MA2 - Advanced Macroeconomics: Homework 3

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The assignment is due on Thursday, March 22 in class.

Exercise 1
Using Blackwell’s sufficient conditions show that the function:

\[ T(x) = (1 - \beta)b + \beta \max(w, x) \]

is a contraction mapping. Why is this result useful?

Exercise 2
Define the problem of an unemployed worker searching for a job in a labor market where job offers are distributed with \( F(w) \). Once a job and a worker are matched, the match might break down every period at an exogenous rate \( \lambda \), in which case the worker becomes unemployed again. Obtain the reservation wage equation in this case. Compute and interpret \( \frac{\partial w}{\partial \lambda} \).

Exercise 3
Consider the standard search model we studied in class. We want to analyze the effect of a mean-preserving spread in the wage offer distribution, i.e. an increase in the weight of the tails of the distribution, for a given mean. This comparative statics exercise captures a situation where there is an increase in the uncertainty of being unemployed. Let \( F(w, \sigma_1) \) and \( F(w, \sigma_2) \) be two distributions parametrized by \( \sigma \). Then, \( F(w, \sigma_2) \) is said to be a mean-preserving spread of \( F(w, \sigma_1) \) if, for every \( w^* \in (0, \infty) \),

\[
\int_{0}^{\infty} \left| F(w, \sigma_2) - F(w, \sigma_1) \right| \, dw > 0.
\]

If we divide both sides by \( \sigma_2 - \sigma_1 \) and take the limit as \( \sigma_2 \to \sigma_1 \), we obtain the condition

\[
\int_{0}^{\infty} F'_u(w, \sigma) \, dw > 0.
\]

Hence, a change in \( \sigma \) induces a mean preserving spread in \( F \) whenever the condition (1) above holds. Suppose that \( F(w, \sigma) \) is the wage offer distribution in the economy and analyze the search problem of a worker. Obtain the reservation wage equation and compute the total derivative \( \frac{\partial w}{\partial \sigma} \). What is its sign when an increase in \( \sigma \) leads to a mean-preserving spread? Explain the economic intuition behind this result.