Frictional Wage Dispersion in Search Models: A Quantitative Assessment

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“Residual” wage distribution for janitors in Philadelphia

Mean / first percentile = 2.24
Why are similar workers paid differently?

- Unobserved heterogeneity
  - Human capital
    (e.g., “innate” ability)
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- Compensating differentials
  - Job characteristics
    (e.g., non-pecuniary amenities)
Why are similar workers paid differently?

• Unobserved heterogeneity
  ▶ Human capital
    (e.g., “innate” ability)

• Compensating differentials
  ▶ Job characteristics
    (e.g., non-pecuniary amenities)

• Labor market luck
  ▶ Search/matching frictions ➔ “frictional wage dispersion”
Outline of the talk

1. **New tool** to study frictional wage dispersion in search models

   Mean-min ratio
Outline of the talk

1. **New tool** to study frictional wage dispersion in search models
   
   **Mean-min ratio**

2. Calibrate the “textbook” model ⇒ *frictional* dispersion is *very small*

3. Data detour: *residual* wage dispersion is *larger by a factor of 20!*

4. Extensions of the textbook model: risk-aversion, volatile wages, OJS, OJS with endogenous search effort

5. Relation to empirical search literature and to Shimer-Hall puzzle
McCall-Mortensen search model (continuous time)

- Homogeneous workers, infinitely lived, risk-neutral
- Discount rate $r$, flow value of unemployment $b$
- Wage remains constant on the job
- Exogenous job separation (into unemployment) at rate $\sigma$
- Search only during unemployment
- At rate $\lambda_u$, unemployed workers encounter wage offers drawn from the exogenous distribution $F(w)$
Solution of the model

- Flow values of employment and unemployment

\[ rW(w) = w - \sigma [W(w) - U] \]

\[ rU = b + \lambda_u \int_{w^*}^{w_{\text{max}}} [W(w) - U] dF(w) \]
Solution of the model

- Flow values of employment and unemployment

\[ rW (w) = w - \sigma [W (w) - U] \]

\[ rU = b + \lambda_u \int_{w^*}^{w_{\text{max}}} [W (w) - U] dF (w) \]

- The reservation wage equation is:

\[ w^* = rU = b + \frac{\lambda_u}{r + \sigma} \int_{w^*}^{w_{\text{max}}} [w - w^*] dF (w) \]
Solution of the model

- **WLOG**, let \( b \equiv \rho \bar{w} \), with \( \bar{w} = E[w | w \geq w^*] \), then:

\[
\begin{align*}
    w^* &= \rho \bar{w} + \frac{\lambda_u [1 - F(w^*)]}{r + \sigma} \int_{w^*}^{w_{\text{max}}} \left[w - w^*\right] \frac{dF(w)}{1 - F(w^*)} \\
    &= \rho \bar{w} + \frac{\lambda^*_u}{r + \sigma} [\bar{w} - w^*]
\end{align*}
\]

★ where \( \lambda^*_u \equiv \lambda_u [1 - F(w^*)] \)
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&= \rho \bar{w} + \frac{\lambda_u^*[\bar{w} - w^*]}{r + \sigma}
\end{align*}
\]

- where \( \lambda_u^* \equiv \lambda_u[1 - F(w^*)] \)

- Last step eliminates \( F(w) \) which is **unobservable** through the job finding rate \( \lambda_u^* \)
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& = \rho \bar{w} + \frac{\lambda_u^*}{r + \sigma} [\bar{w} - w^*]
\end{align*}
\]

▷ where \( \lambda_u^* \equiv \lambda_u[1 - F(w^*)] \)

• Last step eliminates \( F(w) \) which is unobservable through the job finding rate \( \lambda_u^* \)

• Equation relates average wage \( \bar{w} \) and lowest wage paid \( w^* \) only through observables \( (r, \sigma, \lambda_u^*, \rho) \)
Mean-min ratio (Mm)

\[ Mm \equiv \frac{\bar{w}}{w^*} = \frac{\lambda_u^*}{r + \sigma} + 1 + \rho \]

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 8/40
Mean-min ratio (Mm)

\[ Mm \equiv \frac{\bar{w}}{w^*} = \frac{\lambda_u^*}{r + \sigma} + 1 + \frac{\rho}{\lambda_u^* + \rho} \]

- New measure of wage dispersion in search models
- “Distribution-free” measure [i.e., does not depend on \( F(w) \)]
- Derived naturally from reservation wage equation
- Increasing in \((\sigma, r)\), decreasing in \((\rho, \lambda_u^*)\)
Equilibrium search models

- Lucas-Prescott (1974) island-model
  - Random search across islands
  - Competitive labor market on each island
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- Pissarides (1990) matching model
  - Aggregate matching function
  - Free entry of firms
  - Nash bargaining
Equilibrium search models

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  ▶ Nash bargaining

• Both models imply that same expression for \( M_m \)
Quantitative implications for $Mm$

\[ Mm = \frac{\lambda^*_u}{r+\sigma} + 1 \]

- **Calibration** of model to U.S. economy (monthly frequency)
  - Interest rate: $r = 0.0041$ (Cooley, 1995)
  - Separation rate: $\sigma = 0.020$ (Shimer, 2005a)
  - Job finding rate: $\lambda^*_u = 0.39$ (Shimer, 2005a)
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$$Mm = \frac{\frac{\lambda_u^*}{r+\sigma}}{\frac{\lambda_u^*}{r+\sigma} + \rho} + 1$$

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  - Flow value of unemployment: $\rho = 0.40$ (Shimer, 2005b)
Quantitative implications for $Mm$

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$$Mm = 1.036$$
Some unpleasant search arithmetic

\[ Mm = \frac{\frac{\lambda^u}{r+\sigma}}{\frac{\lambda^u}{r+\sigma}} + 1 \]

\[ = \frac{\frac{0.39}{0.0041+0.02}} {\frac{0.39}{0.0041+0.02}} + \rho \]

\[ = 16.2 + 1 \]

\[ \leq \frac{17.2}{16.2} = 1.062 \]

... if we are willing to accept that \( \rho \geq 0 \)

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 11/40
Interpretation: “good things come to those who wait”

• Unemployed workers search longer (and turn down jobs) if there is a large option value of waiting for better offers

• Option value is determined by wage dispersion

• Short unemployment duration, as in U.S. data, reveals that dispersion is small!!
Interpretation: “good things come to those who wait”

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- Option value is determined by wage dispersion.

- Short unemployment duration, as in U.S. data, reveals that dispersion is small!!

- How large is frictional wage dispersion in the data?
Measurement of residual wage dispersion

• Empirical counterpart of theory: *wage observations for ex-ante similar workers searching in the same labor market*

• Three key data issues:

  1. Definition of *labor market*: occupation/geographical area
Measurement of residual wage dispersion

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- Three key data issues:
  1. Definition of labor market: occupation/geographical area
  2. Control for ex-ante individual heterogeneity
     - observable: e.g., education, experience, gender, race
     - unobservable: e.g., innate workers’ characteristics
Measurement of residual wage dispersion

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  2. Control for ex-ante individual heterogeneity
     ▶ observable: e.g., education, experience, gender, race
     ▶ unobservable: e.g., innate workers’ characteristics
  3. Measurement error in hourly wages
Three sources of micro data

1. 5% sample of 1990 US Census (~ 4,600,000 obs.)
   - 487 occupations, 799 geographical areas
   - controls: gender, race, edu, exp, marital status
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   - 637 occupations, 337 metropolitan areas
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Three sources of micro data

1. 5% sample of 1990 US Census (≈ 4,600,000 obs.)
   - 487 occupations, 799 geographical areas
   - controls: gender, race, edu, exp, marital status

2. Occupational Employment Statistics (≈1,200,000 establishments)
   - 637 occupations, 337 metropolitan areas
   - no controls available, but no reporting errors

3. PSID (≈ 80,000 obs.)
   - controls: gender, race, edu, exp, marital status, region, occ, occ × region
   - panel data ⇒ control for fixed individual effects
## Empirical Findings

### Table 1: Dispersion measures for hourly wage from various data sources

<table>
<thead>
<tr>
<th>Min. obs. per cell</th>
<th>Number of labor mkts</th>
<th>Ratio of mean wage to min.</th>
<th>1st pct.</th>
<th>5th pct.</th>
<th>10th pct.</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel Study of Income Dynamics 1990-1996</td>
<td>2nd stage residuals</td>
<td>–</td>
<td>3.11</td>
<td>1.90</td>
<td>1.46</td>
<td>1.32</td>
</tr>
<tr>
<td>5% sample of 1990 US Census</td>
<td>Occ./Geog. Area (N≥50)</td>
<td>13,246</td>
<td>2.94</td>
<td>2.66</td>
<td>2.04</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>Full time/Full year (N≥50)</td>
<td>7,195</td>
<td>2.74</td>
<td>2.49</td>
<td>1.92</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Unskilled Occ. (N≥50)</td>
<td>1,152</td>
<td>2.51</td>
<td>2.37</td>
<td>1.98</td>
<td>1.77</td>
</tr>
</tbody>
</table>

- We target $Mm = 1.70$ and $CV = 0.30$
Distribution of Mp5 across US labor markets (Census)

13,246 cells
Median Mm=2.04
Matched employer-employee data (Denmark)

Private Firm Hourly Wage Dispersion Measures
Danish IDA Data, 1994-1995

<table>
<thead>
<tr>
<th>Occupation</th>
<th># of firms</th>
<th>1pct</th>
<th>5pct</th>
<th>10pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>113,325</td>
<td>1.62</td>
<td>1.39</td>
<td>1.29</td>
</tr>
<tr>
<td>Management</td>
<td>49,667</td>
<td>2.09</td>
<td>1.65</td>
<td>1.45</td>
</tr>
<tr>
<td>Salaried</td>
<td>57,513</td>
<td>1.60</td>
<td>1.38</td>
<td>1.30</td>
</tr>
<tr>
<td>Skilled</td>
<td>44,527</td>
<td>1.56</td>
<td>1.32</td>
<td>1.27</td>
</tr>
<tr>
<td>Unskilled</td>
<td>70,886</td>
<td>1.66</td>
<td>1.40</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 17/4
How far is the model from the data?

Pairs of \((\rho, r)\) consistent with \(Mm=1.70\)

Reasonable pairs

Net value of leisure as a fraction of \(w\) \((\rho)\)

Monthly interest rate \((r)\)

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 18/40
Why is $\rho = -3$ implausible?

• In order to avoid a week of unemployment, a worker would be willing to:
  
  ▶ work for free for a week
  
  ▶ pay $1,500 (3 times the average weekly salary)
  
  ▶ and, at the end of the week, draw a wage offer from the same distribution he would face if unemployed
Resolving the data-model discrepancy

1. Nothing wrong with the model \( \Rightarrow \) frictional wage dispersion is **negligible** compared to unobserved workers’ heterogeneity

   • Bad news for search theory?
Resolving the data-model discrepancy

1. Nothing wrong with the model $\Rightarrow$ frictional wage dispersion is negligible compared to unobserved workers’ heterogeneity
   - Bad news for search theory?

2. Nothing wrong with the model + workers truly hate unemployment $\Rightarrow$ frictional wage dispersion is large
   - Bad news for the rest of macro-labor?
Resolving the data-model discrepancy

1. Nothing wrong with the model ⇒ frictional wage dispersion is negligible compared to unobserved workers’ heterogeneity
   - Bad news for search theory?

2. Nothing wrong with the model + workers truly hate unemployment ⇒ frictional wage dispersion is large
   - Bad news for the rest of macro-labor?

3. Model fails quantitatively ⇒ it needs to be augmented
   - We take this route here, but first...
In defense on the textbook model

1. "Model designed to study unemployment, not wage dispersion"

   • Given $Mm = 1.7$, model predicts expected unemployment duration of 91 months!!
In defense on the textbook model

1. "Model designed to study unemployment, not wage dispersion"
   
   • Given $Mm = 1.7$, model predicts expected unemployment duration of 91 months!!

2. "The model will perform better on European data"
   
   • In steady-state: $\lambda_u^* u = \sigma(1 - u)$

$$Mm = \frac{\lambda_u^*}{r+\sigma} + 1 \quad S.S. \quad \frac{\sigma}{r+\sigma} \frac{1-u}{u} + 1 \quad r \overset{\text{small}}{\sim} \frac{1-u}{u} + 1 \quad \rho > 0 \quad < \frac{1}{1-u}$$
In defense on the textbook model

1. "Model designed to study unemployment, not wage dispersion"
   - Given $Mm = 1.7$, model predicts expected unemployment duration of 91 months!!

2. "The model will perform better on European data"
   - In steady-state: $\lambda^*_u u = \sigma (1 - u)$

   $Mm = \frac{\lambda^*_u}{r+\sigma} + \rho \underset{S.S.}{=} \frac{\sigma}{r+\sigma} \frac{1-u}{u} + 1 \quad r \quad \text{small} \quad \approx \quad \frac{1-u}{u} + 1 \quad \rho > 0 \quad < \quad \frac{1}{1-u}$

3. "Mm ratio is an unconventional measure of dispersion"
Using the “Mm tool” in richer search models

1. Risk-aversion

2. Volatile wages during employment

3. On-the-job search

The critical “rule of the game” is not adding unobserved heterogeneity
Risk Aversion

- Let preferences be $u(c) = c^{1-\theta} / (1-\theta)$, and let $c = w$

- Upper bound for effects of risk-aversion: no self-insurance

- Second-order Taylor expansion of reservation wage equation:

$$ Mm \simeq \left[ \frac{\left(1 + \frac{1}{2} \theta (\theta - 1) CV(w)^2\right) \frac{\lambda_u^*}{r+\sigma} + \rho^{1-\theta}}{\frac{\lambda_u^*}{r+\sigma} + 1} \right]^{\frac{1}{\theta-1}} $$

- **Calibration**: only new number we need is $CV(w) = 0.30$
Numerical results for the model with risk-aversion

Pairs $(\rho, \theta)$ consistent with $M_m = 1.70$

Relative risk aversion ($\theta$)
Wage shocks during employment (M-P, 1994)

- Wages fluctuate randomly along the employment spell
- At rate $\delta$, employed workers draw new wage from $F(w)$
- Endogenous separations at rate $\sigma^* = \delta F(w^*)$
Wage shocks during employment (M-P, 1994)

- Wages fluctuate randomly along the employment spell
- At rate $\delta$, employed workers draw new wage from $F(w)$
- Endogenous separations at rate $\sigma^* = \delta F(w^*)$
- Solve the model and obtain...

$$Mm = \frac{\lambda_u - \delta + \sigma^*}{r + \delta} + 1 + \rho$$

- As $\delta \to \lambda_u$, $Mm \to 1/\rho$

  $\triangleright \quad 1 - \delta$: autocorrelation coefficient of the wage process
Numerical results for the model with stochastic wages

Pairs of $(1-\delta, \rho)$ consistent with $M_m=1.70$

Reasonable pairs

Net value of leisure as a fraction of $w (\rho)$

Annual autocorrelation coefficient of wage process $(1-\delta)$
On-the-job search (Burdett, 1978)

• Workers draw wage offers from $F(w)$ at rate $\lambda_u$ if unemployed, and at rate $\lambda_w$ if employed

  ▶ $F(w)$ can be any wage offer distribution

• When employed, accept offer $w'$ if $w' > w$: $F(w) \Rightarrow G(w)$
On-the-job search (Burdett, 1978)

- Workers draw wage offers from \( F(w) \) at rate \( \lambda_u \) if unemployed, and at rate \( \lambda_w \) if employed

  - \( F(w) \) can be any wage offer distribution

- When employed, accept offer \( w' \) if \( w' > w \): \( F(w) \Rightarrow G(w) \)

- Solve the model for the realized steady-state wage distribution \( G(w) \) and, for \( r \) small, obtain...

\[
Mm \approx \frac{\lambda_u^* - \lambda_w}{r + \sigma + \lambda_w} + 1 + \frac{\lambda_u^* - \lambda_w}{r + \sigma + \lambda_w} + \rho
\]

  - As \( \lambda_w \rightarrow \lambda_u \), \( Mm \rightarrow 1/\rho \)
Restricting the value of $\lambda_w$

- **Average separation rate** $\chi$ in the model is:

  $$\chi = \sigma + \lambda_w \int_{w^*} [1 - F(w)] dG(w) = \frac{\sigma (\lambda_w + \sigma) \log \left( \frac{\sigma + \lambda_w}{\sigma} \right)}{\lambda_w}$$

- **BLS (JOLTS):** monthly separation rate $\chi = 0.04$
Restricting the value of $\lambda_w$

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- **BLS (JOLTS):** monthly separation rate $\chi = 0.04$

- **Set $\lambda_w$** to be consistent with labor mobility facts and explore implications for **Mm ratio**
Numerical results for the model with on the job search

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 29/40
Numerical results for the model with on the job search

Net value of leisure as a fraction of \( w (\rho) \)

-8
-7
-6
-5
-4
-3
-2
-1
0
1

Monthly offer arrival rate on the job \( (\lambda_w) \)

-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0

-8
-7
-6
-5
-4
-3
-2
-1
0
1

Iso-Mm curves

Blue: Mm=1.7
Green: Mm=1.2
Purple: Mm=2.2

Hornstein-Krusell-Violante, “Frictional Wage Dispersion” – p. 30/40
Endogenous search effort (CLMNW, 2005)

- Unemployed and employed workers choose search effort, i.e., contact rate $\lambda$

- Every worker faces the same search effort cost function $c(\lambda)$, with $c'>0$, $c''>0$

- Reservation wage: $w^* = b \Rightarrow Mm = \frac{1}{\rho} = 2.5$
Endogenous search effort (CLMNW, 2005)

- Unemployed and employed workers choose search effort, i.e., contact rate $\lambda$

- Every worker faces the same search effort cost function $c(\lambda)$, with $c' > 0$, $c'' > 0$

- Reservation wage: $w^* = b \Rightarrow Mm = \frac{1}{\rho} = 2.5$

- Disutility of unemployment is now: $b - c(\lambda^*)$

- Assume $c(\lambda) = \lambda^\gamma$, with $\gamma = 2$. Then, for the unemployed worker:

$$\rho = \frac{b - c(\lambda^*)}{\bar{w}} \in (-0.5, -0.3)$$

...still negative!
A journey through the empirical search literature
A journey through the empirical search literature

• Either accept implausible parameter estimates...
  ▶ Postel-Vinay and Robin (ECA, 2002): for unskilled occupations, $r = 57\%$ per year
  ▶ Flinn (ECA, 2006): for realistic values of $r, \rho = -4$
A journey through the empirical search literature

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• ...or need large unobserved heterogeneity
Cross-section vs. time-series

- **Shimer-Hall**: search model does not generate enough business-cycle volatility in vacancies and unemployment

- **Hagedorn-Manovskii**: we can save the standard search model as long as $\rho \approx 1$

- **Our paper**: if $\rho \approx 1$, no hope to get any frictional wage dispersion from search models

- Tension between time-series and cross-sectional implications of search model
Concluding remarks

**Mm ratio**: new, powerful tool to analyze wage dispersion in a large class of search models ⇒ **three possible conclusions**:

1. Data still contain **unobserved worker heterogeneity**
   - Frictional wage dispersion is **small** in both data and model
Concluding remarks

**Mm ratio**: new, powerful tool to analyze wage dispersion in a large class of search models ⇒ three possible conclusions:

1. Data still contain *unobserved worker heterogeneity*
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2. Data OK, but workers *hate* unemployment
   - $\rho < 0$ *implausible*, plus watch for *business-cycle* implications!
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2. Data OK, but workers **hate** unemployment
   - $\rho < 0$ **implausible**, plus watch for **business-cycle** implications!

3. Data OK, and $\rho$ is moderately high
   - Basic search model fails, **latest OJS models** more promising
How far is the model from the data?

Net value of leisure as a fraction of $w$ ($\rho$) vs. Monthly interest rate ($r$).

Iso-$M_m$ curves for $M_m = 1.7$, $M_m = 1.2$, and $M_m = 2.2$.
Distribution of individual Mm (PSID)

14,572 observations

Median Mm=1.57
Relation between $Mm$ and CV

- Assume **Gamma distribution**

\[
g(w; w^*, \beta, \gamma) = \frac{(\frac{w-w^*}{\beta})^{\gamma-1} \exp(-\frac{w-w^*}{\beta})}{\beta \Gamma(\gamma)}
\]
Relation between $Mm$ and CV

• Assume Gamma distribution

$$g(w; w^*, \beta, \gamma) = \frac{(\frac{w-w^*}{\beta})^{\gamma-1} \exp(-\frac{w-w^*}{\beta})}{\beta \Gamma(\gamma)}$$

• Then, we can show that:

$$CV(w) = \frac{1}{\sqrt{\gamma}} \frac{Mm(w) - 1}{Mm(w)}$$

• Data: $Mm(w) = 1.70$ and $CV(w) = 0.30 \Rightarrow \hat{\gamma} = 1.88$

• Search model: $Mm = 1.036 \Rightarrow CV = 0.025$
Numerical results for the model with risk-aversion

Net value of leisure as a fraction of $w$ ($\rho$)

Relative risk aversion ($\theta$)

Iso–Mm curves

Hornstein-Krusell-Violante, “Frictional Wage Dispersion” – p. 38/40
Numerical results for the model with stochastic wages

Annual autocorrelation coefficient of wage process \((1-\delta)\)

Net value of leisure as a fraction of \(w\) \((\rho)\)

Iso-Mm curves

- Mm=1.7
- Mm=1.2
- Mm=2.2
Reallocation shocks (Nagypal, 2005)

- Baseline OJS model, plus...

- At rate $\phi$, employed workers make a wage draw from $F(w)$ which either they accept, or they separate

\[
Mm \approx \frac{\lambda^*_u - \lambda_w - \phi}{r + \sigma + \lambda_w + \phi} + \frac{1}{\lambda^*_u - \lambda_w - \phi + \rho} + \rho
\]

- Restrict $(\lambda_w, \phi)$ to match: (i) $Mm = 1.70$ and (ii) sep. rate of 4%
Reallocation shocks (Nagypal, 2005)

- Baseline OJS model, plus...

- At rate $\phi$, employed workers make a wage draw from $F(w)$ which either they accept, or they separate

$$Mm \simeq \frac{\lambda^*_u - \lambda_w - \phi}{r + \sigma + \lambda_w + \phi} + 1 \frac{\lambda^*_u - \lambda_w - \phi}{r + \sigma + \lambda_w + \phi} + \rho$$

- Restrict $(\lambda_w, \phi)$ to match: (i) $Mm = 1.70$ and (ii) sep. rate of 4%

- Monthly rate at which workers are subject to a wage cut is:

$$\kappa = \phi \int F(w) dG(w) = 0.095$$