1 Social Security

In the U.S. economy, and in the majority of developed countries, there is a Pay-As-You-Go (PAYG) social security system in place: all young workers alive in a given period pay into a general fund administered by the Government (Trust Fund) through a payroll tax on earnings. The old retirees alive in the same period receive pensions from the same general fund. The PAYG system involves an intergenerational transfer (from young to old alive at the same time). This system is unfunded, in the sense that payments to retiree were not funded by savings of those same individuals when young.

A fully funded system is different: young workers are taxed, with the proceed going to an individual retirement account (IRA) administered by the Government and invested in some securities. When workers are old and retire, they draw down the accumulated stock of savings on their individual accounts.

Note that the PAYG is more exposed to adverse demographic trends. As the population get older, the size of the young cohorts is small compared to the size of old cohorts, so to pay a defined amount of pension benefits to the elderly, the payroll tax needs to be raised. Due to the retirement of the baby-boom between 2010-2020, in absence of reduction of benefits (unlikely for political reasons), the payroll tax is projected to raise from the current 12% to 25%.

We want to study the effects of the two systems on savings and consumption, and we’d like to try to understand under which circumstances one can be better than the other.

1.1 The Model

Consider a two period overlapping generations (OLG) model. Households live for two periods, so at every time there are two overlapping generations (young and old) in the economy. Preferences are defined over consumption for the young $c_t^y$ and for the old $c_{t+1}^o$. 


and given by
\[ U(c^y_t, c^\rho_{t+1}) = 2 \sqrt{c^y_t} + 2 \beta \sqrt{c^\rho_{t+1}}, \]
where \( \beta \) is the discount factor. For simplicity, set \( \beta = 1 \).

In period \( t \) there are \( N_t \) new young worker born, and each generation is larger than the previous one by a factor \( (1 + n) \). Hence, the population evolves as
\[ N_{t+1} = (1 + n)N_t, \]
where \( n \) is the population growth rate.

Households produce (and earn) income \( y \) in their youth, and they are retired when old. Productivity grows at rate \( g \), which means that
\[ y_{t+1} = (1 + g)y_t. \]
Households can save (borrow) privately an amount \( a \geq 0 \) (\( < 0 \)) in the asset market which pays a constant exogenous interest rate \( r > 0 \) on savings.

We now compare a fully-funded system with a PAYG system.

### 1.2 Fully-Funded Pension System

Under a funded system, the government levies a proportional tax rate \( \tau \) on young workers income. Since \( y \) is exogenous, this tax is the same as lump-sum tax on income. Young workers pay \( \tau y \) every period. The government, then, invests \( \tau y \) at the market rate \( r \), and returns \( (1 + r)\tau y \) to the workers when they are retired.

Given taxes, savings and income, we can express the budget constraints for the young and old worker of any given generation as
\[
\begin{align*}
    c^y + a &= (1 - \tau)y \\
    c^\rho &= (1 + r)(\tau y + a)
\end{align*}
\]
When the agent is young, she has disposable income \((1 - \tau)y\) to split between current consumption \(c^y\) and savings \(a\) (future consumption). When old, the worker has no
We now solve the household’s problem to characterize the savings-consumption decision. Substituting out the pair \((c^y, c^o)\) using equations (1) into the objective function, we arrive at the simplified unconstrained problem

\[
\max_a \left\{ 2\sqrt{(1-\tau)y - a} + 2\sqrt{(1+r)(\tau y + a)} \right\}.
\]

Taking the FOC with respect to savings \(a\), we have

\[
\frac{1}{\sqrt{(1-\tau)y - a}} = \frac{\sqrt{(1+r)}}{\sqrt{(\tau y + a)}},
\]

and simplifying we obtain

\[
\tau y + a = (1+r)[(1-\tau)y - a],
\]

\[
a = \frac{1+r}{2 + ry - \tau y}.
\]  \hspace{1cm} (2)

Note first that private savings \(a\) increase in the interest rate, as expected. Also, private savings \(a\) are reduced by the tax \(\tau\). The higher is the “mandated” savings imposed by the Government through the funded Social Security system, the lower are the private savings. Indeed, \(\tau\) and the mandated savings can be so large that \(a\) could be negative, i.e. the households borrows on the market.

What is the effect of \(\tau\) on consumption? Substituting the expression for \(a\) into the two budget constraints in (1), we obtain

\[
c^y = (1-\tau)y - \frac{1+r}{2 + ry}y + \tau y = \frac{1}{2 + ry}y, \hspace{1cm} (3)
\]

\[
c^o = (1+r)^2 \frac{1}{2 + ry}y = (1+r)^2 c^y.
\]

The main result we find is that a fully funded social security system has no effect on aggregate consumption: neither \(c^y\) nor \(c^o\) depend on the payroll tax \(\tau\). This is
not surprising: with a fully funded system, the Government is acting like a bank. It taxes earnings and invests them at the ongoing market rate \( r \), the same rate at which individuals would invest them, if they could fully dispose of their earnings. Through private savings, the household can “undo” any imposition of the Government, to restore its individually optimal level of consumption.

It is important to remark that this is true as long as there are no borrowing constraints, for example. With borrowing constraints, the private saving decision in (2) could be suboptimal if it involves borrowing \( \alpha < 0 \), but borrowing is prohibited, or limited.

1.3 Unfunded PAYG Pension system

The government taxes the income \( y \) of all young workers alive at \( t \) (a cohort of size \( N_t \)) at rate \( \tau \), raising an amount \( B_t \)

\[
B_t = (\tau y_t) \cdot N_t
\]

Then, it distributes this amount among the old workers alive in period \( t \) (a cohort of size \( N_{t-1} \)). Hence, each old worker receives

\[
\frac{B_t}{N_{t-1}} = (\tau y_t) \cdot \frac{N_t}{N_{t-1}} = \tau y_t (1 + n) = \tau y_{t-1} (1 + g) (1 + n)
\]

as a pension. The second equality shows that the PAYG system is, effectively like a fully funded system for the young at \( t - 1 \) which forces the households to save \( \tau y_{t-1} \) and compounds savings at a rate of return \( (1 + g)(1 + n) \) instead of \( (1 + r) \). So \( n + g \) is (approximately) the implicit interest rate on the social-security contributions in the PAYG system (as opposed to \( r \) in the fully funded system) This is the key observation of this derivation.

Let’s continue with the derivation. The budget constraints of young and old are

\[
\begin{align*}
  c^y + a &= (1 - \tau)y \\
  c^o &= (1 + r)a + (1 + n)(1 + g)\tau y.
\end{align*}
\]
We now solve the household’s problem to characterize the savings-consumption decision. Substituting out the pair \((c^y, c^\alpha)\) using equations (4) into the objective function, we arrive at the simplified unconstrained problem

\[
\max_a \{2\sqrt{(1 - \tau)y - a} + 2\sqrt{(1 + r)a + (1 + n)(1 + g)\tau y}\}
\]

Taking the FOC with respect to savings \(a\), we have

\[
\frac{1}{\sqrt{(1 - \tau)y - a}} = \frac{1 + r}{\sqrt{(1 + r)a + (1 + n)(1 + g)\tau y}}
\]

Simplifying the expression above, we get

\[
(1 + r)a + (1 + n)(1 + g)\tau y = (1 + r)^2[(1 - \tau)y - a]
\]

and collecting terms, we arrive at

\[
a(1 + r)(2 + r) = (1 + r)^2y - [(1 + n)(1 + g) + (1 + r)^2]\tau y \quad (5)
\]

\[
a = \frac{1 + r}{2 + r}y - \left[\frac{(1 + n)(1 + g)}{(1 + r)(2 + r)} + \frac{1 + r}{2 + r}\right]\tau y, \quad (6)
\]

\[
= \frac{1 + \gamma}{2 + r}\tau y \quad (7)
\]

where

\[
1 + \gamma = \frac{(1 + n)(1 + g)}{(1 + r)} + (1 + r).
\]

Substituting this last equation into the two budget constraints in (4) we obtain,

\[
c^y = y - \tau y - a = y - \tau y - \frac{1 + r}{2 + r}y + \frac{1 + \gamma}{2 + r}\tau y
\]

\[
= \frac{1}{2 + r}y + \left(\frac{1 + \gamma}{2 + r} - 1\right)\tau y \quad (8)
\]

And, after some tedious algebra, we also arrive at

\[
c^\alpha = (1 + r^2)c^y,
\]

as in the fully funded case.
1.4 Comparison between the Two Systems

Household welfare is determined by consumption. Comparing (8) and (3), we see that the PAYG delivers higher consumption to households in both periods, and hence it is a better system, as long as

\[
\frac{1 + \gamma}{2 + r} > 1
\]

\[
1 + \gamma > 2 + r
\]

\[
\frac{(1 + n)(1 + g)}{(1 + r)} + (1 + r) > 2 + r
\]

\[
\frac{(1 + n)(1 + g)}{(1 + r)} > 1
\]

which yields the key condition

\[
(1 + n)(1 + g) > 1 + r.
\]

The intuition for this result is that the existence of a PAYG system forces the workers to save at the implicit rate \(n + g\) (population plus productivity growth) which is higher than \(r\), the market interest rate at which they would invest their additional savings, if they were not taxed by the government.

Therefore, in an economy with fast population and productivity growth, with \(n + g > r\), the PAYG is beneficial and dominates a fully funded system (in the sense of generating higher consumption for the households) due to its higher implicit return. This is why the PAYG was introduced, after the Great Depression. At that time population growth was still quite high and income per capita in the economy was growing at a fast pace. In the U.S., though, for the past 30 years, \(n + g\) has been, on average, lower than \(r\), which suggests the need to move towards a fully-funded system.

Finally, notice that this simple setup has nothing to say about the fact that the interest rate \(r\) is much more risky than \(n + g\). The private IRA are more exposed to interest rate risk: there is uncertainty about the future path of interest rates and this
uncertainty will be reflected in how much retirees find in their account at the end of their working life. For example, if the mandatory savings in the IRA are invested in stocks and the stock market crashes again, like in 2008, there will be a lot of retirement wealth burned. This phenomenon would not happen with the PAYG.

The model has also nothing to say about the fact that, in reality, the PAYG system is progressive and redistributes from rich to poor, i.e., the benefits are more generous for the low income households.