Statistics **NYU - Fall 1999**
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ANSWERS TO MIDTERM EXAM

Question 1.

According to the empirical rule 68% of observations are 1 standard deviation away from the mean. This leaves 32% of observations in the two tails. Given that the distribution is symmetric, there will be 16% of observations in each tail.

Therefore, 68+16=84% of observations will be below the mean plus one standard deviation, i.e. below 71+8=79.

Question 2.

(a) From the text of the question we know that:

P(P) = P(the date is Puertorican) = 48%
P(R) = P(the date is Russian) = 22%
P(I) = P(the date is Italian) = 30%

P(C|P) = P(the date is Catholic give that he is Puertorican) = 40%
P(C|R) = P(the date is Catholic give that he is Russian) = 12%
P(C|I) = P(the date is Catholic give that he is Italian) = 80%

Therefore

P(C) = P (the date is Catholic) = P(C|P)*P(P) + P(C|R) * P(R) + P(C|I)*P(I) =
.40*.48+.12*.22+.80*.30 = .46

Applying Bayes's rule

P(P|C) = P(C|P) * P(P) / P(C) = .42

(b)

P(I'|C) = 1 – P(I|C) = 1 – P(C|I)*P(I) / P(C) = 1 – 0.52 = 0.48

Question 3.

(a) The z-score equals

$z = (18-30) / 4 = -3$

(b) The salesman's claim is likely to be false. According to the empirical rule, if the claim were true, there would be less than 0.15% chances that I buy a batteries that lasts for 18 hours or less.

Question 4.

(a) x, the number of patients admitted in a day, is distributed as a Poisson random variable with λ= 5.2. The probability that the hospital will not have enough beds is:

$P(x>8)=1-P(x\leq 8)$

From the table of the Poisson distribution:
$P(x\leq 8)=0.918$

Therefore,

$P(x>8)=1-P(x\leq 8)= 1 – 0.918 = 0.082$

(b) We now that, for a Poisson distribution,

$\sigma^2 = \sqrt{\lambda} = \sqrt{5.2} = 2.28$

Question 5 (exact solution)

(a) P(all three computer are infected) = (30/200) * (29/199) * (28/198) = 0.003
(b) P(at least one is infected) = 1 - P(all non infected) = 1 – (170/200)*(169/199)*(168/198) = 1 – .61 = .39

Question 5 (approximated solution. It assumes that the probability of having an infected computer is the same in all the three draws).

(a) P(all three computer are infected) = $(30/200)^3$ = 0.003
(b) P(at least one is infected) = 1 - P(all non infected) = 1 – $(170/200)^3$ = 1 – .61 = .39

Question 6

(a)

$P(J) = P(\text{the stock market is up for January}) = .70$
$P(Y) = P(\text{the stock market is up for the year}) = .80$
$P(J \wedge Y) = .63$

Therefore, by the multiplicative law,

$P(Y|J) = P(J \wedge Y) / P(J) = .63/.70 = .90$

(b)

$P(Y \wedge J') = .17$

Therefore

$P(Y|J') = P(J \wedge Y) / P(J') = .17/.30 = .57$

(c)

$P(Y) = .80$
$P(Y|J) = .90$

Therefore, being up for January and being up for the year are not independent events.