Macroeconomic Theory and Analysis
Problem Set 6
Suggested Solutions

1 Problem 1: Ricardian Equivalence

1.1 Government Budget Constraint

The government budget constraints in the two periods are

\[ g_1 = b \]  
\[ g_2 + (1 + r)b = sr_2 \]  

Notice that government is forced to issue debt since there are no revenues from taxation in the first period. The intertemporal budget constraint is:

\[ g_1 + \frac{g_2}{1+r} = \frac{sr_2}{1+r} \]  

Equation 3 tells us that the present value of government expenditure must be equal to the present value of government revenues from taxation.

1.2 Household's problem

The household's problem is

\[ \max_{\{c_1, c_2, s}\} u(c_1, c_2) = \ln c_1 + \beta \ln c_2 \]

subject to:

\[ c_1 + s = y_1 \]  
\[ c_2 = (1 + (1 - \tau_2)r)s + y_2 \]  

Combining equations 5 and 6 we get the intertemporal budget constraint:

\[ c_1 + \frac{c_2}{(1 + (1 - \tau_2)r)} = y_1 + \frac{y_2}{(1 + (1 - \tau_2)r)} \]  

You can easily write down the lagrangean for this problem and get the following Euler equation:

\[ c_2 = \beta (1 + (1 - \tau_2)r)c_1 \]  

The Euler equation depends on taxation.
1.3 Ricardian Equivalence

The euler equation depends on taxes. That means that the timing of taxes do affect decisions, and not only does government expenditures. In this economy there is no room for fiscal policy in the sense that the government cannot choose between debt and taxation to finance its expenditure. This is due to the fact that at time 1 the government can only use debt. The Ricardian equivalence is an irrelevance result stating that the timing of taxes and debt does not matter for agents decisions, but only the timing of taxes does. The Ricardian Equivalence does not hold in this economy since the taxation is not lump-sum.

2 Problem 2: Ricardian Neutrality

2.1 Intertemporal budget constraints

The household’s intertemporal budget constraint is:

\[ c_1 + \frac{c_2}{1 + r} = w_1 (1 - \tau_1)(1 - l_1) + \frac{w_2(1 - \tau_2)(1 - l_2)}{1 + r} \] (8)

The government’s budget constraint is:

\[ g_1 + \frac{g_2}{1 + r} = w_1 \tau_1 (1 - l_1) + \frac{w_2 \tau_2 (1 - l_2)}{1 + r} \] (9)

We can combine the above equations to get:

\[
\begin{align*}
  c_1 + \frac{c_2}{1 + r} &= w_1 (1 - l_1) + \frac{w_2(1 - l_2)}{1 + r} - w_1 \tau_1 (1 - l_1) - \frac{w_2 \tau_2 (1 - l_2)}{1 + r} \\
  &= w_1 (1 - l_1) + \frac{w_2(1 - l_2)}{1 + r} - g_1 - \frac{g_2}{1 + r}
\end{align*}
\]

2.2 Household’s problem

The household’s problem is:

\[
\max_{\{c_1,c_2,l_1,l_2\}} u(c_1, c_2, l_1, l_2) = \ln c_1 + \phi \ln(l_1) + \beta \ln c_2 + \beta \phi \ln(l_2) \] (10)
subject to:

\[ c_1 + \frac{c_2}{1 + r} = w_1(1 - \tau_1)(1 - l_1) + \frac{w_2(1 - \tau_2)(1 - l_2)}{1 + r} \]  

(11)

The first order conditions are given by:

\[ c_1 : 1/c_1 = \lambda \]  

(12)

\[ c_2 : \beta/c_2 = \lambda/(1 + r) \]  

(13)

\[ l_1 : \phi/l_1 = \lambda w_1(1 - \tau_1) \]  

(14)

\[ l_2 : \beta\phi/l_2 = \frac{\lambda w_2(1 - \tau_2)}{1 + r} \]  

(15)

The Euler equation can be obtained by combining equations (13) and (14):

\[ c_2 = \beta(1 + r)c_1 \]  

(16)

So, the Euler equation does not depend on taxes.

The first order conditions for labor depend on taxes. Combining equations (15) and (16) we get:

\[ MRS_{l_2,l_1} = \frac{\beta l_1}{l_2} = \frac{w_2(1 - \tau_2)}{w_1(1 - \tau_1)(1 + r)} \]  

(17)

Moreover, by combining equations (13) and (15) we get the following:

\[ MRS_{l_1,c_1} = w_1(1 - \tau_1) \]  

(18)

### 2.3 Competitive Equilibrium

In order to define a CE we must first solve the problem of the firm. In this economy, for each time \( t \) \((t = 1, 2)\) the firm chooses \( h_t \) to maximise \( Ah_t - w_th_t \). In a CE the firm earns zero profits and the condition \( A = w_t \) holds for each \( t = 1, 2 \).

A CE is an allocation \( \{c_1, c_2, l_1, l_2, h_1, h_2, b, a\} \), a price system \( \{w_1, w_2, r\} \) and a government policy \( \{\tau_1, \tau_2\} \) such that, given the price system and the government expenditures \( \{g_1, g_2\} \):

- household’s problem is solved
- firm’s problem is solved
• government budget constraint is balanced
• bonds market clears \( a = b \)
• labor market clears \((1 - l_t) = h_t \) and \( w_t = A \) for each \( t = 1, 2 \)
• consumption goods market clears \( y_t = c_t + g_t \) for each \( t = 1, 2 \)

3 Ricardian Equivalence and Labor Taxes

(a) There are two periods, \( t = 1, 2 \). Preferences are given by,

\[
U(c_1, l_1, c_2, l_2) = \ln (c_1) + \phi \ln (1 - n_1) + \beta \ln (c_2) + \beta \phi \ln (1 - n_2).
\]

The households’ budget constraints in the two periods are,

\[
\begin{align*}
c_1 + a &= w_1n_1(1 - \tau_1), \\
c_2 - \frac{w_2n_2(1 - \tau_2)}{(1 + r)} &= a.
\end{align*}
\]

Combining both budget constraints, we obtain:

\[
\begin{align*}
c_1 + \frac{c_2}{(1 + r)} &= w_1n_1(1 - \tau_1) + \frac{w_2n_2(1 - \tau_2)}{(1 + r)} \\
&= w_1n_1 + \frac{w_2n_2}{(1 + r)} - \tau_1 w_1n_1 - \tau_2 \frac{w_2n_2}{(1 + r)}
\end{align*}
\]

The firm’s problem is,

\[
\max_{n_t} An_t - w_1n_1.
\]

The governments expenditures are \((g_1, g_2)\) financed with labor income taxes \((\tau_1, \tau_2)\) and debt \( b \) in the first period. The budget constraint’s are:

\[
\begin{align*}
g_1 &= w_1n_1\tau_1 + b, \\
g_2 + b(1 + r) &= w_2n_2\tau_2.
\end{align*}
\]

Combining both conditions one obtains:

\[
\begin{align*}
g_1 + \frac{g_2}{(1 + r)} &= w_1n_1\tau_1 + \frac{w_2n_2\tau_2}{(1 + r)}
\end{align*}
\]
b) The household’s Euler equation satisfies:

\[ \frac{c_2}{c_1} = \beta (1 + r) . \]

The intertemporal condition satisfies:

\[ \frac{(1 - n_1)}{c_1} = \frac{\phi}{(1 - \tau_1) w} \quad \text{and} \quad \frac{(1 - n_2)}{c_2} = \frac{\phi}{(1 - \tau_2) w} \]

(c) Definition: A competitive equilibrium is a set of allocations \((c_1, c_2, n_1, n_2, g_1, g_2)\) and price \((w)\) such that (a) it is feasible, (b) given the wage, and rate of return, firms maximize profits, households maximize utility and the government satisfies its budget constraint.

d) The Ricardian equivalence states that the timing of taxes and debt does not matter for allocations. Does Ricardian Neutrality holds in this economy? No. Clearly, government taxes do distort leisure consumption. Since the work decisions are altered, output is not Ricardian neutral.

\section*{4 Multiple Generations}

The economy lasts for two periods, \(t = 1, 2\). It is populated by two types of individuals, \(S\) and \(L\) who are both born in period \(t = 1\). \(L\) live for both periods whereas the \(S\) only the first period. A fraction \(\phi\) of types \(S\) and a fraction \((1 - \phi)\) of types \(L\). \(S\)'s preferences are \(u(c_1) = \ln c_1\) and income endowment \(y\). Type \(L\) has preferences \(u(c_1, c_2) = \ln c_1 + \ln c_2\) and income endowments of \(y\) in each period. The interest rate is fixed at \(r\).

The government implements a social security system as follows. In period 1, it taxes all (young) households at rate \(\tau\), stores the revenues in a bank and then, in period two, it pays back the capitalized tax revenues equal to \((1 + r) \tau y\) to all the old households alive in that period in a lump-sum fashion.

a) The government budget constraint in \(t=1\) is

\[ \tau y = b \]

and in \(t=2\) is

\[ T = (1 + r) b \]

so the intertemporal condition is:
\[ T = (1 + r) \tau y \]

where \( T \) are lump-sum transfers in period 2.

b) Assume that \( \phi = 0 \). The household's budget constraint is

\[
\begin{align*}
    c_1 + b &= y - \tau y \\
    c_2 &= y + b(1 + r) + T
\end{align*}
\]

The intertemporal constraint is:

\[
\begin{align*}
    c_1 + \frac{c_2}{1 + r} &= y - \tau y + \frac{y + T}{1 + r} \\
    &= y + \frac{y}{1 + r} - \tau y + \frac{T}{1 + r} \\
    &= y + \frac{y}{1 + r}
\end{align*}
\]

The Euler equation is given by:

\[
\frac{c_2}{c_1} = \beta (1 + r)
\]

Does Ricardian Equivalence hold in this case? Yes, notice that the allocation is a function of \( y, r \) but not a function of \( \tau \).

c) Let’s start with the government’s problem. The government’s problem is:

\[ T (1 - \phi) = (1 + r) \tau y \]

Because the term in the government devides tax revenues between the L only. In this case, the BC, of the L’s is given by:

\[
\begin{align*}
    c_1 + \frac{c_2}{1 + r} &= y + \frac{y}{1 + r} + \frac{\phi}{1 - \phi} \tau y
\end{align*}
\]

The problem of the S types is trivial. They consume \( c_1 = (1 - \tau) y \). The Ricardian equivalence here does matter. Although it does not affect resources, and in that sense is neutral, it has redistributive implications. By increasing taxes, the government is transferring resources from the short lived to the old lived.