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Author(s): Albert Ando and Franco Modigliani
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THE "LIFE CYCLE" HYPOTHESIS OF SAVING: AGGREGATE IMPLICATIONS AND TESTS

By Albert Ando and Franco Modigliani*

The recent literature on the theory of the consumption function abounds with discussions of the permanent income hypothesis of Friedman and other related theories and attempts at their empirical verification. Friedman's formulation of the hypothesis is fairly well suited for testing against cross-section data, though numerous difficulties are associated with this task, and there is now a rapidly growing body of literature on this subject [5] [8] [11] [12] [14] [16] [17] [21] [25]. Friedman's model, on the other hand, does not generate the type of hypotheses that can be easily tested against time series data.

More or less contemporaneously with Friedman's work on the permanent income hypothesis, Modigliani and Brumberg developed a theory of consumer expenditure based on considerations relating to the life cycle of income and of consumption "needs" of households [34] [35]. Several tests of the Modigliani-Brumberg theory using cross-section data have been reported in the past including a comparative analysis of the cross-section implications of this hypothesis as against the Friedman model [8] [12] [32] [33].

Modigliani and Brumberg have also attempted to derive time series implications of their hypothesis in an as yet unpublished paper [34], and their theory appears to generate a more promising aggregative consumption function than does Friedman's. However, at the time of their writing the unavailability of data on net worth of consumers made empirical verification exceedingly difficult and indirect [6] [7] [20]. Since then, this difficulty has been partially eliminated as a result of the work of Goldsmith [18] [19].

In Part I of this paper, we give a brief summary of the major aggregative implications of the Modigliani-Brumberg life cycle hypothesis of saving. In Part II, we present the results of a number of empirical tests

* The authors are, respectively, assistant professor of economics and professor of industrial management, Massachusetts Institute of Technology. They are indebted to a number of their colleagues and students for comments and suggestions, particularly to Robert Ferber of the University of Illinois, Fred Westfield of Northwestern University, Franklin M. Fisher, Ralph Beals, and Stephen Goldfeld of the Massachusetts Institute of Technology, who read an earlier draft of this paper and suggested a number of improvements. The final phase of the research for this paper was partially supported by a grant from the National Science Foundation.

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for the United States which appear to support the hypothesis.[1] The reader who is not interested in the derivation and statistical testing of the aggregate Modigliani-Brumberg consumption function may proceed directly to Part III, where we develop some features of the model which, in our view, make it particularly suitable for the analysis of economic growth and fluctuations, as indicated in our past and forthcoming contributions [1] [2] [3] [4] [29] [31].

I. *Theory*

A. *Derivation of the Aggregate Consumption Function*[2]

The Modigliani and Brumberg model starts from the utility function of the individual consumer: his utility is assumed to be a function of his own aggregate consumption in current and future periods. The individual is then assumed to maximize his utility subject to the resources available to him, his resources being the sum of current and discounted future earnings over his lifetime and his current net worth. As a result of this maximization the current consumption of the individual can be expressed as a function of his resources and the rate of return on capital with parameters depending on age. The individual consumption functions thus obtained are then aggregated to arrive at the aggregate consumption function for the community.

From the above brief description, it is quite apparent that the most crucial assumptions in deriving the aggregate consumption function must be those relating to the characteristics of the individual's utility function, and the age structure of the population. The basic assumptions underlying the shape of the utility function are:

Assumption I: The utility function is homogeneous with respect to consumption at different points in time; or, equivalently, if the individual receives an additional dollar's worth of resources, he will allocate it to consumption at different times in the same proportion in which he had allocated his total resources prior to the addition.[3]

Assumption II: The individual neither expects to receive nor desires to leave any inheritance. (This assumption can be relaxed in either of

[1] Since this paper was submitted and accepted for publication, a model that bears some similarity to the one proposed here has been presented ➡ Alan Spiro in "Wealth and Consumption Function," *Jour. Pol. Econ.*, Aug. 1962, *70*, 339–54.

[2] The theory summarized here is essentially the same as that developed by Modigliani and Brumberg in [7] and [34]. Because of the untimely death of Richard Brumberg in August 1954, the original paper by Modigliani and Brumberg [34] has never been published, and it is not likely to be published in the near future. Because of this, we present here a summary. However, the aggregation procedure developed here is different from that followed by Modigliani and Brumberg. This is because they are largely concerned with numerical prediction of parameters while we are interested only in exhibiting the conditions for existence of a particular form of the aggregate consumption function.

[3] This equivalence holds on the assumption that consumers deal in perfect markets.

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two ways. First, we may assume that the utility over life depends on planned bequests but assume that it is a homogeneous function of this variable as well as of planned consumption. Alternatively, we may assume that the resources an individual earmarks for bequests are an increasing function of the individual's resources relative to the average level of resources of his age group, and that the relative size distribution of resources within each age group is stable over time. It can be shown that either of these generalized assumptions implies an aggregate consumption function similar in all essential characteristics to the one obtained from the stricter assumption stated here.)

These two assumptions can be shown to imply (cf. [35, pp. 390 ff.]) that, in any given year t, total consumption of a person of age T (or, more generally, of a household headed by such a person) will be proportional to the present value of total resources accruing to him over the rest of his life, or:

$$c_t^T = \Omega_t^T v_t^T . \tag{1.1}$$

In this equation[4] Ω_t^T is a proportionality factor which will depend on the specific form of the utility function, the rate of return on assets, and the present age of the person, but not on total resources, v_t^T. The symbol c_t^T stands for total consumption (rather than for consumer's expenditure) in the year t. It consists of current outlays for nondurable goods and services (net of changes if any in the stock of nondurables) plus the rental value of the stock of service-yielding consumer durable goods. This rental value in turn can be equated with the loss in value of the stock in the course of the period plus the lost return on the capital tied up. Finally the present value of resources at age T, v_t^T, can be expressed as the sum of net worth carried over from the previous period, a_{t-1}^T, and the present value of nonproperty income the person expects to earn over the remainder of his earning life; i.e.,

$$v_t^T = a_{t-1}^T + y_t^T + \sum_{\tau=T+1}^{N} \frac{y_t^{eT\tau}}{(1 + r_t)^{\tau-T}} \tag{1.2}$$

where y_t^T denotes current nonproperty income; $y_t^{eT\tau}$ is the nonproperty income an individual of age T expects to earn in the τth year of his life; N stands for the earning span and r_t for the rate of return on assets.[5]

In order to proceed further, it is convenient to introduce the notion of "average annual expected income," y_t^{eT}, defined as follows:

[4] For the sake of simplicity, we shall not display the stochastic component of these relations explicitly in this section.

[5] To be precise $y_t^{eT\tau}$ is the income the person expects to earn at age τ, measured in prices prevailing in the year t, and r is the "real" rate of return on assets. In (1.2) the expected real rate is assumed to remain constant over time, but the formula can be generalized to allow for changing rate expectations.

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$$(1.3) \qquad y_t^{eT} = \frac{1}{N-T} \sum_{\tau=T+1}^{N} \frac{y_t^{eT_\tau}}{(1+r_t)^{\tau-T}}.$$

Making use of this definition and of (1.2) we can rewrite equation (1.1) as:

$$(1.4) \qquad c_t^T = \Omega_t^T y_t^T + \Omega_t^T (N-T) y_t^{eT} + \Omega_t^T a_{t-1}^T.$$

To obtain an expression for aggregate consumption we proceed to aggregate equation (1.4) in two steps, first within each age group and then over the age groups.

If the value of Ω_t^T is identical for all individuals in a given age group T, then it is a simple matter to aggregate equation (1.4) over an age group, obtaining:

$$(1.5) \qquad C_t^T = \Omega_t^T Y_t^T + (N-T)\Omega_t^T Y_t^{eT} + \Omega_t^T A_{t-1}^T$$

where C_t^T, Y_t^T, Y^{eT}, and A_{t-1}^T are corresponding aggregates for the age group T of c_t^T, y_t^T, y_t^{eT}, and a_{t-1}^T. If Ω_t^T is not identical for all individuals in the age group, however, the meaning of the coefficients in equation (1.5) must be reinterpreted. It has been shown by Theil [41] that under a certain set of conditions the coefficients of (1.5) can be considered as weighted averages of the corresponding coefficients of (1.4).[6]

Next, taking equation (1.5) as a true representation of the relationship between consumption and total resources for various age groups, we wish to aggregate them over all age groups to get the consumption function for the whole community. Consider the equation:

$$(1.6) \qquad C_t = \alpha_1' Y_t + \alpha_2' Y_t^e + \alpha_3' A_{t-1}$$

where C_t, Y_t, Y_t^e and A_{t-1} are obtained by summing respectively C_t^T, Y_t^T, Y_t^{eT} and A_{t-1}^T over all age groups T, and represent therefore aggregate consumption, current nonproperty income, "expected annual nonproperty income," and net worth.

The theorems given by Theil again specify the conditions under which the coefficients in equation (1.6) are weighted averages of the corresponding coefficients of equation (1.5). In this case, it is likely that the conditions specified by Theil are not satisfied, because both net worth and its coefficient in equation (1.5) are positively correlated with age up to the time of retirement. However, a much weaker set of conditions can be specified which are sufficient to insure stability over time

[6] See Theil [41, pp. 10–26]. More precisely, the least squares estimates of the parameters of equation (1.5) will be weighted averages of the least squares estimates of the corresponding parameters of equations (1.4) only if the set of conditions specified by Theil in the reference cited above is satisfied. Roughly speaking, these conditions require that there be no systematic relations between parameters and variables of equation (1.4) over individuals.

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of parameters in equation (1.6). In particular one such set of conditions is the constancy in time of (i) the parameters of equation (1.5) for every age group, (ii) the age structure of population, and (iii) the relative distribution of income, of expected income, and of net worth over the age groups.

B. *A Priori Estimates of the Coefficients of the Aggregate Consumption Function*

Modigliani and Brumberg [34], in order to obtain a priori estimates of the order of the magnitude of the coefficients of equation (1.6) implied by their model, introduced a number of rather drastic simplifying assumptions about the form of the utility function and life pattern of earnings, to wit:

Assumption III: The consumer at any age plans to consume his total resources evenly over the remainder of his life span.

Assumption IV: (a) Every age group within the earning span has the same average income in any given year t. (b) In a given year t, the average income expected by any age group T for any later period τ, within their earning span, is the same. (c) Every household has the same (expected and actual) total life and earning spans, assumed to be 50 and 40 respectively for the purpose of numerical computation.

Assumption V: The rate of return on assets is constant and is expected to remain constant.

Under these assumptions, if aggregate real income follows an exponential growth trend—whether due to population or to productivity growth—the sufficient conditions for the constancy in time of the parameters of (1.6) are satisfied. The value of these parameters depends then only on the rate of return on assets and on the over-all rate of growth of income, which in turn is the sum of population growth and the rate of increase of productivity.[7]

Table 1 gives some examples of the numerical value of the coefficients under the assumptions described above.

It should be emphasized that assumptions III to V have been introduced only for the sake of numerical estimation of the coefficients and are by no means necessary to insure the approximate constancy in time of the parameters in (1.6). A change in the assumptions would lead to somewhat different values of the parameters. But both a priori considerations and rough numerical calculations suggest that these values would not be drastically affected, and that it is generally possible to

[7] Strictly speaking the values of the parameters would vary somewhat depending on whether the growth of income results from population or from productivity growth. However, for rates of growth within the relevant range, say 0 to 4 per cent per year, the variation turns out so small that it can be ignored for present purposes.

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TABLE 1—COEFFICIENTS OF THE CONSUMPTION FUNCTION (1.6) UNDER STATED ASSUMPTIONS[a]

Yield on assets (per cent)	0	0	0	3	5	5	5
Annual rate of growth of aggregate income (per cent)	0	3	4	0	0	3	4
$\alpha_1+\alpha_2$	.61	.64	—	.69	.73	—	—
α_3	.08	.07	.07	.11	.13	.12	.12

[a] Missing values have not been computed because of the complexity of calculation.

infer the direction in which these values would move when a specific assumption is changed. The recognition of the estate motive would tend to yield lower values for both coefficients, especially that of assets.[8]

On the whole, then, the values shown in Table 1 should be regarded as a rough guide to the order of magnitude of the coefficients consistent with the basic model; i.e., radically different values would cast serious doubts on the adequacy of the life cycle hypothesis.

C. *The Measurement of Expected Income*

The last point that must be clarified before we proceed to the discussion of the empirical tests is the measurement of expected nonproperty income, Y^e, which, at least at present, is not directly observable. A "naive" hypothesis is to assume that expected nonproperty income is the same as actual current income, except for a possible scale factor. Thus, we have:

$$Y_t^e = \beta' Y_t; \qquad \beta' \simeq 1.$$

Substituting the above expression into (1.6), we obtain the aggregate consumption function

$$\begin{aligned} C_t &= (\alpha_1' + \beta'\alpha_2')Y_t + \alpha_3' A_{t-1} = \alpha_1 Y_t + \alpha_3 A_{t-1} \\ \alpha_1 &= \alpha_1' + \beta'\alpha_2' \simeq \alpha_1' + \alpha_2'. \end{aligned}$$

We designate this formulation as hypothesis I.

A similar but somewhat more sophisticated formulation is to assume that expected income is an exponentially weighted average of past income, weights adding up to one, or slightly more than one in order to reflect the expected growth [15] [16]. But it is quite difficult to deter-

[8] On the other hand, if we assume (i) that the preferred pattern of allocation of consumption and the pattern of income over life are the type suggested by the available cross-section data, (ii) that income expectation is consistent with the prevailing pattern of income, again suggested by the cross-section data over age groups, then the resulting coefficients of income and assets in equation (1.6) would be somewhat higher than those reported in Table 1.

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mine the weights from the data we have at our disposal, and Friedman, who favors this formulation, has acknowledged its shortcomings [15].

The third possible formulation is a slight modification of the first. Under our definitions, Y, and expected income, Y^e, are nonproperty or labor income, excluding, for instance, profits. We may hypothesize that for those currently employed, average expected income, y_t^e, is current income adjusted for a possible scale factor, i.e.,

$$y_t^e = \beta_1 \frac{Y_t}{E_t} \tag{1.7}$$

where E_t is the number of persons engaged in production. We should expect β_1 to be quite close to unity.

For those individuals who are currently unemployed, we hypothesize that expected income is proportional to the average current income of those who are employed. The proportionality constant in this case represents three factors. First, as before, there may be some influence from expected growth. Second, and probably most important, the incidence of unemployment is likely to be smaller for higher-paid occupations than for lower-paid, less-skilled workers; hence, the average earnings the unemployed can look forward to, if reemployed, are likely to be lower than the average earnings of those currently employed. Third, it seems reasonable to suppose that some of the currently unemployed persons would expect their current unemployment status to continue for some time and, possibly, to recur. We shall therefore assume:

$$y_t^{eu} = \beta_2 \frac{Y_t}{E_t} \tag{1.8}$$

where y_t^{eu} is the average expected income of unemployed persons; and, for the reasons given above, we expect the constant β_2 to be substantially smaller than β_1. The aggregate expected income is then given by:

$$\begin{aligned} Y_t^e &= E_t y_t^e + (L_t - E_t) y_t^{eu} = E_t \beta_1 \frac{Y_t}{E_t} + (L_t - E_t)\beta_2 \frac{Y_t}{E_t} \\ &= (\beta_1 - \beta_2) Y_t + \beta_2 \frac{L_t}{E_t} Y_t \end{aligned} \tag{1.9}$$

where L_t denotes the total labor force.[9]

Substituting (1.9) into (1.6), we obtain the following variant of hypothesis I,

$$C_t = \alpha_1 Y_t + \alpha_2 \frac{L_t}{E_t} Y_t + \alpha_3 A_{t-1} \tag{1.10}$$

[9] See Ando [1]. Mincer in [28] relied on a similar device, except that he used population in place of labor force.

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where

$$\alpha_1 = \alpha_1' + \alpha_2'(\beta_1 - \beta_2)$$
$$\alpha_2 = \alpha_2'\beta_2; \qquad \alpha_3 = \alpha_3'.$$

We designate the formulation embodied in equation (1.10) above as hypothesis II.

Since β_1 is thought to be close to unity, we have

$$\alpha_1 + \alpha_2 = \alpha_1' + \beta_1\alpha_2' \simeq \alpha_1' + \alpha_2'. \tag{1.11}$$

The individual values of the observable coefficients α_1 and α_2 are, however, dependent on the nonobservable value of β_2, about which there is little we can say a priori.

II. *Empirical Verification and Estimation*

In this section we report results of a number of tests of our model for the United States.[10] Unless otherwise stated, the period of observation is 1929 through 1959 excluding the Second World War years 1941–46.[11] Consumption, C, labor income net of taxes, Y, and net worth, A, are all measured in billions of current dollars as called for by our hypothesis.[12]

In recent years, economists have become increasingly aware of the many sources of bias, inconsistency, and inefficiency that beset prevailing estimation procedures, e.g., the existence of simultaneous relations, errors of observations in the "independent" variables, spurious correlation, multicollinearity, and heteroscedascity.[13] As a result, the simple-minded and straightforward least-squares approach is being replaced by a host of alternative procedures. Unfortunately most of these alternative procedures are designed to cope with one specific source of difficulty, and they often do so at the cost of increasing the difficulties arising from other sources. Under these conditions, we feel that the best course is to utilize a variety of procedures, exploiting our knowledge of the structure of the model and the nature of data to devise methods whose biases are likely to go in opposite directions. By following such a procedure, we can at least have some confidence that the

[10] The data and the procedure by which they have been obtained will be found in Ando, Brown, Kareken, and Solow [3, Data App.]. The derivation of labor income after taxes is particularly troublesome and is based in part on methods suggested by [13] and [38].

[11] A few experiments were made using data including the Second World War years, and equation (1.6) appears to explain consumption behavior during these years better than any other consumption function to our knowledge. However, the fit is still not very good, and, at any rate, we do not feel that these years are relevant because of their obviously special characteristics.

[12] In this section, the time subscript will be omitted whenever there is no danger of confusion.

[13] See, for instance, Theil [40].

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estimates obtained by different methods will bracket the value of the unknown parameters being estimated.

The main alternative procedures used and the estimates obtained are summarized in Table 2. Row (1) shows the results of a straightforward least-squares fit of hypothesis I.[14] The coefficients of both independent variables are highly significant and R^2 extremely high. But in other respects, the results are not altogether satisfactory. The coefficient of Y, which is an estimate of $\alpha_1+\alpha_2$, is somewhat higher and that of A appreciably lower than our model would lead us to expect. Furthermore, the Durbin-Watson statistic [10] falls considerably short of 2, suggesting the presence of pronounced serial correlation in the residuals.

As can be seen from row (2), the results do not change appreciably if we replace hypothesis I with II by introducing an additional variable Y^{L}_{E}. Although the coefficient of Y^{L}_{E} has the right sign it does not appear to contribute significantly to the explanation of C. Meanwhile, it reduces still further the estimate of the coefficient of net worth, and increases the estimate of $\alpha_1+\alpha_2$ which, it will be recalled, is approximately given by the sum of the coefficients of Y and Y^{L}_{E}. Also, the serial correlation of the residuals does not change at all. As will soon become apparent, much of the difficulty with hypothesis II can be traced back to multicollinearity, which makes it rather hard to obtain reliable estimates of the individual coefficients.

Note also that in both (1) and (2) the constant term is very significantly different from zero by customary standards, a result which would seem inconsistent with the hypothesis tested. In our view, however, this result is not as serious as might appear at first glance. The constant term is numerically rather small, amounting to only about 5 per cent of the mean value of the dependent variable. Furthermore, we know that the least-squares estimate of the constant term is upward-biased in the present instance because of the simultaneous-equations bias as well as because of errors of measurement in the independent variables.[15] While the size of these biases cannot be directly estimated, we suspect it to be appreciable. Accordingly, on the basis of presently available evidence, we see no compelling reason to reject the hypothesis that consumption is in fact roughly homogeneous in income and assets. Under these circumstances, a more reliable estimate of the coefficients of these variables might be obtained by suppressing the constant term in accordance with the specification of our model.

The constrained estimation results in the equations reported in rows (3) and (5) of Table 2. A comparison of row (1) and row (3) shows that this procedure leads to estimates which are more nearly of the order

[14] In this section, we shall refer to equations by the rows in Table 2.

[15] See footnote 17.

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TABLE 2

ESTIMATES OF THE COEFFICIENTS OF THE CONSUMPTION FUNCTION

Rows	Hypothesis Tested	Mode of Regression[a]	Coefficients and Their Standard Errors of Estimates[b]						$\alpha_1+\alpha_3$	Standard Error of Dependent Variable	Standard Error of Estimate	R^{2c}	Durbin-Watson Statistic
			Constant	Y (α_1)	XY (α_{1x})	$Y(L/E)$ (α_2)	A (α_3)	XA (α_{3x})					
(1)	I	A	8.1 (1.0)	.75 (.05)	—	—	.042 (.009)	—	.75	88.289	2.233	.998	1.26
(2)	II	A	7.3	.65 (.11)	—	.13 (.13)	.037 (.011)	—	.78	88.289	2.188	.998	1.26
(3)	I	A	—	.56 (.09)	—	—	.081 (.015)	—	.56	88.289	4.414	.997	.33
(4)	I	A	—	.87 (.08)	—	—	.046 (.012)	−.018 (.003)	.87	88.289	2.826	.998	1.13
(5)	II	A	—	.59 (.16)	—	.19 (.12)	.036 (.017)	—	.78	88.289	3.443	.998	.74
(6)	I	B	—	.52 (.11)	—	—	.072 (.018)	—	.52	8.292	2.208	.929	1.85
(7)	I	B	—	.60 (.16)	−.11 (.13)	—	.074 (.018)	—	.60[d]	8.292	2.177	.931	1.92
(8)	I	B	—	.51 (.12)	—	—	.089 (.031)	−.017 (.024)	.51	8.292	2.184	.930	1.91
(9)	II	B	—	.44 (.12)	—	.24 (.15)	.049 (.022)	—	.68	8.292	2.090	.936	1.74
(10)	II	B	—	.39 (.13)	—	.31 (.16)	.076 (.029)	−.033 (.024)	.70	8.292	1.994	.942	1.91
(11)	I	C	—	.44 (.05)	—	—	.105 (.008)	—	.44	.092	.030	.899	.34
(12)	I	C	—	.69 (.07)	−.07 (.02)	—	.071 (.009)	—	.69[d]	.092	.021	.948	.93
(13)	I	C	—	.69 (.06)	—	—	.071 (.009)	−.012 (.003)	.69	.092	.020	.953	.95
(14)	II	C	—	.62 (.09)	−.06 (.02)	.10 (.08)	.065 (.011)	—	.71[d]	.092	.021	.948	.93
(15)	II	C	—	.63 (.09)	—	.09 (.08)	.066 (.010)	−.011 (.003)	.71	.092	.020	.953	1.03

[a] A: Regressions in which variables are used in the original form.
B: Regressions in which variables are used in the first-difference form.
C: Regressions in which variables are used in the form of ratios to labor income.

[b] Figures in parentheses underneath the estimates are estimated standard errors of respective estimates. Number of observations =25. Where no estimate is shown, the variable is excluded from the equation.

[c] The ratio of the variance of the residual to the variance of the dependent variable.

[d] Figures given do not include the coefficient of XY. In other words, the estimate of the coefficient pertains to the period 1929–40.

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of magnitude suggested by our model. Unfortunately the serial correlation is now so high that the reliability of the estimate is open to serious question. From row (5) it also appears that the addition of the variable Y_E^L is again not very helpful. Though its contribution is still hardly significant, it again lowers the coefficient of A, and the serial correlation remains high.

A common procedure in time-series analysis when serial correlation of errors is high is to work with first differences. In the present instance this procedure also serves to reduce drastically the degree of multicollinearity and provides a more meaningful test for the adequacy of the hypothesis as a causal explanation of consumption. The results, reported in rows (6) and (9) and in Figure 1, appear quite favorable to the hypothesis. The multiple correlation remains quite high and the coefficient of net worth is highly significant. Also the Durbin-Watson statistic improves considerably and there is no longer any reason to suspect that the reliability of the estimate is seriously affected by serial correlation of residuals. Furthermore, a comparison of row (6) with row (3) reveals a relatively minor change in the estimates of the coefficients of both Y and A, tending to increase our confidence in these estimates. Also from row (9), we see that the introduction of the variable Y_E^L produces now a much less drastic change in the estimates of the coefficients: the estimate of α_3 in particular is reduced only from .07 to .05, while that of $\alpha_1+\alpha_2$ is raised from .52 to .68. On the other hand, a comparison of row (5) and row (9) reveals that the relatively small change in the estimate of $\alpha_1+\alpha_2$ from .78. to .68 is accompanied by a marked shift in the relative size of the coefficients of Y and Y_E^L, the first falling and the second rising appreciably.

An interpretation and explanation of these various results is readily found. When we deal with actual values the movements of all the variables are dominated by their common trend. On the other hand, when dealing with first differences we are primarily focusing on short-run or cyclical variations. Now with respect to such variations, consumption is much more stable than income, even labor income. This stability is accounted for in our model by the fact that both net worth and expected labor income are more stable than current income in the short run, even though all the variables move together in the longer run. These considerations help to explain why in row (9) the coefficient of current income is appreciably lower than in row (5), while the coefficients of the two remaining, cyclically more stable, variables are correspondingly increased. Equation (9) suggests that consumption may be less sensitive to purely cyclical and temporary swings in current labor income than the estimates reported in row (5) indicate. At the same time the fact that both Y_E^L and A perform a similar function in stabilizing consump-

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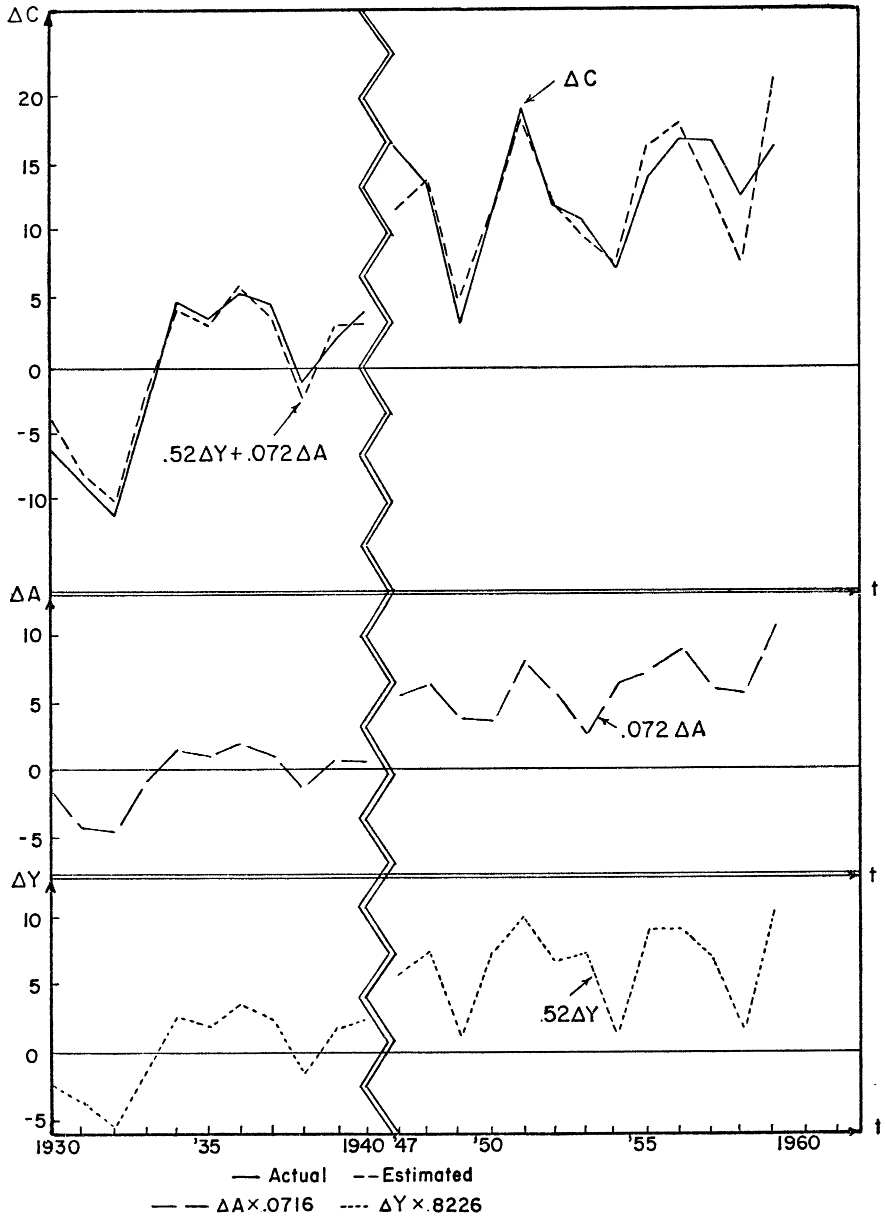

FIGURE 1. TIME PROFILE OF (6) AND ITS COMPONENTS

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tion with respect to short-run variations in Y helps to explain why the addition of the latter variable generally tends to reduce not only the coefficient of Y but also that of A, even though its own contribution is statistically not very significant. It would thus seem that from the available data we cannot obtain a very reliable estimate of the role of each variable separately.[16]

All the estimates reported so far are based on the least-squares method applied to a single equation. As is well known this method leads to estimates which are biased, even in the limit, when one or more of the "independent" variables are related to the dependent variable by other simultaneous relations. In the present instance the variable A can be taken as predetermined, but the same is not true of labor income which is related to consumption via total income. That is, the true error component of the consumption function cannot be assumed to be uncorrelated with Y, and hence the least-squares estimates of its parameters are not consistent. Specifically, it can be shown that, asymptotically, the estimator of the coefficient of Y is upward-biased and that of the coefficient of A downward-biased.[17]

The only really adequate way of resolving this difficulty would be to construct a complete model of the U. S. economy and then apply an appropriate simultaneous-equations estimation procedure. This approach would lead at least to consistent estimates, except to the extent that the model was incomplete or misspecified and the exogenous variables were subject to errors of measurement. Furthermore, the effi-

[16] The simple correlation between ΔA and $\Delta(Y_E^L)$ is .93, higher than that between ΔY and $\Delta(Y_E^L)$, .89.

[17] Let us denote the *true* error term in equation (1.6) by ϵ. Then, under the assumption that the correlation between Y and A and that between Y and ϵ are positive, while A and ϵ are uncorrelated with each other, it can be readily shown that

$$\text{plim}\ \hat{\alpha}_1 = \alpha_1 + \text{plim}\frac{\sum A^2 \sum Y\epsilon}{\sum A^2 \sum Y^2 - (\sum AY)^2} \geq \alpha_1$$

$$\text{plim}\ \hat{\alpha}_3 = \alpha_3 - \text{plim}\frac{\sum Y\epsilon \sum AY}{\sum A^2 \sum Y^2 - (\sum AY)^2} \leq \alpha_3$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_3$ are the least squares estimates of α_1 and α_3, respectively.

The above formulae are stated for the case in which the constant is suppressed, but for the case where the constant is not suppressed, it is only necessary to reinterpret the symbols as deviations from respective means. The asymptotic bias for the constant term can then be expressed in terms of the above limits and the means of the variables involved. Denoting the constant term and its least squares estimate by γ and $\hat{\gamma}$ respectively, it can be shown that

$$\text{plim}\ (\hat{\gamma} - \gamma) = \text{plim}\frac{\sum Y'\epsilon}{\sum A'^2 \sum Y'^2 - (\sum A'Y')^2}(\bar{A}\sum A'Y' - \bar{Y}\sum A'^2)$$

where the primed symbols denote the deviations from the mean. The estimate from our data of the probability limit of the expression inside the parenthesis is positive and fairly large, so that the asymptotic bias of the least squares estimate of the constant term is most likely to be positive.

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ciency of the estimates might be reduced, particularly if the theory and data relating to other sectors of the economy were less reliable than those relating to the consumption sector. Whatever the merits of this approach, however, we regard the specification of a complete model beyond the scope of this paper.

A compromise followed by some authors is to introduce an accounting identity relating consumption, saving, and income, note that saving is equal to investment, assume that investment is autonomous, and estimate the parameters of the consumption function from the regression of consumption on saving [39] [43]. Now in our view this procedure is likely to lead to bias in the estimates of the parameters which is more serious than that resulting from the conventional regression on income. The arguments supporting this conclusion are developed formally in Appendix, section B and can be summarized here as follows:

1. When the "independent" variable is subject to errors of measurement, the resulting estimate of the regression coefficient is biased toward zero, the more so the greater the variance of the error of measurement of the independent variable relative to its true variance. Now since personal saving is in the order of one-tenth of disposable income, it is reasonable to suppose that the variance of true (as distinguished from measured) personal saving is a good deal smaller than the variance of disposable income. At the same time, personal saving, as actually measured in the national income accounts, represents the difference between largely independent estimates of disposable income and of personal consumption. Hence the error of measurement of personal saving is likely to be even larger than that of disposable income. We can therefore be rather confident that the bias toward zero due to errors of measurement will be a good deal more serious when consumption is regressed on personal saving than when it is regressed on disposable income. Furthermore, given the estimating procedure, the error of measurement of consumption is likely to be negatively correlated with the error of measurement of saving, and this negative correlation will produce a further downward bias in the estimate of the true regression coefficient.

2. Personal saving is *not* identically equal to investment either conceptually or in terms of actual measurement, and investment is *not* exogenous, or independent of consumption, even in the short run, especially when account is taken of investment in inventories. At least over a very short period of time, such as a quarter or less, it is quite likely that random variations in consumption behavior will be accompanied by random variations in personal saving in the opposite direction, and, to the extent that this is true, an estimate of the propensity to consume based on the regression of consumption on saving will be seriously downward-biased, even in the absence of errors of measurement.

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For the problem at hand, there exists an alternative and relatively simple way of securing consistent estimates of the parameters in hypothesis I. Relying on the specification that the constant term is zero and that the true error is uncorrelated with A, this procedure consists in (i) regressing income on assets, and obtaining computed values of Y from this regression, say Y_c; (ii) then regressing C on Y_c and on A. The coefficients of Y_c and A so obtained can be shown to be consistent estimators of the coefficients of Y and A respectively.[18]

Unfortunately the application of this procedure to our problem yields rather meaningless results (the point estimate of the coefficient of Y turns out to be negative!). This outcome is not entirely surprising in view of the high degree of multicollinearity present and the fact that this procedure is probably not a very efficient one. Since multicollinearity is reduced by first-differencing data, we have applied this procedure to the first differences, though the assumptions which assure the consistency of the estimates are no longer strictly tenable in this case.[19] The resulting estimates of α_1 and α_3 turned out to be .67 and .05.

Another possible way of coping with the problem of bias in the estimates resulting from a known cause but of unknown magnitude is to construct an alternative estimation procedure in which the same cause may be expected to produce a bias in the opposite direction. If this can be done, then the unknown parameters may be bracketed by the estimates generated by the two alternative procedures, and if they are close together, then we may conclude with some confidence that the bias in either procedure is not too serious. For this purpose, suppose that

[18] Suppose that we estimate the parameters of the following equation by the method of least squares:

$$Y = b_y A + a_y + \eta_y \tag{a}$$

where η_y is the error term.

The substitution of the value of Y estimated by (a) into equation (1.6) yields

$$C = \alpha_1(b_y A + a_y) + \alpha_3 A + \epsilon = (\alpha_1 b_y + \alpha_3)A + \alpha_1 a_y + \epsilon. \tag{b}$$

Comparison of equation (b) with the regression of C on A

$$C = b_c A + a_c + \eta_c \tag{c}$$

results in

$$\alpha_1 b_y + \alpha_3 = b_c \tag{d}$$

$$\alpha_1 a_y = a_c \tag{e}$$

(d) and (e) can then be solved to give the estimated values of α_1 and α_3. These estimates are identical with those resulting from the two-stage least-squares procedure described in the text.

[19] Since $A_{t-1} = A_{t-2} + S_{t-1}$ and S_{t-1} depends on Y_{t-1} and C_{t-1}, it must be admitted that the possibility of a positive correlation between A_{t-1} and ϵ_{t-1}, and hence between ΔA_{t-1} and $\Delta\epsilon_t$, cannot be ruled out. However, it seems safe to assume that this correlation, if it exists, is reasonably small.

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we divide both sides of equation (1.6) by Y, obtaining

$$\frac{C}{Y} = \alpha_1 + \alpha_3 \frac{A}{Y}$$

and then proceed to estimate the parameters of this equation by the conventional least squares method. It can be shown that, in so far as the bias due to the positive correlation between Y and the true error of the consumption function is concerned, the least-squares estimate of α_3 will be upward-biased, and hence, that of α_1 will be downward-biased. Thus, for both coefficients, the bias is in the direction opposite to that resulting from other procedures reported so far. This approach has also other desirable properties: it eliminates the difficulties arising from the presence of strong multicollinearity, and the homoscedasticity condition is more likely to be satisfied. Furthermore, it eliminates altogether the common trend in the variables, thus providing a rather stringent test of the relevance of net worth as a determinant of consumption. Its main drawback is that the above-mentioned bias in the estimates may be appreciably reinforced by error of measurement in the variable Y, although error of measurement in A will tend to work in the opposite direction.[20]

The results of this test reported in row (12) are rather mixed. The still remarkably high value of R^2 provides strong support for the hypothesis. Also, as expected, the procedure yields a higher estimate of α_1 and lower estimate for α_3. But the gap between the estimates obtained in this manner and those reported earlier is so wide as to provide little useful information as to the true value of these coefficients. Furthermore, the very high serial correlation of errors casts serious doubt on the reliability of the estimates.

The high serial correlation in the estimated errors, in this and to a lesser extent in other procedures, suggests the desirability of testing for evidence of a significant change in the parameters of our consumption function as between the prewar and the postwar period. Individual tastes as well as the demographic structure and the rate of return on assets, on which the theoretical values of the coefficients depend, may have changed sufficiently over the two periods to cause a measurable change in the parameters. In addition, there exists a statistical problem arising from the fact that the data, particularly the net worth estimates, are based on somewhat different estimating procedures for the two periods. To test the hypothesis of a shift in parameters we have computed a number of regressions involving a "dummy" variable X, with value zero for the years 1929–40 and value one for the years 1947–59.

The results given in rows (4), (7), (8), (10), and (12) to (14), which

[20] For possible problems arising from the use of ratios, see [26] and [37].

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constitute a representative sample of the tests we have carried out, show that the coefficient of the dummy variable is consistently negative, suggesting a moderate downward shift in the "measured" consumption function in the postwar period.[21] In the first-difference test the shift is not statistically significant and the coefficients of the other variables are altered only slightly. In the constrained and the ratio estimates, the downward shift is generally significant, and the serial correlation of the observed errors is reduced appreciably though it still remains high;[22] there also occur some sizable changes in the coefficients of the other variables. At least in the case of the ratio estimates, these changes tend on the whole to increase the agreement between the various results, which we may now endeavor to summarize.

In the first place, all of the tests seem, by and large, to support the basic hypothesis advanced in this paper, and in particular, the importance of net worth as a determinant of consumption. Unfortunately, serious difficulties arise in the attempt to secure reliable estimates of the coefficients of the "independent" variables, although some tentative conclusions seem to stand out. First, the different estimation procedures when applied to hypothesis I generally yield a similar estimate of the coefficient of net worth for the period as a whole, somewhere between .07 and .08. [Cf. rows (3), (6), (7), and (12), but note (11).]

At the same time, the estimate of the coefficient of income is definitely

[21] Our model provides one possible clue to this apparent downward shift. An examination of the figures reported in [3] reveals a distinct decline in the ratio of nonlabor income to net worth, which can be taken as a measure of the rate of return on capital. This is presumably attributable in large measure to increases in corporate taxes and in the extent and progressiveness of personal income taxation. An examination of the values of the coefficients of income and net worth implied by our model, reported in Table 1, suggests that both coefficients should tend to decline as the rate of return on assets declines. However, we do not wish to press this point, especially since we cannot even be sure whether the apparent decline in the coefficients reflects anything more than error of measurement.

[22] Another possible explanation for the high serial correlation in some of the tests is that consumption does not adjust fully to changes in income and assets within the arbitrary time unit of one year. To allow for this possibility hypothesis I might be written as

$$C_t - C_{t-1} = \delta(\alpha_1 Y_t + \alpha_3 A_{t-1} - C_{t-1}), \text{ or } C_t = (1-\delta)C_{t-1} + \delta\alpha_1 Y_t + \delta\alpha_3 A_{t-1},$$

where the constant δ, with the dimension 1/time, measures the speed of adjustment of consumption and may be expected to approach unity as the time unit increases. This hypothesis was tested in ratio form (to avoid increasing further the already extremely high multicollinearity prevailing in the direct form) with the following results:

$$\frac{C_t}{Y_t} = .46 + \underset{(.077)}{.177}\ \frac{C_{t-1}}{Y_t} + \underset{(.016)}{.072}\ \frac{A_{t-1}}{Y_t}$$

implying $\delta = .82$, $\alpha_1 = .56$, $\alpha_3 = .087$.

The relatively low coefficient of C_{t-1} (which is only moderately significant) suggests that a span of one year is long enough for most of the adjustment to take place. Also the estimates of the remaining coefficients are not greatly affected and move closer to the first difference estimates. However, somewhat surprisingly, the serial correlation is increased still further.

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unstable [see rows (3), (4), (6), (8), (10), and (12)]. This instability is appreciably reduced under hypothesis II, where the third variable, Y_E^L, apparently helps to disentangle the effect of purely cyclical and transitory changes in nonproperty income from that of long-run or permanent changes. The various estimates for the long-run marginal propensity, $\alpha_1+\alpha_2$, are fairly consistent—between .68 and .71 for the first-difference and the ratio estimates [rows (9), (11), and (15)], and only moderately higher for the straight estimate with constant suppressed [row (5)]. There is however much less consistency in the values of α_1 and α_2 separately and hence in the estimates of the short-run marginal propensity to consume with respect to labor income, α_1. The first-difference approach yields figures of .39 and .44 [rows (9) and (11)], while the estimates from straight regression with the constant suppressed and from regression on ratios [rows (5) and (15)] are about .6. On the whole we are inclined to regard the first estimates as somewhat more reliable, in part because of the high serial correlation present in rows (5) and (15), but no firm conclusion seems warranted with the available data and methods. At the same time we observe that the introduction of the third variable, Y_E^L, tends to reduce somewhat the estimate of the coefficient of net worth. It would appear that the value .07 to .08 obtained for hypothesis I, where Y_E^L is not present, may be somewhat too high—since the cyclically sluggish variable A acts partly as a proxy for expected nonlabor income—and the true value may be closer to .06 or even somewhat lower.

Our tests also agree in suggesting a moderate downward shift in both parameters, $\alpha_1+\alpha_2$, and α_3 of the measured consumption function from the prewar to the postwar period, although it does not seem possible at present to estimate reliably the distribution of this downward shift between $\alpha_1+\alpha_2$ and α_3.

As indicated earlier, a few tests of hypothesis I have also been carried out for the period 1900–28. Because the data for this period are mostly obtained from different sources and are subject to very wide margin of error, we have seen little point in combining them with the series relating to the period since 1929. In fact, we are inclined to attach rather little significance to the results of these tests, which are accordingly confined to Appendix section A. For whatever they are worth, these results do not appear grossly inconsistent with those for the period after 1929, especially when account is taken of error of measurement and its likely effects on different estimation procedures. In particular, the contribution of net worth appears again to be significant, its coefficient being of the same order of magnitude as in the later period except in the first-difference test which is obviously most seriously affected by the error of measurement.

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Finally, our empirical results are also roughly consistent with the a priori numerical predictions reported in Table 1. The fact that the coefficients of both variables, especially that of net worth, are on the low side may be accounted for by reference to the estate motive which was ignored in the numerical calculations for Table 1, while it probably plays a nonnegligible role at least for the high-income and/or self-employed groups.[23]

III. *Some Implications*

A. *Relation to the Standard Keynesian Consumption Functions*

The standard Keynesian consumption function [23] is usually written in the form:

$$C = \gamma Y^* + \gamma_0 \tag{3.1}$$

where Y^* denotes personal income net of taxes or disposable income and the γ's are constants.[24] A more sophisticated variant of this hypothesis, which has become quite popular of late, consists in separating income into two parts, disposable labor income Y, and disposable nonlabor or property income, which we shall denote by P. Thus,

$$C = \gamma_1 Y + \gamma_2 P + \gamma_0. \tag{3.2}$$

This variant, which reduces to (3.1) when $\gamma_1 = \gamma_2$, is usually advocated on the ground that property income accrues mostly to higher-income and/or entrepreneurial groups who may be expected to have a lower marginal propensity to consume. Accordingly, γ_2 is supposed to be smaller than γ_1 and this supposition appears to be supported by empirical findings.

It is immediately apparent that (3.2) bears considerable similarity to hypothesis I discussed in this paper, i.e.,

$$C = (\alpha_1 + \alpha_2) Y + \alpha_3 A. \tag{3.3}$$

The main difference lies in the constant term which appears in (3.2) but not in (3.3), and in the fact that the wealth variable A in (3.3) is replaced in (3.2) by a closely related variable, income from wealth, P. We can avoid dealing with the first source of discrepancy by working with both hypotheses in first-difference form,

$$\Delta C = \gamma_1 \Delta Y + \gamma_2 \Delta P \tag{3.2a}$$

$$\Delta C = (\alpha_1 + \alpha_2) \Delta Y + \alpha_3 \Delta A. \tag{3.3a}$$

Equations (3.2a) and (3.3a) are quite useful since they allow a straightforward test of the usefulness of the Modigliani-Brumberg hypothesis

[23] See for instance the results of cross-section studies reported in [24] and [32].

[24] Keynes' own formulation (See [23, Book 3]) was considerably more general than that contained in equation (3.1).

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as compared with the standard Keynesian one. We have already exhibited in Table 2, row (6), the results obtained by fitting (3.3a) to the data. In order to complete the test we need to estimate the parameters of (3.2a). If the standard Keynesian version is correct, the net worth variable in (3.3) and (3.3a) is merely a proxy variable for the return from wealth, P, and hence substitution of ΔP for ΔA should improve the fit. On the other hand, if (3.3) and (3.3a) are closer to the truth than (3.2), then the substitution of ΔA by a proxy variable ΔP should reduce the correlation.

The estimate of P needed for this test is given in [3].[25] The definition of consumption on which we rely, however, is somewhat different from that customarily used in the standard Keynesian formulation in that it includes the current consumption—depreciation—of the stock of consumer durables, while excluding expenditure for the purchase of such goods.[26] The results obtained for hypothesis (3.2a) are as follows:

$$\text{(3.4)} \qquad \Delta C = \underset{(.07)}{.93}\Delta Y + \underset{(.29)}{.07}\Delta P \qquad R^2 = .86.$$

Comparison of this result with those reported in Table 2, row (6), strongly suggests that net worth is definitely not a mere proxy for current property income. While the coefficient of P is positive and smaller than that of Y as expected, this variable is much less useful than A in explaining the behavior of consumption. In fact, its contribution is not significantly different from zero.[27]

[25] Our estimates of Y and P do not add up exactly to disposable personal income as usually defined because we include in disposable personal income contributions to, instead of benefits from, the social security system. However, this discrepancy is quite minor.

[26] Also, our data are in current dollars, while the standard Keynesian version of the consumption function is usually stated in terms of constant dollars.

[27] For the sake of completeness several other variants of (3.2a) were tested by adding variables that were included in the test of our hypothesis and which are consistent with the spirit of the Keynesian model. The addition of the variable ΔY_E^L, which might help to sort out the effect of long run from that of purely cyclical variations in income, yields

$$\text{(3.4a)} \qquad \Delta C = \underset{(.18)}{.47}\Delta Y + \underset{(.13)}{.49}\Delta (Y_E^L) + \underset{(.23)}{.17}\Delta P \qquad R^2 = .921.$$

If we also include the dummy variable X to allow for possible shifts from the prewar to the postwar period, its coefficient is uniformly less than its standard error, and in general hypothesis (3.2) does not fare any better. This conclusion can be illustrated by the following result which is the most favorable to that hypothesis among the battery we have run:

$$\text{(3.4b)} \qquad \Delta C = \underset{(.14)}{.46}\Delta Y + \underset{(.14)}{.51}\Delta (Y_E^L) + \underset{(.28)}{.26}\Delta P - \underset{(.39)}{.21}X\Delta P \qquad R^2 = .921.$$

The fact that in (3.4a) and (3.4b) the coefficient of the variable ΔY_E^L is a good deal higher and statistically more significant than in the corresponding tests reported in Table 2 is readily accounted for by the high correlation between this variable and A which, in the absence of A, makes this variable act partly as a proxy for A. (The correlation in question is .93. See also footnote 16.)

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These results, besides supporting our hypothesis, serve also to cast serious doubts on the conventional interpretation of the empirical coefficients of ΔY and ΔP in (3.2); namely, that incremental labor income is largely consumed while incremental property income is largely saved. For our tests of the Modigliani-Brumberg model indicate that consumption is quite responsive to variations in the market value of wealth, which, in turn, must largely reflect the capitalization of property income. Note, however, that the market valuation of assets will be controlled by expected long-run returns, say $\overline{P}$, which will tend to be a good deal more stable than *current* property income, P. We suggest therefore that the coefficient of P in (3.2) is small not because property income is largely saved but because short-run changes in P are dominated by transitory phenomena and hence are a poor measure of changes in the relevant long-run, or permanent, property income, which will be reflected far more reliably in the market valuation of assets. Put somewhat differently, the low coefficient of P does not imply a low marginal propensity to consume out of property income but merely a low propensity to consume out of transitory income. Correspondingly the extremely high coefficient of labor income in (3.3) is equally misleading, reflecting the fact that Y acts partly as a proxy for the permanent component of property income, $\overline{P}$.

One might be tempted to estimate the marginal propensity to consume with respect to permanent property income $\overline{P}$ by relying on the estimates of the coefficient of net worth in (3.3a) provided in Table 2, and on the relation

$$\overline{P} \simeq rA \quad \text{or} \quad A \simeq \frac{\overline{P}}{r} \cdot$$

Following this reasoning, the coefficient of $\overline{P}$ in the consumption function would be given by $\frac{\alpha_3}{r}$, where r is the rate at which the market capitalizes the return from assets. If we are willing to approximate r with the average realized rate of return on assets, then, from the figures given in [3], we find that r was about .04 in the prewar period and somewhat lower (around .03) in more recent years.[28] Combining this estimate with our estimate of α_3, which is in the order of .06, we seem to be led to the conclusion that the marginal propensity to consume with respect to permanent property income $\frac{\alpha_3}{r}$, far from being low, is actually well above unity.

This result may appear preposterous if judged in terms of the standard Keynesian framework underlying (3.2), with its emphasis on the relation

[28] See, however, our comment below on the shortcomings of our estimate of P given in [3] as a measure of return on assets.

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between flows. It is however possible to interpret this result in terms of the Modigliani-Brumberg framework. For, in this model, wealth affects consumption not only through the stream of income it generates but also directly through its market value which provides a source of purchasing power to iron out variations in income arising from transitory developments as well as from the normal life cycle. It is therefore not surprising that this model implies a marginal propensity to consume with respect to assets, α_3, larger than the rate of return r (cf. Table 1), an inference which, as we have just seen, is supported by empirical tests. It should be noted however that $\frac{\alpha_3}{r}$ should not be interpreted as the marginal propensity to consume with respect to permanent property income in the same sense in which $(\alpha_1+\alpha_2)$ can be said to measure the propensity with respect to permanent nonproperty income, for it measures the *joint* effect on consumption of a change in property income, r constant, and of the accompanying change in assets. It is not possible to infer the two effects separately from knowledge of α_3 and of the average value of r. Although we cannot pursue this subject here, we wish to point out that the effect on C of a change in $\overline{P}$ will be quite different depending on the behavior of A and hence r, as $\overline{P}$ changes.

B. *Cyclical versus Long-Run Behavior of the Consumption-Income Ratio —Relation to the Duesenberry-Modigliani Consumption Function*

As is well known, one of the major difficulties encountered with the standard Keynesian consumption functions (3.1) or (3.2) lies in the constant term γ_0. This constant term is needed to account for the observed cyclical variability in the saving-income ratio, but it also implies a long-run tendency for the saving ratio to rise with income, which is contradicted by empirical findings. The lack of any positive association between income and the saving-income ratio in the long run, at least for the U. S. economy, was first uncovered by Kuznets, and has more recently been confirmed by the extensive investigation of Goldsmith [19], focusing on the years 1896–1949. In his summary recapitulation, he lists as the first item: "Long-term stability of aggregate personal saving at approximately one-eighth of income, and of national saving at approximately one-seventh."[29]

The consumption function proposed here is capable of accounting both for the long-run stability and the cyclical variability of the saving-income ratio. In order to exhibit its long-run properties, let us suppose that Y were to grow at a constant rate n, in which case Y^e can be taken as equal or proportional to Y. Suppose further that the rate of return on assets r is reasonably stable in time. Then the consumption function

[29] Goldsmith [19, Vol. 1, p. 22].

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(1.6) implies that the income-net worth ratio, Y_t^*/A_{t-1} will tend to a constant h, related to the parameters of the consumption function by the equation:[30]

$$h = \frac{n + \alpha_3 - \alpha r}{1 - \alpha}; \qquad \alpha = \alpha_1 + \alpha_2 \cdot \tag{3.5}$$

When the ratio $\frac{Y_t^*}{A_{t-1}}$ is in fact equal to h, then income and net worth grow at the same rate, n, and the saving-income ratio will be a constant given by:

$$\frac{S_t}{Y_t^*} = \frac{A_t - A_{t-1}}{A_{t-1}} \frac{A_{t-1}}{Y_t^*} = n \frac{1}{h} \cdot \tag{3.6}$$

Similarly, we find:

$$\frac{Y_t}{A_{t-1}} = \frac{Y_t^* - rA_{t-1}}{A_{t-1}} = h - r \tag{3.7}$$

$$\frac{C_t}{Y_t} = \frac{Y_t^* - S_t}{A_{t-1}} \frac{A_{t-1}}{Y_t} = \frac{h - n}{h - r} \cdot \tag{3.8}$$

Thus the model implies that if income fluctuates around an exponential trend the income-net worth ratio will tend to fluctuate around a constant level h, and the saving-income ratio around a constant $\frac{n}{h}$.

The empirical estimates reported in Section II suggest that α is around .7, and α_3 close to .06. The average rate of return, r, is much more difficult to guess. If we are willing to rely on the ratio $\frac{P}{A_{t-1}}$ for this purpose, then r would be around or somewhat lower than .04. But this

[30] Under the stated assumptions we have $Y_t^* = Y_t + P_t$, and $P_t = rA_{t-1}$. Hence, saving can be expressed as

$$S_t = Y_t^* - C_t = Y_t + P_t - C_t = (1 - \alpha)Y_t^* - (\alpha_3 - \alpha r)A_{t-1}. \tag{a}$$

We also have $S_t = A_t - A_{t-1}$. Substituting this definition for S_t in (a), dividing through by A_{t-1}, adding and subtracting n, and then rearranging terms, we obtain

$$\frac{A_t - A_{t-1}}{A_{t-1}} = n + (1 - \alpha) \left[\frac{Y_t^*}{A_{t-1}} - \frac{n + \alpha_3 - \alpha r}{1 - \alpha}\right]. \tag{b}$$

Comparison of (b) above with equation (3.5) shows that if Y_t^*/A_{t-1} were larger than h, the second term in the right-hand side would be positive, and hence net worth would grow at a rate larger than that of income, n, causing Y_t^*/A_{t-1} to fall toward h: and conversely if Y_t^*/A_{t-1} were smaller than h.

This argument is oversimplified and incomplete, particularly since it ignores the interaction between the behavior of consumers and the production process in the economy. A more complete analysis of this growth process is given in Ando [2].

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ratio is very likely to understate the true value for r, since the estimate of P given in [3] corresponds to the conventional definition of personal income and omits a number of items whose exclusion is appropriate for the standard Keynesian model but not for the Modigliani-Brumberg model. Among those items, the more important are imputed net rent on consumer durables and undistributed corporate profits.[31] These adjustments suggest an average value for r slightly over .04. If we further take for n, the rate of growth of income, a value in the order of .03, then from (3.6), (3.7), and (3.8) we obtain the following estimates for the various ratios under discussion: (i) total income to net worth, $h \simeq .2$; (ii) nonproperty income to net worth, $h-r \simeq .16$; (iii) saving to income, $\frac{n}{h} \simeq .15$. It can be seen that the first two of the above figures are in fact close to the values around which the ratios

$$\frac{Y_t^*}{A_{t-1}} \quad \text{and} \quad \frac{Y_t}{A_{t-1}}$$

fluctuate according to the data given in [3] while the third, the saving-income ratio, is consistent with the findings of Goldsmith reported earlier.

Needless to say, these calculations are very crude and are given here primarily to bring out certain interesting testable implications of the consumption function discussed in this paper. Among these implications the long-run stability of the ratio of net worth to income is particularly significant, for it paves the way for an explanation of the historical stability of the capital-output ratio in terms of the supply of capital, thereby challenging the prevailing notion that the behavior of this ratio is explained by technological requirements [1] [2] [5] [29].

As for the cyclical implications of our model, we need only observe that at any given point in time net worth A_{t-1} is a given initial condition. Hence, retaining for the moment the assumption that $Y^e \simeq Y$, (1.6) implies that the aggregate consumption function for any given year is a straight line in the C-Y plane with slope α and intercept $\alpha_3 A_{t-1}$. It is shown in Figure 2 for the year 0 as the line labeled $\overline{C}_0$, and looks like the orthodox Keynesian version. Yet, it differs from the latter in one essential respect, namely, that its intercept will change in time as a result of the accumulation (or decumulation) of wealth through saving. As we have shown in preceding paragraphs, so long as income keeps rising on its exponential trend, the growth in net worth will shift the function in such a way that the observed consumption-income points will trace out the long-run consumption function (3.8) represented in

[31] The rationale for including corporate saving in property income and personal saving is given in Modigliani and Miller [36].

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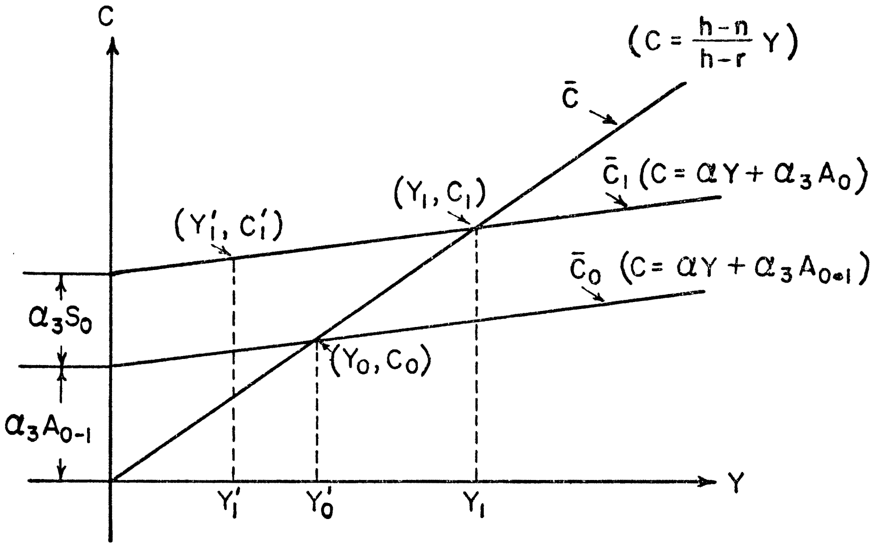

FIGURE 2. CONSUMPTION INCOME RELATIONS: LONG-RUN AND SHORT-RUN

our graph by the line $\overline{C}$ through the origin. This point is illustrated in Figure 2 for two years, 0 and 1. However, suppose that a cyclical disturbance caused income to fall short of Y_1, say to the level Y'_1. Then the consumption C'_1 given by the short-run consumption function $\overline{C}_1$ implies a higher consumption-income ratio and a lower saving-income ratio.

Thus, cyclical swings in income from its long-run trend will cause swings in the saving-income ratio in the same direction,[32] especially since the position of the function will not change appreciably when income is cyclically depressed below its previous peak due to the small or negative saving that would prevail.[33] After income has recovered beyond the previous peak, it may for a while rise rapidly as it catches up with

[32] This phenomenon will be further accentuated when we recognize the possibility that a cyclical fall in Y is likely to bring about a smaller change in Y^e. Also, because property income may be expected to fluctuate cyclically even more than labor income, the ratio of saving to total income will fluctuate even more than the ratio of saving to labor income. See footnote 30, equation (a).

[33] Some downward shift of the consumption function might occur even in the absence of dissaving, if there is some downward revaluation of the market value of assets, as the depressed level of property income tends to bring about less favorable evaluation of the long-run prospects for return from assets.

Because of this dependence of the value of assets on property income, the statement made earlier to the effect that A_{t-1} can be taken as a given initial condition in the year t is only approximately true. Also the relation $\Delta A_t = S_t$ does not hold in the presence of capital gains and losses. Note however that what is relevant in the present connection is the change in the value of assets in terms of purchasing power over consumption goods and not the change in terms of money value, which may be considerably more severe.

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its trend, running ahead of the slowly adjusting wealth. In this phase we may observe points to the right of $\overline{C}$, and the corresponding high saving will tend to make A catch up with Y.

Thus the model advanced here may be expected to generate a behavior of consumption and saving which is very similar to that implied by the earlier Duesenberry-Modigliani type of hypothesis, in which consumption was expressed as a function of current income and the highest previous peak income (or consumption) [9] [30]. If we interpret the role of the highest previous income as that of a proxy for net worth, then the Duesenberry-Modigliani consumption function can be considered as providing a good empirical approximation to the consumption function discussed in this paper, and to this extent the empirical support provided for the Duesenberry-Modigliani type of hypothesis can also be considered as empirical support for the consumption function advanced here, and vice versa. At the same time the present model has the advantage that the hypotheses on which it rests are explicitly stated as specifications of the consumer's utility function. It is also analytically more convenient as a building block in models of economic growth and fluctuations, as we have endeavored to demonstrate in various contributions [1] [2] [3] [29] [31].[34]

Appendix

A. *Some Statistical Results for Earlier Years*

The following are the estimates obtained using data for 1900–28, excluding years 1917, 1918, and 1919. As stated in the text, the data used are very rough, and may not be compatible with the data for the period since 1929. The data and their derivation are described in Ando [2, App.], except for the adjustments needed for different treatments of the government sector. This adjustment is self-explanatory from the description given in [2]. The data presented in [2] are in turn based largely on [18] [19] [22] [27] and [42].

$$\text{(a)}\qquad C = \underset{(.134)}{.755Y} + \underset{(.020)}{.073A} \qquad R^2 = .995 \qquad DW = 1.63$$

$$\text{(b)}\qquad \Delta C = \underset{(.180)}{.731\Delta Y} + \underset{(.037)}{.047\Delta A} \qquad R^2 = .44 \qquad DW = 2.48$$

$$\text{(c)}\qquad \frac{C}{Y} = \underset{(.144)}{.505} + \underset{(.021)}{.112}\,\frac{A}{Y} \qquad R^2 = .51 \qquad DW = 1.05$$

[34] On the other hand, because up-to-date estimates of wealth are not readily available, at least for the present, some variant of the Duesenberry-Modigliani model may well be more useful for short-run forecasting.

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B. *Biases in Estimating the Consumption Function by Regressions on Saving*

Suppose that true consumption c^* and true income y^* are related by a linear function (all variables being measured from their means),

$$c^* = \alpha y^* + \epsilon \tag{a}$$

and measured income and measured consumption are related to their respective true values by

$$c = c^* + \eta \tag{b}$$

$$y = y^* + \xi \tag{c}$$

where ϵ, η, and ξ are random variables. For simplicity, let us assume that ϵ is uncorrelated with y^*, η, and ξ; η and ξ are uncorrelated with c^* and y^*. We also have the definitions

$$s^* = y^* - c^* \tag{d}$$

$$s = y - c = y^* - c^* + \xi - \eta \tag{d$'$}$$

Using (a) and (d), we have

$$c^* = \frac{\alpha}{1-\alpha} s^* + \frac{\epsilon}{1-\alpha} \equiv \beta s^* + \epsilon'. \tag{e}$$

In order to concentrate first on the effect of errors of measurements, let us momentarily accept the unwarranted assumption that saving is equal to investment which in turn is truly exogenous. Under this assumption s^* can be taken as independent of ϵ' and therefore, if we could actually observe s^* and c^*, by regressing c^* on s^* we could secure an unbiased estimate of β from which we could in turn derive a consistent estimate of α. If however we estimate β by regressing c on s, then remembering that s is obtained as a residual from y and c, we obtain the estimate

$$\begin{aligned} \hat{\beta} &= \frac{\sum cs}{\sum s^2} = \frac{\sum (\beta s^* + \epsilon' + \eta)(s^* + \xi - \eta)}{\sum (s^* + \xi - \eta)^2} \\ &= \frac{\beta \sum s^{*2} + \sum \eta(\xi - \eta)}{\sum s^{*2} + \sum (\xi - \eta)^2}. \end{aligned} \tag{f}$$

The term $\sum \eta(\xi - \eta)$ arises from the fact that when there is a statistical error η in measuring consumption, there will be an error $-\eta$ in measuring saving, except in so far as this is offset by an error ξ in measuring income. This term will tend to be negative and introduces a downward bias into β. The term $\sum(\xi-\eta)^2$ in the denominator is the well-known result of an error of measurement of the independent variable, and it introduces an unambiguous bias towards zero into β.

However, as pointed out in the text, the assumption that personal saving is exogenous is completely unwarranted. In order to bring out the nature of the bias resulting from this misspecification, let us make the other extreme and equally unwarranted assumption that disposable income is truly exogenous. We can then regard y^* as independent of ϵ and by substituting (a) and (d) into (d$'$) we find

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$$s = s^* + \xi - \eta = (1 - \alpha)y^* - \epsilon + \xi - \eta. \tag{g}$$

Equation (f) is then replaced by

$$\beta = \frac{\alpha(1-\alpha)\sum y^{*2} - \sum \epsilon^2 + \sum \eta(\xi - \eta)}{(1-\alpha)^2 \sum y^{*2} + \sum \epsilon^2 + \sum (\xi - \eta)^2}. \tag{h}$$

The presence of the term $\sum\epsilon^2$ in both numerator and denominator is the result of the fact, discussed in the text, that when there is residual error ϵ in the consumption-income relationship, there will be residual error $-\epsilon$ in the saving-income relationship. Because of the signs, this effect, too, will bias β downward.

Since all these biases are downward and there is no offsetting upward bias of any significance, it is not surprising that recent applications of this approach [39] [43] lead in a number of cases to a negative estimate of the marginal propensity to consume.

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