FINAL EXAM 1997. ANSWERS

Question 1

According to the CLT the distribution of the sample mean is:

$$\bar{x} \sim N(65, \frac{8}{\sqrt{8}} = 1)$$

Therefore,

$$\Pr(\bar{x} \geq 71) = \Pr(\frac{\bar{x} - \mu}{\sigma/\sqrt{N}} \geq \frac{71 - \mu}{\sigma/\sqrt{N}}) = \Pr(z \geq \frac{71 - 65}{1} = 6) \approx 0$$

Question 2

a) By the CLT,

$$\bar{x} \sim N(\mu = 500, \sigma = \frac{200}{\sqrt{36}} = \frac{200}{6} = 33.3)$$

b)

$$\frac{\sigma}{\sqrt{N}} = 25 \Rightarrow \sqrt{N} = \frac{200}{25} = 8 \Rightarrow n = 64$$

Question 3

a) In repeated sampling, the population parameter would fall in the given interval 95% of the time.

b) To reduce the width you have either to increase the sample size or to reduce the confidence level.

Question 4

a)

$$\hat{p} = \frac{x}{N} = \frac{380}{600} = 0.63$$

Now we know that:

$$\hat{p} \sim N(p, \sigma_{\hat{p}} = \sqrt{\frac{pq}{N}} = \sqrt{\frac{0.63 * 0.37}{600}} = 0.0197)$$

Therefore,

$$CI_{1-\alpha} = [\hat{p} \pm \sigma_{\hat{p}} * z_{\frac{\alpha}{2}}] = [0.63 \pm 1.645 * 0.0197] = [0.60, 0.66]$$

b)

We want to choose N so that

$$CI_{90} = [\hat{p} \pm 0.01]$$

$$0.01 = z_{\frac{\alpha}{2}} * \sigma_{\hat{p}} = 1.645 * \sqrt{\frac{p * q}{N}} = 1.645 * \sqrt{\frac{0.63 * 0.37}{N}}$$

$$\sqrt{N} = \frac{1.645 * \sqrt{0.63 * 0.37}}{0.01}$$

$$N = \left(\frac{1.645}{0.01}\right)^2 * 0.63 * 0.37 = 6{,}308$$

Question 5

a)

$$H_0: \mu = 35$$
$$H_1: \mu > 35$$

We set up the null and the alternative hypothesis so that for the claim to be supported we have to reject the null. This set up allows us to know the probability of mistakenly supporting the claim (i.e. the type one error, α).

b) The test statistics is:

$$z = \frac{\bar{x} - \mu}{s/\sqrt{N}} = \frac{\bar{x} - 35}{6/6} = \bar{x} - 35$$

Rejection region is:

$z \geq z_{\alpha} = z_{0.05} = 1.645$

c) In our case,

z=35.8-35=1.8>1.645

So, we reject the null.

d) The p-value is given by:

$$P(z \geq 1.8) = \frac{1}{2} - 0.4641 = 0.036 = 3.6\%$$

We can reject the null with up to 96.4% confidence level.

Question 6

a)

$H_0 : p = 0.15$

$H_{\cdot} : p < 0.15$

Also here we set up the null and the alternative hypothesis so that for the claim to be supported we have to reject the null. In this case, to claim that the new method works we should be able to claim that μ is lower than 0.15, i.e. we should be able to reject the null.

b) The test statistics is:

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{pq}{N}}}$$

The rejection region is:

$z \leq z_{\alpha} = -2.326$

c)

$$z = \frac{0.086 - 0.15}{\sqrt{\frac{0.15 * 0.85}{70}}} = \frac{-0.064}{0.0427} = -1.499 > z_{\alpha}$$

Therefore, we cannot reject the null.

Question 7

a)

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_0: \mu_1 - \mu_2 \neq 0$$

The test statistics is:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{N} + \frac{S_2^2}{N}}}$$

We reject the null if

$$|z| > z_{\frac{\alpha}{2}} = 1.96$$

$$z = \frac{20.6 - 23.5}{\sqrt{\frac{(5.3)^2}{55} + \frac{(4.1)^2}{200}}} = \frac{-2.9}{\sqrt{0.51 + 0.084}} = -3.76 < -1.96$$

Therefore, we reject the null.

c) We would do a matched-pairs experiment.