Exercise 1

Consider a pure exchange economy, where time is discrete, indexed by \( t = 0, 1, 2, \ldots \) and continues forever. The economy is populated by 2 individuals with logarithmic preferences and discount factor equal to \( \beta \in (0, 1) \) who trade a nonstorable consumption good \( c_t \). Agents have deterministic endowment streams \( \{ e^i_t \}_{t=0}^{\infty} \) of the consumption good given by

\[
e^i_t = \begin{cases} 
0 & \text{if } (t + i) \text{ is even} \\
2 & \text{if } (t + i) \text{ is odd}
\end{cases}
\]

Agents behave competitively. All markets open at time zero and contracts are exchanged specifying how many units of consumption good will be exchanged at each time \( t \) between the two agents. Contracts are perfectly enforceable.

1) Cast the economy into the Arrow-Debreu language. Define an Arrow-Debreu competitive equilibrium and verify that the Welfare Theorems hold.

2) Using the same notation above, solve for the Arrow-Debreu equilibrium allocations, i.e. characterize the equilibrium sequences of prices and allocations of consumption good among agents.

3) Define a Pareto Optimal allocation. Write down the Social Planner’s problem where the Planner gives weight \( \alpha \) to agent 1 and \( (1 - \alpha) \) to agent 2. Solve the Planner’s problem for arbitrary weights \( \alpha \in (0, 1) \). Characterize the transfer function that allows to map any Pareto Optimum, for a given weight \( \alpha \), into a competitive equilibrium with transfers among agents.

4) How can you characterize the competitive equilibrium of the original economy by using only the Social Planner problem?

Exercise 2

After having read chapters 7-8-9 of Stokey-Lucas, consider the stochastic growth model of Section 10.1. Write down the Social Planner Problem as a Stochastic Dynamic Programming problem and use the Euler Equation to characterize the allocations. Next, define a Recursive Competitive Equilibrium for this economy and use the Euler Equation and the other equilibrium conditions to characterize the allocations. Show the equivalence between decentralized equilibrium allocations and Planner’s solution.

**Remark:** throughout the exercise, use Dynamic Programming. No “time subscripts” are admitted!