Consider the standard one sector Real Business Cycle (RBC) model with aggregate technology
\[ c_t + i_t = y_t = F(z_t, k_t, h_t) \equiv e^{z_t} k_t^\alpha h_t^{1-\alpha}, \]
and law of motion for capital
\[ k_{t+1} = (1 - \delta) k_t + i_t. \]

The aggregate technology shock \( z_t \) follows an AR(1) process with parameters \( (\rho_z, \sigma_z) \). There is a measure \( \mu \) of identical agents with instantaneous utility function:

\[ u(c_t, l_t) = \left( \frac{c_t^{1-\eta} l_t^{1-\eta}}{1-\theta} \right)^{1-\theta} - 1 \]

who discount the future at rate \( \beta \in (0, 1) \). They divide their time endowment between work and leisure, i.e. \( l_t + h_t = 1 \). The government levies proportional taxes on wages at rate \( \tau_t \) and redistributes the tax revenues to each agent in a lump sum fashion. The tax rate \( \tau_t \) is stochastic and follows an AR(1) process with parameters \( (\rho_\tau, \sigma_\tau) \). The government balances its budget every period. The representative firm maximizes profits by choosing labor and capital in competitive input markets.

1. Could you use the social planner problem to solve for the equilibrium allocations? Explain.
2. Define the individual and aggregate states of this economy and define a Recursive Competitive Equilibrium.
3. Using Dynamic Programming, derive the first order conditions for the agent’s problem and interpret them.
4. Obtain a system of 3 nonlinear equations which implicitly characterize the steady-state levels of consumption, capital and hours worked.
5. Describe how you would calibrate the parameters of the model \( \{ \theta, \beta, \eta, \alpha, \delta, \rho_z, \sigma_z, \rho_\tau, \sigma_\tau \} \) and give some plausible number for each parameter.
6. One of the problems of RBC models is that they generate a too high correlation between wages and hours worked across the cycle. Suppose that in the economy taxes \( \tau_t \) are correlated (positively or negatively) with the technology shock \( z_t \). Derive supply and demand for labor in the model and explain graphically what sign in the correlation between taxes and shocks can help most in reducing the correlation between hours worked and wages generated by the model.