The Demographic Transition in Closed and Open Economy: A Tale of Two Regions*

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Abstract 

This paper constructs a general equilibrium overlapping generation model to evaluate quantitatively how a demographic transition (declining mortality and fertility rates) affects aggregate variables (wages, interest rate, output), and inter-generational welfare in closed and open economy. We perform this analysis for two economies calibrated to resemble the North (US and Europe) and Latin America. When we assume that the two regions will not open to capital flows, the main finding is that while the beneficial effects for the North will quickly fade away in the next decade, Latin America should still benefit from higher than average growth rates for the next half century at least. In terms of welfare, the demographic transition in closed economy is costly for the North, while in Latin America it will generate welfare gains up to 20% of lifetime consumption. When we allow for perfect capital mobility across the two regions, starting from the mid 1990’s, the key result is that international capital flows accelerate the adjustment process in Latin America by exacerbating income growth in the short-run and reducing it in the long-run. The largest relative welfare gains from opening the economy (relative to the closed economy transition) accrue to the baby-boom generations in the North (+2%), and to the cohorts born around the opening in Latin America (+6%). The reason is that the implied capital flows across regions raise the interest rate in the North and the wage rate in Latin America.

Keywords: Baby-Boom, Demographic Transition, Latin America, Open Economy.

JEL Classification: E21, F43, J11, O54.

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1 Introduction

Many regions of the world are currently in the middle of significant demographic transitions. The main elements characterizing these transitions, namely a reduction in fertility rates and an increase in life expectancy, are common across many regions. However, the pace and timing of these changes are not, as different regions of the world are going through different stages of the demographic change.

Most of the developed world, and in particular the US, Europe and Japan have already reached very high levels of dependency ratios, as measured by the proportion of individuals aged over 65 to those aged between 16 and 64.\(^1\) On the other hand, Latin America has just experienced a dramatic drop in fertility rates: in the last 30 years, they have declined from over five children per woman to less than three. Even after such a sharp decline, however, fertility is still well above the low levels observed in Europe and the US.\(^2\)

The implications of these massive demographic changes are far reaching. The debate on the demographic trends is centered on the sustainability of defined-contributions social security schemes financed on a pay-as-you-go (PAYG) basis, common in developed countries. In the US, for instance, it is projected that the Social Security trust fund, which is currently running a substantial surplus thanks to the presence of the large baby-boom generation in the labor force, will exhaust around the year 2040 (OASDI Board of Trustees Report, 2004). The literature on computational experiments using OLG models of the kind pioneered by Auerbach and Kotlikoff (1987) is vast. Some recent examples for the US are Conesa and Krueger (1998), Huggett and Ventura (1999), DeNardi, Imrohoroglu and Sargent (1999), Kotlikoff, Smetters and Walliser (1999). Miles (1998) and Miles and Timmermann (1999) applied similar models to European countries. See Diamond (2004) for a recent comprehensive essay on social security reform in the US.

Surprisingly, much less attention has been devoted in the literature to another important aspect of demographic trends in developed countries: the implications for the “baby-boomers”, i.e. the welfare dynamics of a relatively “large” generation followed by a relatively smaller one,\(^1\) There are some differences even within the developed world. For example, thanks mainly to the larger immigration flow, the US have a higher population growth rate than Europe. See Storesletten (2000) for an quantitative analysis of the role of migration for fiscal policy in the US.

\(^2\) Other regions of the world are at different stages of the transition. Some countries of South East Asia have passed a few years ago through the phase Latin America is undergoing now, while some regions of Africa are still much younger and experiencing remarkably high growth rates of the population. Even China witnessed a dramatic decline in fertility rate in the past two decades, mostly due to coercive family planning government policies, such as the “one-child” policy implemented in the late 70’s.
even in situations where the pension system is fully funded and possibly private. In general equilibrium, there will be effects on factor prices: when the capital-labor ratio is high, as it will be after the retirement of the baby-boom cohorts, the return on capital is bound to be low. As a consequence, the consumption that can be sustained by the “large” generation when it retires might be small. This turns out to be true even when such a generation understands and forecasts the prevailing demographic trends and can save to provide for its retirement. To make this point starkly, we work in a model without social security where all expenditures during retirement are financed through lifetime savings.\(^3\)

For the developing regions of the world, the implications of the current demographic trends are different. Since the demographic transition is less advanced, some authors argue that the next decades constitute a *window of opportunity* for these regions. For example, Behrman et al. (1999) speculate that the demographic structure that will be prevailing in the next decades in Latin America might be associated to high saving rates and, therefore, to faster capital accumulation. Other authors have uncovered a strong empirical nexus between changes in the demographic structure and aggregate macroeconomic variables. Some recent contributions includes Taylor (1995) on saving rates in Latin America, Bloom and Williamson (1998) on income per capita growth in South-East Asia, Higgins and Williamson (1997) on capital inflows also in South-East Asia.

Methodologically, the results in this literature are vastly based upon estimation of reduced-form saving and output equations with demographic variables as regressors. Even though this empirical approach has yielded useful insights on the economic association between demographic dynamics and macroeconomic outcomes, it has a number of limitations. First, it ignores the general equilibrium interactions between factor prices and capital accumulation. Second, although it allows to quantify the impact of demographic changes on aggregate output, it is silent on the inter-generational welfare consequences, which is ultimately a crucial question.\(^4\)

In this paper we contribute to the ongoing debate on the economic impact of the demographic transition in two ways. First, we calibrate a general equilibrium overlapping-generations models to reproduce some basic facts of both developed and developing economies, in terms of basic demographic trends and in terms of equilibrium steady state values. We choose US and Europe to represent the North and Latin America to represent the South. We then simulate

\(^3\)This effect can be even more dramatic if such a generation has to pay, at least in part, the cost of the transition from an un-funded to a funded scheme.

\(^4\)Another limit is that the demographic transition is always assumed to be exogenous, but this is true in our model as well.
the transition of our two economies from an initial steady state to a new one characterized by much lower fertility rates and higher life expectancy, under the assumption of no capital mobility.

Second, we recognize that a key aspect of this debate is whether the right benchmark for these analyses is a closed or an open economy. The different global demographic trends induce diverging dynamics of the implicit rates of returns in the North and the South of the world. Thus, in an open economy, with capital mobility, these rates of return differential will command large capital flows. Potentially, the answers to the questions above can be very different according to the assumption made about factor mobility. Thus, we repeat our simulations under the assumptions of perfect capital mobility.

The focus is, for the North, on quantifying the welfare loss of the baby boomers, the generations that most suffer from the demographic transition and on studying whether opening the economy can alleviate their losses. For Latin America, the interest lies in establishing the impact of the process of capital mobility on income growth and whether the welfare effects on the generations undergoing the transition are attenuated or exacerbated.

There are several exercises in the literature closest to ours in spirit. Storesletten (2000) has quantified the effect that migration flows into the US can have on the welfare of the baby-boom generations. In a sense, our exercise complements Storesletten’s by looking at the mobility of capital rather than labor. Brooks (2000), Abel (2003), and Lim and Weil (2003) have studied the effect of the baby-boom on the stock market. We abstract from fluctuations in the price of capital (constant in our model), and focus on the fact that the arrival of an unusually large cohort will lead to fluctuations in the capital-labor ratio and the rate of return on capital. Brooks (2003) has solved a multi-region two-period equilibrium OLG model of the world to study whether the diverging demographic trends can explain the observed patterns of international capital mobility over the past 50 years and predict future trends. We are not so interested in the size or the direction of the capital flows per se, but rather on its impact on the welfare of the generations in Latin America, in the open economy experiment.

The rest of the paper is organized as follows. In Section 2 we document the current and projected demographic trends in Europe, the US and Latin America. In Section 3 we present the overlapping generation model whose basic structure we use in most simulations. In section 4 we explain how we parameterize and calibrate our model economies. In Section 5 we present the benchmark simulations and the welfare calculations by cohort for the two closed economies. In section 6 we repeat this exercise under the assumption of perfect capital mobility across
regions. Section 7 concludes the paper.

2 Demographic trends in the North and Latin America

In this section we briefly illustrate the main demographic trends that motivate the exercise we undertake below. For this purpose, rather than focusing on a few countries or detailing the trends in all countries in the regions of interest, we construct two wide aggregates. The first, which we label the “North”, is composed of the United States and the European Countries. The second is composed of South and Central American countries (including the Caribbeans). The source of information on the demographic variables and projections is the *World Population Prospects: The 2000 Revision*, a United Nations data set.

In Figure 1 we plot, from 1950 to 2050 actual and projected data on fertility rates, life expectancy, population growth and dependency ratios in the two regions. In the first panel we plot average fertility rates (measured as the average number of children of women aged 15 to 45) in the two regions: notice that even though fertility has decreased dramatically in Latin America, it is still substantially above the levels observed in the North. Convergence of fertility rates is expected to happen around 2040. Life expectancy, on the other hand, is still much higher in the North (as of today, it is still 7 years longer) although there is some tendency for the two series to slowly converge.

As can be seen from the upper-right panel of Figure 1, while population growth in the North is slowing down and even expected to be slightly negative after 2025, the population of Latin America is still increasing considerably: in the next two decades population in Latin America will be growing above 1% per year, thus the relative size of the two regions in terms of population is projected to reach 1 around the year 2020.

In the lower-left panel we plot the elderly dependency ratio, defined as the ratio between the number of individuals aged more than 64 to the number of individuals aged 15 to 64. The elderly dependency ratio in the North, after decelerating during the 1970s and 1980s, when the bulk of the baby boomers reached maturity, is now increasing faster and it is projected to increase dramatically over the next 50 years. This sharp acceleration is caused by the dramatic drop in fertility and the increase in longevity illustrated above. The acceleration is particularly pronounced in Europe where the decline in fertility was greater. In Latin America, instead, the dependency ratio is still fairly constant and is not projected to start climbing until the year 2010. The absolute difference between dependency ratios across regions (7% in Latin America and
22% in the North, in the year 2000) is striking and shows that there is a substantial potential for capital accumulation in Latin America as a large number of working age individuals will have to support a relatively small number of retirees. Only towards the end of the period considered (2050) there is a slight tendency for the dependency ratio in Latin America to catch up with that of the North. But even then, the difference will be substantial.

The simple demographic trends illustrated so far show the enormous potential for factor mobility that might exist between the two regions. However, these trends are only part of the story. There are some important factors they neglect. The three more important ones are: (i) labor force participation rates, (ii) human capital accumulation and (iii) differences in the existing stock of capital. While an detailed discussion of these factors is beyond the scope of this paper, it is worth to mention briefly some of their implications for the issues at hand.

(i) Labor force participation rates have changed dramatically over the last 25 years and keep changing. In the North, men’s participation has decreased and women’s participation has increased so that, in many countries, nowadays the two are quite similar. The large increase in female labor force participation means that in the North there is a limited scope for a further rise in the workforce that could come from this source. In Latin America, female labor force participation has also increased. However, it is still considerably lower than that of men or that of women in the North. Therefore an additional change in the relative sizes of the labor forces could result from historical trends towards higher female participation into the labor force in Latin America. We will not investigate this phenomenon because it would require a considerably richer model with an endogenous participation decision. Nevertheless, we reckon that neglecting this issue leads us to underestimating the effects we stress in the paper.

(ii) We will also neglect the issue of human capital accumulation. This is a very important topic of research that, however, is beyond the scope of this paper. What is important is not so much the different level of human capital across regions, which we somehow capture, but rather the different evolution of the stock over time. For example, higher schooling levels are associated to steeper wage income profiles, and in turn to different lifetime patterns of savings. However, modeling this transition in the income profile is far from the objective of this paper, where we wish to focus solely on the demographic transition. It must be kept in mind that the quantitative analysis might be affected by this omission.

(iii) In addition to the differences in number of workers, the initial per-capita capital stock in the two regions are also very different. We will calibrate the two model economies so to start them with the observed different levels of capital stocks and capital/labour ratio. To give an
idea of the magnitude involved, the *Penn World Tables* show that capital per worker for the
US in 1965 was about 3 times that of Mexico, Colombia and Chile. The US value relative to
other Latin American countries look quite similar.\(^5\)

Finally, in Figure 2 we have plotted the growth rate of GDP per capita in the past 50
years in the North and Latin America. The sample averages are 2.4\% for the North and 1.8\%
for Latin America. This difference is entirely due to the recessions post-1980 which hit Latin
America particularly strongly: until the late 70’s both regions were growing at around 2.9\%
per year. It is useful to keep these numbers in mind because, when we simulate our model,
we’ll be in the position to assess how much of the observed growth in income can be attributed
to the demographic transition.

### 3 The Model Economy

To quantify the effects that the demographic trends we discussed above have on the variables
of interest (and ultimately on the welfare of different generations), we calibrate a relatively
simple general equilibrium model in the spirit of Auerbach and Kotlikoff (1987), DeNardi,
Imrohoroglu and Sargent (1999, 2001) and others. While the construction of a manageable
general equilibrium model will require some strong and at time questionable assumptions, such
a model is very useful to quantify rigorously the effects under study. The model we propose
necessarily leaves out a number of important factors. With some exception discussed below,
however, we do not believe that the assumptions made affect our results in any crucial way.

As mentioned above, we present two versions of the model. The first assumes that the two
regions (North and Latin America) are completely isolated. In the second, we still assume that
capital can freely move from the North to Latin America. In this section we discuss those
blocks of the model that are common to the two versions, and then we define the two concepts
of equilibrium for the closed and the two-region open economy model.

#### 3.1 Demographics

The economy consists of overlapping generations of ex-ante identical individuals who live for
at most \(I_1\) periods. Therefore, at any point in time, there are \(I_1\) different generations alive.
Individuals remain children until age \(I_0\) and as children they don’t make independent decisions,
they do not consume nor work. After period \(I_0\) they become adults and, for the rest of their

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life, they make decisions. An individual of age $j$ at time $\tau$ faces a time-varying probability $\pi_{\tau,j}$ of surviving into age $j+1$. Furthermore, in each period these individuals give birth (without mating) to a certain number of children, according to a time and age-specific fertility rate $\phi_{\tau,j}$. Following Lee (1974), Rios-Rull (2001) and Storesletten (2000), the evolution of the population structure can be described by a simple matrix of dimension $(I_1 \times I_1)$. If we denote with $\Gamma_\tau$ such a matrix for time $\tau$ and with $\mu_\tau$ the $(I_1 \times 1)$ vector containing the number of individuals in each age group at time $\tau$, the evolution of the population structure is given by:

$$\mu_{\tau+1} = \Gamma_\tau \mu_\tau,$$

where the first row of $\Gamma_\tau$ contains the relevant fertility rates at each age, and each element $(j+1, j)$, with $j = 1, \ldots, I_1 - 1$, contains the terms $\pi_{\tau,j}$, i.e. the conditional probabilities, at time $\tau$, of surviving from age $j$ to age $j+1$. The remaining elements of the $\Gamma_\tau$ matrix are all zero.

Since the economic problem is relevant only during adulthood, it is useful to introduce the following normalization. Let us denote by $t$, the time when adulthood begins $\tau + I_0$, by $i$ the numbers of years spent in adulthood $j - I_0$, and by $I$ the length of adulthood $I_1 - I_0$. Hereafter, when we talk about an individual of age $i$ born at $t$, it is understood that we mean an individual who has been adult for $i$ years and became adult at time $t$.

### 3.2 Preferences and Household Optimization

Individuals derive utility from a homogeneous consumption good. They do not derive utility from leisure, nor from their children. Each individual is endowed with one unit of labor that she supplies inelastically to the market. The productivity of each individual, however, changes with age according to a deterministic pattern. We denote the vector of efficiency units of labor with $\{\epsilon_i\}_{i=1}^{I}$. As typical in these models, we assume that labor productivity is zero at very early ages (until age 16) and late in the life cycle (after age 70). This is an admittedly simple way to capture (exogenous) retirement.\(^6\) Instantaneous utility is assumed to be of the CES family:

$$u(c t,i) = \frac{c_{t,i}^{1-\gamma} - 1}{1 - \gamma},$$

\(^6\)In many Latin American countries child labor is not uncommon. As for the retirement issue, a richer model would endogenize the choice between schooling and work for children. We exogenously assume that individuals start being productive at age 16 (when adulthood starts).
where $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution, and $c_{t,i}$ denotes consumption of an individual aged $i$ and born at $t$. An individual born at time $t$ solves the following problem:

$$
\max_{\{c_{t,i}\}} \sum_{i=1}^{I} \beta^{i-1} \Pi_{t+i-1,i} u(c_{t+i-1,i})
$$

s.t.

$$
c_{t,i} + a_{t,i+1} = (1 + r_t)a_{t,i} + w_t \epsilon_{t,i} + b_t
$$

$$
a_{t,1} = a_{t,i} = 0, \text{ for all } t.
$$

(1)

The term $\Pi_{t,i}$ denotes the unconditional probability of surviving until age $i$ at time $t$, obtained through the product of all the past conditional probabilities. Thus, an individual of (adult) age $i$ at time $t$ discounts next period utility at rate $\beta \pi_{t,i}$, where $\beta$ is a constant discount factor. Asset holdings $a_{t,i}$ represent claims on the capital stock used in production. We assume that initial wealth is zero for each agent and we impose no borrowing constraints, except for a no-Ponzi scheme condition that requires every agent to hold non-negative assets when they reach the terminal period of life, $i = N$.

Mortality risk is the only uncertainty faced by individual consumers. In the event of early death, wealth is divided equally among the survivors. The variable $b_{t,i}$ denotes the fraction of the aggregate accidental bequests $B_t$ received by each individual alive at time $t$:

$$
b_{t,i} = \frac{B_t}{\sum_{i=1}^{I} \mu_{t,i}},
$$

where the aggregate accidental bequest $B_t$ is defined in equation (5) below. The wage and interest rates at time $t$, taken as given by the individuals, are denoted respectively by $w_t$ and $r_t$. From the individual’s Euler Equation, we obtain:

$$
\frac{c_{t+1,i+1}}{c_{t,i}} = [\beta \pi_{t,i} (1 + r_{t+1})]^{\frac{1}{\gamma}}
$$

which determines the slope of the household consumption profile. The initial level of consumption is determined by the present discounted value of lifetime resources $W_t$:

$$
W_t = \sum_{j=1}^{I} (w_{t+j-1} + b_{t+j-1}) \prod_{i=1}^{j} \left( \frac{\pi_{t+i-1,i}}{1 + r_{t+i-1}} \right).
$$

### 3.3 Technology and Firm Optimization

Output is produced by competitive firms operating a constant returns to scale Cobb-Douglas production function with capital share equal to $\alpha$ and total factor productivity equal to $\theta$. 


We do not consider technical progress so that in steady-state the growth rate of the economy is normalized to zero.\(^7\) The firms rent capital and labor every period from competitive spot markets, thus firms maximize static profits by taking factor prices as given. The implied optimality conditions are:

\[
\begin{align*}
    r_t + \delta &= \alpha \theta K_t^{\alpha-1} L_t^{1-\alpha}, \\
    w_t &= (1 - \alpha) \theta K_t^{\alpha} L_t^{-\alpha},
\end{align*}
\]

where \(L_t\) denotes aggregate efficiency units of labor and \(K_t\) the aggregate capital stock used in production at time \(t\). The firms sets marginal product of labor equal to the wage rate (per efficiency unit) \(w_t\) and the marginal product of capital equal to the rate of return on capital \(r_t\).

At this point we need to distinguish between closed and open economy. In a closed economy, the aggregate capital stock in each region \(r\) evolves according to the law of motion:

\[
K_{r,t} = (1 - \delta) K_{r,t-1} + S_{r,t},
\]

where \(S_{r,t}\) are aggregate national savings. In the open economy model, we denote the variables for the North with the subscript \(N\) and the ones for Latin America with the subscript \(A\). The transition equations for aggregate capital stock are, respectively:

\[
\begin{align*}
    K_{N,t} &= (1 - \delta) K_{N,t-1} + S_{N,t} - F_t, \\
    K_{A,t} &= (1 - \delta) K_{A,t-1} + S_{A,t} + F_t,
\end{align*}
\]

where \(F_t\) is the capital flowing from the North to Latin America in period \(t\), and \(S_{r,t}\) is the flow of capital invested domestically in region \(r = N, A\) at time \(t\).

### 3.4 Equilibrium

An equilibrium for the closed economy is a sequence for the variables \(\{c_{r,t,i}, a_{r,t,i}, K_{r,t}, B_{r,t}, w_{r,t}, r_{r,t}, \mu_{r,t,i}\}\) in each region \(r = N, A\) such that: the population shares are determined by the transition matrix \(\Gamma_{r,t}\); households and firms solve optimally their problems taking prices as given; factor prices are set to marginal productivities as in (2); the aggregate resource constraint in the economy of region \(r\) requires:

\[
\theta_r K_r^{\alpha_r} L_r^{1-\alpha_r} = S_{r,t} + \sum_{i=1}^{I} c_{r,t,i} \mu_{r,t,i},
\]

\(^7\)During the transition, the ratio of income per capital across regions will change due to the different dynamics in demographics, but in the long-run it will be constant again. The introduction of productivity growth would require imposing the same rate for both regions, otherwise asymptotically one of the two vanishes. Hence, even in the presence of productivity growth, the model would display a constant ratio of income per capita across regions.
Market clearing in the labor and asset markets imply the conditions:

\[ L_{r,t} = \sum_{i=1}^{I} \epsilon_{r,t,i} \mu_{r,t,i}, \]

\[ K_{r,t+1} = \sum_{i=1}^{I} a_{r,t,i+1} \pi_{r,t,i} \mu_{r,t,i}, \]

and the aggregate accidental bequest is determined by:

\[ B_{r,t} = \sum_{i=1}^{I} a_{r,t,i+1} (1 - \pi_{r,t,i}) \mu_{r,t,i}. \]

An *equilibrium for the two-region open economy* is a sequence for the variables \( \{ c_{r,t,i}, a_{r,t,i}, K_{r,t}, B_{r,t}, w_{r,t}, \mu_{r,t,i} \} \) in each region \( r = N, A \), an international capital flow \( F_t \) and a world interest rate \( r_t \) such that: the population shares are determined by the region-specific transition matrix \( \Gamma_{r,t} \); households solve optimally their problems taking prices as given; wages in each country are set to marginal productivity of labor as in (2); the aggregate resource constraint in the two regions requires conditions

\[ \theta_N K_{N,t}^{\alpha} L_{N,t}^{1-\alpha} = S_{N,t} - F_t + \sum_{i=1}^{I} c_{N,t,i} \mu_{N,t,i}, \]

\[ \theta_A K_{A,t}^{\alpha} L_{A,t}^{1-\alpha} = S_{A,t} + F_t + \sum_{i=1}^{I} c_{A,t,i} \mu_{A,t,i}, \]

to be satisfied; market clearing in the labor market imply the condition in the first line of (4); equalization of rate of return in the two countries implies the no-arbitrage condition:

\[ r_t + \delta = \alpha \theta_N \left( \frac{L_{N,t}}{K_{N,t}} \right)^{1-\alpha} = \alpha \theta_A \left( \frac{L_{A,t}}{K_{A,t}} \right)^{1-\alpha}, \]

while the equilibrium flow of capital across regions satisfies equation (11) in the Appendix.\(^8\)

For a given parametrization of the model economies, described in the next section, we compute numerically the steady states of the model (before and after the demographic transition) as well as the equilibrium transition between the two steady states. We describe some of the details of the numerical solution method in the Appendix.

4 Parametrization of the Model Economy

In this section, we describe the choice of the preference, demographics and technology parameters used in the baseline simulations. We calibrate the original steady state using average

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\(^8\)As customary, a *steady-state* equilibrium requires the population shares, the capital-labor ratio (hence, factor prices), the accidental bequest, and (in the open economy model) the international capital flows to be constant over time.
fertility rates, mortality rates and population shares in the 1950s and 1960s in the two regions of interest. We choose some of the preference and technology parameters to match a number of properties of the steady state solution (such as the capital-output ratio), and others in accordance with related studies.

The main exercise we perform consists in perturbing the original stationary equilibrium of each economy by changing fertility and mortality rates. In particular, we anchor our final steady state to one that roughly matches the long-run UN projections discussed above. We then move our economy from the initial steady state to the next by imposing a path of mortality and fertility rates that roughly matches the data and projections of the UN over a 100 year period. We assume that even though the shock initiating the transition takes households by surprise, the whole future convergence path is fully anticipated.

We assume that the two regions converge to a common steady-state. The reason is that in the long run of the open economy version, if one region grows more than the other, it will become the only relevant one and the steady state values will only reflect the features of that region.

4.1 Preferences and Endowments

We chose to model differences between the North and Latin America only in terms of demographic structure and technology. Preferences, and the age profile of efficiency units of labor are common across regions. Thus, in both regions the coefficient of risk aversion $\gamma$ is set at 2, corresponding to an elasticity of intertemporal substitution of $.5$. The discount factor $\beta$ is set to an annual value of 1.011, based on the study of Hurd (1989) for the US.

The age-profile of efficiency units of labor $\{\epsilon_i\}_{i=1}^{N}$ is chosen to roughly match the lifetime wage profile, as documented for example in Hansen (1993). It peaks at age 50, with an increase of about 100% vis-a-vis the initial level, and then it declines modestly in the years close to retirement. Beyond age 70 it is set to zero, which is equivalent to a retirement status.

4.2 Demographics

We calibrate the first steady state of the model economy to the 50’s and 60’s, and the second steady state to the period 2030-2050, for which we have demographic forecasts. We divide the lifetime of an household in the model in 20 5-year periods, hence the unit of time in the model is 5 years. For the fertility rate, we have used the data described in Figure 1, assuming that
women are fertile between ages 15 and 45 with a peak between 25 and 30 years old. The fertility rates in the initial (SS1) and final (SS2) steady state for the two regions are given in Table 1 below.

<table>
<thead>
<tr>
<th>Age</th>
<th>LA North</th>
<th>SS2</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;15</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
<tr>
<td>15-19</td>
<td>1.06</td>
<td>.50</td>
<td>.30</td>
</tr>
<tr>
<td>20-24</td>
<td>1.24</td>
<td>.60</td>
<td>.46</td>
</tr>
<tr>
<td>25-29</td>
<td>1.60</td>
<td>.80</td>
<td>.60</td>
</tr>
<tr>
<td>30-34</td>
<td>1.40</td>
<td>.60</td>
<td>.44</td>
</tr>
<tr>
<td>35-39</td>
<td>.54</td>
<td>.20</td>
<td>.26</td>
</tr>
<tr>
<td>40-44</td>
<td>.16</td>
<td>.10</td>
<td>.14</td>
</tr>
<tr>
<td>&gt;44</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
<tr>
<td>TOT</td>
<td>6.00</td>
<td>2.80</td>
<td>2.20</td>
</tr>
</tbody>
</table>


The survival probabilities for the initial and final steady states in the two regions were computed on the basis of actual data and projections of the United Nations on the age structure of the population in US, Europe and Latin America. After constructing the shares of the population by 5-year age groups in the two periods, we obtained the implied surviving probabilities, exploiting that the population is in steady-state. For the U.S., we have also used the Life Tables for the US Social Security from Bell, Wade and Goss (1992).

<table>
<thead>
<tr>
<th></th>
<th>SS1</th>
<th>SS2</th>
</tr>
</thead>
<tbody>
<tr>
<td>population growth</td>
<td>0.033</td>
<td>0.01</td>
</tr>
<tr>
<td>life exp. (years)</td>
<td>56</td>
<td>63</td>
</tr>
<tr>
<td>dependency ratio</td>
<td>0.055</td>
<td>0.16</td>
</tr>
<tr>
<td>average age (years)</td>
<td>21</td>
<td>31.5</td>
</tr>
</tbody>
</table>

4.3 Technology

As mentioned above, the production function is Cobb-Douglas with share of capital $\alpha$ set at 0.36 in both countries. For Latin America we set the shift parameter $\theta_A$ in order to normalize

---

9We have also attempted to model the observed shift of child bearing towards later stages in life. As clear from Table 1, we have increased, in the second steady-state, the fertility rates for women older than 35.
income per capita to 1 in the first steady state. Based upon the Summers-Heston dataset, income per capita in the North in 1950 was approximately 3 times that of the South, thus $\theta_N$ for the North is set accordingly. The depreciation rate of capital is set to 5% per year.

These assumptions about preferences, labor productivity, demographics and technology imply a capital-output ratio of roughly 3.5 on an annual basis in the North in the first steady-state, and a lower level of about 2.4 in Latin America, which roughly match the corresponding estimates from the Penn World Tables for the 1950’s-1960’s. As a result, the initial equilibrium return to capital is 4% in the North and 8.5% in Latin America.\textsuperscript{10}

5 The computational experiment

5.1 Methodology

We assume that changes in the demographic structure are exogenous to households’ decisions. The experiment we run is of an unexpected change in the steady-state demographic variables (fertility and mortality rates) of the 1950s and 1960s in the two regions of interest. We assume that the change in mortality happens gradually and monotonically, whilst that in fertility follows an inverse $U$ shape in order to generate the baby-boom observed in both economies during the early 60’s. We parameterize the speed of adjustment towards the new steady-state level of fertility and mortality rates in order to replicate the actual demographic transition.

Technically, what changes (unexpectedly) across steady states is the matrix $\Gamma_t$ governing the demographic structure of the population. Although the initial change is, as of 1950, unexpected, we assume that the all generations alive during the change can perfectly forecast the future path of population shares, and factor prices. In other words, in the initial steady state the matrix $\Gamma$ is (and is expected to be) constant. Once the change occurs in the form of a sequence of matrices $\Gamma_t$ that converges slowly towards the new steady state, all generation alive are assumed to forecast perfectly the new path. We discuss the likely implications of this assumption below.

In the next sections we analyze several simulations. In describing the results, we first start with the closed economy and analyze separately what we label the “fertility transition”, and the “surviving rate transition”. That is, in order to disentangle the macroeconomic effects of the two types of shocks, we first assume that only fertility changes, and then that only longevity does. We then proceed to study the full demographic transition. At every step, we compare

\textsuperscript{10}Because the interest rate exceeds the aggregate growth rate in both regions, the two economies are dynamically efficient.
Latin America with the North. Next, we analyze the demographic transition for the North and Latin America in open economy, using our two-region model with international capital flows. In that experiment, we assume that a few periods after the demographic transition in the closed economy, capital markets are suddenly and unexpectedly open. The period we choose for the opening is the year 1995.

5.2 The Fertility Transition

In Figure 3, we plot the simulated fertility transition which matches very closely the actual data, as clear from a comparison of the first panel across Figures 1 and 3. The fertility transition accounts for a very large rise in the average age of the population, especially in Latin America. The fertility transition generates an acceleration in the dependency ratio between 2020 and 2040 in the North which is clearly visible in the data of Figure 1: it is the crest of the baby-boom entering the retirement age. In Latin America the dependency ratio picks up earlier, around year 2010, as in the data. Overall, the effect of the baby boom in Latin America is negligible.\footnote{Recall that in this experiment life expectancy does not vary, as we keep the surviving rates unchanged.}

The simulated transition path of some of the most important aggregate variables during the fertility transition are plotted in Figure 4. In the long-run, the fertility transition brings about a reduction in the growth rate of the labor force, and an increase in the share of the elderly population. The key consequence of the decline in the population growth rate is a decrease in the capital-labor ratio, hence an increase in wages and a reduction of interest rates. Aggregate saving rates move because of changes in individual saving profiles and because of changes in the composition of the population. Aggregation effects increase aggregate saving rates, as during the transition the age groups with relatively high saving rates are more numerous as a consequence of the baby boom. The fall in the interest rate and the increase in wages have both income and substitution effects on individual savings. Which of the two sets of effects prevails depends on a variety of factors ranging from the curvature of the utility function, the discount factor, the slope of the life time earning profile, the length of retirement. In our calibrated model, it is clear that the overall effect on aggregate savings is strongly positive. The generations alive at the time of the shock are forced to make a fairly dramatic adjustment in their consumption plans, and this explains the spike in the saving rate in the first period, particularly strong for Latin America, where the fertility shock is larger.

The impact of the demographic change on the growth rate of wages is also quite substantial. From Figure 4 one can infer that, in the first decade of the fertility transition, the fertility shock
can explain 1.8% of the average wage growth in Latin America and about 0.5% in the North. However, much of this result is due to the surprise effect. Interestingly, the shock is extremely persistent, especially in Latin America where, according to the model, even in the year 2000, 50 years after the initial shock, the slowly unfolding decrease in fertility trends can generate wage growth of about 0.2% per year for another 50 years.\textsuperscript{12} Turning to the interest rate, the model suggests that the fertility transition could still lead to a reduction of the annual interest rate in the next 50 year of around 1 basis point.

Finally, it is worth analyzing the effects on the average efficiency level of the labor force. In the North, after a labor productivity slowdown in the 1970’s (which interestingly appears in the data as well) related to the entry of the young baby-boomers into the labor force, the average efficiency units of labor increase, and a rise is expected for the next two decades as the baby-boomers go through their most productive years. The rise of average efficiency of labor in Latin America is much bigger, around 10% in the long run. The model suggests that, looking ahead, Latin America can still exploit half of the rise in labor productivity associated to the fertility transition in the next 50 years.\textsuperscript{13} This is the size of the “window of opportunity”, referred to by Berhman et al. (1999), estimated by our model.

\subsection{The Surviving Rate Transition}

The demographic changes caused by the rise in the surviving probabilities are represented in Figure 5. The rising longevity manifests itself mainly through a decline in infant mortality rates and in the mortality rates of the oldest generations. In the North the latter is more pronounced, which generates a considerable rise in the average age from 32 to 36, and a rise in the dependency ratio from 16% to 27%. In Latin America child mortality falls sharply during the transition. This force counteracts the aging effects of an increase in the surviving rates of the elderly. As a result, the effects on average age and on the dependency ratio are quantitatively fairly small.

Figure 6 shows that from the point of view of the household, an improved longevity (especially during the retirement period) calls for more life-cycle savings during the productive years. The consequent rise in the capital stock increases wages and lowers the rate of return.

\begin{itemize}
\item \textsuperscript{12}It should be remembered that we have normalized our model so that in steady state, per capita income growth is zero. The observed rate of growth in our simulations are therefore induced completely by the demographic trends.
\item \textsuperscript{13}The rise in the average efficiency units of labor counteracts the decline in population growth, but the latter effects dominates and the total labor input falls.
\end{itemize}
In the North the saving rate increases by 5%. The impact on wage growth is .3% per year in the 60’s to decline to .12% in the year 2000. The effects on Latin America are fairly small: no change in the saving rate, and a small growth effect on wages. In both economies the rate of return falls, and average efficiency units of labor increase very mildly, since most of the increase in longevity is enjoyed by retirees.

5.4 The Full Demographic Transition

Figure 7 displays the simulated demographic transition in the model which, as clear from a quick comparison with Figure 1, reproduces the data very closely. The simulations of the key macroeconomic variables for the full demographic transition are pictured in Figure 8. It is easy to see the combined effects of the falling fertility rate and the rising surviving rates. In particular, notice the increase in the aggregate saving rate at impact and the decline in the interest rate.

In Figure 9, we have summarized the impact of the full demographic transition on income per capita. Given that our initial steady state is benchmarked to have zero per-capita growth, the growth in per-capita income generated during the transition is solely due to the observed demographic trends. It is remarkable that, according to our model, the observed changes in the age structure can account for 0.55% per year of income per capita growth in Latin America and .2% per year in the North in the past 50 years. Moreover, while the beneficial effects for the North are quickly fading away, the simulation suggests that Latin America should still benefit from higher than average growth rates for the next half century.

To summarize, the closed economy model suggests that:

1. the North has largely exhausted the macroeconomic benefits of the demographic transition, in particular:

   (a) only an additional 2% increase in the average efficiency units of the labor force should be expected in the next few decades.\(^\text{14}\)

   (b) in the next two decades the contribution to wage growth will stand at around 0.2%. After a positive echo effect due to the retirement of the baby-boom generation around 2020, it will fall quickly towards 0.1%.

\(^\text{14}\)The size of this effect depends, obviously, on the assumed slope of the earning profile.
(c) the additional effect on income per capita will be around 0.1% per year in the next two decade, but it will decline quickly to zero afterwards.

(d) The North experiences a dramatic drop in the rate of return to capital that goes from 4% to 1% between the two steady states. Notice that by the time the baby boomers retire, most of the transition in the rate of return has occurred.

2. Latin America can still enjoy large gains from the demographic transition, in particular:

(a) a rapid increase in average efficiency units of labor (+8%) should be expected in the next 30 years,

(b) the demographic transition could contribute up to .45% in the yearly growth of real wages in the next 30 years, with a declining, but persistent effect for further 50 years,

(c) income per capita growth associated purely to capita accumulation and improvements in efficiency units of labor will be fairly substantial in the next 50 years: from .6% per year in 2000, down to .1% in 2050,

(d) however, most of the rise in the aggregate saving rate seems, according to our model, to have taken place already.

5.5 Welfare Analysis in Closed Economy

A structural general equilibrium model has the advantage, compared to reduced-form analyses, that it allows to determine what are the welfare gains (or losses) for each generation alive during the demographic transition. This is probably the most relevant criterion to evaluate the effects of the demographic transition.

To perform our welfare analysis, we compute for each cohort a measure of compensated variation in consumption, using the first-steady-state as a benchmark. In other words, we compute how much we would have to rescale (up or down) the consumption profile of the representative household in cohort \( t \) in order to make her indifferent between being born at time \( t \) or in the first steady-state.\(^{15}\) In Figure 10 we plot this compensating variation against the year of birth of the individuals living around the transition: a positive (negative) number corresponds to a welfare loss (gain) compared to the initial steady-state.

\(^{15}\)Different cohorts face different effective discount factors due to the change in the surviving rate. To avoid our welfare computation from being contaminated by this effect, we discount the utility of every cohort with the same sequence of age-specific surviving rates – in particular, that of the initial steady-state. Thus, differences in welfare only reflect different consumption profiles.
The largest losses are borne by the baby-boomers since they face both a sharply falling rate of return and a slower rise in wage due to the size of their cohort. This loss stands at 8% for the North, while it is more modest for Latin America (just above 2%), where the baby-boom occurred more mildly. In the long-run there is a small gain in the North, and a staggering welfare gain in Latin America, around 22% of lifetime consumption in the new steady-state. The size of the welfare gain in Latin America is explained by the large change in fertility and longevity between the two steady states which results in a two-fold increase in the saving rate. The higher capital-labor ratio reflects, in turn, in higher levels of lifetime labor earnings.

By examining Figure 10 one can observe that the generations which are expected to be born in the next 30 years in Latin America will still improve their welfare level considerably with respect to the existing ones. The same is not true for the North.

It should be stressed that the welfare losses for the baby boomers we have estimated here are probably a lower bound on the actual loss that these generations could observe. The main reason for this assertion lies in the assumption that the baby boomers fully anticipate and act upon the decline in fertility. As many of them are born after the demographic transition has started, they are not subject, in our model, to any demographic surprise.

6 The Demographic Transition in Open Economy

The previous experiments in our paper were performed under the assumption of closed economy, thus no interaction was allowed between the two regions. Our two-country model allows us to analyze the macroeconomics of the demographic transition in an open economy context.

The general equilibrium effects on the welfare of different generations going through the demographic transition were calculated under the assumption of a closed economy. This computation neglects the important point that demographic trends are not perfectly synchronized and that interactions between different regions could change substantially the effects we have identified. Our two-country model allows us to analyze the macroeconomics of the demographic transitions that differ in their timing and size across regions, when these regions interact through frictionless capital markets.

Data on international capital flows between Latin America and the rest of the world suggest that on average, between 1970 and 1990 these flows were below 3% of GNP. By the year 1997 this ratio had doubled to 6%. Therefore if one had to choose a single date to model the opening
of capital markets, somewhere in the mid 90’s would seem reasonable.\footnote{The numbers on capital flows are derived from the World Bank Development Indicators.}

In our experiment, we assume that until 1995 the two regions were in autarchy, with no capital flowing across the borders. In 1995 we shock the two regions by allowing capital to flow. Once again, the shock is unexpected.

The lower-right panel of Figure 11 displays the transition for interest rate. As we have imposed no restrictions on capital flow, in the period immediately following the opening the rates of return on capital across regions must be equalized. Equalization requires a fall from 5.5\% to 4.5\% in Latin America, and an upward jump from 2.5\% to 4.5\% for the North. After the shock, the interest rate slowly decays towards its long-run value (equal to the closed economy value) of roughly 1\% per year.\footnote{This number might be considered low in comparison with current returns on investments. However, it should be remembered that we have completely abstracted from risk in this model, so it is a plausible number for a risk-free rate of return.}

The upper-right panel of Figure 11 pictures the flows of capital into Latin America as a percentage of output in Latin America. The model predicts that a flow equal to 18\% of output in Latin America would be necessary on impact. The flow would slowly decrease until reaching almost zero in 2100, however notice that in 2040 it would still stand at a remarkable 6\% of GDP.\footnote{The model implies that in the long run the two economies have no capital flows among them. This is because they converge to the same rate of growth of the population. Capital flows adjust capital labor ratios in the transition.} The sudden spike in the capital flow implies a shift of savings towards Latin America and a rise in the capital-labor ratio in Latin America, with beneficial effects on wage growth: following the opening, Latin America enjoys labor income growth rates up to 2\% per year.

Turning to the saving rate, in the closed economy (see Figure 8) the transition of the saving rate was almost concluded around 2020, while in the open economy model, in 2020 the saving rate still stands quite far from its long-run value. For Latin America, the reason is that the saving rate falls sharply below its long-run value after the openness, because of a “crowding out” effect due to the inflow of capital from the North which substitutes for domestic savings and allows more resources to be devoted to consumption. After this fall, it takes over 60 years for the saving rate to reach its new steady-state level.

Figure 12 shows the differential impact of the demographic transition on output for Latin America, in closed and open economy. Upon impact, the large inflow of capital could generate growth rates of output around 2\%, levels three times larger than those observed in the closed economy. After a couple of decades, however, the rate of growth of per capita output in the open
economy decreases to values lower than those in the closed economy. In other words, opening capital markets concentrates the impact on the growth rate of income per-capita associated to the demographic transition. Interestingly, the cumulative effect of income per capita growth is not too different between open and closed economy, but under the open economy assumption, the bulk of the gains is concentrated in just a couple decades.\(^{19}\)

To summarize, the simulations suggests that for Latin America, opening capital markets makes the gains in income growth implied by the demographic transition sharper, but short-lived compared to the closed-economy case. The reason is that international capital flows accelerate the adjustment process by exacerbating income per capita growth in the short-run and reducing it in the long-run.

### 6.1 Welfare Analysis in Open Economy

This conclusion suggests that the intergenerational redistributive impact of the demographic trends can be very different across the two benchmarks. Welfare is ultimately the best measure of the consequences of the demographic shock for households and allows to answer the following question: if, in the middle of the demographic transition, these two economies were opened to capital flows, who would be the winners and the losers in each region?

Figure 13 gives us the answer for the North. Notice first that, as in the closed economy, essentially all generations lose from the demographic transition. In relative terms, however, the winners are the baby-boom generations who, once the economy is open are near retirement and therefore do not suffer from the drop in the wage rate, but enjoy the higher (than expected) returns on their savings. The losers are all those generations who will still supply labor after the opening, because of the lower wage rate. Figure 14 contains the answer for Latin America. Here the logic is exactly reversed: the relative winners are those generations which will be productive in the labor market after capital has massively flown into Latin America from the North, boosting the wage rate. The biggest relative welfare gains from financial liberalization accrue to those cohorts born around the opening and are approximately equivalent to 6\% of lifetime consumption.

How to reconcile the findings on income growth with those on welfare? The answer is that welfare is largely determined by the level of the wage rate, and the fast growth of the years

\(^{19}\)An important remark at this point is that the nature of our open-economy results is largely driven by the extreme assumption of perfect mobility of capital across borders. This explains the fact that the model overpredicts the magnitude of capital flows.
following the opening increases the capital-labor ratio above the closed economy level. This generates a further welfare gain even for the future generations.

Overall, our model suggests that by opening to financial flows Latin America will compress the growth benefits of the demographic transition into a shorter time horizon, but this will generate a significant welfare gain for future generations as well. Although the future generations will face an economy with slower growth compared to a closed economy, their level of income per capita will be higher on average, thanks to the higher stock of capital.

7 Concluding Remarks

In this paper we have used a calibrated general equilibrium model to study and quantify the effects of the demographic transition on factor returns, income growth and, ultimately, on the welfare of the generations alive during the transition. While the model is somewhat stylized, the results we present are interesting and far-reaching for at least two reasons.

First, while the demographic trends discussed in this paper have been the main determinant of the current policy debate on the sustainability of the pension systems currently in place in the US and several European countries (see Diamond, 2004, for an essay on social security reform), the implications of the same trends for the dynamics of the return to capital—and the role of the latter for the welfare of the baby boomers—has almost been absent from the debate.20 In our simulations, we show that the effect of the current demographic trends on the welfare of a “large” generation are substantial even in a situation in which the saving for retirement happens in a completely funded and private way. The welfare decrease, that we quantify for the baby boomers of the North at around 8% of lifetime consumption, is caused mainly by shifts in factor prices and, in particular, of rates of returns, which decline considerably as a consequence of the aging of the population.

Second, the fact that the current demographic trends are not synchronized in different regions of the world, puts in place strong incentives to capital mobility, in open economy. The capital flow from the North to the South would generate an increase in labor productivity and wages in Latin America and a rise in the rate of return in the North. While for the North baby-boomers the welfare effect of opening the economy is positive, as it alleviates the effect of the demographic transition the dramatic reduction in rates of return, the effects for all other

20There is a recent literature on the effect of the baby-boom on the stock market. See, for example, Abel (2003), Brooks (2000), and Lim and Weil (2003).
generations are negative because of the decline in the future growth of wage rates associated
to the capital outflow. In Latin America, because of the inflow of capital and the associated
surge in labor productivity, large gains accrue to all future generations, of the order of 15%-20% of lifetime consumption. Another interesting result for Latin America is that opening
the economy will concentrate the growth-effects induced by the demographic transition over a
couple of decades only.

Several important caveat to our conclusions should be kept in mind. First, the capital flows
implied by our exercise are larger than those currently observed. Our baseline simulations
implied, on opening the capital markets, a flow of capital equal to 18% of the Latin American
GDP. Currently, capital flows, are much lower: for the Latin American and Caribbean region,
they were just above 6% of GNP in 1997. Our model cannot match this number for several
reasons. First, we start from a situation in which capital does not flow across regions. Sec-
ond, and more importantly, in our simulations, we abstract from all sources of aggregate risk,
including political uncertainty and sovereign default. Third, when the international capital
flows are allowed, we do not consider any impediment to capital mobility. All these aspect
should be addressed carefully in future work, if one wants to quantify realistically the extent to
which differences in demographic trends constitute an opportunity to be exploited, by fostering
capital mobility.

Second, in our model we do not consider labor supply choices. The increase in female labor
force participation that is realistic to expect in Latin America (while it has already happened
to a large extent in the North), would reinforce the asymmetries between the two regions that
we have stressed and that are at the basis of our argument: even more capital will be flowing
into Latin America to compensate the growth in labor supply and equalize interest rates in
open economy. Moreover, the trend towards early retirement in the North would go in the
same direction, insofar as they are not mirrored by a similar phenomenon in Latin America.

Third, we abstract from the presence of a large unfunded pension system that is currently
in place in the US and in many countries in Europe. This allowed us to focus on the impact
of the demographic trends on the economy even in a situation where the “pension system” is
private and funded, which seems to be the aim of several reforms currently on the table.

In Attanasio, Kitao and Violante (2005), we extend this framework in all these directions.
We compare several projects of social security reform in the North under the assumption that
the economy is open, and that the global demographic trends in act create strong incentives
for capital flows, but financial market frictions limit perfect capital mobility.
A Appendix: Description of the Algorithm

A.1 Steady-State of the closed economy

The absence of aggregate and idiosyncratic uncertainty greatly simplifies the solution of this model, which can be solved in two steps. The model can be expressed in terms of a pair of aggregate state variables, the capital stock and the aggregate accidental bequests, thus the steady state computations are very simple and involve the solution of two nonlinear equations. Given an initial guess for the capital stock and the accidental bequests, it is possible to construct factor prices and from these one can construct age specific consumption and saving decisions which, in turn, by aggregating through steady-state population shares will imply a value for the aggregate capital stock that, in equilibrium, coincides with the original guess.

A.2 Transition of the closed economy

We fix the length of the transition to $NT$ periods. We guess a path of $\{K_t\}_{t=1}^{NT}$ and $\{B_t\}_{t=1}^{NT}$ such that $K_1$ and $B_1$ and $K_{NT}, B_{NT}$ are respectively the initial and the final steady-state values computed as described above. We linearly interpolate these two points to obtain first guess. Given this path we can compute factor prices along the transition and then, for every period $t$ of the transition we can compute the assets and consumption profiles. From these and the population shares implied by the demographic transition, we can then aggregate to compute the implied capital stocks and accidental bequests. If the path for the implied capital stock and accidental bequests does not coincide with the original guess we adjust the latter. Otherwise, the solution has been found.

A.3 Steady State of the open economy

In the final steady state, the demographic variables in the two countries must be equal. This implies a unique final growth rate of population, call it $\eta$.

The law of motion for aggregate capital in the North and in Latin America in steady state:

$$K_N \delta = S_N - F$$
$$K_A \delta = S_A + F$$

No arbitrage between the two countries requires a unique rate of return, so it implies the condition:

$$\left( \frac{L_N}{K_N} \right)^{\frac{1}{1-a}} = \left( \frac{L_A}{K_A} \right)^{\frac{1}{1-a}} = \left( \frac{r + \delta}{\alpha} \right)^{\frac{1}{1-a}}$$
Hence, using (7) and (8) we obtain:

\[
{S_N - F \over S_A + F} = \left( {L_N \over L_A} \right) \left( {\theta_N \over \theta_A} \right)^{\frac{1}{\alpha}} \equiv \phi,
\]

which yields:

\[
F = {S_N - \phi S_A \over 1 + \phi}. \tag{9}
\]

The algorithm that we use to compute the steady-state is the following. First, we guess \(K_{r,t}\), \(B_{r,t}\), and we obtain the wages in each region from marginal productivities, and the world rate of return from (8). Given prices, we can compute permanent income in steady-state, and through the Euler equation the vector of consumption profiles \(\{c_{i,r}\}_{i=1}^{N}\). Using the steady-state population shares, we obtain aggregate savings \(S_r\). Given \(S_r\), we can compute \(F\) from (9) and then use \(F\) into (7) to check whether the guess of \(K_r\) and \(B_r\) is verified, otherwise we update our guess and continue the iteration.

### A.4 Transition of the open economy

We start from a vector of guesses of length \(NT\) for \(B_{r,t}\) and \(K_{r,t}\). In each country, for every period \(t\), from the Euler equation we obtain the age profile for consumption and asset holdings. We can therefore derive the implied value for \(B_{r,t}\). We then compute savings of every individual of age \(i\) in period \(t\) in region \(r\) from the budget constraint, and we aggregate to obtain aggregate savings \(S_{r,t}\). From the law of motion for capital, we obtain with some simple algebra:

\[
\frac{K_{N,t+1}}{L_{N,t+1}} = (1 - \delta) \frac{K_{N,t}}{L_{N,t}} \left( \eta_{N,t+1} \right)^{-1} + \frac{S_{N,t}}{L_{N,t}} \left( \eta_{N,t+1} \right)^{-1} - \frac{F_t}{L_{N,t}} \left( \eta_{N,t+1} \right)^{-1},
\]

\[
\frac{K_{A,t+1}}{L_{A,t+1}} = (1 - \delta) \frac{K_{A,t}}{L_{A,t}} \left( \eta_{A,t+1} \right)^{-1} + \frac{S_{A,t}}{L_{A,t}} \left( \eta_{A,t+1} \right)^{-1} - \frac{F_t}{L_{A,t}} \left( \eta_{A,t+1} \right)^{-1}. \tag{10}
\]

Using (10) into the no-arbitrage condition (6), it is easy to obtain an expression for the flow of capital:

\[
F_t = L_{N,t} \left( 1 - \delta \right) \left[ \frac{K_{N,t}}{L_{N,t}} \tilde{\eta}_{t+1} - \frac{K_{A,t}}{L_{A,t}} \right] + \frac{S_{N,t}}{L_{N,t}} \tilde{\eta}_{t+1} - \frac{S_{A,t}}{L_{A,t}}, \tag{11}
\]

where \(\tilde{\eta}_{t+1} = {\eta_{N,t+1} \over \eta_{A,t+1}} \left( {\theta_A \over \theta_N} \right)^{1/\alpha} \). One can now use \(F_t\) into one of the equations of (10) to obtain the implied value of \(K_{r,t+1} \over L_{r,t+1}\) from which it is immediate to derive the implied value for \(K_{r,t+1}\). Once again comparing the vector of guesses with the vector of implied values, we can update the guesses until convergence is reached.
References


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CLOSED ECONOMY

DEMOGRAPHIC TRANSITION: WELFARE LOSS BY COHORT

![Graph showing welfare loss by cohort.]
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