Redistributive Taxation in a Partial Insurance Economy

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Redistributive Taxation

• How progressive should earnings taxation be?
Redistributive Taxation

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• Arguments in favor of progressivity:
  1. Social insurance of privately-uninsurable shocks
  2. Redistribution from high to low innate ability

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Redistributive Taxation

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• Arguments in favor of progressivity:
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  2. Redistribution from high to low innate ability

• Arguments against progressivity:
  1. Distortion to distribution of labor supply
  2. Distortion to human capital investment
  3. Redistribution from low to high taste for leisure
  4. Inefficient financing of G expenditures
Ramsey Approach

Government/Planner takes policy instruments and market structure as given, and chooses the CE that yields the largest social welfare

• CE of an heterogeneous-agent, incomplete-market economy

• Nonlinear tax/transfer system

• Utilitarian social welfare function

• Valued public expenditures also chosen by the government
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*Tractable equilibrium framework* clarifies economic forces shaping the optimal degree of progressivity

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Overview of the model

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- **Steady-state analysis**
Demographics and preferences

- **Perpetual youth** demographics with constant survival probability $\delta$

- **Preferences** over consumption ($c$), hours ($h$), publicly-provided goods ($G$), and skill-investment effort ($s$):

$$ U_i = v(s_i) + \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta \delta)^t u_i(c_{it}, h_{it}, G) $$

$$ v(s_i) = -\frac{1}{\kappa_i} \frac{s_i^2}{2\mu} $$

$$ u_i(c_{it}, h_{it}, G, s_{it}) = \log c_{it} - \exp(\varphi_i) \frac{h_{it}^{1+\sigma}}{1 + \sigma} + \chi \log G $$

$$ \kappa_i \sim \text{Exp}(\eta) $$

$$ \varphi_i \sim N\left(\frac{\nu \varphi}{2}, \nu \varphi\right) $$

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Technology

• **Output** is CES aggregator over continuum of skill types:

\[
Y = \left[ \int_0^\infty N(s) \frac{\theta - 1}{\theta} ds \right]^{\theta \over \theta - 1}, \quad \theta \in (1, \infty)
\]

• Aggregate **effective hours** by skill type:

\[
N(s) = \int_0^1 I_{\{s_i = s\}} z_i h_i di
\]

• Aggregate **resource constraint**:

\[
Y = \int_0^1 c_i di + G
\]
Individual efficiency units of labor

\[ \log z_{it} = \alpha_{it} + \varepsilon_{it} \]

- \( \alpha_{it} = \alpha_{i,t-1} + \omega_{it} \) with \( \omega_{it} \sim N \left( -\frac{v_\omega}{2}, v_\omega \right) \)
  \[ \alpha_{i0} = 0 \quad \forall i \]

- \( \varepsilon_{it} \) i.i.d. over time with \( \varepsilon_{it} \sim N \left( -\frac{v_\varepsilon}{2}, v_\varepsilon \right) \)

- \( \varphi \perp \kappa \perp \omega \perp \varepsilon \) cross-sectionally and longitudinally

Heathcote-Storesetten-Violante, "Redistributive Taxation in a Partial Insurance Economy"
Individual efficiency units of labor

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- \(\varphi \perp \kappa \perp \omega \perp \varepsilon\) cross-sectionally and longitudinally

- Pre-government earnings:

\[
y_{it} = p(s_i) \times \exp(\alpha_{it} + \varepsilon_{it}) \times h_{it}
\]

determined by skill, fortune, and diligence

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Markets

- Competitive good and labor markets

- Competitive asset markets (all assets in zero net supply)
  - Non state-contingent bond
Markets

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• Competitive asset markets (all assets in zero net supply)

  ▶ Non state-contingent bond

  ▶ Full set of insurance claims against $\varepsilon$ shocks

  ■ If $v_{\varepsilon} = 0$, it is a bond economy

  ■ If $v_{\omega} = 0$, it is a full insurance economy

  ■ If $v_{\omega} = v_{\varepsilon} = v_{\phi} = 0$ & $\theta = \infty$, it is a RA economy

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- Perfect annuity against survival risk

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Budget constraints

1. **Beginning of period:** innovation $\omega$ to $\alpha$ shock is realized

2. **Middle of period:** buy insurance against $\varepsilon$:

   \[
   b = \int_{E} Q(\varepsilon)B(\varepsilon)d\varepsilon,
   \]

   where $Q(\cdot)$ is the price of insurance and $B(\cdot)$ is the quantity

3. **End of period:** $\varepsilon$ realized, consumption and hours chosen:

   \[
   c + \delta qb' = \lambda(wh)^{1-\tau} + B(\varepsilon)
   \]
Government

- Runs a two-parameter tax/transfer function to redistribute and finance publicly-provided goods $G$

- Disposable (post-government) earnings:
  \[ \tilde{y}_i = \lambda y_i^{1-\tau} \]

- Government budget constraint (no government debt):
  \[ G = \int_0^1 \left[ y_i - \lambda y_i^{1-\tau} \right] di \]

  Government chooses $(G, \tau)$, and $\lambda$ balances the budget residually
Our model of fiscal redistribution

\[ T(y_i) = y_i - \lambda y_i^{1-\tau} \]

- The parameter \( \tau \) measures the rate of progressivity:
  - \( \tau = 1 \): full redistribution \( \rightarrow \tilde{y}_i = \lambda \)
  - \( 0 < \tau < 1 \): progressivity \( \rightarrow \frac{T'(y)}{T(y)/y} > 1 \)
  - \( \tau = 0 \): no redistribution \( \rightarrow \) flat tax \( 1 - \lambda \)
  - \( \tau < 0 \): regressivity \( \rightarrow \frac{T'(y)}{T(y)/y} < 1 \)
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• Marginal tax rate monotone in earnings

• Negative average tax rates below \( y^0 = \lambda^{\frac{1}{\tau}} \)
Our model of fiscal redistribution

- CPS 2005, Nobs=52,539: $R^2 = 0.92$ and $\tau = 0.18$
Our model of fiscal redistribution

Marginal and average tax rates

- US Marginal ($\tau_{US} = 0.18$)
- US Average ($\tau_{US} = 0.18$)

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Recursive stationary equilibrium

- **Given** $(G, \tau)$, a stationary RCE is a value $\lambda^*$, asset prices $\{Q(\cdot), q\}$, skill prices $p(s)$, decision rules $s(\varphi, \kappa, 0)$, $c(\alpha, \varepsilon, \varphi, s, b)$, $h(\alpha, \varepsilon, \varphi, s, b)$, and aggregate quantities $N(s)$ such that:

  - households optimize
  - markets clear
  - the government budget constraint is balanced
Recursive stationary equilibrium

- Given \((G, \tau)\), a stationary RCE is a value \(\lambda^*\), asset prices \(\{Q(\cdot), q\}\), skill prices \(p(s)\), decision rules \(s(\varphi, \kappa, 0)\), \(c(\alpha, \varepsilon, \varphi, s, b)\), \(h(\alpha, \varepsilon, \varphi, s, b)\), and aggregate quantities \(N(s)\) such that:
  
  ▶ households optimize
  
  ▶ markets clear
  
  ▶ the government budget constraint is balanced

- The equilibrium features no bond-trading
  
  ▶ \(b = 0\) → allocations depend only on exogenous states
  
  ▶ \(\alpha\) shocks remain uninsured, \(\varepsilon\) shocks fully insured
No bond-trade equilibrium

- Micro-foundations for Constantinides and Duffie (1996)
  - CRRA, unit root shocks to log disposable income
  - In equilibrium, no bond-trade $\Rightarrow c_t = \tilde{y}_t$
No bond-trade equilibrium

• Micro-foundations for Constantinides and Duffie (1996)
  ➤ CRRA, unit root shocks to log disposable income
  ➤ In equilibrium, no bond-trade \( \Rightarrow c_t = \tilde{y}_t \)

• Unit root disposable income micro-founded in our model:
  1. Skill investment+shocks: \( \rightarrow \) wages
  2. Labor supply choice: wages \( \rightarrow \) pre-tax earnings
  3. Non-linear taxation: pre-tax earnings \( \rightarrow \) after-tax earnings
  4. Private risk sharing: after-tax earnings \( \rightarrow \) disp. income
  5. No bond trade: disposable income = consumption
Equilibrium risk-free rate $r^*$

$$\rho - r^* = (1 - \tau) ((1 - \tau) + 1) \frac{\nu_\omega}{2}$$

- Intertemporal dissaving motive = precautionary saving motive
- Key: precautionary saving motive common across all agents
- $\frac{\partial r^*}{\partial \tau} > 0$: more progressivity $\Rightarrow$ less precautionary saving $\Rightarrow$ higher risk-free rate
Equilibrium skill choice and skill price

• **FOC** → \[ \frac{s}{\kappa \mu} = (1 - \beta \delta) \frac{\partial U_0(\varphi, s)}{\partial s} = (1 - \tau) \frac{\partial \log p(s)}{\partial s} \]
Equilibrium skill choice and skill price

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- Skill price has **Mincerian shape**: \( \log p(s) = \pi_0 + \pi_1 s \)

\[ \pi_1 = \sqrt{\frac{\eta}{\theta \mu (1 - \tau)}} \] (return to skill)
Equilibrium skill choice and skill price

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\[ \text{var}(\log p(s)) = \frac{1}{\theta^2} \]

Offsetting effects of \( \tau \) on \( s \) and \( p(s) \) leave pre-tax inequality unchanged.
Equilibrium skill choice and skill price

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Offsetting effects of \(\tau\) on \(s\) and \(p(s)\) leave pre-tax inequality unchanged

- Distribution of skill prices (in level) is **Pareto with parameter** \(\theta\): \[\frac{E[p(s) | s > s^*]}{p(s^*)} = \frac{\theta}{\theta - 1}\]
Upper tail of wage distribution

Top 1pct of the Wage Distribution

- Model Wage Distribution
- Lognormal Wage Distribution

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Equilibrium consumption allocation

\[
\log c^*(\alpha, \varphi, s; G, \tau) = \log \lambda^*(G, \tau) + \frac{1}{1 + \hat{\sigma}} \log (1 - \tau)
\]

\[\text{C of representative agent}\]
\[+ \quad \mathcal{M}(v_\varepsilon)\]
\[\text{level effect from ins. variation}\]
\[+ (1 - \tau) \log p(s; \tau)\]
\[\text{skill price}\]
\[-(1 - \tau) \varphi + (1 - \tau) \alpha\]
\[\text{pref. het.} \quad \text{unins. shock}\]

- Response to variation in \((p(s), \varphi, \alpha)\) mediated by progressivity
- Invariant to insurable shock \(\varepsilon\)
Equilibrium hours allocation

$$\log h^*(\varepsilon, \varphi; G, \tau) = \frac{1}{(1 - \tau)(\hat{\sigma} + 1)} \log(1 - \tau)$$

- H of representative agent
- $M(v_\varepsilon)$ level effect from ins. variation
- $\varphi$ pref. het.
- $\frac{1}{\hat{\sigma}} \varepsilon$ ins. shock

- Response to $\varepsilon$ mediated by tax-modified Frisch elasticity $\frac{1}{\sigma} = \frac{1 - \tau}{\sigma + \tau}$

- Invariant to skill price $p(s)$ and uninsurable shock $\alpha$

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Utilitarian Social Welfare Function

- Steady states with constant \((G, \tau)\)

\[
W(G, \tau) \propto \sum_{k=-\infty}^{\infty} \mu_k \int_{0}^{1} U_{i,k}(\cdot; G, \tau) \, di
\]

- Government sets weights: \(\mu_k = \beta^k \times \text{cohort size}\)

  ▶ SWF becomes average period utility in the cross-section

  ▶ Skill acquisition cost for those currently alive imputed to SWF proportionally to their remaining lifetime
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  - SWF becomes average period utility in the cross-section
  - Skill acquisition cost for those currently alive imputed to SWF proportionally to their remaining lifetime

- WLOG, government chooses \(g = G/Y\)
Exact expression for SWF

\[
\mathcal{W}(g, \tau) = \log(1 + g) + \chi \log g + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \\
+ (1 + \chi) \left[ \frac{-1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu(1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \\
- \frac{1}{2\theta} (1 - \tau) - \left[ - \log \left( 1 - \left( \frac{1 - \tau}{\theta} \right) \right) - \left( \frac{1 - \tau}{\theta} \right) \right] \\
- (1 - \tau)^2 \frac{v_\varphi}{2} \\
- \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{v_\omega}{2} - \log \left( \frac{1 - \delta \exp \left( \frac{-\tau(1 - \tau)}{2} v_\omega \right) \right) \right] \\
- (1 + \chi) \sigma \frac{1}{\hat{\sigma}^2} \frac{v_\varepsilon}{2} + (1 + \chi) \frac{1}{\hat{\sigma}} v_\varepsilon
\]
Representative Agent component

\[ \mathcal{W}(g, \tau) = \log(1 + g) + \chi \log g + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

Representative Agent Welfare = \( \mathcal{W}^{RA}(g, \tau) \)

\[ + (1 + \chi) \left[ -\frac{1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu (1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \]

\[ - \frac{1}{2\theta} (1 - \tau) - \left[ -\log \left( 1 - \left( \frac{1 - \tau}{\theta} \right) \right) - \left( \frac{1 - \tau}{\theta} \right) \right] \]

\[ - (1 - \tau)^2 \frac{v_\varphi}{2} \]

\[ - \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{v_\omega}{2} - \log \left( 1 - \delta \exp \left( \frac{-\tau (1 - \tau)}{2} v_\omega \right) \right) \right] \]

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Representative Agent

\[
\max_{C,H} U = \log C - \frac{H^{1+\sigma}}{1 + \sigma} + \chi \log G \\
\text{s.t.} \quad C + G = Y = H \\
G = Y - \lambda Y^{1-\tau}
\]

\[
\mathcal{W}(g, \tau) = \log(1 + g) + \chi \log g + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})}
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Representative Agent

\[
\max_{C,H} \quad U = \log C - \frac{H^{1+\sigma}}{1 + \sigma} + \chi \log G
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\[s.t.
C + G = Y = H
\]

\[
G = Y - \lambda Y^{1-\tau}
\]

\[
\mathcal{W}(g, \tau) = \log(1 + g) + \chi \log g + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})}
\]

- Welfare maximizing \((g, \tau)\) pair:

\[
g^* = \frac{\chi}{1 + \chi}
\]

\[
\tau^* = -\chi
\]

- Allocations are first best

Heathcote-Storesletten-Violante, "Redistributive Taxation in a Partial Insurance Economy"
Exact expression for SWF

\[ W(\tau) = \chi \log \chi - (1 + \chi) \log(1 + \chi) + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

\[ + (1 + \chi) \left[ \frac{-1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu (1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \]

\[ - \frac{1}{2\theta} (1 - \tau) - \left[ - \log \left( 1 - \left( \frac{1 - \tau}{\theta} \right) \right) - \left( \frac{1 - \tau}{\theta} \right) \right] \]

\[ - (1 - \tau)^2 \frac{v_\varphi}{2} \]

\[ - \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{v_\omega}{2} - \log \left( \frac{1 - \delta \exp \left( \frac{-\tau(1-\tau)}{2} v_\omega \right)}{1 - \delta} \right) \right] \]

\[ -(1 + \chi) \sigma \frac{1}{\hat{\sigma}^2} \frac{v_\varepsilon}{2} + (1 + \chi) \frac{1}{\hat{\sigma}} v_\varepsilon \]
Skill investment component

\[ \mathcal{W}(\tau) = \chi \log \chi - (1 + \chi) \log(1 + \chi) + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

\[ + (1 + \chi) \left[ -\frac{1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu (1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \]

productivity gain = log \( E [(p(s))] = \log (Y/N) \)

avg. education cost

\[ - \frac{1}{2\theta} (1 - \tau) \]

consumption dispersion across skills

\[ - \left[ -\log \left( 1 - \left( \frac{1 - \tau}{\theta} \right) \right) - \left( \frac{1 - \tau}{\theta} \right) \right] \]

\[ - (1 - \tau)^2 \frac{v_\varphi}{2} \]

\[ - \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{v_\omega}{2} - \log \left( \frac{1 - \delta \exp \left( \frac{-\tau(1-\tau)}{2} v_\omega \right)}{1 - \delta} \right) \right] \]

\[ -(1 + \chi) \sigma \frac{1}{\hat{\sigma}^2} \frac{v_\epsilon}{2} + (1 + \chi) \frac{1}{\hat{\sigma}} v_\epsilon \]

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Skill investment component

Skill Investment Component

(A) Prod Gain − Edu Cost
(B) Btw−Skill Cons Ineq
(A)+(B) Net Effect

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Uninsurable component

\[ W(\tau) = \chi \log \chi - (1 + \chi) \log(1 + \chi) + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

\[ + (1 + \chi) \left[ -\frac{1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu (1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \]

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\[ - \frac{(1 - \tau)^2 v_\varphi}{2} \]

\[ \text{cons. disp. due to prefs} \]

\[ - \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{v_\omega}{2} - \log \left( \frac{1 - \delta \exp \left( \frac{-\tau(1 - \tau)}{2} v_\omega \right)}{1 - \delta} \right) \right] \]

\[ \text{consumption dispersion due to uninsurable shocks} \]

\[ -(1 + \chi)\sigma \frac{1}{\hat{\sigma}^2} \frac{v_\varepsilon}{2} + (1 + \chi) \frac{1}{\hat{\sigma}} v_\varepsilon \]
Uninsurable component

\[ \mathcal{W}(\tau) = \chi \log \chi - (1 + \chi) \log(1 + \chi) + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

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\[ - (1 - \tau)^2 \frac{v_{\varphi}}{2} \]

cons. disp. due to prefs

\[ -(1 - \tau)^2 \frac{\delta}{1 - \delta} \frac{v_{\omega}}{2} \]

\[ \approx (1 - \tau)^2 v_{\alpha} \]

\[ -(1 + \chi) \sigma \frac{1}{\hat{\sigma}^2} \frac{v_{\varepsilon}}{2} + (1 + \chi) \frac{1}{\hat{\sigma}} v_{\varepsilon} \]

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Insurable component

\[ \mathcal{W}(\tau) = \chi \log \chi - (1 + \chi) \log(1 + \chi) + (1 + \chi) \frac{\log(1 - \tau)}{(1 + \hat{\sigma})(1 - \tau)} - \frac{1}{(1 + \hat{\sigma})} \]

\[ + (1 + \chi) \left[ \frac{-1}{\theta - 1} \log \left( \sqrt{\frac{\eta \theta}{\mu (1 - \tau)}} \right) + \frac{\theta}{\theta - 1} \log \left( \frac{\theta}{\theta - 1} \right) \right] \]

\[ - \frac{1}{2\theta} (1 - \tau) - \left[ - \log \left( 1 - \left( \frac{1 - \tau}{\theta} \right) \right) - \left( \frac{1 - \tau}{\theta} \right) \right] \]

\[ - (1 - \tau)^2 \frac{\nu_\varphi}{2} \]

\[ - \left[ (1 - \tau) \frac{\delta}{1 - \delta} \frac{\nu_\omega}{2} - \log \left( 1 - \delta \exp \left( \frac{-\tau(1 - \tau) \nu_\omega}{2} \right) \right) \right] \]

\[ -(1 + \chi) \sigma \frac{1}{\hat{\sigma}^2} \frac{\nu_\epsilon}{2} + (1 + \chi) \frac{1}{\hat{\sigma}^2} \nu_\epsilon \]

hours dispersion prod. gain from ins. shock = \log(N/H)
Parameterization

- Parameter vector \( \{ \chi, \sigma, \delta, \theta, v_\varphi, v_\omega, v_\varepsilon, \} \)
Parameterization

- Parameter vector \( \{\chi, \sigma, \delta, \theta, v_\varphi, v_\omega, v_\varepsilon, \} \)

- To match \( G/Y = 0.20 \):
  \[ \rightarrow \chi = 0.25 \]
Parameterization

- Parameter vector \( \{\chi, \sigma, \delta, \theta, v_\varphi, v_\omega, v_\varepsilon, \} \)

- To match \( G/Y = 0.20 \):
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- Frisch elasticity (micro-evidence):
  \[ \rightarrow \sigma = 2 \]

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\[
\begin{align*}
cov(\log h, \log w) &= \frac{1}{\hat{\sigma}}v_\varepsilon \\
v_\varphi (\log h) &= v_\varphi + \frac{1}{\hat{\sigma}^2}v_\varepsilon \\
v_\varphi(\log c) &= v_\varphi + \frac{1}{\hat{\theta}^2} \\
v_\varphi(\log w) &= \frac{1}{\theta^2} + v_\alpha + v_\varepsilon \\
\Delta v_\varphi(\log w) &= v_\omega
\end{align*}
\]
Parameterization

- Parameter vector \{\chi, \sigma, \delta, \theta, v_\varphi, v_\omega, v_\varepsilon, \}

- To match \(G/Y = 0.20\):
  \[\rightarrow \chi = 0.25\]

- Frisch elasticity (micro-evidence):
  \[\rightarrow \sigma = 2\]

\[
cov(\log h, \log w) = \frac{1}{\hat{\sigma}} v_\varepsilon
\]

\[
\rightarrow v_\varepsilon = 0.18
\]

\[
var(\log h) = v_\varphi + \frac{1}{\hat{\sigma}^2} v_\varepsilon
\]

\[
\rightarrow v_\varphi = 0.06
\]

\[
var^0(\log c) = v_\varphi + \frac{1}{\theta^2}
\]

\[
\rightarrow \theta = 3
\]

\[
var(\log w) = \frac{1}{\theta^2} + v_\alpha + v_\varepsilon
\]

\[
\rightarrow v_\alpha = 0.13
\]

\[
\Delta var(\log w) = v_\omega
\]

\[
\rightarrow v_\omega = 0.005, \delta = 0.963
\]
Optimal progressivity

Social Welfare Function

Welfare gain = 0.82 pct

τ = 0.087

τ^US = 0.18

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Optimal progressivity: decomposition

Social Welfare Function

welf change rel. to baseline optimum (% of cons.)

Progressivity rate ($\tau$)

(1) Rep. Agent $\tau = -0.25$

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Optimal progressivity: decomposition

Social Welfare Function

(1) Rep. Agent $\tau = -0.25$

(2) + Skill Inv. $\tau = -0.066$

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Optimal progressivity: decomposition

Social Welfare Function

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Optimal progressivity: decomposition

Social Welfare Function

(1) Rep. Agent \( \tau = -0.25 \)
(2) + Skill Inv. \( \tau = -0.066 \)
(3) + Pref. Het. \( \tau = 0.00 \)
(4) + Unins. Shocks \( \tau = 0.102 \)

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Optimal progressivity: decomposition

Social Welfare Function

Heathcote-Storesletten-Violante, "Redistributive Taxation in a Partial Insurance Economy"
Alternative SWF

Utilitarian SWF embeds desire to insure and to redistribute wrt $(\kappa, \varphi)$

Isolate desire to insure against $\omega$ shocks
Alternative SWF

Utilitarian SWF embeds desire to insure and to redistribute wrt $(\kappa, \varphi)$

Isolate desire to insure against $\omega$ shocks

- Economy with heterogeneity in $(\kappa, \varphi)$, and $\chi = \nu_\omega = \tau = 0$
- Compute CE allocations
- Compute Negishi weights s.t. planner’s allocation = CE
- Use these weights in the SWF

Heathcote-Storesletten-Violante, "Redistributive Taxation in a Partial Insurance Economy"
## Alternative SWF

<table>
<thead>
<tr>
<th></th>
<th>Utilitarian</th>
<th>$\kappa$-neutral</th>
<th>$\varphi$-neutral</th>
<th>Insurance-only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redist. wrt $\kappa$</td>
<td>Y</td>
<td>$N$</td>
<td>$Y$</td>
<td>$N$</td>
</tr>
<tr>
<td>Redist. wrt $\varphi$</td>
<td>Y</td>
<td>$Y$</td>
<td>$N$</td>
<td>$N$</td>
</tr>
<tr>
<td>Insurance wrt $\omega$</td>
<td>Y</td>
<td>$Y$</td>
<td>$Y$</td>
<td>$Y$</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>0.087</td>
<td>0.046</td>
<td>0.030</td>
<td>-0.012</td>
</tr>
<tr>
<td>Welf. gain (pct of $c$)</td>
<td>0.82</td>
<td>1.33</td>
<td>1.66</td>
<td>2.67</td>
</tr>
</tbody>
</table>

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Optimal progressivity: alternative SWF

Heathcote-Storesletten-Violante, "Redistributive Taxation in a Partial Insurance Economy"
Alternative assumptions on G

1. G endogenous and valued: \( \chi = 0.25 \), \( G^* = \chi/(1 + \chi) = 0.2 \)
Alternative assumptions on G

1. G endogenous and valued: \( \chi = 0.25, G^\ast = \chi/(1 + \chi) = 0.2 \)

2. G endogenous but non valued: \( \chi = 0, G^\ast = 0 \)

3. G exogenous and proportional to \( Y \): \( G/Y = \bar{g} = 0.2 \)

4. G exogenous and fixed in level: \( G = \bar{G} = 0.2 \times Y^{US} \)
Alternative assumptions on $G$

1. $G$ endogenous and valued: $\chi = 0.25, G^* = \chi/(1 + \chi) = 0.2$

2. $G$ endogenous but non valued: $\chi = 0, G^* = 0$

3. $G$ exogenous and proportional to $Y$: $G/Y = \bar{g} = 0.2$

4. $G$ exogenous and fixed in level: $G = \bar{G} = 0.2 \times Y^{US}$

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<tr>
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<th>Insurance-only SWF</th>
</tr>
</thead>
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<tr>
<td>$G$ endogenous</td>
<td>$\chi = 0.25$</td>
<td>$G/Y(\tau^*)$</td>
</tr>
<tr>
<td></td>
<td>0.200</td>
<td>0.087</td>
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<tr>
<td>$G$ endogenous</td>
<td>$\chi = 0$</td>
<td>$\tau^*$</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.209</td>
</tr>
<tr>
<td>$g$ exogenous</td>
<td>$\bar{g} = 0.2$</td>
<td>$\tau^*$</td>
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<td></td>
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<td>0.209</td>
</tr>
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<td>$G$ exogenous</td>
<td>$\bar{G} = 0.2 \times Y(\tau^{US})$</td>
<td>$\tau^*$</td>
</tr>
<tr>
<td></td>
<td>0.188</td>
<td>0.095</td>
</tr>
</tbody>
</table>

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Progressive consumption taxation

\[ c = \lambda \tilde{c}^{1-\tau} \]

where \( c \) are expenditures and \( \tilde{c} \) are units of final good

- Implement as a tax on total (labor plus asset) income less saving
- Consumption depends on \( \alpha \) but not on \( \varepsilon \)
- Can redistribute wrt. uninsurable shocks without distorting the efficient response of hours to insurable shocks
- Higher progressivity and higher welfare

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