Problem 1. Pareto Optimality and Competitive Equilibrium with Taxes

Consider a static economy where agents have preferences
\[ U(c, l) = \ln(c) + \alpha \ln(1 - h) \]
over consumption \( c \) and hours worked \( h \), with \( u_c > 0, u_\infty < 0, v_h < 0, v_{hh} > 0 \). Let \( w \) be the wage per hour worked. Production takes place through the aggregate technology
\[ y = z h, \]
Aggregate output is all consumed, i.e. \( y = c \). The government taxes households with the lump-sum tax \( \tau \) and redistributes tax revenues through a subsidy \( s \) to hours worked. I.e., let \( \omega h \) be the earnings of the household without the subsidy, with the subsidy they would be \((1 + s) \omega h\).

a) Write down the household problem, including the first order condition for hours worked.

b) Write down the social planner problem for this economy. Do the competitive equilibrium allocations coincide with the planner’s allocations? Explain your answer.

Problem 2. Keeping Up with the Joneses

Consider a static (one-period) economy inhabited by two households indexed by \( i = 1, 2 \). Household \( i \) can either work \((h_i)\) or enjoy leisure \((l_i)\). Each household \( i \) is endowed with one unit of time, \( l_i + h_i = 1 \). When working, households earn the wage \( w \) which they can use to purchase consumption goods whose price is normalized to 1. Households derive utility from their own consumption \( c_i \) only relative to the other type’s consumption, i.e., they get more utility if they know they’re richer than their neighbor. Preferences for agent 1 are therefore
\[ U(c_1, c_2, l_1) = \ln \left( \frac{c_1}{(c_2^\theta)} \right) + \psi \ln l_1, \]
with \( \theta \in (0, 1) \), and similarly for household 2. Firms hire labor in a competitive labor market, and produce with technology \( y = f(h_1 + h_2) \).

a) Define the competitive equilibrium for this economy. Derive the first-order conditions of household’s \( i \) problem, and those of the firm’s problem.

b) Solve the social planner’s problem for a planner who maximizes the sum of the welfare of household 1 and household 2, deriving the first order conditions with respect to \((c_i, l_i)\) for each type of agent. Compare the Planner’s allocations with the competitive equilibrium allocations. Are they the same, i.e., do the Welfare Theorems hold? Explain your answer.

c) Is there any need for government intervention in this economy? What policy would you suggest to the government, if any, to restore social optimality?