

**FINAL EXAM**

**December 15<sup>th</sup>, 1997**

**Answer each of the questions in the exam books provided. There are 90 total points. Read the questions carefully and keep your answers *brief and to the point*. PLEASE WRITE NEATLY. Good luck!**

**Question 1. [10 points]**

The amount of time it takes to take an exam has a left-skewed distribution, with a mean of 65 minutes and a standard deviation of 8 minutes. If 64 students were randomly sampled, find the probability that the sample mean of the sampled students exceeds 71 minutes.

**Question 2. [10 points]**

A random sample of size  $n = 36$  is to be drawn from a population with  $\mu = 500$  and  $\sigma = 200$ .

- (a) What is the distribution of the sample mean  $\bar{x}$ ?
- (b) What sample size would be necessary in order to reduce the standard deviation of the sample mean to 25?

**Question 3. [10 points]**

Suppose a 95% confidence interval for  $\mu$  turns out to be [1000, 2100].

- (a) Give a definition of what it means to be "95% confident" in this inference.
- (b) Suppose it is desired to reduce the width of the confidence interval. How would you achieve that?

**Question 4. [10 points]**

The U.S. Commission on Crime wishes to estimate the fraction of crimes related to firearms in a certain city. The commission randomly selects 600 files of recently committed crimes in the area, and finds 380 in which a firearm was reportedly used.

- (a) Find a 90% confidence interval for  $p$ , the true fraction of firearm-related crimes in the area.
- (b) Keeping the same confidence level, how many files should the commission select in order to estimate  $p$  to within .01?

**Question 5. [20 points]**

The Environmental Protection Agency (EPA) estimated that the 1991 XYZ- car obtains a mean of 35 miles per gallon. The car manufacturer claims that the car exceeds the EPA estimate. To support its assertion, the company randomly selects 36 1991 XYZ cars and records the mileage obtained by each car, under the same condition as the EPA experiment. The following data resulted:  $\bar{x} = 36.8$  mpg,  $s = 6.0$  mpg.

- (a) Set up the null and the alternative hypotheses that the auto manufacturer wishes to test, and explain why they should be defined this way.
- (b) Write down the test statistic you would use, and define the rejection region for  $\alpha = .05$ .
- (c) State whether or not you reject the null hypothesis at this significance level.
- (d) What is the  $p$ -value of this test? How do you interpret it?

**Question 6. [15 points]**

A method currently used by doctors to screen men for possible prostate cancer fails to detect cancer in 15% of the men who actually have the disease. A new method has been developed that should detect cancer more accurately. A random sample of 70 men known to have prostate cancer were screened using the new method. Of these, the new method failed to detect cancer in six.

- (a) Specify the null and alternative hypotheses that the researchers wish to test.
- (b) Calculate the test statistic used by the researchers for this test of hypothesis.
- (c) If the significance level  $\alpha$  is .01, do you reject your null hypothesis?

**Question 7. [15 points]**

An experiment is conducted at a university to compare the mean number of study hours for student athletes, with the mean number of study hours for non-athletes. A random sample of 55 athletes produced a sample mean equal to 20.6 hours studied per week, and a standard deviation of 5.3 hours. A second random sample of 200 non-athletes produced a sample mean equal to 23.5 hours per week, and a standard deviation equal to 4.1 hours.

- (a) Conduct a test of hypothesis to determine if a difference exists between the average study hours of athletes and non-athletes (fix the significance level  $\alpha$  to .05).
- (b) What is the  $p$ -value of the test?
- (c) Suppose you suspect that the students' enrollment year (freshman, sophomore, ...) also affects the average study time: how would you design an experiment to test whether study hours differ between athletes and non-athletes, controlling for the enrollment year?