 1980s" in Sweden.

(d) Comparison between production function and cost function estimates

One important question which arises from these studies is if one is able to derive a "single" estimate of the effect of public infrastructure, in terms of magnitude, on the productivity of the private sector. Another important question is how the estimates from the cost function approach compare with the estimates of the production function approach. Both questions are difficult to answer because of the diverse data, assumptions employed, level of aggregation and available information provided by the authors.

First, the direct magnitude of the productivity effect in terms of the elasticity of
cost with respect to public infrastructures is unfortunately not reported in many studies except in Morrison and Schwartz (1991) and Nadiri and Mamuneas (1993a) who report an elasticity of -.1 to -.3 and 0 to -.2 respectively. Therefore, if any comparison should be made it must be based only on those two studies. Second, in comparing these elasticities which are based on a disaggregated level with the output elasticity generated from an aggregate production function, reported by Aschauer (1989a), there are two problems involved. One is the problem of proper aggregation and second is that the elasticity of cost with respect to output has to be known. It is easy to show that the public capital output elasticity is equivalent to the negative of the ratio of the elasticity of cost with respect to public capital over the cost elasticity of output.6

6 Under cost minimization the Lagrangian is given by

\[ L (Y, P, S, T; \lambda) = C (Y, P, S, T) + \lambda [F(\cdot) - Y], \]

Applying the envelope theorem, it is

\[ \frac{\partial L}{\partial S_k} = \frac{\partial C}{\partial S_k} + \lambda F_j = 0, \forall j \]

\[ \frac{\partial L}{\partial Y} = \frac{\partial C}{\partial Y} - \lambda = 0, \]

where \( F_j = \frac{\partial Y}{\partial S_k} \) and \( \lambda \) the Lagrangian multiplier. Multiplying the second condition by \( S_k / Y \) and using the third, the relationship between public capital output elasticity and public capital cost elasticity is given by

\[ \frac{\partial \ln Y}{\partial \ln S_k} = -\left( \frac{\partial \ln C}{\partial \ln S_k} \right) / \left( \frac{\partial \ln C}{\partial \ln Y} \right), \forall k, \]

which provides the linkage between production function approach and cost function approach. This condition can be used to recover the public capital output elasticities from the public capital cost elasticities.

If there are constant returns to scale, then it is well known (see for instance, Champers (1988)) that aggregation of cost functions over firms or industries is consistent with an aggregate cost function and also note that there would be one to one relationship between output and cost elasticity \( (\partial \ln C / \partial \ln Y = 1) \). Then a straightforward comparison between public capital cost elasticities and output elasticities would be correct; otherwise it is not.

The cost elasticity of public capital of industry \( f \) is given by

\[ \varepsilon_{ck}^f = \frac{\partial C^f}{\partial S_k} \left( \frac{S_k}{C^f} \right) \]
Based on the public capital cost elasticities of Nadiri and Mamuneas (1993a) and noting that the output of the twelve industries of their sample corresponds to about one sixth of private sector output, we can deduce from (9) that Nadiri and Mamuneas' estimates will imply a public capital output elasticity of about -.12. Similarly, the corresponding public capital output elasticity based on the cost elasticities of Morrison and Schwartz (1991) will be a weighted average from -.15 to -.27. Both these estimates are much lower than the elasticities based on the production function approach and reported by Aschauer (1989a) and Munnell (1990a). Note however that since Nadiri and Mamuneas estimation is based on gross output rather than value added the output elasticities are not directly compatible with the other elasticities and is likely to be downwards biased.

In a yet unpublished study, Nadiri and Mamuneas (1993b) have estimated a cost function using the aggregate non-farm business sector data for the US for the period 1950-1990. They estimate a translog cost function which includes three types of public sector capital, i.e., infrastructure capital, publicly financed educational capital, and publicly financed R&D capital. The results indicate an output elasticity of about 0.16 for infrastructure capital, 0.05 for educational capital, and 0.08 for the R&D capital. These elasticities suggest that other types of public capital have important and more or less the same degree of impact on productivity and output growth of the private sector. More importantly, the estimate from this study is comparable to those reported by Aschauer (1989a) and Munnell (1989a) for the US aggregate economy. The output elasticity of 0.16 in this study is about one third of the estimate suggested by Aschauer.

The cost elasticity of public capital for the economy is

\[
\epsilon_{ck} = \left[ \sum_t \frac{\partial C^t}{\partial S_k} \right] S_k / \sum_t C^t = \sum_t \epsilon_{ck}^t C^t / \sum_t C^t
\]

That is, a cost weighted average of individual industries. Assuming now that there are constant returns to scale, the cost share weights will be proportional to output shares and the left hand side of this condition will provide the output elasticity of public capital at the economy level from industry level elasticity estimates.
V. General Analytical Framework: The Roles of Demand, Relative Price, and Infrastructure Capital

In order to avoid attributing productivity slowdown to a single or a few causes, it is necessary to develop a general framework which will allow an interplay of both supply and demand considerations in determining TFP growth. In the context of such a framework we can appropriately measure the relative contribution of public infrastructure to growth of output and productivity.

There are several advantages to such an analytical framework: (i) Almost no studies of productivity analysis, whether based on production or cost functions, explicitly allow for the effect of aggregate demand on productivity growth; (ii) We can explicitly account for the contribution of an increase in real factor prices, such as real wages and rental prices of capital, that may generate downward pressure on productivity growth; (iii) The direct and indirect effects of publicly-financed infrastructure on the productivity growth of an industry can be properly delineated; (iv) We can isolate the role of autonomous technological change and assess its influence on output and productivity; and (v) We can assess the impact of public infrastructure and technology on demand for inputs such as demand for employment and private sector physical capital demand.

Our analytical framework, following the previous work of Nadiri and Schankerman (1981a,b), distinguishes between the contributions of output demand, relative input prices, technical change and publicly financed capital to total factor productivity growth (TFP). Analyzing the relative contribution of these types of capital in the context of a comprehensive framework may provide reasonable answers to policy questions.

Let the production function of an industry be given by

\[ Y = F(X, S, T) \] (1)

where \( Y \) is the output of the industry, \( X \) is an \( n \)-domain vector of traditional private inputs, \( S \) is an \( m \)-dimensional vector of infrastructure capital services, and \( T \) is the disembodied technology level.
The traditional measure of Total Factor Productivity growth ($T\hat{F}P$) is defined by the path-independent Divisia index:

$$
T\hat{F}P = \hat{Y} - \sum_{i} \Pi_{i} \hat{X}_{i} \quad i = L, K, M
$$

where $\hat{X}$ denotes the rate of growth and $\Pi_{i} = P_{i} \delta_{i} / \delta_{y} Y$ is the value share of the $i$th private input.

Differentiating (1) with respect to time, and dividing by the output, we obtain

$$
\hat{Y} = \sum_{i} \frac{\partial F}{\partial X_{i}} \frac{X_{i}}{Y} \hat{X}_{i} + \sum_{k} \frac{\partial F}{\partial S_{k}} \frac{S_{k}}{Y} \hat{S}_{k} + \frac{1}{Y} \frac{\partial F}{\partial T}
$$

Assuming cost minimization of all inputs, public included, and letting $P_{i}$ be the price of $i$th private input and $Q_{k}$ the shadow price of public input $k$, we get the following first-order conditions:

$$
\frac{\partial F}{\partial X_{i}} = \frac{P_{i}}{\mu} \quad \forall \ i
$$

$$
\frac{\partial F}{\partial S_{k}} = \frac{Q_{k}}{\mu} \quad \forall \ k
$$

where $\mu$ is the Lagrangian multiplier, together with the envelope conditions

$$
\frac{\partial C^{*}}{\partial Y} = \mu
$$

$$
-\frac{\partial C^{*}}{\partial T} = \mu \frac{\partial F}{\partial T}
$$
where \(C^* = \sum_i P_iX_i + \sum_k Q_kS_k = C^* \) (\(Y, P, Q, T\)) is the total cost function which includes the shadow cost of the two types of public capital. Eliminating \(\mu\) from (4) and (5) and substituting (4)-(5) in (3), we get

\[
\dot{Y} = \sum_i \frac{P_i}{\frac{\partial C^*}{\partial Y}} \dot{X}_i + \sum_k \frac{Q_k}{\frac{\partial C^*}{\partial Y}} \dot{S}_k + \frac{\partial C^*}{\partial T} \frac{\partial T}{\partial Y}
\]

Firms, however, do not adjust the public capital stocks. They are exogenously given. What actually is observed is that firms minimize their private production cost subject to the production function (1). Let the optimal private cost of production, given the output level and public capital, be \(C = \sum_i P_iX_i = C(Y, P, S, T)\). Then the marginal benefit of an increase of public capital at equilibrium will be given by

\[
-\frac{\partial C}{\partial S_k} = Q_k.
\]

It is not difficult to show by using comparative statistics that the total cost elasticity, \(\eta^*\), is given by

\[
\eta^* = \frac{\partial \ln C^*}{\partial \ln Y} = \frac{\partial \ln C}{\partial \ln Y} / B = \eta / B
\]

where \(B = 1 - (\sum \ln C)/(\sum \ln S_k) = 1 - \Sigma \eta_{ck}\) and \(\eta_{ck}\) is the private cost elasticity with respect to public inputs, and \(\eta\) is the private cost elasticity. And the cost diminution due to technical change is
Following Caves et al. (1981), the total return to scale of the production function is defined as the proportional increase in output due to equiproportional increase of all inputs (private and public, holding technology fixed), and is given by the inverse of \( \eta \). The private returns to scale, i.e., the proportional increase in output due to equiproportional increase of private inputs holding public inputs and technology fixed, is given by the inverse of \( \eta \).

Thus, we identify two scale effects in our study, one internal and the other total, which is the sum of internal and external scale effects. Substituting (7) in (6) and then in (2) we have

\[
\text{TFP} = \left( \frac{k - \eta^*}{k} \right) \dot{Y} - \frac{1}{k B} \sum_k \eta_{\alpha k} \dot{S}_k - \frac{1}{k B} \ddot{T}
\]

where \( \kappa = (P_Y Y)/C^* = P_Y/AC^* \) is the ratio of output price, \( P_Y \), to average total cost, \( AC^* \). According to equation (8), TFP growth can be decomposed into three components: a gross total scale effect given by the first term, a public capital stock effect given by the second term, and the technological change effect given by the last term.

The next step is to further decompose the scale effect. We assume that the price of output is related to private marginal cost according to

\[
P_Y = (1 + \theta) \frac{\partial C}{\partial Y}
\]

where \( \theta \) is a markup over the marginal cost. The markup will depend on the elasticity of demand as well as on the conjectural variations held by the firms within an industry. Using the definition of the output elasticity, \( \eta \), along the private cost function, we obtain
\[(9) \quad P_Y = (1 + \theta) \eta \frac{C}{Y} \]

Time differentiating (9), the pricing rule implies

\[(10) \quad \dot{P}_Y = (1 + \theta) \dot{\eta} + \dot{C} - \dot{Y} \]

where \((\cdot)\) denotes growth rate. Differentiating the private cost function with respect to time and using Shephard’s lemma, we have

\[(11) \quad \dot{C} = \eta \dot{Y} + \sum_i \dot{n}_i \dot{p}_i + \sum_k \eta_{\alpha_k} \dot{s}_k + \ddot{\tau} \]

where \(\dot{n}_i = \frac{P_i x_i}{\sum_i P_i x_i}\) is the share of the ith input in private cost, \(C\).

In order to obtain the equilibrium of \(\dot{Y}\) we assume a log linear demand function (see Nadiri and Schankerman (1981a)) in the growth rate form

\[(12) \quad \dot{Y} = \lambda + \alpha \dot{P}_Y + \beta \dot{Z} + (1 - \beta) \dot{N} \]

where \(\dot{Z}\) and \(\dot{N}\) are the growth of aggregate income and population, respectively, and \(\lambda\) reflects a demand time trend. Substituting (11) in (10) and the result in (12), we obtain the reduced form function for the growth rate of total factor productivity.

\[
\text{TFP} = A \left[ a \dot{\eta} + a (1 + \theta) \right] + A a \sum_i \dot{n}_i \dot{p}_i + A \left[ \lambda + b \dot{Z} + (1 - b) \dot{N} \right]
\]

\[(13) \]
\[ + A\sigma \sum_{k} \eta_{ck} \dot{S}_k - \frac{1}{KB} \sum_{k} \eta_{ck} \dot{S}_k + A\sigma \dot{T} - \frac{1}{KB} \dot{T} \]

where \( A = \frac{K - \eta^*}{K} / [1 - \sigma(\eta - 1)] \).

Equation (24) decomposes TFP into components:

(i) a factor price effect \( A\sigma \sum_{i} \dot{N}_i \dot{\hat{p}}_i \);

(ii) an exogenous demand effect \( A [ \lambda + b \bar{Z} + (1 - b) \bar{N} ] \);

(iii) a public capital effect \( [ A\sigma - \frac{1}{KB} ] \sum_{k} \eta_{ck} \dot{S}_k \); and

(iv) disembodied technical change \( [ A\sigma - \frac{1}{KB} ] \dot{T} \).

The underlying model is an equilibrium model in which there is minimization over all inputs, the level of public capital is adjusted at its optimal level by the government until it earns its social rate of return. The public capital and disembodied technical change effects can be further decomposed into direct and indirect effects. The direct effect of infrastructure \( k \), for instance, is given by \( (\eta_{ck} / KB) \dot{S}_k \) while the indirect effect is given by \( A\sigma \eta_{ck} \dot{S}_k \). Thus an increase of public infrastructure initially increases total factor productivity by reducing the private cost of production, which in turn leads to lower output price and higher output growth. Change in output growth in turn leads to changes in TFP growth.

The important parameters in (13) are the price and income elasticities of demand and the cost elasticities of the private cost function. Note that if the demand function is completely inelastic \( (\sigma = 0) \) then shifts in the cost function due to real factor price changes, public capital, or disembodied technical change have no effect on output and hence TFP. Second if there are constant returns to scale including the public inputs, \( \eta^* = \kappa = 1 \), then (24) collapses to \( TFP = -\frac{1}{B} \sum_{k} \eta_{ck} \dot{S}_k - \frac{1}{B} \dot{T} \).
VI. Proposed Econometric Model Specification and Data Requirement

To carry out the decomposition of TFP growth into its various components as indicated by (13), two sets of parameter estimates are needed. We need to estimate the parameters of the demand function given by equation (12), which relates growth of output demand to changes in price of output and per capita income. We also need estimates of the cost elasticities of infrastructure capital and the two scale parameters. The output demand equation for each state or region, $r$, can be written as

\begin{equation}
\dot{Y}_r = \lambda_r + a_r \dot{P}_r + \beta_r \dot{Z}_r + (1 - \beta_r) \dot{N}_r
\end{equation}

where $\dot{Y}_r$, $\dot{P}_r$, $\dot{Z}_r$, and $\dot{N}_r$ are state specific rate of change of output, price of output, total income, and population. It is possible, and perhaps desirable, to include the effect of infrastructure investment in the demand equation as well. The notion will be that states that invest in public infrastructure, i.e., have a higher propensity to invest in public sector capital, may induce growth of demand. This is a testable hypothesis that could be easily introduced. If, however, the influence of infrastructure capital on demand for output turns out to be statistically significant, it would be necessary to modify the decomposition of TFP shown in equation (13) to incorporate the direct demand effect of public infrastructure capital.

For the estimation of the effect of infrastructure on the cost side, suppose that industry $f$ in each region $r$ produces output ($Y$) with private inputs $X = (X_1, ..., X_n)$ and rental prices $P = (P_1, ..., P_n)$. In addition, industries of the region utilize infrastructure services which are provided by all levels of the government. Let $S = (S_1, ..., S_m)$ be an $m$-dimensional vector of the infrastructure services that might be provided to the industries in the region free from user charges. We assume that the firms in the industry chose the private inputs to minimize their production cost subject to their production functions. The technology of the industry in a given region can be represented by a cost function which can be approximated by a continuous, twice differentiable, and linearly homogeneous function in private input prices of the following form:
\[ C(P, Y, S) = \left\{ .5 \sum_i \sum_j a_{ij} P_i P_j / \left[ \sum_i \theta_i P_i \right] + \sum_i b_{ii} P_i + b_{yy} \left[ \sum_i \alpha_i P_i \right] Y \\
+ \sum_i \sum_k c_{ik} P_i S_k + \sum_i \sum_t d_{it} \left[ \sum_i \beta_i P_i \right] S_k S_t \right\} Y \\
+ \sum_i b_i P_i + \sum_k c_k \left[ \sum_i \gamma_i P_i \right] S_k, \quad i, j = 1, \ldots, n, k, t = 1, \ldots, m, \]

where \( a_{ij} = a_{ji}, d_{kt} = d_{tk}, \) and the parameters \( \theta_i, \alpha_i, \beta_i, \gamma_i \) are assumed to be exogenously given. This functional form is the symmetric generalized MacFadden cost function introduced by Diewert and Wales (1987), augmented to include the infrastructure services. The cost function is dual to a well-behaved production function if it is nonnegative, monotonically increasing, homogeneous of degree one, and concave in input prices. If in addition, for some reference point \( P^* > 0, Y^* > 0, S^* > 0, \) the following restrictions are satisfied

\[ \sum_i a_{ij} P_j^* = 0, \]
\[ \sum_i \theta_i P_j^* \neq 0, \sum_i \alpha_i P_j^* \neq 0, \sum_i \beta_i P_j^* \neq 0, \text{and} \sum_i \gamma_i P_j^* \neq 0; \]

then \( C(\cdot) \) is a flexible, linearly homogeneous in input prices, cost function. The advantage of this functional form over the translog cost function is that if the estimated matrix \( A = [a_{ij}] \) is negative semidefinite, then the cost function will be concave in input prices. However, if the \( A \) is not negative semidefinite, we can impose concavity in input prices globally by a Cholesky factorization, without destroying the flexibility property of the cost function (See Diewert and Wales (1987) for a further discussion).

The system of estimating equations, introducing, region and time subscripts \( r, t \) respectively, can be derived by applying Shephard's Lemma \( (X_i = \partial C / \partial P_i) \)
\( \frac{X_{it}}{Y_{rt}} = \sum_i a_i P_{ir} / \left[ \sum_i \theta_i P_{ir} \right] - 0.5 \sum_i \sum_a a_i P_{ir} P_{pt} \theta_i / \left[ \sum_i \theta_i P_{ir} \right]^2 \\
+ b_i + b_{i, y} \sigma_i Y_{rt} + \sum_k c_{ik} S_{ikrt} + \sum_k \sum_l d_{i, k, l} \beta_i S_{ikrt} S_{tln} \\
+ b_j / Y_{rt} + \sum_k c_{i, k} \gamma_i S_{ikrt} / Y_{rt} + \epsilon_{irt}, \quad i, j = 1, \ldots, N, k, l = 1, \ldots, M, \\
r = 1, \ldots, R, \ t = 1, \ldots, T \)

where \( \epsilon_{irt} = (\epsilon_{i, rt}, \ldots, \epsilon_{i, rt}) \) have zero mean and constant covariance matrix \( \Sigma \) for all \( r \) and \( t \). This assumption seems reasonable enough since by dividing each input by the output reduces the degree of heteroskedasticity of errors. In addition in order to capture regional and time specific effects, we can assume that the parameters \( b_i \) and \( c_{ik} \) are state and time specific. Furthermore, to ensure invariant elasticity estimates the chosen parameters, \( \theta, \sigma, \beta, \gamma \), should be measured in units of input. One candidate could be the average amounts of inputs over all samples. This system of conditional input demand equations (16) can be estimated by using an iterative seemingly unrelated regression technique. By introducing subscript \( f \) to (16) to index different industries, the model becomes general enough to identify the effect of different types of infrastructures provided by each state to each industry and over time.

However, potential problems might arise during the estimation process, due to unavailability of data or multicollinearity. The above model could then be modified to meet specific requirements. Suppose we were interested in estimating the contribution of publicly financed infrastructure capital in different industries across various states to the growth of output and productivity in a particular year. In such a case, no time series data are needed; the time subscript \( t \) in equation (16) can be dropped and the estimation can proceed with a cross-sectional analysis of data on industries and states.

One potential estimation problem which needs to be mentioned is the dimensionality of vector \( S \). Since the different infrastructure services move together, this might create multicollinearity problems, and the parameters of different infrastructure services might be very difficult to identify. An alternative is to construct a pool of public capital variables by each region or state; that is
\[ S_r = \sum_{k=1}^{m} W_{rk} S_k \]

where \( W_{rk} \) are some unknown parameters which can be specified by the investigator or estimated. For instance, if \( W_{rk} \) is assumed to be equal to one, then \( S_r = \sum_{k=1}^{m} S_k \), and it corresponds to the total capital services provided by the region or state \( r \).

The marginal benefit of infrastructure services can be calculated by taking the derivative of the cost function with respect to infrastructure service \( S_{kr} \).

\[
(17) \quad \frac{\partial C_r}{\partial S_{kr}} = -\left\{ \sum_{i} c_{ik} P_{irt} + 2 \sum_{i} d_{kr} \left[ \sum_{i} \beta_i P_i \right] S_{irt} \right\} Y_{rt} - \left[ \sum_{i} \gamma_i P_{irt} \right] c_k
\]

Note that if the estimated matrix \( D = [d_{kr}] \) is positive semidefinite, then condition (17) can be interpreted as the demand of infrastructures. Also, if the user fee of infrastructure is known, say equal to \( Q_{rk} \), then condition (17) can be imposed on the estimation. Condition (17) is the shadow value or marginal benefit of each type of public capital by region. Index condition (17) by \( f \) to refer to industry \( f \). This condition can then be used to estimate (i) the rate of return of each type of public capital by region; (ii) the rate of return of total public capital by region; (iii) the economy-wide rate of return, with proper aggregation, of different types of public capital and total public capital. By knowing the marginal cost of public capital (ignoring consumption), we can also directly estimate the optimal amount of different public capital that will equate the sum of marginal benefits to its marginal cost. That is,

\[
\sum_{f} - \frac{\partial C_r^f}{\partial S_{kr}} = P_{kr}
\]

where \( P_{kr} \) is the marginal cost of public capital of type \( k \) of region \( r \).
Finally, the indirect effects of the different types of infrastructure services on private inputs like capital and employment is given by

\begin{equation}
\frac{\partial X_{ikt}}{\partial S_{kt}} = \left\{ c_{ik} + 2 \sum_t d_{kt} \beta_i S_{tkt} \right\} Y_{kt} + y_i c_k.
\end{equation}

Thus we can test the so-called "public capital hypothesis" and estimate the effect of public infrastructure on private capital and labor.

Data Requirements

Apogee has undertaken to collect desegregated data along several dimensions; by private two digit industries, by state, and by functional use of public infrastructure investment across states and over time.

In order to estimate the above model the following data are required: Data on quantities and price indices, across states and over time, on output, labor, and private capital stock for each 2-digit industry and state aggregates; data on public capital stock, their price deflators and depreciation rates by function and state and also data of state real and current income and state population characteristics. Gross State Product (GSP) data are available from Bureau of Economic Analysis (BEA) from 1963 on approximately the two-digit SIC level. Estimates of employment and level of labor earning at the same level of detail are also available from the same source. Data on state income and population characteristics are available from published sources.

Data on output and labor state income and population can be relatively easy obtained; however, difficulties might arise for the construction of capital stock and user cost of capital. Currently Apogee is in the process of obtaining data on output, employment, hours worked, wage rate, construction of private capital stocks for the two-digit industry level in each state and public infrastructure capital by function (seven categories) for each state. Construction of infrastructure by functional categories are generated by the perpetual inventory method, which requires careful estimation for each category of capital of the benchmark capital stock, depreciation rate, investment expenditure and investment deflator. There are a number of data sets already available. Holtz-Eakin (1991) has constructed measures of public infrastructure by function of state
and local governments in 50 states; Munnell (1990) has constructed the private sector capital by states, including estimates for manufacturing industries. Other alternative sources are being explored by Apogee in its attempt to construct the disaggregated data by industry, state and function.

An important challenge in data construction may arise in constructing the user cost of private capital. To construct the user cost (rental price) of private capital it is necessary to have data on the acquisition prices of different types of capital investment, as well data of the depreciation rates of the private capital stock. In addition it is required that we have information about the discount rate and tax rate of private capital faced by each industry in each state. For instance the rental price of physical capital can be measured as $p_k = q_k (r + \delta_k) [(1 - \lambda_k - u_c)z/(1-u_c)] + PT$, where $q_k$ is the price deflator of investment, $r$ is the discount rate which can be proxied by the rate of ten-year Treasury notes and be obtained from Citibase, $\delta_k$ is the physical capital depreciation rate, $\lambda_k$ is the investment tax credit, $u_c$ is the corporate income tax, $z$ is the present value of capital consumption allowances, and PT is the property tax rate. The capital consumption allowances is defined as $z = \rho (1 - \omega \lambda_k)/(r + \rho)$ (see Bernstein and Nadiri (1987, 1988, 1991)), where $\rho$ is the capital consumption allowance rate obtained by dividing the capital consumption allowances by the capital stock, and $\omega$ takes value 0, except for the periods in which the firms have to reduce the depreciable base of the assets. Not all of the required data may be available to construct the user cost of capital or some of the other variables. Some reasonable compromise and approximation will be necessary to construct the variables of the model. In particular, data on business tax rates for two-digit industries at the state level may not be readily available. Data for the construction of the corporate income tax rate can be developed by extending the data given in Auerbach (1983) and Jorgenson and Sullivan (1981). Also data on business tax collection at the state level are available from Government Finances that can be used to compute average tax rates; statutory tax rates are published annually in Facts and Figures in Government Finance.

It will also be desirable to have data on the usage of infrastructures and user fees if they are available.
VI. Potential Research Output and Some Concluding Comments

The proposed research can be carried out in several phases subject to the availability of time and resources. The data and estimation requirements are very demanding. However, the potential results of the research project is also very promising. We can apply the model to any of the following possibilities:

1. One industry across all states
2. State level analysis, no industry disaggregation
3. Two or more industries across all states
4. All two-digit industries across all states

The most promising would be to concentrate on option (1) or option (2) as a pilot study, and, if time and resources permit, proceed with options (3) and (4). Each of these possibilities can be carried out using data on a cross-section of industries and states, or for cross-section time series data on these units of analysis. The preferable strategy would be to use the cross-section and time series data if the necessary data are available. In the estimation procedure, careful attention must be given to control for state- and industry-specific characteristics that have been pointed out by Holtz-Eakin (1992), as noted earlier. Deciding which estimation strategy to follow requires further discussion.

The potential research output that we hope to generate includes some of the following:

1. The impact of increase in national income and/or state income on productivity growth.
2. The effect of state population growth on demand for output and productivity growth.
3. The role of real wages and other relative input prices on productivity growth in different industries across various states.
4. The direct effect of major types of infrastructure capital on cost of structure of different industries and enhancement of productivity growth of different industries across states or regions.
5. The effect of infrastructure investment on employment and capital formation in the private sector industries across different states.
6. The contribution of various types of infrastructure capital to regional or state economic growth as measured by the growth of output, employment, and capital formation.

7. We may be able to test, by modifying our model, the effect of infrastructure investment on industry demand for output across states or regions. This would provide some evidence about whether there are direct demand inducement effects of infrastructure capital on the regional or state level.

8. We can measure the benefits of additional infrastructure in each state and calculate the rate of return on infrastructure capital in each state. We can compare the rate of return on public and private capital at the industry, state or regional level.

9. Furthermore, it is possible to measure productivity spillovers that may be generated from aggregate or "cross-border" spillovers from one state's provision of infrastructure (particularly highways) to neighboring states. The methodology for estimating such an effect has already been developed in a series of papers by Bernstein and Nadiri (1988, 1991) who have studied the spillover effect of R&D capital among various industries. We can calculate the "social" rate of return to a state infrastructure investment.

10. In this research, emphasis will be on disaggregate measure of the benefits of infrastructure capital. The disaggregations will be carried out along several dimensions: by private industry, by functional use of public infrastructure capital, and by level of state or region. The impact of different types of infrastructure capital is likely to differ across states or by industrial classification.

Severall challenging tasks require careful attention: First, the appropriate data to estimate the model need to be checked for accuracy and consistency. Second, the results of the estimated model should be checked for robustness and reasonableness. Third, potential policy implications of the results need to be explored. Lastly, using the final version of the estimated model, a limited number of simulations need to be performed to evaluate potential effects of alternative policy options.
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