

1 Optimal Taxation of Labor Income

Until now, we have assumed that government policy is exogenously given, so the government had a very passive role. Its only "concern" was balancing the intertemporal budget. In this section, we let the government be more proactive, and choose tax rates.

Consider a two period economy where some government expenditures \( g \) (given exogenously) needs to be financed in the first period. The government can use taxes on labor income in the two periods at rates \( \{\tau_1, \tau_2\} \) and is free to set these taxes with the only constraint that the intertemporal budget constraint must hold, i.e., the DPV of tax revenues must equal \( g \). However, the government can choose whether to tax agents more in the first period, more in the second, or equally. We also assume that the government is benevolent, in the sense that it maximizes households’ utility. We leave considerations about corruption and self-interest of politicians out of the picture for now: that is the realm of political economics.

We start by describing the problem of the household and solve it for given tax rates, next we describe the problem of the government. Finally, we define an equilibrium and we solve for the equilibrium tax rates and consumption-labor allocations.

**Households**— To simplify the analysis, we assume that households have quasi-linear utility, specifically, linear utility over consumption and quadratic disutility of hours worked. Their problem reads

\[
\max_{\{c_1, c_2, h_1, h_2\}} c_1 - \frac{1}{2} h_1^2 + \beta \left( c_2 - \frac{1}{2} h_2^2 \right) \\
\text{s.t.
}
\frac{c_1}{1+r} = (1-\tau_1) w_1 h_1 + \frac{(1-\tau_2) w_2 h_2}{1+r}.
\]

Linear utility means that households’ consumption allocation between periods 1 and 2 can be of three types: 1) if \( \beta (1+r) < 1 \), then they want to consume all in period 1; 2) if \( \beta (1+r) > 1 \), then they will consume all in period 2; 3) if \( \beta (1+r) = 1 \) then , they’re exactly indifferent and only care about the total resources they consume. Let’s assume we are in this latter case. Moreover, let’s assume that the aggregate production technology is \( y_t = An_t \), so that in equilibrium the wage equals the exogenous constant \( A \) in both periods. As a normalization, let’s set \( A = 1 \) and so \( w_1 = w_2 = 1 \).

From the budget constraint, and the assumptions on wages and interest rates, we
obtain
\[ c_1 + \beta c_2 = (1 - \tau_1) h_1 + \beta (1 - \tau_2) h_2, \]
which we can substitute in the objective function and simplify the problem. Now, the only choice of the households is labor supply in the two periods. The household problem simplifies to the unconstrained problem
\[
\max_{\{h_1, h_2\}} (1 - \tau_1) h_1 - \frac{1}{2} h_1^2 + \beta \left[ (1 - \tau_2) h_2 - \frac{1}{2} h_2^2 \right],
\]
with solution
\[ h_t^* = (1 - \tau_t), \quad t = 1, 2. \quad (1) \]
Because of the quasi-linearity of preferences, there is no income effect, only substitution effects in labor supply and optimal hours worked are always decreasing in the tax.

**Government**— The government maximizes households’ welfare, i.e., it solves
\[
\max_{\{\tau_1, \tau_2\}} (1 - \tau_1) h_1^* - \frac{1}{2} (h_1^*)^2 + \beta \left[ (1 - \tau_2) h_2^* - \frac{1}{2} (h_2^*)^2 \right], \quad (GP)
\]
s.t.
\[
g = \tau_1 h_1^* + \beta \tau_2 h_2^*
\]
\[ h_t^* = (1 - \tau_t), \quad t = 1, 2. \]
where the first equation is the government budget constraint. The fact that all \( h \) variables have a star is not by chance! The star refers to the optimal household decision in equation (1). The government in solving this problem will choose a pair of tax rates \( \{\tau_1, \tau_2\} \) being aware that these tax rates will induce the household to change its labor supply behavior through (1). Households react to government policies (e.g. work less if taxes are high) and governments take this into account. The second equation in (GP) represents precisely this reaction function.

**Equilibrium**— A competitive equilibrium with endogenous tax rates is called a *Ramsey Equilibrium*. A Ramsey equilibrium for this economy is a pair of choices for hours \( \{h_1^*, h_2^*\} \) and a pair of taxes \( \{\tau_1^*, \tau_2^*\} \) such that: (i) given taxes, households solve problem (HP); (ii) given the reaction function of the households, the government solves problem (GP); (iii) markets clear, i.e., \( w_t = 1 \); and (iv) the government budget is balanced.
Solution— To solve the government problem (GP), it is convenient to substitute out \( h^* \) using (1) and then write down the Lagrangian only as a function of tax rates. Doing this, we obtain

\[
L(\tau_1, \tau_2, \lambda) = \max_{\{\tau_1, \tau_2, \lambda\}} \frac{1}{2} (1 - \tau_1)^2 + \beta \frac{1}{2} (1 - \tau_2)^2 + \lambda [\tau_1 (1 - \tau_1) + \beta \tau_2 (1 - \tau_2) - g]
\]

with solution, from the FOC’s,

\[
\tau_1^* = \tau_2^* = \frac{\lambda - 1}{2\lambda - 1}. \tag{2}
\]

Let \( \tau^* \) be the constant tax rate. Substituting \( \tau^* \) into the government budget constraint, we obtain

\[
g = (1 + \beta) \tau^* (1 - \tau^*) \Rightarrow (\tau^*)^2 - \tau^* + g/(1 + \beta) = 0, \tag{3}
\]

which is a quadratic equation in \( \tau^* \) with two solutions. We must choose the one (perhaps there will be two) between zero and one, as a tax rate should be.

1.1 Discussion

What is the main conclusion of this exercise? Equation (3) says that the level of taxes is then determined by the level of \( g \) and the interest rate (\( \beta \) in this case). But the key part of the result is in equation (2): the government chooses the same tax rate across the two periods. This important result is called “tax smoothing” and is a typical outcome of optimal taxation problems.

The government wants to minimize the total disutility of effort for the agent over the two periods. Let the disutility of labor effort be denoted by \( v \). Since the disutility of labor is convex (quadratic), by Jensen’s inequality we have that

\[
\frac{v(h_1) + v(h_2)}{2} \geq v\left(\frac{h_1 + h_2}{2}\right)
\]

\[
\Rightarrow v(h_1) + v(h_2) \geq 2v(\bar{h})
\]

where \( \bar{h} = 1/2 (h_1 + h_2) \) is mean hours. This means that, in order to maximize welfare, it is better to induce the agent to supply the same number of hours in the two periods than inducing her to work different amounts in the two periods with the same average hours worked (see below).
Another way to look at this result is that the government wants to minimize distortions in households’ decisions. If the households could decide freely, in the first-best without government expenditures and taxes \((g = r_1 = r_2 = 0)\) they would set \(h_1^* = h_2^* = w = 1\) from equation (1). By setting equal taxes in the two periods, the government still let the agent work the same hours in the two periods, as in the first-best, just a little less every period, because taxes need to be positive in order to finance expenditures.

Notice a key implication of this tax-smoothing result: when the government faces unexpected temporary expenditure shocks (like unexpected wars or bail-outs), it should use \(debt\) to finance them in the short run—rather than inducing a spike in tax rates—and should increase taxes smoothly over time to avoid large distortions.

### 2 Optimal Taxation of Capital

We now extend the model to allow the government to levy also a tax on capital income. Reconsider the household problem:

\[
\max_{\{c_1,c_2,h_1,h_2\}} \quad c_1 - \frac{1}{2} h_1^2 + \beta \left( c_2 - \frac{1}{2} h_2^2 \right) \quad \text{(HPK)}
\]

\[
s.t.
\]

\[
c_1 + k = (1 - \tau_1) h_1
\]

\[
c_2 = (1 - \rho) (1 + r) k + (1 - \tau_2) h_2
\]

where \(k\) is saving and \(\rho\) is the capital income tax.

How will the government set \(\rho\) optimally? We can use an heuristic argument without a formal derivation. Suppose the government sets a value of \(\rho\) high enough that \(\beta (1 + r) (1 - \rho) < 1\). Then, by linearity of preferences, all consumption is made at date \(t = 1\). The household borrows exactly the amount that will be able to repay next period with his labor income net of taxes at date \(t = 2\), and does all his consumption in the first period. Saving is zero and so is the tax revenue from capital income taxation. Suppose, instead, the government sets \(\rho\) low enough that \(\beta (1 + r) (1 - \rho) > 1\). Then, all the consumption is done at date \(t = 2\) and the household saves all his labor income in period 1.

Therefore, it is easy to see that the government can increase \(\rho\) without affecting tax
revenues until the relationship
\[
\beta (1 + r) (1 - \rho^*) = 1 \rightarrow \rho^* = 1 - \frac{1}{\beta (1 + r)}
\]
holds exactly.

The problem of optimal labor income taxation is exactly the same as before, with the difference that the amount to be financed by labor income tax is now less than \( g \): it is only \( g - \rho^* (1 + r) k \). The optimal choice for labor income taxes will still be to set \( \tau_1 \) and \( \tau_2 \) at the same level across the two periods.

3 Time Inconsistency of Government Policy

Let \( \rho^* (t + n) \) be the optimal policy (the capital income tax rate) announced by the government at date \( t \) for a future period \( t + n \). For example, at date \( t \) the government announces that at \( t + n \) it will reduce the capital income tax rate. Now, let’s move forward one period and consider \( \rho^* (t + n) \), i.e., the best policy for \( t + n \) of a government that, after its announcement at \( t \), reoptimizes at \( t + 1 \). Suppose that \( \rho^* (t + n) \neq \rho^* (t + n) \), for example at \( t + 1 \) the best policy for the government is to raise the tax rate instead of decreasing it. Then, a so called time-inconsistency problem arises. The government would like to renege its promise, deviate from its original plan and change its policy.

The example of the previous section represents indeed a time-inconsistent policy. At date \( t = 1 \) the government would announce that its capital income tax rate for \( t = 2 \) is \( \rho^* \). But at \( t = 2 \) the optimal policy is different: capital is installed already, and by taxing capital heavily the government would not induce any distortion. Taxing existing capital (not future capital) is like a lump-sum tax which much better than the alternative distortionary labor income tax \( \tau_2 \). The optimal policy at \( t = 2 \) is then to tax capital as high as possible (maybe even 100% which means expropriation) until \( g \) is entirely financed. Only if \( (1 + r) k < g \) the government would resort to labor income taxation.

However, the households at \( t = 1 \) would understand this issue. They would correctly predict that, if the government could change its policy at \( t = 2 \) would do it. Their optimal response at \( t = 1 \) is not to save and set \( k = 0 \). So, in equilibrium, \( k = 0 \) and no revenue is raised from capital taxes.
If the government could commit ex-ante, via a constitution or another device, not to change its policy later it should do it. In other words, commitment devices always lead to better economic outcomes. Indeed, if the government can induce some saving, total consumption would be higher because saving accumulate a return \((1 + r)\) and increase output.

To sum up: optimal policies often suffer from a time inconsistency problem when the government is unable to commit, since the temptation to deviate along the original plan is strong (note that this deviation is in the households’ interests at that point). In presence of commitment, the economy can achieve better outcomes.