Insurance and Opportunities: 
The Welfare Implications of Rising Wage Dispersion

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Motivation

• Welfare analysis in heterogeneous agents models with incomplete insurance against idiosyncratic risk is central to macroeconomics
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- Examples: welfare effects of a...
  
  1. change in the amount of risk (*technology*)
    
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  2. change in the **insurability** of risk (*markets*)
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• Examples: welfare effects of a...

  1. change in the *amount* of risk (*technology*)

  2. change in the *insurability* of risk (*markets*)

  3. change in redistributive *policies* (*government*)
     - Long list... related to welfare costs of business cycles (Lucas, 2003)
Contributions

1. Tractable framework delivering *transparent mapping* between primitives of economy (preferences, risk, insurance market structure) and welfare expressions
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2. Role of flexible labor supply: insurance vs. opportunities
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2. Role of flexible labor supply: insurance vs. opportunities

3. Alternative representation of welfare effects based on changes in observable cross-sectional moments
Outline of the Talk

1. Baseline economy with Cobb-Douglas preferences and simple statistical representation of individual risk
   - Equilibrium allocations
   - Welfare expressions for 3 thought experiments
   - Alternative representation for welfare effects

2. Some illustrative calculations

3. Extension to richer process for individual risk
   - Tractability preserved by no-bond-trade equilibrium (Constantinides-Duffie, 1996)

4. Separable preferences
The Economy

- **Demographics and preferences**: Continuum of agents with time-separable preferences

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^n (1 - h_t)^{1-\eta}^{1-\theta} - 1}{1 - \theta} \right)
\]
The Economy

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  \]

- **Endowments**: initial wealth is zero for all agents and assets are in zero net supply
The Economy

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\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^n (1 - h_t)^{1-n} (1 - \eta)_{1-\theta}}{1 - \theta} - 1 \]

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- **Technology**: linear in aggregate hours weighted by efficiency-units of labor
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- **Endowments**: initial wealth is zero for all agents and assets are in zero net supply

- **Technology**: linear in aggregate hours weighted by efficiency-units of labor

- **Labor market**: competitive, hourly wages equal individual labor productivities
Individual Productivity Shocks

• Two orthogonal log-Normally distributed components

\[ \log w = \alpha + \varepsilon_t \]

\[ \alpha \sim N \left( -\frac{v_\alpha}{2}, v_\alpha \right), \quad \varepsilon_t \sim N \left( -\frac{v_\varepsilon}{2}, v_\varepsilon \right) \text{ i.i.d.} \]

• Hence:

\[ \log w(\alpha, \varepsilon_t) = (\alpha + \varepsilon_t) \sim N \left( -\frac{v}{2}, v \right), \text{ with } E[w] = 1 \]

• We model \( \alpha \) as a permanent individual effect and \( \varepsilon_t \) as i.i.d. shock (Gottschalk-Moffitt, 1994)
Asset Market Structure

• Three distinct structures:

  1. **Autarky**: no financial instruments
Asset Market Structure

- Three distinct structures:
  
  1. **Autarky**: no financial instruments

  2. **Complete markets**: complete insurance against either component of the wage shock

     - Trade opens before the realization of $\alpha$
Asset Market Structure

- Three distinct structures:

1. **Autarky**: no financial instruments

2. **Complete markets**: complete insurance against either component of the wage shock
   - *Trade opens before the realization of* $\alpha$

3. **Incomplete markets**: no insurance against the permanent component of wages, complete insurance against transitory shocks
   - *Trade opens after the realization of* $\alpha$
Households literally have access to insurance against some shocks, but not others

Incomplete-Markets Economy: Interpretations

- Households literally have access to insurance against some shocks, but not others

- Ex-post complete markets with ex-ante heterogeneous agents
Incomplete-Markets Economy: Interpretations

- Households literally have access to insurance against some shocks, but not others

- Ex-post complete markets with ex-ante heterogeneous agents

- Economy with a non-contingent bond (i.e., “Bewley economy”) where precautionary saving and borrowing allow smoothing shocks that aren’t too persistent
  - Numerical comparison of two economies → good approximation
Properties of Cobb-Douglas Preferences

\[ u(c, h) = \frac{c^\eta (1 - h)^{1-\eta}^{1-\theta} - 1}{1 - \theta} \]
Properties of Cobb-Douglas Preferences

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- Coefficient of relative risk aversion: \( \tilde{\gamma} \equiv 1 - \eta + \eta \theta \)
Properties of Cobb-Douglas Preferences

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- Coefficient of relative risk aversion: \( \bar{\gamma} \equiv 1 - \eta + \eta \theta \)

- Frisch labor supply elasticity: \( \phi \equiv \lambda \frac{1-h}{h} \)

  ▶ where \( \lambda \equiv \frac{1-\eta+\eta \theta}{\theta} \) is the Frisch elasticity of leisure

▶ Non-stochastic Frisch labor supply elasticity:

\[ \bar{\phi} = \lambda \cdot \frac{1-\eta}{\eta} > 1 - \eta \]
Properties of Cobb-Douglas Preferences

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▷ Non-stochastic Frisch labor supply elasticity:

\[ \bar{\phi} = \lambda \cdot \frac{1-\eta}{\eta} > 1 - \eta \]

• \((c, 1 - h)\) substitutes when \( \theta > 1(\lambda < 1) \)
Equilibrium Allocations

• Autarky

\[
\log c_{AUT}(\alpha, \varepsilon) = \log(\eta) + \alpha + \varepsilon
\]

\[
\log l_{AUT}(\alpha, \varepsilon) = \log(1 - \eta)
\]
Equilibrium Allocations

• Autarky

\[ \log c_{AUT}(\alpha, \varepsilon) = \log(\eta) + \alpha + \varepsilon \]
\[ \log l_{AUT}(\alpha, \varepsilon) = \log(1 - \eta) \]

• Complete markets

\[ \log c_{CM}(\alpha, \varepsilon) = \log(\eta) + \lambda(1 - \lambda)\frac{v}{2} + (1 - \lambda)(\alpha + \varepsilon) \]
\[ \log l_{CM}(\alpha, \varepsilon) = \log(1 - \eta) + \lambda(1 - \lambda)\frac{v}{2} - \lambda(\alpha + \varepsilon) \]
Equilibrium Allocations

- **Autarky**

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  \log c_{AUT}(\alpha, \varepsilon) = \log(\eta) + \alpha + \varepsilon \\
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  \]

- **Incomplete markets**

  \[
  \log c_{IM}(\alpha, \varepsilon) = \log(\eta) + \lambda(1 - \lambda)\frac{\nu_{\varepsilon}}{2} + \alpha + (1 - \lambda)\varepsilon \\
  \log l_{IM}(\alpha, \varepsilon) = \log(1 - \eta) + \lambda(1 - \lambda)\frac{\nu_{\varepsilon}}{2} - \lambda\varepsilon
  \]

- **Individual wealth is always zero**
Welfare Analysis

- $\omega_m$: welfare change of a change in labor market risk $(\Delta v_\alpha, \Delta v_\varepsilon)$

$$\int_{A \times \mathcal{E}} u \left((1 + \omega_m) c_m, h_m\right) d\mathcal{f}(\alpha, \varepsilon) = \int_{A \times \mathcal{E}} u \left(\hat{c}_m, \hat{h}_m\right) d\hat{\mathcal{f}}(\alpha, \varepsilon)$$
Welfare Analysis

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- $\chi_m$: welfare change from completing markets
  $(\Delta v_\alpha = -v_\alpha, \Delta v_\varepsilon = v_\alpha)$

$$\int_{A \times \mathcal{E}} u ((1 + \chi_m) c_m, h_m) \, df(\alpha, \varepsilon) = \int_{A \times \mathcal{E}} u (c_{CM}, h_{CM}) \, df(\alpha, \varepsilon)$$
Welfare Analysis

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- $\chi_m$: welfare change from completing markets $(\Delta v_\alpha = -v_\alpha, \Delta v_\varepsilon = v_\alpha)$

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\int_{A \times E} u((1 + \chi_m) c_m, h_m) \, df(\alpha, \varepsilon) = \int_{A \times E} u(c_{CM}, h_{CM}) \, df(\alpha, \varepsilon)
\]

- $\kappa_m$: welfare change from eliminating risk $(\Delta v_\alpha = -v_\alpha, \Delta v_\varepsilon = -v_\varepsilon)$

\[
\int_{A \times E} u((1 + \kappa_m) c_m, h_m) \, df(\alpha, \varepsilon) = u(\bar{c}, \bar{h})
\]
Welfare Expressions

• Welfare effect of a change in labor market risk

\[ \omega_{AUT} \simeq -\bar{\gamma} \frac{\Delta v}{2} \]
\[ \omega_{CM} \simeq \bar{\phi} \frac{\Delta v}{2} \]
\[ \omega_{IM} \simeq \bar{\phi} \frac{\Delta v_\varepsilon}{2} - \bar{\gamma} \frac{\Delta v_\alpha}{2} \]
Welfare Expressions

• Welfare effect of a change in labor market risk

\[
\omega_{AUT} \simeq -\bar{\gamma} \frac{\Delta v}{2} \quad \omega_{CM} \simeq \bar{\phi} \frac{\Delta v}{2} \quad \omega_{IM} \simeq \bar{\phi} \frac{\Delta v_\varepsilon}{2} - \bar{\gamma} \frac{\Delta v_\alpha}{2}
\]

• Welfare gain from completing markets

\[
\chi_{AUT} \simeq (\bar{\phi} + \bar{\gamma}) \frac{v}{2} \quad \chi_{IM} \simeq (\bar{\phi} + \bar{\gamma}) \frac{v_\alpha}{2}
\]
Welfare Expressions

• Welfare effect of a change in labor market risk

\[ \omega_{AUT} \simeq -\gamma \frac{\Delta v}{2} \quad \omega_{CM} \simeq \phi \frac{\Delta v}{2} \quad \omega_{IM} \simeq \phi \frac{\Delta v_\varepsilon}{2} - \gamma \frac{\Delta v_\alpha}{2} \]

• Welfare gain from completing markets

\[ \chi_{AUT} \simeq (\phi + \gamma) \frac{v}{2} \quad \chi_{IM} \simeq (\phi + \gamma) \frac{v_\alpha}{2} \]

• Welfare change from eliminating risk, e.g. through progressive taxation system \( \tau(w) = 1 - 1/w \)

\[ \kappa_{AUT} \simeq \gamma \frac{v}{2} \quad \kappa_{CM} \simeq -\phi \frac{v}{2} \quad \kappa_{IM} \simeq \gamma \frac{v_\alpha}{2} - \phi \frac{v_\varepsilon}{2} \]
A Caveat

- Fix $\eta = 1/3$, vary $\theta$
- $\bar{\gamma} = 1 - \eta + \eta \theta$
- $\bar{\phi} = \frac{\bar{\gamma}}{\theta}$
Equilibrium cross-sectional moments

- **Closed-form** cross-sectional variances and covariances of the joint distribution of \((w, h, c)\) in IM economy

\[
\begin{align*}
\text{cov} (\log w, \log h) &= \bar{\phi} v_{\varepsilon} \\
\text{var} (\log h) &= \bar{\phi}^2 v_{\varepsilon} \\
\text{var} (\log c) &= v_\alpha + (1 - \lambda)^2 v_{\varepsilon} \\
\text{cov} (\log c, \log h) &= (1 - \lambda) \bar{\phi} v_{\varepsilon}
\end{align*}
\]
Alternative representation of welfare change

- Use cross-sectional moments to map our expression for $\omega_m$ into an alternative representation based on “observables”

$$
\omega_m \simeq \Delta cov (\log w, \log h) \\
- \frac{\bar{\gamma}}{2} \Delta var (\log c) - \frac{1}{2\phi} \Delta var (\log h) \\
+ \frac{\bar{\gamma} - 1}{2} \Delta cov (\log c, \log h)
$$

- Alternative approach to welfare calculations
Alternative representation of welfare change

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\omega_m \simeq \Delta \text{cov} (\log w, \log h) \\
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+ \frac{\bar{\gamma} - 1}{2} \Delta \text{cov} (\log c, \log h)
$$

• Alternative approach to welfare calculations

• Percentage change in aggregate labor productivity

$$
\Delta \log(Y/H) = \Delta \text{cov} (\log w, \log h)
$$
Welfare calculations I

- Preferences: $\eta = 1/3, \theta = 4$
  - Risk aversion coefficient $\bar{\gamma} = 2$
  - Frisch labor supply elasticity $\bar{\phi} = 1$
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- Individual Risk
  - PSID (1968-1997)
  - Residual wage dispersion $v_w : 0.25 \rightarrow 0.35$
  - Transitory component $v_\varepsilon : 0.08 \rightarrow 0.13$
  - Permanent component: $v_\alpha : 0.17 \rightarrow 0.22$
Welfare calculations I

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- “Level” component: increase in aggregate productivity that mitigates the loss
- Bounds: $\omega_{AUT} = -10\%$ and $\omega_{CM} = +5\%$
### Welfare calculations I

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- Productivity gain twice as big as insurance gain
# Welfare calculations I

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- Policies eliminate also the “good risk”
Welfare calculations II

• Same preference parameters: $\tilde{\gamma} = 2, \tilde{\phi} = 1$
Welfare calculations II

• Same preference parameters: \( \hat{\gamma} = 2, \hat{\phi} = 1 \)

• From PSID data:

\[
\Delta \text{cov} (\log w, \log h) \simeq +0.012
\]

\[
\Delta \text{var} (\log h) \simeq +0.010
\]
Welfare calculations II

• Same preference parameters: $\bar{\gamma} = 2, \bar{\phi} = 1$

• From PSID data:

\[ \Delta \text{cov} (\log w, \log h) \simeq +0.012 \]
\[ \Delta \text{var} (\log h) \simeq +0.010 \]

• From CEX data:

  – Slesnick (2001), Krueger-Perri (2005), Attanasio-Battistin-Ichimura (2005): $\Delta \text{var} (\log c) \in (0.01, 0.05)$

  – Krueger-Perri (2005): $\Delta \text{cov} (\log c, \log h) \simeq -0.007$

• Result: $\omega = -2.65\%$
Economy with permanent shocks

- Individual risk process: \( \log w_t = \alpha_t + \varepsilon_t \)

- Uninsurable component:

\[
\begin{align*}
\alpha_t &= \pi_t + \psi, \quad \psi \sim N \left( -\frac{v_\psi}{2}, v_\psi \right) \\
\pi_t &= \pi_{t-1} + \omega_t, \quad \omega_t \sim N \left( -\frac{v_\omega}{2}, v_\omega \right) \quad \text{and} \quad \pi_0 = 0
\end{align*}
\]
Economy with permanent shocks

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  \end{align*}
  \]

- **Insurable component:**
  \[
  \begin{align*}
  \varepsilon_t &= \pi_t^* + \zeta_t, \quad \zeta_t \sim N \left( -\frac{v_\zeta}{2}, v_\zeta \right) \text{ i.i.d.} \\
  \pi_t^* &= \pi_{t-1}^* + \omega_t^*, \quad \omega_t^* \sim N \left( -\frac{v_{\omega^*}}{2}, v_{\omega^*} \right) \text{ and } \pi_0^* = 0
  \end{align*}
  \]
Market Structure: An Island-Economy Interpretation

- At $t$, agents born onto islands indexed by $\psi, \{\omega_s\}, s = t, \ldots, \infty$

- **Within island:** agents can trade full set of Arrow securities paying one unit of consumption at $t + 1$, for each $(\omega_{t+1}, \omega^*_{t+1}, \zeta_{t+1})$
• At $t$, agents born onto islands indexed by $(\psi, \{\omega_s\}), s = t, \ldots, \infty$

• **Within island:** agents can trade full set of Arrow securities paying one unit of consumption at $t + 1$, for each $(\omega_{t+1}, \omega_{i+1}^*, \zeta_{t+1})$

• **Between islands:** agents can trade non-contingent bond

• Model nests complete markets $(v_\psi = v_\omega = 0)$ and Bewley economy $(v_{\omega^*} = v_\zeta = 0)$

• Perfect annuity markets
No-bond-trading Equilibrium

• Expected growth in marginal utility of consumption is the same across all islands

• Let $\beta \delta = 1/(1 + \rho)$, then the equilibrium interest rate satisfies:

$$\rho - r^* \simeq \bar{\gamma}(1 + \bar{\gamma})\frac{v_\omega}{2} + \bar{\gamma}(\lambda - 1)\frac{v_\omega^*}{2}$$
No-bond-trading Equilibrium

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- Constantinides and Duffie (1996) extended to:
  - flexible labor supply
  - groups of population allowed to perfectly pool risk
No-bond-trading Equilibrium

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- Constantinides and Duffie (1996) extended to:
  - flexible labor supply
  - groups of population allowed to perfectly pool risk

- Same allocations: $c(\alpha_t, \varepsilon_t)$, $l(\alpha_t, \varepsilon_t)$ and zero wealth
Welfare Effect of Change in Labor Market Risk

\[ \omega_{IM} = \bar{\phi} \left( \frac{\Delta v_\zeta}{2} + \mu \frac{\Delta v_{\omega^*}}{2} \right) - \bar{\gamma} \left( \frac{\Delta \psi}{2} + \mu \frac{\Delta v_\omega}{2} \right) \]

where the lifetime multiplier \( \mu \) is given by:

\[ \mu = \frac{1}{1 - \beta \delta \exp \left( (\bar{\gamma} - 1) \left( \bar{\phi} \frac{v_{\omega^*}}{2} - \bar{\gamma} \frac{v_\omega}{2} \right) \right)} \]
Welfare Effect of Change in Labor Market Risk

\[ \omega_{IM} = \tilde{\phi} \left( \frac{\Delta v\xi}{2} + \mu \frac{\Delta v \omega^*}{2} \right) - \tilde{\gamma} \left( \frac{\Delta \psi}{2} + \mu \frac{\Delta v \omega}{2} \right) \]

where the lifetime multiplier \( \mu \) is given by:

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- As households age:
  - Expected labor productivity grows thanks to \( v_{\omega^*} > 0 \)
  - Size of uninsurable uncertainty grows due to \( v_{\omega} > 0 \)
Welfare Effect of Change in Labor Market Risk

\[
\omega_{IM} = \phi \left( \frac{\Delta v_\zeta}{2} + \mu \frac{\Delta v_\omega^*}{2} \right) - \tilde{\gamma} \left( \frac{\Delta \psi}{2} + \mu \frac{\Delta v_\omega}{2} \right)
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\]

• As households age:
  
  ▶ Expected labor productivity grows thanks to \( v_\omega^* > 0 \)
  
  ▶ Size of uninsurable uncertainty grows due to \( v_\omega > 0 \)

• With \( \beta = 1 \) and separability (\( \tilde{\gamma} = \lambda = 1 \)): \( \mu = \frac{1}{1-\delta} \)
Separable Preferences

\[ u(c, h) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{\varphi h^{1+\sigma}}{1+\sigma} \]

- \(1/\sigma\) is the Frisch (compensated) elasticity of labor supply

- \(\frac{1-\gamma}{\gamma+\sigma}\) is the Marshallian (uncompensated) elasticity of labor supply

- \(\varphi\) measures the relative taste for leisure: WLOG, \(\varphi = 1\)
Incomplete Markets Allocations

\[
\log c(\alpha) = \left( \frac{1 + \sigma}{\sigma + \gamma} \right) \frac{v_\epsilon}{2\sigma} + \left( \frac{1 + \sigma}{\sigma + \gamma} \right) \alpha
\]

\[
\log h(\alpha, \epsilon) = -\left( \frac{\gamma}{2\sigma^2} \frac{1 + \sigma}{\sigma + \gamma} \right) v_\epsilon + \left( \frac{1 - \gamma}{\sigma + \gamma} \right) \alpha + \frac{1}{\sigma} \epsilon
\]

- Individual consumption is increasing in \(v_\epsilon\) and \(\alpha\)

- Insurable shock \(\epsilon\) and uninsurable shock \(\alpha\) have different effects on labor supply decision
Welfare Effects of Change in Labor Market Risk

\[ \omega_{CM} \approx \frac{1}{\sigma} \frac{\Delta v}{2} \]

\[ \omega_{AUT} \approx -\frac{(\gamma - 1) + \gamma (1 + \sigma)}{\sigma + \gamma} \frac{\Delta v}{2} \]

\[ \omega_{IM} \approx \frac{1}{\sigma} \frac{\Delta v_{\varepsilon}}{2} - \frac{(\gamma - 1) + \gamma (1 + \sigma)}{\sigma + \gamma} \frac{\Delta v_{\alpha}}{2} \]
Separable vs. CD Preferences

1. **Productivity gain**: with CD taken only as higher average leisure, with separability taken also as higher average consumption
Separable vs. CD Preferences

1. **Productivity gain**: with CD taken only as higher average leisure, with separability taken also as higher average consumption

2. **Autarky**: with CD there is always a welfare loss, with separability there could be a welfare gain

\[
\omega_{AUT} \simeq - \frac{(\gamma - 1) + \gamma (1 + \sigma)}{\sigma + \gamma} \frac{\Delta v}{2}
\]

- \(\gamma \in [0, 1/(2 + \sigma)] \rightarrow \omega_{AUT} > 0\)

- by continuity, since as \(\gamma \rightarrow 0, \omega_{AUT} \rightarrow \omega_{CM} > 0\)
Welfare Calculations with Separability

Welfare cost of change in wage dispersion
\( (\gamma = 2) \)

-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
-0.0

Fraction of lifetime consumption

Inverse of Frisch elasticity (\(\sigma\))

0
1
2
3
4
5
6
7
8
9
10

Based on estimates of individual risk
Based on changes in cross-sectional moments

Heathcote-Storesletten-Violante, “Insurance and Opportunities” – p. 29/30
• Tractable equilibrium framework to study consumption and labor supply with partial insurance → analytical solution

• Extensions include:
  ▶ preference heterogeneity/shocks
  ▶ time-varying risk and aggregate uncertainty
  ▶ measurement error in \((c, h, w)\)

• Given panel data on \((w, h)\) (from PSID) and cross-sectional data on consumption (from CEX), we can identify and estimate all the structural parameters of model