1 The Permanent Income Hypothesis

1.1 A two-period model

Consider a two-period model where households choose consumption \((c_1, c_2)\) to solve

\[
\max_{\{c_1, c_2\}} \log c_1 + \beta \log c_2 \\
text{subject to} \\
\frac{c_2}{1 + r} = \left(1 + \frac{1}{1 + r}\right) y^P + y^T
\]

where \(\beta\) is the discount factor, \(r\) the interest rate. The term \(y^P\) is the permanent component of income, i.e., income that accrues to the individual in every period, whereas \(y^T\) is the transitory component, i.e., the component that is earned only in period \(t = 1\) but not in period \(t = 2\). Think of \(y^P\) as a base salary and \(y^T\) as commissions or as a lottery win. Clearly, \(y^T\) could be negative, e.g., a capital loss on a financial investment.

Let’s solve this problem.

\[
L(c_1, c_2, \lambda) = \log c_1 + \beta \log c_2 + \lambda \left[\left(1 + \frac{1}{1 + r}\right) y^P + y^T - c_1 - \frac{c_2}{1 + r}\right]
\]

\[
FOC(c_1) : \frac{\partial L}{\partial c_1} = 0 \rightarrow \frac{1}{c_1} = \lambda
\]

\[
FOC(c_2) : \frac{\partial L}{\partial c_2} = 0 \rightarrow \beta \frac{1}{c_2} = \frac{\lambda}{1 + r}
\]

which yields the Euler equation

\[
\frac{c_2}{c_1} = \beta (1 + r).
\]

And using the Euler equation into the lifetime budget constraint, we arrive at the optimal consumption \(c_1^*\) (and similarly we arrive at \(c_2^*\))

\[
c_1^* + \frac{c_1^* \beta (1 + r)}{1 + r} = \left(1 + \frac{1}{1 + r}\right) y^P + y^T
\]

\[
c_1^* (1 + \beta) = \left(1 + \frac{1}{1 + r}\right) y^P + y^T
\]

\[
c_1^* = \frac{1 + \frac{1}{1 + \beta}}{1 + \beta} y^P + \frac{1}{1 + \beta} y^T.
\]

Now, let’s make the simplifying assumption that \(\beta (1 + r) = 1\). From the Euler equation, this means that the offsetting effects of patience and interest rate on saving exactly cancel
out and the household optimally chooses to perfectly smooth consumption over the two periods and \( c_1 = c_2 = c \). Note that \( y \) could be very large, which means that the income profile could be very steep, and yet consumption is perfectly smoothed. The individual will borrow enough in the first period against future income to equalize consumption across dates \( t = 1 \) and \( t = 2 \).

Imposing the parametric restriction \( \beta (1 + r) = 1 \) or \( \beta = 1/(1 + r) \), we arrive at

\[
c^* = y^p + \frac{1 + r}{2 + r} y^T.
\]

This equation explains how consumption reacts differently to permanent and transitory changes in income. Suppose \( y^p \) and \( y^T \) both change by an amount \( \Delta Y \), then consumption will change, respectively, by

\[
\Delta c^* = \Delta Y
\]

if the change in income is permanent and

\[
\Delta c^* = \frac{1 + r}{2 + r} \Delta Y \approx \Delta Y
\]

if the change in income is transitory where the approximation sign holds for \( r \approx 0 \).

In other words, consumption responds more to permanent relative to transitory changes in income. This result is the heart of the so called “permanent income hypothesis”.

### 1.2 The infinite horizon case

The infinite horizon assumption is made in macroeconomics for two main reasons. First, it is a more convenient assumption than the one of a finite horizon (of length \( T \)), since the problem is easier to solve. Second, the finite horizon problem corresponds to an individual without any form of altruism for her offsprings, i.e. she puts zero weight on the utility after her death. Consider instead the case of an agent who lives for, say, two periods and discounts her own future utility at rate \( \beta \) and the utility of her children at rate \( \beta \delta \leq \beta \). The objective function of this agent is

\[
U = u(c_0) + \beta u(c_1) + \beta^2 \delta \left\{ u(c_2) + \beta u(c_3) + \beta^2 \delta \left( u(c_4) + \beta u(c_5) + \beta^2 \delta (u(c_6) + \beta u(c_7) + \ldots) \right) \right\} + \delta [\beta^2 u(c_2) + \beta^3 u(c_3)] + \delta^2 [\beta^4 u(c_4) + \beta^5 u(c_5)] + \delta^3 [\beta^6 u(c_6) + \beta^7 u(c_7)] + \ldots
\]
Now, suppose that $\delta = 1$, i.e., we are in the case of perfect altruism. Then, this individual’s preferences become:

$$u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + \beta^3 u(c_3) + \beta^4 u(c_4) + \beta^5 u(c_5) + \beta^6 u(c_6) + \beta^7 u(c_7) + \ldots = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

In other words, the infinite horizon corresponds to finite horizon agents who have perfect altruism towards their descendants.

Consider then an agent with an infinite planning horizon who faces an infinite sequence of earnings $\{y_t\}_{t=0}^{\infty}$ which is perfectly known already at time $t = 0$. And it is equal to

$$y_t = \begin{cases} 
y^P + y^T & \text{for } t = 0 
y^P & \text{for } t > 0
\end{cases}$$

This agent solves the following consumption/saving problem:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t \log (c_t) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t = y^T + \left(\frac{1+r}{r}\right) y^P$$

How did we obtain that budget constraint? Consider the two period model budget constraint

$$c_0 + \frac{c_1}{1+r} = y^T + \left(1 + \frac{1}{1+r}\right) y^P$$

and add one extra period

$$c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} = y^T + \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}\right] y^P$$

and so on. It is easy that the left hand side becomes $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t$. Consider the right hand side. The infinite geometric series

$$1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^t} + \ldots = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r}.$$ 

Now, the Euler equation is always

$$\frac{c_{t+1}}{c_t} = \beta (1+r).$$
Imposing $\beta (1 + r) = 1$ we arrive at the same result: consumption $c_t$ is constant and equal to $c^*$ in both periods. Therefore, from the budget constraint
\[
\frac{c^*}{r} = \frac{y^r + y^p}{r} \quad \text{and} \quad c^* = \frac{y^p}{1 + r} y^r.
\]
This solution shows the permanent income hypothesis at work in an even more extreme way. Permanent changes in income are reflected “one for one” into consumption, whereas transitory changes are reflected proportionally to the factor $r/ (1 + r)$ which for $r \simeq 0$ is also close to zero. Hence, in the infinite horizon model, consumption basically does not respond to transitory changes in income.

Note that in the two-period model changes in transitory income translated into consumption with the coefficient $1/2$ and in the infinite horizon model with the coefficient $1/\infty = 0$. So, it is easy to generalize this as a rule of thumb. In a $T$ period model, a change in income that takes place in only one of the $T$ periods affects consumption proportionately to $1/T$. If the change occurs for $N$ periods, the transmission coefficient to consumption is approximately $N/T$ and if $N = T$ (and hence the change in income is permanent), this transmission coefficient is one.

## 2 Saving motives

This is a good time to make a short remark about “saving motives”, i.e. reasons why households save:

1. The saving motive associated to $\beta R > 1$ in the deterministic model without uncertainty, which pushes the individual to postpone consumption because of high patience and/or high returns to savings is called *intertemporal motive*.

2. The saving motive pushing households to smooth consumption through income shocks is called *smoothing motive*.

3. The saving motive which pushes the individual accumulate assets as a precaution against future income uncertainty is called *precautionary motive*.

4. We add that in a life-cycle model where the individual faces a retirement period, during the working stage of the life-cycle the individual would have a *life-cycle*
motive for saving associated to the desire of smoothing consumption throughout her life, across the working-life where she earns income and retirement where she does not.

5. Bequest motive that induces altruistic households to leave part of their assets as bequest to their offsprings upon death.

**FIGURE HERE**