Insurance and Opportunities: The Welfare Implications of Rising Wage Dispersion*

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Abstract

This paper develops a series of heterogeneous-agents economies in order to characterize analytically the welfare implications of (changes in) the uncertainty in individual labor productivity. We perform two types of welfare calculations. First we compute the welfare consequences of a rise in labor market risk, given a particular asset market structure (complete markets, incomplete markets, autarky). Second, we compute the expected gain from completing markets (starting from autarky or incomplete markets) for a given level of wage dispersion. We are able to derive intuitive closed-form expressions for these welfare effects which depend only on preference parameters and the size of (the change in) wage dispersion. A higher level of productivity dispersion has always two effects: it tends to reduce welfare in presence of partial insurance, but it also tends to improve production opportunities in presence of flexible labor supply. We can separate, analytically, the “insurance” and the “opportunity” components. We quantify the welfare consequences of the rise in wage inequality observed in the United States over the past three decades as, roughly, 3% of lifetime consumption. Moreover, we calculate that completing the asset markets, given the current level of inequality, would lead to a welfare improvement equivalent to 35% of lifetime consumption.

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1 Introduction

There has been a sharp increase in wage inequality in the United States over the past thirty years. A large literature is devoted to investigating the causes of this phenomenon (for surveys, see Katz and Autor, 1999; Acemoglu, 2002; Aghion, 2003; Hornstein, Krusell and Violante, 2004). In this paper we explore its implications for welfare. We view this as an important exercise, since cross-sectional wage inequality and individual wage fluctuations over the life-cycle are large. For example, in the United States the cross-sectional variance of the growth rate of wages for male workers is over \textit{200 times} bigger than the time variance of the growth rate of average wages for the same group.\footnote{This number is calculated from the PSID, 1967-1996. See section 6 for details on the sample selection. In particular, the variance of the mean wage growth over the period is 0.008 and the cross-sectional variance of individual wage growth, averaged over the period, is .16.} Thus the welfare implications of idiosyncratic labor market risk are likely to be much larger than the welfare costs of aggregate business-cycle risk.

In this paper we develop a series of simple models which serve as transparent laboratories for isolating the welfare implications of rising wage inequality and for understanding the source of these welfare effects. Agents in our models are subject to stochastic idiosyncratic shocks to their labor market productivity and thus to their market wage. Welfare costs will depend crucially on the set of instruments available to insure against this risk. In order to understand the role of access to explicit insurance against labor market risk, we consider three alternative asset market structures. In the first - complete markets – all wage inequality is insurable. At the other extreme – autarky – we rule out all asset trade between agents. Finally, as an intermediate case, we put some structure on the idiosyncratic wage generating process and assume that it has two components: a fully insurable piece, and an uninsurable component. We label this the incomplete markets economy.

We assume that agents are infinitely-lived, and are free to choose labor supply in addition to making consumption / savings decisions. Incorporating a labor supply choice is essential to the analysis, since the ability to adjust hours can potentially mitigate the welfare cost of rising wage inequality via two channels: on the one hand, it represents a margin of self-insurance against shocks to wages; on the other, it allows agents to concen-
trate work effort in periods of relatively high productivity. We consider two widely-used specifications for preferences: one in which preferences are separable between consumption and hours worked, and one in which consumption and leisure enter the utility function in a Cobb-Douglas fashion.

Given our particular market structures, we are able to derive intuitive analytic expressions for equilibrium allocations, for cross-sectional moments, and for expected lifetime utility. We use these expressions to ask two sets of questions relating to welfare. First, what are the welfare costs of rising wage dispersion? To address this question, we hold the asset market structure constant and increase the variance of one or both components of the wage-generating process. A key finding of the paper is that with flexible labor supply, an increase in idiosyncratic wage risk increases the need for insurance, but also presents an opportunity to increase productivity, measured as output per hour worked, by concentrating work effort among more productive workers. When markets are complete, a rise in wage dispersion always leads to a welfare gain, since insurance is perfect and thus the opportunity effect dominates. In this case the welfare gain is proportional to the labor supply elasticity under both preference specifications.

Second, we compute the welfare costs of market incompleteness, which we think of as the difference between welfare in the autarky or incomplete markets economy on the one hand, and the complete markets economy on the other. In this case, we hold the wage-generating process constant across economies, and think about welfare comparisons from the point of view of an agent being dropped into different economies from behind a “veil of ignorance”.

Consistently with the discussion above, we find that the welfare costs of market incompleteness stem both from missing insurance markets and from wasted opportunities for workers to specialize according to their comparative advantage in market work or leisure. In particular, when markets are incomplete, agents tend to use labor supply to smooth earnings and consumption, instead of changing hours in tune with productivity. Thus introducing additional insurance instruments reduces the welfare cost of wage inequality, both by allowing for better consumption insurance, and also by permitting more efficient exploitation of cross-sectional heterogeneity in productivity.

For each preference specification we report results for a range of alternative preference
parameters, defining agents’ willingness to substitute consumption and leisure intertemporally.\textsuperscript{2} For the incomplete markets economy, the fraction of wage inequality that is insurable is also important for welfare calculations. Using our preferred parameter values and our preferred (Cobb-Douglas) specification for preferences, we find that the welfare cost of the observed rise in labor market risk in the incomplete-market economy corresponds to 2.8\% of lifetime consumption, and that, ex-ante, households would be willing to give up over 40\% of their expected lifetime consumption to be able to fully insure all labor market risk and fully exploit their productive opportunities in complete markets. To put these magnitude in perspective, one has to think that these numbers are 2-3 orders of magnitude larger than commonly estimated welfare losses of business cycles fluctuations (for a survey, see Lucas 2003).

The rest of the paper is organized as follows. Section 2 describes the model economies. Section 2.1 characterizes the equilibrium allocations. Section 3 computes the implied welfare costs associated with rising wage inequality and market incompleteness under separable preferences, and Section 4 extends the analysis and the welfare. Finally, in Section 7, we make our concluding remarks.

2 The Economy

Demographics and preferences: The economy is populated by a unit mass of infinitely-lived agents. Each agent has the same time-separable utility function $U (c, h)$ over streams of consumption $c = \{c_t\}_{t=1}^{\infty}$ and hours worked $h = \{h_t\}_{t=1}^{\infty}$,

$$
U (c, h) = (1 - \beta) \sum_{t=1}^{\infty} \beta^{(t-1)} u (c_t, h_t),
$$

where $\beta \in (0, 1)$ is the agents’ discount factor.\textsuperscript{3} We will consider two alternative specifications for the period utility function. In the first consumption and leisure enter in a Cobb-Douglas fashion. In the second, period utility is separable between consumption and hours worked.

\textsuperscript{2}Note that these elasticities cannot be varied independently under the Cobb-Douglas specification.

\textsuperscript{3}Later we will introduce heterogeneity in the population regarding the relative taste for consumption versus leisure.
Production and individual labor productivity: Production takes place through a constant-returns-to-scale aggregate production function with labor as the only input. The labor market and the goods market are perfectly competitive, so individual wages equal individual productivity. Since we do not focus on growth or short-term fluctuations, we assume the hourly rental rate per efficiency unit to be constant and normalized to unity.

Individuals’ wage rates vary stochastically over time and are independently and identically distributed across the agents in the economy. We assume that an individual’s wage at a point in time has two orthogonal components: an ‘uninsurable’ component \( \alpha \in A \subseteq \mathbb{R} \), and an ‘insurable’ component \( \varepsilon \in E \subseteq \mathbb{R} \):

\[
\log w(\alpha, \varepsilon) = \alpha + \varepsilon.
\]

In order to fix ideas and simplify the expressions for our subsequent welfare results, we will interpret the uninsurable component as a non-stochastic fixed effect for the agent, and we will interpret the insurable component as a purely transitory shock that is \( iid \) over time. Thus, at the beginning of period \( t = 1 \), each agent draws a pair \((\alpha, \varepsilon)\). Then for every \( t > 1 \), each agent draws a new value for \( \varepsilon \). All shocks are publicly observable. At the end of the paper we explain how the analysis can be extended to allow for a much richer specification of the process for productivity, incorporating permanent uninsurable shocks to wages, as well as persistent (insurable) shocks.

We assume that \( \varepsilon \) and \( \alpha \) are drawn from normal distributions, with

\[
\varepsilon \sim N\left(-\frac{v_\varepsilon}{2}, v_\varepsilon\right), \quad \alpha \sim N\left(-\frac{v_\alpha}{2}, v_\alpha\right).
\]

As a result,

\[
\log w \sim N\left(-\frac{v}{2}, v\right),
\]

where \( v = v_\varepsilon + v_\alpha \) is the variance of the log-normal productivity shock. Note that equation (2) implies that the mean population wage (in levels) is equal to one, i.e. \( E[w] = 1 \). One important implication is that mean hourly wage per worker will be invariant to dispersion when we study comparative statics with respect to the variance of wages, \( v \). Let \( \phi_v \) denote the normal density function with mean \(-\frac{v}{2}\) and variance \( v \).

Market structure: We compare competitive equilibria under three alternative asset market structures. Each market structure decentralizes an equal-weight planner’s problem. There is an intuitive mapping between the number of assets that may be traded in
the particular market structure, and the planner’s ability to transfer resources between agents in the corresponding planner’s problem.

A common assumption for all market structures is that the available assets come in zero net supply and that agents have zero financial wealth before markets open for trade. Hence, aggregate wealth is zero in every period. While this assumption might appear stark, we believe it is a natural complement to assuming perfect insurance against transitory shocks. Indeed, the main role of wealth in a standard buffer-stock model of savings where agents can borrow and lend but not insure shocks directly, is to facilitate self-insurance against risk. It is well known that this self-insurance mechanism provides good insurance against transitory shocks but poor insurance against permanent shocks. With full insurance against transitory (insurable) risk, agents have no precautionary motive for savings.

We will define equilibria sequentially, and assume that all assets traded are one-period-lived. See TBA for a proof that this assumption comes without loss of generality. We now present three alternative market structures and the associated sequential budget constraints.

1. **Autarky (AUT):** In this economy no financial instruments are traded, and households simply consume their labor income every period. The period $t$ budget constraint for a household with fixed effect $\alpha$ and a draw of the transitory shock $\varepsilon$ is simply

$$c_t(\alpha, \varepsilon) = w(\alpha, \varepsilon) h_t(\alpha, \varepsilon).$$

(3)

This corresponds to a planner’s problem in which each agent lives alone on an island, and the planner is unable to transfer resources across islands.

2. **Complete markets (CM):** In the complete-markets economy, households are free to trade contracts contingent on every possible realization of the individual productivity shocks at every date. Insurance markets open in period 0 before $\alpha$ is realized, at which time the household budget constraint is given by

$$\int_A \int_E p_0(\alpha, \varepsilon) b_0(\alpha, \varepsilon) \, d\alpha d\varepsilon = 0,$$

(4)

where $b_0(\alpha, \varepsilon)$ and $p_0(\alpha, \varepsilon)$ are respectively the quantity and the price of one period state-contingent bonds that pay one unit of output in period 1 if the realization of
the individual fixed effect is equal to $\alpha$ and the transitory shock is equal to $\varepsilon$. The right hand side of the period 0 constraint is zero since no labor income is received in that period, consumption starts from period 1 and initial individual wealth is zero. From period 1 onwards there is no more uncertainty regarding the fixed effect $\alpha$ and assets traded need only be contingent on the transitory shock. Hence, the period $t$ budget constraint for an agent with individual state $(\alpha, \varepsilon)$ becomes

$$c_t(\alpha, \varepsilon) + \int_E p_t(\omega) b_t(\omega; (\alpha, \varepsilon)) d\omega = b_{t-1}(\varepsilon; (\alpha, \varepsilon-1)) + w_t(\alpha, \varepsilon) h_t(\alpha, \varepsilon), \quad (5)$$

where $b_t(\omega; (\alpha, \varepsilon))$ denotes the quantity of a bond, purchased by an individual with fixed effect $\alpha$ and transitory shock $\varepsilon$, that pays one unit of output in period $t+1$ if the realization of the transitory shock is $\omega$; $p_t(\omega)$ denotes the price of this state-contingent bond. An arbitrarily loose constraint on borrowing rules out Ponzi schemes. This decentralization corresponds to a planner’s problem in which all agents have equal weights and live on the same island, and the planner is free to dictate hours worked for each agent and to redistribute aggregate output.

3. **Incomplete markets (IM):** This is our intermediate benchmark, which we interpret as a description of the world. Here, households have access to perfect insurance against the transitory $\varepsilon$-shocks and no insurance against the permanent $\alpha$-shocks. In other words, markets open after $\alpha$ is realized. This corresponds to a planner’s problem in which agents are segregated across islands by their fixed effect $\alpha$. Within each $\alpha$-island, the planner is unconstrained regarding how to allocate work effort across agents with different values for the transitory shock $\varepsilon$, and can allocate resulting output freely across agents on the island. However, the planner cannot transfer resources across islands.

In the decentralized version of this economy, budget constraints are exactly as in the complete markets economy for $t > 0$ (see equation 5), except that the initial budget constraint is given by

$$\int_E p_0(\varepsilon) b_0(\varepsilon; \alpha) d\varepsilon = 0.$$ 

We interpret this economy as one in which agents can effectively insure some risks (such as short insurance spells) but cannot insure others (such as being endowed
with low ability). In Heathcote et al. (2004) we explicitly compare this market structure to the framework developed by Bewley, Huggett and Aiyagari in which a single non-contingent bond is traded. In that paper we argue that for reasonably calibrated economies, the allocations are quantitatively similar to the economies explored in this paper. More precisely, the first and second moments of individual consumption and labor supply, as well as the welfare effects of changes in the wage process, are not very different. However, whereas equilibrium allocations in the incomplete markets economy considered here can be characterized analytically, numerical methods are required to approximate allocations in the economy with one risk-free savings instrument.

2.1 Equilibrium allocations

A competitive equilibrium requires individual optimization and market clearing in all markets, i.e. that the net demand for all assets is zero and that aggregate consumption equals aggregate labor earnings.

In order to find the equilibrium allocations of consumption and labor supply we solve the corresponding planner’s problem for each market structure. These problems and the solutions to them are described in detail in the Appendix. Focussing on planner’s problems has the advantage that asset prices do not appear in the constraint set, and one can abstract from portfolio choices.

Since there is no interaction between agents in the autarky economy, it is immediate that allocations in the competitive equilibrium and the solution to the corresponding planner’s problem coincide: irrespective of how the planner weights the welfare of different agents, it is efficient for the planner to equate each agent’s marginal rate of substitution between hours worked and consumption in preferences to the agent’s marginal rate of transformation between hours and output.

In the economies with asset trade, we compute first the equilibrium prices and asset purchases implied by the planner-allocations of consumption and hours. We then verify that all the conditions characterizing equilibrium under the particular market structure are satisfied given these allocations and prices. In particular, we check that (i) agents’ intra-temporal first order conditions for labor supply and inter-temporal first order conditions
for asset purchases are satisfied, (ii) that agents’ budget constraints are satisfied, and (iii) that the goods market and all asset markets clear.

We now characterize the equilibrium allocations. These have the property that under each market structure, consumption and hours worked for a particular agent in period \( t \) depend only the agents fixed effect \( \alpha \) and on the agent’s current draw for the transitory shock \( \varepsilon_t \). The functions \( c_m(\alpha, \varepsilon) \) and \( h_m(\alpha, \varepsilon) \) denote the time-invariant equilibrium expressions for consumption and hours as functions of these parameters under market structure \( m \in \{CM, IM, AUT\} \).

3 Qualitative welfare analysis

We compare and rank allocations using the following utilitarian social welfare function:

\[
W = (1 - \beta) E_0 \left[ \sum_{t=1}^{\infty} \beta^{(t-1)} u(c_m(\alpha, \varepsilon_t), h_m(\alpha, \varepsilon_t)) \right]
\]

\[
= \int_A \int_E u(c_m(\alpha, \varepsilon), h_m(\alpha, \varepsilon)) \phi_{v_\varepsilon}(\varepsilon) \phi_{v_\alpha}(\alpha) d\varepsilon d\alpha
\]

(6)

This expression for welfare has two interpretations. First, it is the value for a utilitarian planner who weights all agents equally. Second, it is expected lifetime utility for an agent at time 0 “under the veil of ignorance”; i.e. expected lifetime utility before uncertainty is realized. This definition of welfare has also the desirable property that it is invariant to the societal degree of inequality-aversion, since all agents are ex-ante equal.

As discussed in the introduction, we assess the welfare costs associated with labor market uncertainty from two distinct perspectives. First, for a given insurance market structure, what are the welfare costs of a rise in labor market risk? Second, for a given level of risk, what are the welfare gains from completing markets?

The welfare implications of rising labor market risk: We begin by fixing the market structure of the economy and measuring the welfare implications from increasing wage dispersion. Suppose the variances of permanent and transitory shocks rise from the baseline values \( v_\alpha \) and \( v_\varepsilon \) respectively to \( \hat{v}_\alpha \) and \( \hat{v}_\varepsilon \). Let \( \Delta v_\alpha = \hat{v}_\alpha - v_\alpha \) and \( \Delta v_\varepsilon = \hat{v}_\varepsilon - v_\varepsilon \). Let \( \chi_m \) denote the associated welfare gain under market structure \( m \), expressed in units
of the “equivalent compensating variation” in lifetime consumption under the baseline wage variance:

$$\int_A \int_E u ((1 + \chi_m) c_m(\alpha, \varepsilon), h_m(\alpha, \varepsilon)) \phi_{\nu_\varepsilon}(\varepsilon) \phi_{\nu_\alpha}(\alpha) d\varepsilon d\alpha$$

$$= \int_A \int_E u (\hat{c}_m(\alpha, \varepsilon), \hat{h}_m(\alpha, \varepsilon)) \phi_{\nu_\varepsilon}(\varepsilon) \phi_{\nu_\alpha}(\alpha) d\varepsilon d\alpha.$$ (7)

Here \((c_m(\alpha, \varepsilon), h_m(\alpha, \varepsilon))\) denotes the optimal choices for consumption and hours worked for an individual with shocks \(\alpha\) and \(\varepsilon\) in an economy with shock variances \(\nu_\alpha\) and \(\nu_\varepsilon\), and \((\hat{c}_m(\alpha, \varepsilon), \hat{h}_m(\alpha, \varepsilon))\) denotes the optimal choices for the same individual in the economy with variances \(\hat{\nu}_\alpha\) and \(\hat{\nu}_\varepsilon\).

Strictly speaking, the solution for \(\chi_m\) allows one to compare the welfare level across two economies with different levels of inequality. One can interpret it as a welfare gain from rising inequality when the change in variances is unforeseen, and the transition between steady states is instantaneous.

**The welfare gains from completing markets:** Similarly, we measure the welfare gain associated with completing insurance markets, for given levels of permanent and transitory risk \(\nu_\alpha\) and \(\nu_\varepsilon\), as the percentage increase in consumption in the incomplete-markets (or autarkic) economy required to achieve the same welfare as in the economy with complete markets. In particular, we define the welfare gain as the value for \(\xi_m\) that solves

$$\int_A \int_E u ((1 + \xi_m) c_m(\alpha, \varepsilon), h_m(\alpha, \varepsilon)) \phi_{\nu_\varepsilon}(\varepsilon) \phi_{\nu_\alpha}(\alpha) d\varepsilon d\alpha$$

$$= \int_A \int_E u (c_{CM}(\alpha, \varepsilon), h_{CM}(\alpha, \varepsilon)) \phi_{\nu_\varepsilon}(\varepsilon) \phi_{\nu_\alpha}(\alpha) d\varepsilon d\alpha.$$ (8)

4 **Cobb-Douglas preferences**

First, we consider preferences that are Cobb-Douglas between consumption and leisure. In this case

$$u (c, h) = \frac{(c^n (1 - h)^{1-\eta})^{1-\gamma}}{1 - \gamma}.$$
where \( \eta \in (0, 1) \) determines the relative taste for consumption versus leisure. Cobb-Douglas preferences are widely used in the macro literature, since they are consistent with balanced growth, irrespective of the choice for \( \gamma \). In labor economics, this specification is often advocated because there is some empirical evidence of non-separability between consumption and leisure; see, for example, Heckman (1974). The intertemporal elasticity of substitution for consumption is given by \( 1/\gamma \). The coefficient of relative risk aversion is \( 
abla = -\frac{\partial u_c}{\partial c} = 1 - \eta (1 - \gamma). \) (9)

The Frisch elasticity of labor supply depends on hours worked, and is given by \( \phi (\gamma, \eta) = \tau (1 - h) \) \( h \)

where \( \tau = (1 - \eta + \eta \gamma) / \gamma \) defines the Frisch elasticity for leisure. We define an “average Frisch elasticity” in terms of hours worked in a non-stochastic representative-agent economy, in which case \( h = \eta \):

\[ \tilde{\phi} (\gamma, \eta) = \frac{\tau (1 - \eta)}{\eta}. \] (10)

4.1 Allocations with Cobb-Douglas preferences

**Autarky (AUT)**—In autarky, consumption equals earnings every period. Individual hours worked are chosen optimally, and using the budget constraint (3) and the appropriate intra-temporal first order condition, it is straightforward to solve for the equilibrium choices for consumption and hours worked. Under the Cobb-Douglas preference specification, allocations in autarky are given by:

\[ c_{AUT} (\alpha, \varepsilon) = \eta \exp(\alpha + \varepsilon), \]

\[ h_{AUT} (\alpha, \varepsilon) = \eta. \] (11)

Note that allocations depend only on the current period wage, \( w (\alpha, \varepsilon) = \exp(\alpha + \varepsilon) \) and not on the two shocks separately. Under the Cobb-Douglas specification, income and substitution effects from uninsurable wage changes exactly offset, so hours are constant.

\[ \text{The Frisch elasticity of labor supply measures the elasticity of hours worked to transitory changes in wages, keeping the marginal utility of consumption constant.} \]

10
Complete markets (CM)—In the complete markets economy equilibrium allocations for consumption and hours are given by:

\[
c_{CM}(\alpha, \varepsilon) = \eta \exp \left( (1 - \tau) \tau \frac{v}{2} \right) \exp \left( (1 - \tau) (\alpha + \varepsilon) \right),
\]
\[
h_{CM}(\alpha, \varepsilon) = 1 - (1 - \eta) \exp \left( (1 - \tau) \frac{v}{2} \right) \exp \left( -\tau (\alpha + \varepsilon) \right).
\]

Because of non-separability, equalizing the marginal utility of consumption across agents in the Cobb-Douglas case does not in general imply equalizing consumption. For \( \tau < 1 \) (equivalently for \( \gamma > 1 \)), consumption and leisure are substitutes in the sense that high productivity individuals who in the constrained-efficient equilibrium allocation enjoy relatively little leisure are compensated with relatively high consumption. When \( \gamma = 1 \) the Frisch elasticity of leisure is also equal to 1, in which case consumption is constant and equal to \( \eta \), while hours respond strongly to wage changes. Since \( \tau \) is declining in \( \gamma \), increasing \( \gamma \) above one implies that consumption responds more strongly and hours less so to idiosyncratic wage shocks.

It is interesting to note that with Cobb-Douglas preferences, average consumption and average hours worked in complete markets are:

\[
E[c_{CM}(\alpha, \varepsilon)] = \eta,
\]
\[
E[h_{CM}(\alpha, \varepsilon)] = 1 - (1 - \eta) \exp (\tau v).
\]

Thus with Cobb-Douglas preferences, it is efficient to take advantage of more variable shocks by reducing average hours and increasing leisure rather than by increasing consumption.

Equilibrium asset prices are given by:

\[
p_0(\alpha, \varepsilon) = \beta \phi_{v_\alpha}(\alpha) \phi_{v_\varepsilon}(\varepsilon) \quad \forall \alpha \in A, \forall \varepsilon \in E,
\]
\[
p_t(\varepsilon) = \beta \phi_{v_\varepsilon}(\varepsilon) \quad \forall \varepsilon \in E, \forall t > 1.
\]

Note that the interest rate on a risk free bond (the price of purchasing a complete set of state contingent claims) is equal to \( \beta \), the discount rate. This reflects the fact that agents have are perfectly able to insure idiosyncratic risk, so there is no motive for precautionary savings.
Equilibrium asset purchases are given by

\[ b_{0,CM}(\alpha, \varepsilon) = c_{CM}(\alpha, \varepsilon) - w(\alpha, \varepsilon) h_{CM}(\alpha, \varepsilon) + \frac{\beta}{1 - \beta} \int \phi_{\varepsilon}(\varepsilon) d\varepsilon \]  

\[ b_{CM}(\varepsilon; (\hat{\alpha}, \varepsilon_{-1})) = b_{0,CM}(\hat{\alpha}, \varepsilon) \]

The expressions for purchases of state contingent claims are such that financial assets plus the expected present value of labor income in each possible realization of the state \((\alpha, \varepsilon)\) is equal to the present value of equilibrium consumption. Note that the market price of the portfolio purchased at date zero is zero, while the price of the portfolio purchased at future dates may not be zero. In particular, if the realization of \(\alpha\) is low, so that the present value of future labor income is lower than the present value of consumption, there will be a large initial payout from the securities indexed to \(\alpha\) — i.e. \(b_{0,CM}(\alpha, \varepsilon)\) will be a large positive number. From this point onwards, the low-\(\alpha\) agent will effectively consume the interest from this initial payoff and maintain a constant wealth level.\(^5\)

**Incomplete markets (IM)**—Equilibrium allocations in the incomplete markets economy are given by

\[ c_{IM}(\alpha, \varepsilon) = \eta \exp(\alpha) \exp \left( (1 - \tau) \left( \varepsilon + \tau \frac{v_{\varepsilon}}{2} \right) \right) \]  

\[ h_{IM}(\alpha, \varepsilon) = 1 - (1 - \eta) \exp \left( \tau \left( (1 - \tau) \frac{v_{\varepsilon}}{2} - \varepsilon \right) \right) \]

In this case, hours worked are increasing in the insurable transitory shock \(\varepsilon\) (as in the complete markets economy) but are independent of the uninsurable permanent shock \(\alpha\) (as under autarky). Because offsetting income and substitution effects imply that cross-sectional differences in the permanent shock do not translate to differences in hours worked, they must show up in consumption, which is directly proportional to \(\alpha\). Current consumption is increasing in current \(\varepsilon\) if and only if \(\tau < 1 \Leftrightarrow \gamma > 1\). The interpretation for this result follows immediately from the discussion for the complete markets case.

\(^5\)By combining all the one-period claims indexed to \(\varepsilon\) one can construct a non-contingent bond in zero net supply, which suffices for agents with low (high) \(\alpha\) to maintain a constant positive (negative) wealth.
Within each group of fixed-effects $\alpha$, the average consumption is constant and equal to $\eta \exp(\alpha)$. Aggregate economy-wide consumption is equal to $\eta$ and invariant to changes in wage inequality, while aggregate hours worked is declining in $v_\varepsilon$.

Equilibrium prices in the first period are $p_0(\varepsilon) = \beta \phi_{v_\varepsilon}(\varepsilon)$, and for $t > 0$ prices are given by (16). Purchases of state contingent bonds are such that for each possible realization for $\varepsilon$, the quantity of bonds that pays out plus the equilibrium value of labor income is equal to the equilibrium value for consumption. Note that the market price of buying this portfolio is zero at each date, so in this sense the agent’s wealth is constant and equal to zero.

$$b_{IM}(\varepsilon; (\hat{\alpha}, \varepsilon_{-1})) = c_{IM}(\hat{\alpha}, \varepsilon) - w(\hat{\alpha}, \varepsilon) h_{IM}(\hat{\alpha}, \varepsilon).$$

### 4.2 Welfare with Cobb Douglas preferences

We now analyze the welfare effects of rising inequality for each of the three market structures described above.

**Proposition 1:** With Cobb-Douglas preferences, the (approximate) welfare gain of a change in labor market risk equal to $\Delta v$, where $\Delta v = \Delta v_{\varepsilon} + \Delta v_{\alpha}$, is given by the following expressions under the three market structures we consider:

$$\chi_{CM} \simeq \bar{\phi}(\gamma, \eta) \frac{\Delta v_{\varepsilon}}{2},$$

$$\chi_{AUT} \simeq -\rho(\gamma, \eta) \frac{\Delta v}{2},$$

$$\chi_{IM} \simeq \bar{\phi}(\gamma, \eta) \frac{\Delta v_{\varepsilon}}{2} - \rho(\gamma, \eta) \frac{\Delta v_{\alpha}}{2}.$$ 

**Proof:** See Appendix.

In the Proof of Proposition 1 we obtain the exact closed-form solution for the welfare effects $\chi$ for all three market structures. However, these expressions are cumbersome and
not particularly transparent. Through a set of log-approximations of the class \( \ln (1 + x) \approx x \) and \( e^x \approx 1 + x \), one obtains the simple and useful solutions stated in Proposition 1. The linearity of \( \chi \) in \( \Delta v_\alpha \) and \( \Delta v_\varepsilon \) is a feature of the approximation.\(^6\)

**Complete markets**— Arguably, the most surprising result in Proposition 1 is that under complete markets, increasing productivity dispersion strictly increases welfare. This gain is proportional to the average Frisch elasticity. The source of this result comes from the endogeneity of labor supply: an unconstrained planner can achieve better allocative efficiency with larger dispersion, without any loss in terms of consumption smoothing, by commanding longer hours from high-productivity workers and higher leisure from less productive workers. This result is closely related to the standard result from classical consumer theory that the indirect utility function of a static consumer is quasi-convex in prices, so a mean-preserving spread of the price distribution raises welfare (see for example, Mas Colell 1995, page 59). In this case the indirect utility function of the planner in the planner’s problem that corresponds to the complete markets equilibrium is quasi-convex in the productivities \( w_{it} \).

Panel (A) in Figure 1 plots \( \chi_{CM} \) as a function of \( \gamma \) for \( \Delta v \) normalized to one. For given \( \eta \), the larger is \( \gamma \), the lower is the Frisch elasticity for labor supply, and the smaller are the welfare gains from increase wage dispersion. Intuitively, the larger the Frisch elasticity the greater the opportunities for exploiting the heterogeneity in labor productivity. Recall that with Cobb-Douglas preferences, welfare gains are enjoyed in the form of higher average leisure, with constant average consumption.

**Autarky**— In autarky, there is always a welfare loss associated with greater wage inequality. This loss is equal the expression computed by Lucas (1987) for the welfare costs of aggregate consumption fluctuations in an economy with inelastic labor supply. Recall that with Cobb-Douglas preferences, hours worked in autarky are always equal to \( \eta \) irrespective of preference parameters or the variance of wage risk. Because income and substitution effects for labor supply exactly offset, the Lucas result extends to our economy with endogenous hours. The larger is the risk aversion parameter, the larger are the welfare costs associated with increased uninsurable risk. See Panel (B) in Figure 1.

\(^6\)In Figure 5, we show that for plausible parameter values the approximation is remarkably good.
Incomplete markets—Realistic insurance market structures lie, arguably, strictly between complete markets and autarky. Hence, one can think of $\chi_{CM}$ and $\chi_{AUT}$ as, respectively, an upper bound and a lower bound on the welfare consequences of a rise in wage inequality. Our intermediate incomplete-markets case has precisely the purpose of reproducing an economy closer to actual ones. In the IM economy, the welfare gain can be re-expressed as

$$\chi_{IM} = \chi_{CM} \frac{\Delta v_\varepsilon}{\Delta v} + \chi_{AUT} \frac{\Delta v_\alpha}{\Delta v},$$

i.e. exactly as a weighted average between the welfare gain in complete markets and the welfare loss in autarky, with weights equal to the share of insurable and uninsurable shocks in the economy. Relative to an economy with inelastic labor supply, flexibility to adjust hours worked can reduce the welfare cost or increases the welfare gain from an increase in wage inequality because it allows for a more efficient division of labor in response to additional insurable risk.

Decomposing welfare gains—When markets are complete, perfect insurance is always achieved, and any welfare gains from greater wage inequality reflect greater opportunities for specialization in time use. We therefore label the welfare gain in the complete markets economy the “opportunity effect” of increased inequality. In autarky, hours are constant, so there is no opportunity effect. In this case we label the welfare loss associated with greater consumption inequality the “insurance effect” of increased inequality. The welfare gain in the incomplete markets economy combines both a positive opportunity effect, and a negative insurance effect.

Floden (2001) extending work by Benabou (2002) decomposes the welfare effects from changes in government policy into two components: a “level” component designed to capture the welfare effect from with the changes in average consumption and average leisure associated with a switch from policy A to policy B, and an “uncertainty” effect, designed to capture the welfare effect of changes in risk associated with the switch. This decomposition into level versus uncertainty effects maps directly into the opportunity versus insurance effects we defined above. In particular, it is possible to show that the Benabou / Floden level effect in complete markets is exactly twice our opportunity effect, while the level effect in autarky is zero and thus equal to our opportunity effect.
Exploiting the welfare analysis for changes to the variance of wages, we now compare the measure of household welfare defined in (6) across economies with different market structures, given the same stochastic process for idiosyncratic labor market risk.

**Proposition 2:** With Cobb-Douglas preferences, the (approximate) welfare gains from increasing the set of assets traded in an economy with labor market risk equal to $v = v_\alpha + v_\epsilon$ are:

$$\xi_{AUT\rightarrow CM} \approx \left[ \bar{\phi} (\gamma, \eta) + \rho (\gamma, \eta) \right] \frac{v}{2},$$

$$\xi_{IM\rightarrow CM} \approx \left[ \bar{\phi} (\gamma, \eta) + \rho (\gamma, \eta) \right] \frac{v_\alpha}{2},$$

$$\xi_{AUT\rightarrow IM} \approx \left[ \bar{\phi} (\gamma, \eta) + \rho (\gamma, \eta) \right] \frac{v_\epsilon}{2}.$$

**Proof:** See Appendix.

These expressions are very intuitive in light of the welfare expressions from the previous section. In particular, one way to think about what it means to complete markets beginning with autarky is (i) there is a change $\Delta v = -v$ in the variance of uninsurable risk with associated welfare gain $-\chi_{AUT}$ and (ii) there is a change $\Delta v = v$ in the variance of insurable risk with associated welfare gain $\chi_{CM}$. Similarly, beginning with incomplete markets, completing markets means (i) a change $\Delta v = -v_\alpha$ in the variance of uninsurable risk, and (ii) a change $\Delta v = v_\alpha$ in the variance of insurable risk. Moving from autarky to incomplete markets means (i) a change $\Delta v = -v_\epsilon$ in the variance of uninsurable risk, and (ii) a change $\Delta v = v_\epsilon$ in the variance of insurable risk.

The parametric expression multiplying the variance in each case has two separate components. The first term $\bar{\phi} (\gamma, \eta)$ captures the opportunity effect associated with increased risk-sharing in the presence of elastic labor supply: more productive households work relatively harder and less productive households enjoy more leisure. This contribution to welfare associated with the opportunity effect is increasing in the average Frisch elasticity. The second term - the insurance effect which is proportional to the coefficient of relative risk aversion $\rho (\gamma, \eta)$ - captures the value of additional insurance provided by increased risk-sharing.
Figure 2 documents how the welfare gain from increasing the set of assets traded varies with the curvature parameter $\gamma$, which implicitly defines both aversion to consumption risk, and the willingness to substitute hours inter-temporally.

5 Separable Preferences

We now consider preferences that are separable between consumption and hours worked. Separability is a common assumption in the micro literature that estimates elasticities for consumption and labor supply (for a survey, see Browning, Hansen and Heckman, 1999). In this case

$$u(c, h) = \frac{c^{1-\gamma}}{1-\gamma} - \psi \frac{h^{1+\sigma}}{1+\sigma},$$

where $\gamma, \sigma \in [0, +\infty)$. In this case, the coefficient of relative risk aversion is simply $\gamma$, while the intertemporal elasticity of substitution for consumption is $1/\gamma$, as in the Cobb-Douglas case. The Frisch elasticity for labor supply is simply $1/\sigma$. The parameter $\psi$ measures the strength of the distaste for work, relative to the taste for consumption.

Note that separability allows for a lot of flexibility in distinguishing between agents’ willingness to substitute consumption and hours intertemporally, as defined respectively by the curvature parameters $\gamma$ and $\sigma$. By contrast, with Cobb-Douglas preferences the share parameter $\eta$ is generally pinned down by the share of disposable time agents devote to market work, implying that the single parameter $\gamma$ governs both the intertemporal elasticity of substitution for consumption and the corresponding elasticity for hours worked.

5.1 Allocations with separable preferences

Autarky (AUT)– When preferences are separable between consumption and hours worked, allocations are given by

$$c_{AUT}(\alpha, \varepsilon) = \psi^{\frac{1}{\sigma+\gamma}} \exp\left(\frac{1+\sigma}{\gamma+\sigma} \cdot (\alpha + \varepsilon)\right),$$

$$h_{AUT}(\alpha, \varepsilon) = \psi^{\frac{1}{\sigma+\gamma}} \exp\left(\frac{1-\gamma}{\gamma+\sigma} \cdot (\alpha + \varepsilon)\right).$$

As with Cobb-Douglas preferences, allocations depend only on the current period wage, $w(\alpha, \varepsilon) = \exp(\alpha + \varepsilon)$ and not on the two shocks separately. Whether hours increase or
decrease with individual productivity depends on the relative strength of income versus substitution effects. With separable preferences, the size of the income effect is determined by the consumption risk aversion parameter, $\gamma$.

**Complete markets (CM)**—Equilibrium allocations for consumption and hours worked in the complete markets economy are given by:

$$c_{CM}(\alpha, \varepsilon) = \bar{c} = \psi^{\frac{1}{\gamma + \sigma}} \exp \left( \frac{1}{\gamma + \sigma} \cdot \frac{v}{2\sigma} \right),$$  \hspace{1cm} (24) \\

$$h_{CM}(\alpha, \varepsilon) = \psi^{\frac{1}{\gamma + \sigma}} \exp \left( \frac{1 + \sigma}{\gamma + \sigma} \cdot \frac{-\gamma v}{2\sigma^2} \right) \exp \left( \frac{\alpha + \varepsilon}{\sigma} \right).$$  \hspace{1cm} (25)

These allocations are easy to interpret. First, since utility is separable in consumption and hours worked, agents insure fully against fluctuations in consumption, so consumption is constant across states and over time. This consumption level is equal to average labor earnings and is increasing in the variance of the shock $v$ (we return to this point below). Hours worked are increasing in individual productivity and there is no distinction between permanent and transitory shocks in the labor supply decision, exactly as in autarky, but for the opposite reason: all shocks are equally insured. The Frisch elasticity $1/\sigma$ determines the responsiveness of individual hours to differences in the individual wage. Note that average consumption is increasing in the variance of wages, in contrast to the Cobb-Douglas preferences case. Expressions for asset purchases and prices are as in the Cobb-Douglas case.

**Incomplete markets (IM)**—In the incomplete markets economy, when preferences are separable between consumption and hours worked, equilibrium allocations are given by:

$$c_{IM}(\alpha, \varepsilon) = \psi^{\frac{1}{\gamma + \sigma}} \exp \left( \frac{1 + \sigma}{\gamma + \sigma} \cdot \frac{v}{2\sigma} \right) \exp \left( \frac{1 + \sigma}{\gamma + \sigma} \cdot \frac{\alpha}{\sigma} \right),$$  \hspace{1cm} (26) \\

$$h_{IM}(\alpha, \varepsilon) = \psi^{\frac{1}{\gamma + \sigma}} \exp \left( \frac{1 + \sigma}{\gamma + \sigma} \cdot \frac{-\gamma v}{2\sigma^2} \right) \exp \left( \frac{1 - \gamma}{\gamma + \sigma} \cdot \frac{\alpha}{\sigma} \right) \exp \left( \frac{\varepsilon}{\sigma} \right).$$  \hspace{1cm} (27)

Equilibrium allocations are closely related to the expressions for the autarkic and complete markets economies. In particular, the logarithm of consumption and hours are, up to a constant, convex combinations of the logarithms of consumption and hours under
complete markets and autarky. Individual consumption is independent of the realization of the transitory shock, since that can be fully insured, but is rescaled by the individual permanent effect $\alpha$, as under autarky. Hours worked are increasing in the transitory shock $\varepsilon$, since these shocks have a substitution effect but no income effect given that they are perfectly insured. Permanent shocks do have an income effect, and whether hours increase or decrease with $\alpha$ depends on whether $\gamma$ is below or above one.

5.2 Welfare with separable preferences

We are now ready to state a pair of Propositions equivalent to Propositions 1 and 2 in which we first characterize the welfare implications of a rise in wage dispersion within a given market structure, and then the welfare gains of completing the markets for a given amount of uninsurable wage dispersion. Note that the approximated expressions, as well as the exact ones, do not depend on $\psi$, but only on the parameters $(\gamma, \sigma, \Delta v_\alpha, \Delta v_\varepsilon)$.

**Proposition 1a:** With separable preferences, the (approximate) welfare gain of a change in labor market risk equal to $\Delta v$, where $\Delta v = \Delta v_\varepsilon + \Delta v_\alpha$, is given by the following expressions under the three market structures we consider:

\[
\chi_{AUT} \approx \left[\frac{1 - \gamma}{\sigma + \gamma} - \frac{1 + \sigma}{\sigma + \gamma}\right] \frac{\Delta v}{2},
\]

\[
\chi_{CM} \approx \frac{1}{\sigma} \frac{\Delta v}{2},
\]

\[
\chi_{IM} \approx \frac{1}{\sigma} \frac{\Delta v_\varepsilon}{2} + \left[\frac{1 - \gamma}{\sigma + \gamma} - \frac{1 + \sigma}{\sigma + \gamma}\right] \frac{\Delta v_\alpha}{2}.
\]

**Proof:** See Appendix.

**Complete Markets**—As with Cobb-Douglas preferences, under complete markets, increasing productivity dispersion strictly increases welfare, as long as the labor supply elasticity is positive ($\sigma$ finite). Once again, the intuition is simply that an unconstrained planner can achieve better allocative efficiency with larger dispersion by having more productive agents specialize in market work.

Panel (A) in Figure 3 plots $\chi_{CM}$ as a function of $\sigma$ for $\Delta v$ normalized to one. The larger the Frisch elasticity, the greater the opportunities for exploiting the heterogeneity
in labor productivity, and the larger the welfare gains from increased wage dispersion. As an example, consider the case with unit elasticity of labor supply ($\sigma = 1$). Then an increase in wage dispersion translates into a rise in welfare of half its size. Note that with inflexible labor supply ($\sigma \to \infty$) rising wage inequality has no welfare implications since hours worked is equal for all agents.

**Autarky**—In autarky, when $\gamma > 1 / (2 + \sigma)$, there is always a welfare loss from raising productivity dispersion. Note that as $\sigma \to \infty$, the welfare cost of rising productivity fluctuations in autarky becomes

$$\chi_{AUT} \approx -\frac{\gamma}{2} \Delta v,$$

the expression computed by Lucas (1987) for the welfare costs of aggregate consumption fluctuations in an economy with inelastic labor supply. Note that $\partial \chi_{AUT} / \partial \sigma < 0$. Thus, introducing flexible labor supply always (weakly) reduces the welfare cost of uninsurable wage fluctuations, in contrast to the Cobb-Douglas case above. Precisely how labor supply effectively substitutes for the presence of missing insurance markets depends on the value for $\gamma$. When $\gamma > 1$, the income effect from a positive wage shock dominates the substitution effect, so agents increase work effort in bad times. In this case, flexible labor supply is used to improve consumption smoothing. When $\gamma < 1$, the substitution effect dominates the income effect, and agents increase work effort in good times. In this case, flexible labor supply actually increases consumption volatility, but it is still beneficial because agents are relatively unconcerned about fluctuations in consumption, and concentrating work effort in high wage periods raises average output per hour. It is only one (knife-edge) case when flexibility fails to mitigate the welfare cost of additional wage risk, namely when $\gamma = 1$ and labor supply is constant across all agents (see equation (23)).

Interestingly, when risk aversion is sufficiently small, a rise in $v$ has a positive welfare effect (see panel (B) in Figure 3). The complete-markets result sheds some light on this. When $\gamma = 0$ (the risk-neutrality case), it is easy to see that $\chi_{AUT} = \chi_{CM} > 0$. For low levels of risk-aversion, $\gamma < 1 / (2 + \sigma)$, agents willingly substitute labor supply intertemporally to raise average productivity, and are relatively unconcerned about the resulting fluctuations in consumption.
We conclude that for flexible labor supply to mitigate the welfare cost of increases in uninsurable wage risk it must be the case that preferences are inconsistent with balanced growth.

**Incomplete Markets**—The welfare loss under incomplete markets is a convex combination of the loss under the other two market structures.

We now revisit the welfare gains associated with expanding the set of insurance assets that may be traded

**Proposition 2a**: With separable preferences, the (approximate) welfare gains from increasing the set of assets traded in an economy with labor market risk equal to \( v = v_\alpha + v_\varepsilon \) are:

\[
\xi_{AUT\rightarrow CM} \approx \left( \left( \frac{1}{\sigma} + \frac{\gamma - 1}{\sigma + \gamma} \right) + \frac{1 + \sigma}{\sigma + \gamma} \right) \frac{v}{2},
\]

\[
\xi_{IM\rightarrow CM} \approx \left( \left( \frac{1}{\sigma} + \frac{\gamma - 1}{\sigma + \gamma} \right) + \frac{1 + \sigma}{\sigma + \gamma} \right) \frac{v_\alpha}{2},
\]

\[
\xi_{AUT\rightarrow IM} \approx \left( \left( \frac{1}{\sigma} + \frac{\gamma - 1}{\sigma + \gamma} \right) + \frac{1 + \sigma}{\sigma + \gamma} \right) \frac{v_\varepsilon}{2}.
\]

**Proof**: See Appendix.

As in the Cobb-Douglas case, the parametric expression multiplying the variance in each case has two separate components. The first is associated to the allocative efficiency gain of risk-sharing in presence of elastic labor supply: more productive households work relatively harder and less productive households enjoy more leisure. This efficient “specialization” improves with the value of the Frisch elasticity, allowing to achieve higher aggregate welfare. The second term is a measure of the value of additional insurance provided by increased risk-sharing.

Figure 4 plots \( \xi_{SEP} \) for different values of \( \sigma \) and \( \gamma \) in their admissible range \((0, \infty)\) and for \( v \) normalized to 1. Notice first that the welfare gain of completing the markets under autarky is always weakly positive and strictly increasing in \( \gamma \), the degree of risk-aversion. A few benchmarks are of interest. First, for \( \gamma = 0 \) (risk-neutrality), the welfare gain is exactly zero, since consumption fluctuations are not costly for individuals. Second, in
absence of flexible labor supply, $\sigma \to \infty$, the welfare gain is $\xi_{SEP} \simeq \gamma \frac{v}{2}$, which is exactly Lucas (1987) formula for the welfare cost of business cycles. This is intuitive, since in both cases the calculation quantifies the gain from eliminating uninsurable consumption fluctuations, and in autarky with inflexible labor supply, labor productivity fluctuations translate one to one into consumption fluctuations. Third, when $\gamma = 1$ and $\sigma = 1$, $\xi_{SEP} \simeq v$, i.e. the welfare gain of completing the markets equals exactly the variance of log-productivity.

Other things equal, greater flexibility in adjusting hours must always be welfare-improving. But is additional labor supply flexibility more useful when markets exist to pool wage risk or when they do not? On the one hand, a higher Frisch elasticity increases the value of the gain via specialization in labor supply that can be achieved when more contingent claims are traded. On the other a high elasticity reduces the (positive) value of increasing explicit insurance through financial markets, since agents can effectively adjust hours to self-insure against ‘uninsurable’ shocks. It turns out that which effect dominates depends on the particular combination of $\gamma$ and $\sigma$.

For large values of the labor supply elasticity ($1/\sigma > 1$), the specialization effect dominates, and $\xi^{AUT}$ is increasing in $1/\sigma$. For $1/\sigma \leq 1$, whether or not $\xi_{SEP}$ is increasing in $1/\sigma$ depends on whether $\gamma \leq 2\sigma/(\sigma - 1)$. For large values of risk-aversion $\gamma$, there is always a region where agents are relatively unwilling to adjust hours inter-temporally in which $\xi_{SEP}$ becomes smaller as the labor supply elasticity rises. The intuition is that given high aversion to consumption fluctuations, an increase in the willingness to substitute hours intertemporally might have a large positive impact on welfare under autarky by effectively improving self-insurance, thereby reducing the gain from expanding insurance markets (recall that when $\gamma > 1$, the income effect dominates the substitution effect in autarky and low-productivity households work harder). By contrast, for $\gamma \leq 2$, $\xi^{AUT}$ is always increasing in $1/\sigma$.\footnote{The condition under which the welfare costs of completing the markets is increasing in the Frisch elasticity has an intuitive interpretation. In particular, when $1/\sigma \leq 1$, the variance of log-hours worked is larger in complete markets relative to autarky if and only if $\gamma \leq 2\sigma/(\sigma - 1)$, the same condition we derived above under which $\xi_{SEP}$ is increasing in the Frisch elasticity.}
As documented by a vast empirical literature (see Katz and Autor, 1999; Eckstein and Nagypal 2004, for surveys), cross-sectional wage inequality has increased substantially in the United States, since the early 1970s. Our simple framework allows to quantify 1) the welfare loss of the recent rise in wage dispersion, and 2) it allows to assess how much U.S. households would benefit, at the current level of labor market risk, from the availability of a full set of insurance markets. Moreover, it allows a natural split between an “insurance” component and a component related to the “opportunities” offered by productivity dispersion in presence of flexible labor supply.

**Measurement of wage dispersion:** The first step of the calibration requires choosing values for the variances of permanent and transitory components before and after the surge in wage dispersion \((v_\alpha, v_\epsilon)\) and \((\hat{v}_\alpha, \hat{v}_\epsilon)\), respectively. From the 1968-1997 waves of the *Panel Study of Income Dynamics* (PSID), we have selected a sample of roughly 2,400 observations/year including all the heads of households (males and females) aged between 20 and 59 with positive earnings (not top-coded and not below half of the current minimum wage). We computed hourly wages as annual earnings divided by annual hours worked and found that the variance of log wages rose from 0.25 to 0.35 over this time period. Next, we estimated a simple permanent/transitory model for the variance of log wages, exactly as the process specified in equations (1). The estimated variance of the transitory/insurable component \(v_\epsilon\) starts around 0.08 and levels off 30 years later at around 0.13. The variance of the permanent/uninsurable component \(v_\alpha\) starts at a value around 0.17 and rises up to 0.22 in the mid 1990s.\(^8\)

In light of these results, when evaluating the welfare implications of rising dispersion in autarky and complete markets we set \(\Delta v = 0.10\), whereas in incomplete markets we set \(\Delta v_\epsilon = \Delta v_\alpha = 0.05\). When evaluating the welfare gains from completing the markets starting from autarky we set \(v = 0.35\), and starting from incomplete-markets we set \(v_\alpha = 0.22\) –in other words, we perform this latter computation for the levels of labor market uncertainty of the 1990s.

\(^8\)Our findings can be summarized as follows: the transitory component accounts for roughly 1/3 of the total dispersion. The rise in wage dispersion is accounted equally by the two components. These results are in line with the existing literature. See, among others, Gottschalk and Moffitt (1994), Katz and Autor (1999), Heathcote, Storesletten and Violante (2003).
Preference parameters: The second step is to choose values for the preference parameters. We report two sets of welfare estimates, one for the Cobb-Douglas case and one for the case of separable preferences.

The welfare expressions for separable preferences discussed in section 5 depend only on two parameters \((\gamma, \sigma)\). Estimates for the risk-aversion coefficient (or, identically, the intertemporal labor supply elasticity) \(\gamma\) between one and three are typical in the empirical consumption literature (see Attanasio 1999, for a survey), so we set \(\gamma = 2\), a value common also in applied macroeconomic research. Estimates for the Frisch labor supply elasticities are much less agreed upon. Recently, Domeij and Floden (2002) sample the empirical literature on male labor supply and conclude that estimates range in the interval \((0.1, 0.5)\).\(^9\) Estimates of labor supply for females can be up to four times larger (see Blundell, MaCurdy, 1999, Table 2). We set the Frisch elasticity to the mid point of that interval, corresponding to a value of \(\sigma = 3.33\). Given that we have both males and females in our wage dispersion sample, this value for \(\sigma\) can be considered a conservative estimate.

With Cobb-Douglas preferences, the Frisch labor supply elasticity and the coefficient of risk-aversion are not independent, as they are both function of the pair of parameters \((\gamma, \eta)\), as discussed in section 4.2. Moreover, the parameter \(\eta\) has a natural counterpart in the fraction of the time endowment devoted to work activities, which requires setting \(\eta = 0.33\), as extensively discussed in the macroeconomic literature on business cycles (see Cooley, 1995).\(^10\) We choose then to set \(\gamma = 4.33\) so that the implied coefficient of risk-aversion equals two, like in the separable case. As a by-product, we obtain a Frisch elasticity exactly equal to one—a not too high number, possibly, in light of our earlier comment on female labor supply being more elastic.

We recognize that our preferred parametrization may be somewhat distant from others in the literature. The key advantage of our approach leading to intuitive closed form expressions for welfare is that one can quickly compute different estimates based on al-

\(^9\)More precisely, the typical estimates of uncompensated wage elasticity for male labor supply range up to around 0.3. However, Domeij and Floden (2002) argue that they are downward biased because estimation methods ignore that borrowing constraints may bind. Simulations show that the unbiased estimates can be up to twice as large.

\(^10\)Precisely, from the first-order condition for hours worked in the non-stochastic version of the model, we obtain:

\[
\frac{1 - h_i}{h_i} = \frac{1 - \eta}{\eta}.
\]
ternative parametrizations. Figures 1-4 allow to gauge the sensitivity of our results to changes in the preference parameters.

Table 1: Welfare Results (% of lifetime consumption)

<table>
<thead>
<tr>
<th></th>
<th>Welfare gain of rise in dispersion</th>
<th>Welfare gain of completing markets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^C$</td>
<td>$\chi^A$</td>
</tr>
<tr>
<td>Cobb-Douglas Preferences</td>
<td>+4.99%</td>
<td>-10.02%</td>
</tr>
<tr>
<td>Separable Preferences</td>
<td>+1.51%</td>
<td>-8.60%</td>
</tr>
</tbody>
</table>

**Results:** We summarize our results in Table 1. In the complete-markets economy there is a sizeable increase in welfare when wage dispersion increases, especially for the Cobb-Douglas case where this gain is close to 5% of lifetime consumption, due to the higher labor supply elasticity with respect to the separable case. In contrast, the welfare loss in autarky is smaller with separable preferences. To understand this result recall that, with Cobb-Douglas preferences, in autarky hours worked are fixed and are not used as a vehicle of self-insurance, like in the separable case when $\gamma > 1$.

Putting together these first two results, and recalling that $\chi^IM$ is a weighted average of $\chi^CM$ and $\chi^AUT$, it is not surprising that the welfare losses of the observed rise in wage dispersion for the incomplete markets economy are quite similar across the two cases, between $-2.8\%$ and $-3.7\%$ of lifetime consumption. This number is quite close to the one computed by Heathcote, Storesletten and Violante (2003), and roughly twice as large than the estimate in Krueger and Perri (2003) who use a very different approach. Interestingly, the opportunity component has a non-trivial role in mitigating this welfare loss.

Let us now turn to the welfare gains from completing the markets. With Cobb-Douglas preferences, a household living in autarky would be willing to give up over 70% of his lifetime consumption to be able to face a complete set of insurance markets against her labor
market risk. Starting from an incomplete-market economy, this number drops to roughly
40% of lifetime consumption. This is the value of being able to smooth completely the
permanent wage component. With separable preferences, these estimates are consider-
abley smaller because the implied labor supply elasticity is also smaller, hence the gain
of completing markets is mostly associated to better insurance possibilities rather than
better productive opportunities. In related work, Pijoan-Mas (2004) estimates the welfare
costs of market incompleteness to be on the order of 15% of lifetime consumption.

7 Concluding Remarks

The main contribution of the paper is the analytical characterization of expressions for the
welfare gains/losses from an increase in the dispersion of labor productivity, for a large
class of heterogeneous-agents economies with various market structures, ranging from
autarky to complete markets. The transparency of the closed-form expressions allows to
identify two major offsetting forces. First, as the uncertainty rises, the value of insurance
grows and, with incomplete markets, welfare falls. Second, in presence of flexible labor
supply, a rise in dispersion represents an improvement in productive opportunities for
households, and welfare rises. Interestingly, the mechanism behind this latter welfare
gain is very different according to whether preferences are Cobb-Douglas or separable in
consumption and leisure.

We applied our framework to the recent observed surge in cross-sectional wage disper-
sion in the United States and quantified the welfare loss associated to higher labor market
uncertainty. We found that improved production opportunities account for a welfare gain
of approximately 1% of lifetime consumption, whereas the lack of insurance induces a loss
of 4%, for a net welfare loss around 3% of lifetime consumption. These numbers are at
least 2 orders of magnitude larger than commonly estimated welfare losses of business
cycles fluctuations (Lucas 2003) and suggest that redistributive policies can, potentially,
have much larger welfare effects than aggregate stabilization policies.

In this paper, we purposefully kept our framework of analysis simple, to highlight
the forces at work in shaping the welfare analysis. In particular, we chose to model the
uninsurable component as a fixed effect drawn at the beginning of the life-cycle. Although
the most recent statistical analyses of the wage process conclude that the variance of the fixed effect is large, permanent shocks to wages also play a major role.

In Heathcote, Storesletten and Violante (2004) we extend the framework studied in this paper to include an uninsurable random-walk component to labor market risk. The framework remains surprisingly tractable and can also easily handle preference heterogeneity in the form of permanent cross-sectional differences in the relative taste for consumption versus leisure. We show that one can obtain exact analytical expressions for the cross-sectional equilibrium means, variances and covariances of wages, hours, earnings and consumption implied by the incomplete markets model.

These moments allow to (over-)identify and estimate all the structural parameters of the model, both the preference parameters and the variances of the insurable and uninsurable shocks. Hence, the framework allows a generalization of the methodology used by Blundell and Preston (1998) and Blundell, Pistaferri and Preston (2004) in two directions: 1) all expressions are exact rather than approximated, and 2) one can also identify and estimate preference parameters, beyond the fraction of the variance of the shocks which is insurable.
References


Appendix

8.1 Equilibrium allocations

Incomplete markets— It is useful to start from the incomplete markets case and then generalize our argument to the complete-markets economy. Consider a typical “component-planner” who chooses efficiently consumption and hours worked for all agents on a particular $\alpha$–island. First, note that only an agent’s current transitory shock $\varepsilon$ (and the island-wide value for $\alpha$) can be relevant for consumption and labor allocations. This follows from the fact that the planner weights all agents identically, and preferences are time-separable. Thus the component planner’s problem is given by

$$\max_{\{c(\alpha,\varepsilon)h(\alpha,\varepsilon)\}_{\varepsilon \in \mathcal{E}}} \int u(c(\alpha,\varepsilon)h(\alpha,\varepsilon)) \phi_{\varepsilon}(\varepsilon)d\varepsilon$$

subject to

$$\int_{\mathcal{E}} (w(\alpha,\varepsilon)h(\alpha,\varepsilon) - c(\alpha,\varepsilon)) \phi_{\varepsilon}(\varepsilon)d\varepsilon = 0$$

Separable preferences: When preferences are separable between consumption and hours, the first-order conditions for the planner imply

$$c(\alpha,\varepsilon) = \lambda^{-1/\gamma},$$

$$h(\alpha,\varepsilon) = \psi^{-1/\sigma} c(\alpha,\varepsilon)^{-\gamma/\sigma} \exp\left(\frac{\alpha + \varepsilon}{\sigma}\right),$$

where $\lambda$ is the Lagrange multiplier on the resource constraint for the component planner. Note immediately from equations 30 that when preferences are separable, the planner gives all agents on an $\alpha$–island the same level of consumption.

Using the planner’s resource constraint (equation 29), it is straightforward to solve for $\lambda$:

$$\lambda = \psi^{1+\sigma/\gamma} \exp\left(-\gamma \left(\alpha + \frac{v}{\sigma} \frac{1 + \sigma}{\sigma + \gamma}\right)\right).$$

Substituting this expression into equations (30) yields efficient allocations only as a function of the primitive parameters, as described in equations 26 in the text.

Cobb-Douglas preferences: In the Cobb-Douglas case, the planner’s first order condition for hours may be written as

$$c(\alpha,\varepsilon) \frac{1 - \eta}{\eta} = w(\alpha,\varepsilon) (1 - h(\alpha,\varepsilon))$$

30
or equivalently
\[ c(\alpha, \varepsilon) \frac{1 - \eta}{\eta} - w(\alpha, \varepsilon) = -w(\alpha, \varepsilon) h(\alpha, \varepsilon) \]

Substituting the left hand side of this latter equation into the resource constraint (equation 29) and collecting terms gives
\[ \int_{\mathcal{E}} c(\alpha, \varepsilon) \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon = \eta \int_{\mathcal{E}} w(\alpha, \varepsilon) \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon = \eta \exp(\alpha) \]  
(32)

The first order condition for consumption is
\[ \lambda = \eta c(\alpha, \varepsilon)^{\eta(1-\gamma)-1} (1 - h(\alpha, \varepsilon))^{(1-\eta)(1-\gamma)} \]
(33)

Using the intratemporal first order condition to substitute out for leisure in equation (33) and rearranging gives
\[ c(\alpha, \varepsilon) = \left( \frac{\eta}{\lambda} \right)^{1/\gamma} \left( \frac{1 - \eta}{\eta} \right)^{(1-\eta)(1-\gamma)/\gamma} w(\alpha, \varepsilon)^{-\eta(1-\gamma)/\gamma} \]
(34)

Integrating across the population
\[ \int_{\mathcal{E}} c(\alpha, \varepsilon) \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon = \left( \frac{\eta}{\lambda} \right)^{1/\gamma} \left( \frac{1 - \eta}{\eta} \right)^{(1-\eta)(1-\gamma)/\gamma} \int_{\mathcal{E}} w(\alpha, \varepsilon)^{-\eta(1-\gamma)/\gamma} \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon \]
(35)

Note that
\[ \int_{\mathcal{E}} w(\alpha, \varepsilon)^{-\eta(1-\gamma)/\gamma} \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon = \int_{\mathcal{E}} (\exp(\alpha) \exp(\varepsilon))^{-\eta(1-\gamma)/\gamma} \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon \]
\[ = \exp \left( \frac{(1 - \eta)(\gamma - 1)}{\gamma} \right) \int_{\mathcal{E}} \exp(\varepsilon)^{-\eta(1-\gamma)/\gamma} \phi_{v_{\varepsilon}}(\varepsilon) d\varepsilon \]
\[ = \exp \left( \frac{(1 - \eta)(\gamma - 1)}{\gamma} \right) \exp \left( (1 - \eta)(1 - \gamma) \frac{1 - \eta + \eta \gamma v_{\varepsilon}}{\gamma^2} \right) \]

where the last step exploits the fact that \( \varepsilon \) is log-normally distributed.

Combining equations (32) and (35) implies
\[ \left( \frac{\eta}{\lambda} \right)^{1/\gamma} \left( \frac{1 - \eta}{\eta} \right)^{(1-\eta)(1-\gamma)/\gamma} = \eta^{-1} \exp \left( \frac{(1 - \eta)(\gamma - 1)}{\gamma} \right) \exp \left( (1 - \eta)(1 - \gamma) \frac{1 - \eta + \eta \gamma v_{\varepsilon}}{\gamma^2} \right) \exp(-\alpha) \]
(36)

Substituting this term into equation (34) to solve for consumption and then using the first order condition for hours (equation 31) to solve for hours yields efficient allocations only as a function of the primitive parameters, as described in equations ?? in the text.
Endowing the planner with a savings technology: Note that \( v_z \) is time invariant, since we are considering a stationary environment. Thus within an \( \alpha \)-island, consumption is constant through time. It follows immediately we could endow the component-planner with the ability to transfer resources through time via a technology offering gross return \( \beta^{-1} \) without affecting allocations.

8.2 Proof of Proposition 1a

Complete markets—The welfare level associated to the complete markets equilibrium with time-invariant log-wage variance \( v = v^* \) can be therefore expressed as

\[
W^{CM} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{I} \phi_i u (c_{it}^*, h_{it}^*)
\]

where the first equality follows from the equal-weights assumption and the time-invariance of the distribution of the labor productivity shocks; the second equality uses the efficient allocations \( ??? \) and the log-Normality assumption.

For convenience, let us restate equation (7) characterizing the welfare gain from a change in inequality from \( v^* \) to \( \hat{v} = v^* + \Delta v \)

\[
\left( \frac{(1 + \chi^{CM})^{1-\gamma}}{1-\gamma} - \frac{1}{1+\sigma} \right) \exp \left( \frac{1+\sigma}{\sigma} \frac{1-\gamma}{1+\sigma} v^* \right) = \frac{\gamma + \sigma}{(1-\gamma)(1+\sigma)} \exp \left( \frac{1+\sigma}{\sigma} \frac{1-\gamma}{1+\sigma} \hat{v} \right),
\]

where the last expression can be rewritten, after some simple algebra, as

\[
1 + \chi^{CM} = \left[ \frac{1-\gamma}{1+\sigma} + \frac{\gamma + \sigma}{1+\sigma} \exp \left( \frac{\kappa \Delta v}{\sigma} \right) \right]^{\frac{1}{1-\gamma}},
\]

where \( \kappa \equiv \frac{(1+\sigma)(1-\gamma)}{\sigma + \gamma} \) is a useful constant that reappears frequently in our welfare analysis. Equation (38) provides the exact welfare gain of a rise in wage dispersion in complete
markets. Using a log-approximation of the type \( \ln (1 + x) \simeq x \) on the left-hand side of that equation, and the approximation \( \exp (x) \simeq 1 + x \) on the right-hand side, we obtain

\[
\chi^{CM} \simeq \frac{1}{1 - \gamma} \ln \left[ \frac{1 - \gamma + \gamma + \sigma}{1 + \sigma} \left( 1 + \frac{(1 + \sigma)(1 - \gamma) \Delta v}{\sigma + \gamma} \right) \right] \\
= \frac{1}{1 - \gamma} \ln \left[ 1 + (1 - \gamma) \frac{\Delta v}{2\sigma} \right] \\
\simeq \frac{\Delta v}{2\sigma},
\]

the expression for \( \chi^{CM} \) in Proposition 2.

**Autarky**—In autarky, at each time \( t \) individual \( i \) solves the static problem

\[
\max_{c_{it}, h_{it}} \left\{ \frac{c_{it}^{1-\gamma}}{1-\gamma} - \psi \frac{h_{it}^{1+\sigma}}{1+\sigma} \right\} \\
\text{s.t.} \quad c_{it} = w_{it} h_{it}
\]

with solution \( h_{it}^* = \psi^{-\frac{1}{\sigma+\gamma}} w_{it}^{\frac{1-\gamma}{\sigma+\gamma}} \) and \( c_{it}^* = \psi^{-\frac{1}{\sigma+\gamma}} w_{it}^{\frac{\sigma+1}{\sigma+\gamma}} \). Once again, to measure welfare we focus on the utilitarian function defined in equation (6) to obtain

\[
\mathcal{W}^{AUT} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{I} \phi_i u (c_{it}^*, h_{it}^*) \\
= \frac{1}{I} \left( \frac{1}{1-\gamma} - \frac{1}{1+\sigma} \right) \psi^{-\frac{1}{\sigma+\gamma}} \sum_{i=1}^{I} w_{i}^{\frac{(\sigma+1)(1-\gamma)}{\sigma+\gamma}} \\
= \psi^{-\frac{1}{\sigma+\gamma}} \frac{1}{\kappa} \exp \left( -\kappa (1 - \kappa) \frac{v^*}{2} \right)
\]

where the first equality uses the time-invariance in the distribution of the shocks, the equal-weights assumption, and the optimal individual choices of consumption and hours worked; the second equality uses the log-Normality of \( w_{it} \) and the definition \( \kappa \equiv \frac{(1+\sigma)(1-\gamma)}{\sigma+\gamma} \).

Thus, equation (7) characterizing the welfare gain of a change in inequality from \( v^* \) to \( v = v^* + \Delta v \) can be restated, after some simple algebra, as

\[
1 + \chi^{AUT} = \left[ \frac{1 - \gamma + \sigma + \gamma}{1 + \sigma} \exp(-\kappa (1 - \kappa) \frac{\Delta v}{2}) \right]^{1/\kappa},
\]

Once again, using standard log-approximations, equation (41) yields the approximated expression for \( \chi^{AUT} \) in the main text.
Incomplete markets—We start by computing the ex-ante expected utility of a household, conditional on $\alpha$. Using the time-invariance and the log-Normality of the distribution for $\varepsilon_{it}$ and the expressions for the optimal choices of consumption and hours worked in (30), we can show that

$$W_{\alpha}^{IM} = \frac{1}{I^\alpha} \sum_{i=1}^{I^\alpha} \left[ (c^*_\alpha)^{1-\gamma} - \psi h^*_\alpha (\varepsilon)^{1+\sigma} \right] = \psi^{-\frac{1-\gamma}{\sigma+\gamma}} \frac{1}{\kappa} \exp \left( \kappa \left( \alpha + \frac{\varepsilon^*_\alpha}{2\sigma} \right) \right),$$

where $\kappa \equiv \frac{(1+\sigma)(1-\gamma)}{\sigma+\gamma}$, exactly as in the autarky case. Averaging over the entire distribution of fixed effects, the expected utility under the veil of ignorance for an individual “dropped randomly” in this economy is then given by

$$W^{IM} = E \left[ W_{\alpha}^{IM} \right] = \psi^{-\frac{1-\gamma}{\sigma+\gamma}} \frac{1}{\kappa} \exp \left( \kappa \left( \frac{\varepsilon^*_\alpha}{\sigma} - \left(1 - \kappa\right) v^*_\alpha \right) \right). \tag{42}$$

Consider now the welfare effects associated with a change in inequality from $v^*$ to $\hat{v}$ where the variance of the fixed-effect increase from $v^*_\alpha$ to $\hat{v}_\alpha = v^*_\alpha + \Delta v_\alpha$, and where the variance of the insurable component $\varepsilon_{it}$ increase from $v^*_\varepsilon$ to $\hat{v}_\varepsilon$. As usual, we focus on the equivalent variation, expressed as a fraction $\chi^{IM}$ of consumption (the percentage increase in consumption in the economy with variance $v^*$ required to let the planner achieve the same welfare level as in the economy with higher productivity dispersion). Using (42) into equation (7) yields

$$1 + \chi^{IM} = \left[ 1 - \gamma + \frac{\gamma + \sigma}{1 + \sigma} \exp \left( \kappa \left( \frac{1}{2} \frac{\Delta v_\varepsilon}{\sigma} - \left(1 - \kappa\right) \frac{\Delta v_\alpha}{2} \right) \right) \right]^{\frac{1}{1-\gamma}}. \tag{43}$$

The usual log-approximations deliver the expression for $\chi^{IM}$ in the main text.

TO BE COMPLETED
Figure 1: Cobb-Douglas Preferences

(A) Welfare gain of a rise in labor market risk under complete markets ($\Delta v$ normalized to 1)

(B) Welfare gain of a rise in labor market risk under autarky ($\Delta v$ normalized to 1)
Figure 2: Cobb-Douglas Preferences

Welfare gain of completing the markets under autarky $\xi$
(variance of uninsurable shock normalized to 1)

$\gamma = 1$
$\gamma = 2$
$\gamma = 3$
$\gamma = 5$
$\gamma = 10$
Figure 3: Separable Preferences

(A) Welfare gain of a rise in labor market risk under complete markets 
($\Delta v$ normalized to 1)

(B) Welfare gain of a rise in labor market risk under autarky 
($\Delta v$ normalized to 1)
Figure 4: Separable Preferences

Welfare gain of completing the markets \( \xi \)
(variance of uninsurable shock normalized to 1)
Figure 5: Cobb-Douglas Preferences

Approximation bias in $\chi^\text{CM}$ ($\Delta v = .1$)

Approximation bias in $\chi^\text{AUT}$ ($\Delta v = .1$)

Approximation bias in $\xi^\text{AUT}$ ($v = .3$)
Figure 6: Separable Preferences

Approximation bias in $\chi^{CM}$ ($\Delta v=0.1$)

Approximation bias in $\chi^{AUT}$ ($\Delta v=0.1$)

Approximation bias in $\zeta^{AUT}$ ($v=0.3$)