

**MIDTERM EXAM**

**March 7, 2001**

**Answer each of the questions in the exam books provided. There are 75 total points. Read the questions carefully and keep your answers brief and to the point. PLEASE WRITE NEATLY. Good luck!**

**Question 1. [10 points]**

Suppose you work in the personnel department of a firm that just finished training a group of its employees. You have been requested to review the performance of one of the trainees on their final test. The mean and standard deviation of the test scores are 80 and 5, respectively.

- (a) (2 points) Suppose the trainee in question scored a 62: compute the trainee's  $z$ -score.
- (b) (4 points) Assuming the distribution of scores is mound-shaped, approximately what percentage of test takers scored better than this trainee?
- (c) (4 points) Assuming nothing is known about the shape of the distribution, what percentage of test takers scored above 90?

**Question 2. [10 points]**

The University Bookstore has received a late shipment of 25 statistics books from the publisher. Each book also contains a copy of the software to be used in the class. Unknown to the bookstore, five of the disks shipped contain an error that will not allow the students to access the software. Suppose you and a friend both purchase books from this shipment of 25.

- (a) (5 points) Find the probability that at least one of your disks does not contain the error.
- (b) (5 points) Find the probability that neither of your disks contains the error.

**Question 3. [25 points]**

Tests for diseases (such as AIDS) involve errors: there are FALSE POSITIVES and FALSE NEGATIVES. A false positive is the case in which a test indicates that the tested individual is HIV-positive when in fact he/she is not; a false negative is the case in which a test indicates that the tested individual is HIV-negative when in fact he/she is. In this problem we want to examine the consequences of such errors when we test a large number of people.

Suppose we are given the following information. The probability of false positives is 5 %: formally we can write this as  $\text{Prob}(\text{TEST+} \mid \text{HIV-}) = .05$ ; the probability of false negatives is 1 % (formally,  $\text{Prob}(\text{TEST-} \mid \text{HIV+}) = .01$ ). So the test is quite accurate. We also know that in a population of 100 million, there are roughly 500,000 cases of HIV-positive individuals. Therefore,  $\text{Prob}(\text{HIV+}) = .005$ .

- (a) (10 points) Using the above information and applying Bayes Rule, calculate the probability of being HIV-positive, given that one has tested positive:  $\text{Prob}(\text{HIV+} \mid \text{TEST+})$ ;
- (b) (10 points) Now calculate (again using Bayes Rule) the probability of being HIV-negative, given that one has tested negative:  $\text{Prob}(\text{HIV-} \mid \text{TEST-})$ ;
- (c) (5 points) Please comment briefly on these results, from the point of view of an individual taking a test, as well as from a public health perspective.

#### Question 4. [10 points]

The Department of Commerce in Wisconsin has determined that the number of small businesses that declare bankruptcy per month follows a Poisson distribution with a mean equal to 6.4.

- (a) (5 points) Find the probability that more than three bankruptcies occur next month.
- (b) (5 points) Find the probability of exactly five bankruptcies occurring next month.

#### Question 5. [10 points]

A brewery has a beer dispensing machine that pours beer into the company's 12 ounce bottles. The distribution of the amount of beer actually dispensed by the machine follows a Normal distribution with a standard deviation of 0.2 ounces. The company can control the mean amount of beer dispensed by the machine.

- (a) (5 points) What value of the mean should the company set if it wants to guarantee that 98.5% of the bottles contain at least 12 ounces (the amount on the label)?
- (b) (5 points) Suppose now that the distribution of pours is Normal with mean 12.1 and standard deviation 0.5. Customers get very angry when their beers have less than the advertised 12 ounces of beer in them. Calculate the proportion of customers that could get angry because of this reason.

#### Question 6. [10 points]

A major department store chain is interested in estimating the average amount its credit card customers spend on their first visit to the chain's new store at the Manhattan Mall. Thirty-six accounts were randomly sampled and analyzed with the following results:  $\bar{x} = \$80.50$  and  $s^2 = \$625$ .

- (a) (5 points) Construct a 95% confidence interval for the average amount spent in the population of first visits to the new store.
- (b) (5 points) How do you interpret your estimate, in words?