Day 5

Monetary Policy

in Models with Heterogeneous Agents

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New York University

Mini-Course on “Policy in Models with Heterogeneous Agents”
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Automatic fiscal stabilizers

- **Fiscal stabilizers**: rules in law that make fiscal revenues and outlays relative to total income change with the business cycle.

- **Question**: are the automatic stabilizers effective at reducing the volatility of macroeconomic fluctuations?

- Estimate how much higher the volatility of aggregate activity would be if some or all of the fiscal stabilizers were removed.

- Investigate the theoretical channels by which the stabilizers may attenuate the business cycle.

- Investigate whether type of monetary policy matters for the results.
Automatic stabilizers in the US

<table>
<thead>
<tr>
<th>Revenues</th>
<th>Outlays</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Progressive income taxes</strong></td>
<td><strong>Transfers</strong></td>
</tr>
<tr>
<td>Personal Income Taxes</td>
<td>Unemployment benefits</td>
</tr>
<tr>
<td>10.98%</td>
<td>0.33%</td>
</tr>
<tr>
<td>Safety net programs</td>
<td>Safety net programs</td>
</tr>
<tr>
<td>1.02%</td>
<td>1.02%</td>
</tr>
<tr>
<td><strong>Proportional taxes</strong></td>
<td><strong>Supplemental nutrition assistance</strong></td>
</tr>
<tr>
<td>Corporate Income Taxes</td>
<td>0.24%</td>
</tr>
<tr>
<td>2.57%</td>
<td>Family assistance programs</td>
</tr>
<tr>
<td>Property Taxes</td>
<td>0.24%</td>
</tr>
<tr>
<td>2.79%</td>
<td>Security income to the disabled</td>
</tr>
<tr>
<td>Sales and excise taxes</td>
<td>0.36%</td>
</tr>
<tr>
<td>3.85%</td>
<td>Others</td>
</tr>
<tr>
<td>0.19%</td>
<td>0.19%</td>
</tr>
<tr>
<td><strong>Budget deficits</strong></td>
<td><strong>Government purchases</strong></td>
</tr>
<tr>
<td>Public deficit</td>
<td>15.60%</td>
</tr>
<tr>
<td>1.87%</td>
<td>Net interest income</td>
</tr>
<tr>
<td>2.76%</td>
<td>2.76%</td>
</tr>
</tbody>
</table>

Figure 1: The personal income tax rate from TAXSIM
Forces at work

• The literature suggests three main channels:
  
  ▶ Stabilization of average disposable income: makes aggregate demand more stable

  ▶ Social insurance: some of these policies reduce individual income fluctuations (e.g., UI)

  ▶ Redistribution: from those with lower MPC to those with higher MPC stabilizes aggregate demand

• Model: Aiyagari meets Calvo-Woodford with 2 types (patient and impatient)
Patient households

- **Patient**: measure 1 of hh with access to complete markets against idiosyncratic risk, thus RA with problem:

\[
\max_{\{c_t, n_t, b_{t+1}\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log c_t - \varphi_1 \frac{n_t^{1+\varphi_2}}{1+\varphi_2} \right]
\]

s.t.

\[
(1 + \tau_t^c) p_t c_t + b_{t+1} - b_t = p_t [x_t - \bar{\tau}^x (x_t)]
\]

\[
x_t = d_t + (i_t/p_t) b_t + w_t \bar{\varepsilon} n_t
\]

\[
\bar{\tau}^x (x_t) = \int_0^{x_t} \tau^x (z) \, dz
\]

- Two assets: (i) trade bonds with the impatient households and the government, and (ii) invest capital in the production firms
Impatient households

- There is a measure $\nu = 4$ of impatient households indexed by $i$ facing problem:

$$\max_{\{c_t, n_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[ \log c_{it} - \varphi_1 \frac{n_{it}^{1+\varphi_2}}{1 + \varphi_2} \right]$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log c_{it} - \varphi_1 \frac{n_{it}^{1+\varphi_2}}{1 + \varphi_2} \right]$$

s.t.

$$(1 + \tau_t^c) p_t c_{it} + b_{i,t+1} - b_{it} = p_t [x_{it} - \tau^x (x_{it})]$$

$$b_{i,t+1} \geq 0$$

$$x_{it} = \begin{cases} 
(i_t/p_t) b_{it} + w_t \varepsilon_{it} n_t & \text{if } e_{it} = 2 \\
(i_t/p_t) b_{it} + T^u_{it} & \text{if } e_{it} = 1 \\
(i_t/p_t) b_{it} + T^s_{it} & \text{if } e_{it} = 0
\end{cases}$$

where $T^u_{it}$ is unemployment insurance and $T^s_{it}$ is safety net transfer, with $T^u_{it} > T^s_{it}$. 
Production sector

- Competitive final good sector combines intermediate goods:

\[ y_t = \left( \int_0^1 y_t(j)^{1/\mu_t} \, dj \right)^{\mu_t} \]

where the subscript \( t \) on \( \mu_t \) allows for mark-up shocks

- A unit continuum of intermediate-goods monopolistic firms, each producing variety \( j \) using a production function:

\[ y_t(j) = a_t k_t(j)^{\alpha} l_t^{1-\alpha}(j) \]

Set prices subject to nominal rigidities a la Calvo (1983)

- Maximize profits with the patient household (owner) SDF \( \lambda_{t+1} \)

- Profits taxed at the flat corporate tax income rate \( \tau_{cor} \)
Production sector

• Representative firm

• It owns the capital stock and solves

\[ v_t (k_t) = \max_{\Delta k_{t+1}} \left( (1 - \tau_{corp}) [r_t k_t - \Delta k_{t+1} - g(k_t, \Delta k_{t+1})] + E_t [\lambda_{t+1} \cdot v_{t+1}(k_{t+1})] \right) \]

and pays off its after-tax profits as dividends to its owner, the patient RA

• Very crude asset ownership structure: stark assumptions
Government

• Progressive income taxes, transfers, corporate income tax, sales taxes

• Government debt (the risk-free asset held by all households) and expenditures linked by a fiscal rule:

$$\log (g_t) = \log (\bar{g}) - \gamma \log \left( \frac{B_t}{p_t} \frac{\bar{B}}{\bar{p}} \right)$$

$\gamma$ measure the speed at which the deficits from recessions are paid back over time through a reduction in expenditures.

• Shocks: technology, markup, and monetary policy

$$i_t = \bar{i} + \phi \Delta \log p_t + \varepsilon_t$$

all following independent AR(1) processes.
In spite of all the heterogeneity, the aggregate responses to shocks are similar to those of the standard NK RA model.
Stationary equilibrium

\[ \beta^{-1} \]

\[ \hat{\beta}^{-1} \]

Interest Rate

Assets

Eq’m capital stock

Impatient household savings

Eq’m impatient household savings

Capital demand
Cyclicality of stabilizers in data and model

<table>
<thead>
<tr>
<th>Fiscal variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenues</td>
<td>0.095</td>
<td>0.044</td>
</tr>
<tr>
<td>Sales tax</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>Property tax</td>
<td>-0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>0.052</td>
<td>0.046</td>
</tr>
<tr>
<td>Corporate income tax</td>
<td>0.041</td>
<td>-0.013</td>
</tr>
<tr>
<td>Purchases</td>
<td>-0.009</td>
<td>0.022</td>
</tr>
<tr>
<td>UI payments</td>
<td>-0.020</td>
<td>-0.010</td>
</tr>
<tr>
<td>Net government savings</td>
<td>0.185</td>
<td>0.136</td>
</tr>
</tbody>
</table>

Quarterly data from 1960:I - 2011:IV and expressed relative to potential output (HP filter trend).

**Problem:** cyclicality of corporate income tax due to countercyclical markups (and dividends)
Main result

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
<td></td>
</tr>
<tr>
<td>output</td>
<td>-0.0229</td>
<td>0.0567</td>
<td></td>
</tr>
<tr>
<td>hours</td>
<td>-0.0296</td>
<td>0.0344</td>
<td></td>
</tr>
<tr>
<td>consumption</td>
<td>0.1232</td>
<td>0.0603</td>
<td></td>
</tr>
</tbody>
</table>

- **Experiment**: eliminate all stabilizers and move to a flat income tax
  - Stabilizers have a small effect on the volatility of the U.S. business cycle for $Y$ and $H$.
  - Removing the stabilizers raises the variance of aggregate consumption because budget deficits (government purchases) would not be as countercyclical (pro-cyclical).
  - By lowering marginal tax rates, it would be a significantly richer economy on average.
## Role of monetary policy

<table>
<thead>
<tr>
<th></th>
<th>Flexible prices</th>
<th>Schmitt-Grohé Uribe</th>
<th>Taylor rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>output</td>
<td>-0.0428</td>
<td>-0.0430</td>
<td>-0.0229</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0390</td>
<td>-0.0408</td>
<td>-0.0296</td>
</tr>
<tr>
<td>consumption</td>
<td>0.1165</td>
<td>0.1256</td>
<td>0.1232</td>
</tr>
<tr>
<td>inflation</td>
<td>-0.4123</td>
<td>-0.2907</td>
<td>-0.2828</td>
</tr>
</tbody>
</table>

### Reduction in volatility

- **Standard Taylor rule not far from optimal policy**

- **With baseline/aggressive monetary policy, small additional room for fiscal policy to stabilize the economy**

- **With passive monetary policy, fiscal stabilizers important**
Welfare gain from stabilizers

- Recall experiment: eliminate all stabilizers and move to a flat income tax
- Patient CEV is +14% and Impatient CEV is -15%
- Do stabilizers reduce costs of recessions?

Table 14: Utilitarian cost of recessions.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Cost with stabilizers</th>
<th>Cost without stabilizers</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>-0.00092</td>
<td>-0.00365</td>
<td>-0.00272</td>
</tr>
<tr>
<td>Mon. pol.</td>
<td>-0.00227</td>
<td>-0.00258</td>
<td>-0.00031</td>
</tr>
<tr>
<td>Markup</td>
<td>-0.00206</td>
<td>-0.00363</td>
<td>-0.00157</td>
</tr>
<tr>
<td>ZLB</td>
<td>-0.02964</td>
<td>-0.03554</td>
<td>-0.00590</td>
</tr>
</tbody>
</table>

- Yes, but costs are small to begin with
Monetary Policy According to HANK

Greg Kaplan
Ben Moll
Gianluca Violante
HANK: Heterogeneous Agent New Keynesian models

- New framework for quantitative analysis of aggregate shocks and macroeconomic policy
HANK: Heterogeneous Agent New Keynesian models

• New framework for quantitative analysis of aggregate shocks and macroeconomic policy

• Three building blocks:
  1. Uninsurable idiosyncratic income risk
  2. Nominal rigidities
  3. Assets with different degrees of liquidity

• Realistic MPC distributions
Questions for HANK

1. Aggregate and Distributional Effects of Shocks
   • Supply shocks, demand shocks, idiosyncratic uncertainty shocks, liquidity shocks, redistributive shocks

2. Fiscal policy
   • Government expenditure
   • Transfers and fiscal stimulus payments

3. Monetary policy
   • Conventional: nominal interest rate
   • Quantitative easing: swap of illiquid assets for liquid assets
   • Forward guidance: announcement of nominal interest rate
Questions for HANK

1. Aggregate and Distributional Effects of Shocks
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2. Fiscal policy
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Literature and contribution

Combine two workhorses of modern macroeconomics:

1. **New Keynesian models** with limited heterogeneity
   
   Campell-Mankiw, Gali-Lopez Salido-Valles, Iacioviello, Challe-Matherau-Ragot-Rubion Ramirez
   
   - Micro-foundation of spender-saver behavior

2. **Aiyagari models** with sticky prices
   
   
   - Assets with different liquidity Kaplan-Violante (2013)
   - New view of individual earnings risk Guvenen-Karahan-Ozkar-Song (2014)

- Continuous time approach Achdou-Lasry-Lions-Moll (2014)
- Literature on inventory management and asset illiquidity
  
  Baumol, Tobin, Romer, Alvarez-Lippi, Alvarez-Guiso-Lippi and many others
Model elements

• Households
  • face uninsured idiosyncratic labor income risk
  • hold two assets: liquid and illiquid
  • consume and supply labor

• Firms
  • monopolistic competition
  • quadratic price adjustment costs à la Rotemberg (1982)

• Assets
  • liquid assets: return determined by monetary policy rule
  • illiquid assets: return determined by profitability of capital

• "MIT shocks"
Households

\[
\max_{\{c_t, \ b_t, \ z_t\}} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t) \ dt \quad \text{s.t.} \quad b_t = r^b - c_t
\]

\[
z_t = \text{some Markov process}
\]

\[
b_t \geq b
\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(d_t\): illiquid deposits
- \(l_t\): hours worked
- \(w_t\): transaction cost function
- \(h_t\): total housing services
- \(c_t\): housing services expend

\(\text{• } c_t, b_t, z_t, d_t, l_t, w_t, h_t, c^t, ch^t, lt, dt, dh^t\)
Households

\[
\max_{\{c_t, d_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-(\rho + \lambda) t} u(c_t) dt \quad \text{s.t.}
\]

\[
\dot{b}_t = r^b \quad b_t + wz_t - d_t - \chi(d_t, a_t) - c_t
\]

\[
\dot{a}_t = r^a \quad a_t + d_t
\]

\[
z_t = \text{some Markov process}
\]

\[
b_t \geq b, \quad a_t \geq 0
\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(a_t\): illiquid assets
- \(d_t\): illiquid deposits
- \(\chi\): transaction cost function
Households

\[
\max_{\{c_t, c^h_t, l_t, d_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t, l_t, h_t) \, dt \quad \text{s.t.}
\]

\[
\begin{align*}
\dot{b}_t &= r^b(b_t)b_t + (1 - \xi)wz_t l_t - d_t - \chi(d_t, a_t) - c_t - c^h_t \\
\dot{a}_t &= r^a(1 - \omega)a_t + \xi wz_t l_t + d_t \\
h_t &= c^h_t + r^h\omega a_t \\
z_t &= \text{some Markov process}
\end{align*}
\]

\[
b_t \geq b, \quad a_t \geq 0, \quad c^h_t \geq 0
\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(a_t\): illiquid assets
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- \(c^h_t\): housing services expend
- \(h_t\): total housing services
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Households

\[
\max_{\{c_t, c^h_t, l_t, d_t\}} \quad \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t, l_t, h_t) dt \quad \text{s.t.}
\]

\[b_t = r^b(b_t) b_t + (1 - \xi) wz_t l_t - d_t - \chi(d_t, a_t) - c_t - c^h_t\]

\[\dot{a}_t = r^a (1 - \omega) a_t + \xi wz_t l_t + d_t\]

\[h_t = c^h_t + r^h \omega a_t\]

\[z_t = \text{some Markov process}\]

\[b_t \geq b, \quad a_t \geq 0, \quad c^h_t \geq 0\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(a_t\): illiquid assets
- \(d_t\): illiquid deposits

- \(\chi\): transaction cost function
- \(c^h_t\): housing services expend
- \(h_t\): total housing services
- \(l_t\): hours worked
Households

- Adjustment cost function:

\[ \chi(d, a) = \chi_0 \mathbf{1}_{\{d \neq 0\}} + \chi_1 \begin{vmatrix} d \\ a \end{vmatrix}^{\chi_2} a \]

- Fixed component implies inaction
- Convex component implies finite deposit rates

- Recursive solution consists of:
  - joint distribution of households \( \mu(da, db, dz; t) \)
  - consumption policy function \( c(a, b, z; t) \)
  - deposit policy function \( d(a, b, z; t) \)
  - labor supply policy function \( l(a, b, z; t) \)
Typical policy functions

Consumption Policy Function

Deposit Policy Function
Firms

Representative final goods producer:

\[ Y = \left( \int_0^1 y_j^{\frac{\epsilon-1}{\epsilon}} d\epsilon \right)^{\frac{\epsilon}{\epsilon-1}} \implies y_j = \left( \frac{p_j}{P} \right)^{-\epsilon} Y \]
Firms

Representative final goods producer:

\[ Y = \left( \int_0^1 y_j^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}} \Rightarrow y_j = \left( \frac{p_j}{P} \right)^{-\epsilon} Y \]

Monopolistically competitive intermediate goods producers:

- Technology: \( y_j = Z k_j^\alpha n_j^{1-\alpha} \)
- Marginal costs:

\[ m = \frac{1}{Z} \left( \frac{r}{\alpha} \right)^\alpha \left( \frac{w}{1-\alpha} \right)^{1-\alpha} \]

- Set prices subject to quadratic adjustment costs:

\[ \Theta \left( \frac{\dot{p}}{p} \right) = \frac{\theta}{2} \left( \frac{\dot{p}}{p} \right)^2 Y \]
New Keynesian Phillips Curve

Lemma

Inflation rate implied by firms' optimal price-setting satisfies

\[
\left( r^b - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left( \frac{\varepsilon}{\varepsilon - 1} m - 1 \right) + \dot{\pi}
\]
New Keynesian Phillips Curve

**Lemma**
Inflation rate implied by firms' optimal price-setting satisfies

\[
\left( r^b - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left( \frac{\varepsilon}{\varepsilon - 1} m - 1 \right) + \dot{\pi}
\]

- In present value form

\[
\pi_t = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-r^b(s-t)} \frac{Y_s}{Y_t} \left( \frac{\varepsilon}{\varepsilon - 1} m_s - 1 \right) ds
\]

- Logic: increase prices when markup < flex price optimum

\[
\frac{\varepsilon}{\varepsilon - 1} m_s - 1 = \frac{M^* - M_s}{M_s}, \quad M^* = \frac{\varepsilon}{\varepsilon - 1}
\]

- Exact NK Phillips curve: no log-linearization
Investment fund sector

- Investment funds receive illiquid assets from households:
  \[ A^p = (1 - \omega) \int_a^d \, ad\mu \]

- Investment funds have three sources of income:
  1. Rent out illiquid asset as capital to intermediate producers
     \[ (r - \delta)K \]
• Investment funds receive illiquid assets from households:

\[ A^p = (1 - \omega) \int a \, d\mu \]

• Investment funds have three sources of income:

1. Rent out illiquid asset as capital to intermediate producers

\[(r - \delta)K\]

2. Dividends from ownership of intermediate producers

\[ qK \]

**Steady state:**

\[ \bar{q} = (1 - m)Y \]

**Outside steady state:**

No dividend smoothing:

\[ q = (1 - m)Y - \frac{\theta}{2} \pi^2 Y \]

Dividend smoothing:

\[ \dot{q} = -\rho^q (q - \bar{q}) \]
Investment fund sector

- Investment funds receive illiquid assets from households:
  \[ A^p = (1 - \omega) \int a d\mu \]

- Investment funds have three sources of income:
  1. Rent out illiquid asset as capital to intermediate producers
     \[ (r - \delta)K \]
  2. Dividends from ownership of intermediate producers
     \[ qK \]
  3. Liquidity transformation: issue liquid assets, reinvest as capital
     \[ K = A^p + (-B^f) \]
Illiquid asset return

- Investment fund can issue deposits up to fraction of its assets

\[-B^f \leq \zeta K\]

\[K \leq \frac{1}{1 - \zeta} A^p\]

- Interpret \(\zeta\) as leverage of financial sector

- Investment fund optimization implies illiquid asset return

\[r^a = \frac{1}{1 - \zeta} \left( r - \delta + q - \zeta r^b \right)\]
Illiquid asset return

- Investment fund can issue deposits up to fraction of its assets

\[ -B^f \leq \zeta K \]

\[ K \leq \frac{1}{1 - \zeta} A^p \]

- Interpret \( \zeta \) as leverage of financial sector

- Investment fund optimization implies illiquid asset return

\[ r^a = r - \delta \]
Illiquid asset return

- Investment fund can issue deposits up to fraction of its assets

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\[K \leq \frac{1}{1 - \zeta} A^p\]

- Interpret $\zeta$ as leverage of financial sector

- Investment fund optimization implies illiquid asset return

\[r^a = r - \delta + q\]
Illiquid asset return

• Investment fund can issue deposits up to fraction of its assets

\[-B^f \leq \zeta K\]

\[K \leq \frac{1}{1 - \zeta} A^p\]

• Interpret \(\zeta\) as leverage of financial sector

• Investment fund optimization implies illiquid asset return

\[r^a = \frac{1}{1 - \zeta} (r - \delta + q - \zeta r^b)\]
Monetary authority and liquid assets

- Taylor rule
  \[ i = \max\{\bar{r}^b + \phi \pi, 0\}, \quad \phi > 1 \]

- Fisher equation \( r^b = i - \pi \)

- Four participants in bond market:
  - Households: \( B^h = \int bd\mu \)
  - Investment fund: \( B^f = -\zeta K \)
  - Government: \( B^g = -\bar{g}Y \)
  - Foreign sector: \( B^w \)

- Bond market clearing:
  \[ B^h + B^f + B^g + B^w = 0 \]
World bond demand

- World has exogenous demand function for domestic liquid assets:

\[ \dot{B}^w = \phi[\eta(r^b - \bar{r}^b) - (B^w - B^w)] \]

- Special cases
  - \( \eta = 0 \) or \( \phi = 0 \): bonds in fixed supply
  - \( \eta = \infty \): small open economy with \( r^b = \bar{r}^b \)

- Three reasons
  1. Realism
  2. Symmetry with illiquid assets
  3. Makes computations easier
Government

- Progressive tax on labor income:
  \[ T(wzl) = -\tau_0 + \tau_1 wl \]

- Exogenous government spending \( G \)

- Issues debt. Steady state debt level is fraction \( \bar{g} \) of GDP:
  \[ B^g = -\bar{g}Y \]

- Government budget constraint
  \[ G - r^b B^g = \int T(wzl(a, b, z)) \, d\mu \]

- Out of steady state we will consider various alternatives
Summary of market clearing conditions

- Liquid asset market
  \[ B^h + B^f + B^g + B^w = 0 \]

- Illiquid asset market
  \[ K = A^p + (-B^f) \]

- Labor market
  \[ N = \int zl(a, b, z) d\mu \]

- Goods market: Walras Law
Calibration

Three particularly important aspects, relatively unique to paper:

1. Continuous time household earnings dynamics

2. Definition and measurement of asset categories
   - Liquid vs illiquid
   - Productive vs non-productive

3. Adjustment cost function
Calibration

Three particularly important aspects, relatively unique to paper:

1. Continuous time household earnings dynamics  [no time today]

2. Definition and measurement of asset categories
   - Liquid vs illiquid
   - Productive vs non-productive

3. Adjustment cost function
## 50 shades of K

<table>
<thead>
<tr>
<th>Non-productive</th>
<th>Liquid</th>
<th>Illiquid</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Household deposits</td>
<td>Net housing</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>Net of revolving debt</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Govt bonds</td>
<td>Net durables</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B^h = 0.22$</td>
<td>$A^d = 1.31$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-productive</td>
<td></td>
<td>Indirectly held equity</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>Deposits at inv fund</td>
<td>Directly held equity</td>
<td>$K$</td>
</tr>
<tr>
<td></td>
<td>$-B^f = 0.55$</td>
<td>Noncorp bus equity</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A^p = 1.61$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B^w + B^g = 0.33$</td>
<td>$A = 2.92$</td>
<td>3.69</td>
</tr>
</tbody>
</table>

- $\zeta = \frac{-B^f}{K} = \frac{0.55}{2.16} = 0.255$
- $\omega = \frac{A^d}{A} = \frac{1.31}{2.92} = 0.449$
- $\bar{g} = 0.25 \Rightarrow B^g = -0.25 \Rightarrow B^w = 0.33 - B^g = 0.58$
- $0.06$ of $B^g$ is from domestic households, $0.19$ from foreign sector
Adjustment cost function calibration

- Three parameter function

\[ \chi(d) = \chi_0 \mathbf{1}_{d \neq 0} + \chi_1 \left| \frac{d}{a} \right|^\chi_2 a \]

- Choose \( \chi_0, \chi_1, \chi_2 \) together with:
  - discount rate \( \rho \)
  - borrowing intermediation wedge: \( \kappa^b = r^{bor} - r^b \)

  to match five features of the household wealth distribution
Adjustment cost function calibration

• Three parameter function

\[ \chi(d) = \chi_0 1_{\{d \neq 0\}} + \chi_1 \left| \frac{d}{a} \right|^{\chi_2} a \]

• Choose \( \chi_0, \chi_1, \chi_2 \) together with:
  • discount rate \( \rho \)
  • borrowing intermediation wedge: \( \kappa^b = r^{bor} - r^b + \)

to match five features of the household wealth distribution

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Illiquid assets (rel to GDP)</td>
<td>11.68</td>
<td>11.68</td>
</tr>
<tr>
<td>Median Illiquid assets (rel to GDP)</td>
<td>2.92</td>
<td>3.03</td>
</tr>
<tr>
<td>Mean Liquid assets (rel to GDP)</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>Fraction with Liquid assets ( \in [0, $1000] )</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>Fraction with Liquid assets &lt; 0</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Adjustment cost function

Adjustment Cost: Median Illiquid Assets

Adjustment Cost: Mean Illiquid Assets
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>λ</strong> Death rate</td>
<td>1/180</td>
<td>Av. lifespan 45 years</td>
</tr>
<tr>
<td><strong>γ</strong> Risk aversion</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td><strong>φ</strong> Frisch elasticity</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td><strong>ψ</strong> Disutility of labor</td>
<td>27</td>
<td>Av. hours worked equal to 1/3</td>
</tr>
<tr>
<td>Weight on housing</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ε</strong> Demand elasticity</td>
<td>10</td>
<td>Profit share 10 %</td>
</tr>
<tr>
<td><strong>α</strong> Capital share</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td><strong>δ</strong> Depreciation rate (p.a.)</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td><strong>θ</strong> Price adjustment cost</td>
<td>100</td>
<td>Bayer et. al. (2014)</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>τ</strong> Proportional labor tax</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td><strong>Τ</strong> Lump sum transfer (rel GDP)</td>
<td>0.073</td>
<td>40% hh with net govt transfer</td>
</tr>
<tr>
<td><strong>g</strong> Govt debt to annual GDP</td>
<td>0.25</td>
<td>Flow of Funds</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>φ</strong> Taylor rule coefficient</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td><strong>r^b</strong> Steady state real liquid return (pa)</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td><strong>Housing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ω</strong> Fraction of illiquid assets in housing</td>
<td>0.449</td>
<td></td>
</tr>
<tr>
<td><strong>r^h</strong> Net housing return (pa)</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td><strong>Illiquid Assets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ξ</strong> Fraction of labor income auto deposited</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td><strong>ζ</strong> Investment fund leverage</td>
<td>0.255</td>
<td></td>
</tr>
<tr>
<td><strong>r^a</strong> Illiquid asset return</td>
<td>11.4%</td>
<td>Equum outcome</td>
</tr>
</tbody>
</table>
Wealth distributions
Marginal propensities to consume

1. Slope of consumption function

\[ \text{MPC} = \frac{\partial}{\partial b} c(a, b, z) \]

2. Marginal propensity to consume over a period \( \tau \)

\[ C_\tau(a, b, z) = \mathbb{E} \left[ \int_0^\tau c(a_t, b_t, z_t) \, dt \bigg| a_0 = a, b_0 = b, z_0 = z \right] \]

\[ \text{MPC}_\tau = \frac{\partial}{\partial b} C_\tau(a, b, z) \]

3. Consumption out of \( x \) dollars over period (empirical estimates)

\[ \text{MPC}_{\tau, x} = \frac{C_\tau(a, b + x, z) - C_\tau(a, b, z)}{x} \]
Marginal propensities to consume

Theoretical MPC distribution

Quarterly MPC distribution

Annual MPC distribution

Quarterly Responses to $500 Rebate
MPC heterogeneity

Theoretical MPC

Quarterly MPC $500
Channels for monetary policy

Innovation $\epsilon < 0$ to the Taylor rule: $i = \bar{r}^b + \phi \pi + \epsilon$
Channels for monetary policy

Innovation $\epsilon < 0$ to the Taylor rule: 

$$i = \bar{r}^b + \phi \pi + \epsilon$$

Direct effects from $r^b \downarrow$

1. Intertemporal substitution: $C \uparrow$ (households on EE)

2. Cash flow effect (or income, or URE): $C \downarrow$ (savers), $C \uparrow$ (borrowers)
Channels for monetary policy

Innovation $\epsilon < 0$ to the Taylor rule: 

\[ i = \bar{r}^b + \phi \pi + \epsilon \]

Direct effects from $r^b \downarrow$

1. Intertemporal substitution: $C \uparrow$ (households on EE)

2. Cash flow effect (or income, or URE): $C \downarrow$ (savers), $C \uparrow$ (borrowers)

Indirect effects

3. Aggregate demand effect $C \uparrow$ (high MPC households)
   - Through labor market: $C \uparrow \rightarrow L^d \uparrow \rightarrow w \uparrow$

4. Asset returns effect $C \uparrow$ (households with high $a$)
   - Through fund leverage $r^b \downarrow \rightarrow r^a \uparrow$
IRF: Monetary policy shock I

- Liquid return ($r^b$), % pa
- Consumption ($C$), % dev
- Output ($Y$), % dev
- Real wage ($w$), % dev
- Inflation ($\pi$), % pa
- Illiquid return ($r^a$), % pa
IRF: Monetary policy shock II

Investment ($I$), % dev

Hours ($H$), % dev

Portfolio ($A, B$), % dev

Markup ($\mu$), %

Rental rate ($r$), % pa

Nominal rate ($i$), % pa
Which channel of monetary policy?

- Intertemporal substitution channel is tiny
- Financial intermediation channel is large
- Aggregate demand effect through labor demand is huge
Consumption responses by wealth

- Consumption response is twice as large for the bottom quartile of net worth distribution compare to the top quartile.
MP channels across the wealth distribution

- Even high net worth households have a small substitution channel
- Q4 households: mean $90k liquid wealth, $1m illiquid wealth
Final thoughts

- Heterogeneous Agent New Keynesian models: a new framework for quantitative analysis of monetary policy

- Consistency with income, wealth and MPC distributions allows:
  - Analysis of broader set of channels for policy
  - Distributional analysis of policies

- Policy implications: transmission of nominal rates to real rates is not enough for real effects, crucial that real rates affect labor demand.
Intermediate good firm pricing problem

$$\max_{\{p_t\}_{t \geq 0}} \int_0^\infty e^{-rt} \left\{ \Pi_t(p_t) - \Theta_t \left( \frac{\dot{p}_t}{p_t} \right) \right\} dt \quad \text{s.t.}$$

$$\Pi(p) = \left( \frac{p}{\overline{P}} - m \right) \left( \frac{p}{\overline{P}} \right)^{-\varepsilon} Y$$

$$m = \frac{1}{Z} \left( \frac{r}{\alpha} \right)^{\alpha} \left( \frac{w}{1-\alpha} \right)^{1-\alpha}$$

$$\Theta(\pi) = \frac{\theta}{2} \pi^2 Y$$
Investment fund optimization problem

**Objective:** Maximize PDV of dividends

\[
\max \int_0^\infty e^{-\int_0^t \lambda s \, ds} D_t \, dt \quad \text{s.t.}
\]

\[
D + I + \dot{A} + \dot{B} = rK + qK + aA + bB
\]

\[
\dot{K} = I - \delta K, \quad -B \leq \zeta K
\]

- \(q\): dividend rate paid by intermediate firms (\(\propto\) no. of shares)

- **Fund's problem (stationary version)**

\[
\Lambda V(W) = \max_{D,K,A,B} D + V'(W) \dot{W}
\]

\[
\dot{W} = (r + q - \delta)K + aA + bB - D
\]

\[
W = K + A + B, \quad -B \leq \zeta K
\]

- **Fund balance sheet:** assets \(K\), liabilities \(-A - B\), net worth \(W\)
THANK YOU FOR ATTENDING!