Consumption and Labor Supply with Partial Insurance

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Introduction

• We want to understand consumption and labor supply choices at the micro level, and the implications of these choices for macro facts:
  – the evolution of within-cohort inequality over the life-cycle
  – the evolution of cross-sectional inequality through time

• We develop a **tractable** version of the life-cycle model with labor supply and partial insurance in which the mapping between structural parameters and moments is transparent

• Framework can be used to
  – Measure fraction of inequality attributable to heterogeneity at birth versus shocks over the life cycle
  – Measure extent to which these shocks are insurable
  – Explore what model ingredients are required to account for various features of the data
Outline

1. Facts
2. Model
3. Identification
4. Estimation results
5. Sensitivity
Macro Facts

- Joint evolution of within-cohort dispersion in wages, hours and consumption contains valuable information about risk and insurance when organized within the life-cycle model of consumption and labor supply. \textit{Storesletten, Telmer and Yaron (2004)}

- Changes in cross-sectional dispersion over time contain valuable information about the nature of changes in the nature of risk over time. \textit{Blundell and Preston (1998), Krueger and Perri (2004)}

- \textbf{Our approach:} both sets of facts are important, one would like to simultaneously account for both within life-cycle framework
Micro Facts

- Labor economists traditionally more concerned with dynamics of wages and hours worked at the individual level

- **Our approach:**
  - Valuable information contained in variances of changes (the micro picture) as well as in changes in variances (the macro picture)
Data

- Wages, hours and earnings from PSID, 1967-1996
  - Wage computed as annual earnings divided by annual hours
  - Observations dropped if wage less than half minimum wage or if earnings top coded

- Consumption from Consumer Expenditure Survey, 1980-1996
  - Focus on measure that excludes services from durables

- Sample Selection
  - Age range 25-54
  - Annual hours $\geq 520, \leq 5096$
  - Same sample selection criteria applied to PSID and CEX data
Variance of Log Wages by Age

Variance of Log Earnings by Age

Variance of Log Hours by Age

Variance of Log Consumption by Age
Model Elements

- Perpetual youth framework with constant survival probability \( \delta \)
- Agents can trade a non-contingent bond (Huggett, 1993, Aiyagari, 1994)
- In addition groups of agents - "islands" - potentially pool some risks
  - Groups may be families, firms, industries or countries
  - Within-group risk-sharing may be achieved by a group planner, via redistributive taxation, or through explicit insurance markets
- Agents face wage risk at the idiosyncratic, group (and aggregate) level
- Agents heterogeneous with respect to preferences and productivity
Preferences

- Lifetime utility for an agent of age $a$ born in year $b$ is

$$\sum_{t=b}^{\infty} (\beta \delta)^{t-b} u(c_t, h_t, \zeta_t, \varphi)$$

$$u(c_t, h_t, \zeta_t, \varphi) = \frac{c_t^{1-\gamma} - 1}{1 - \gamma} - \exp((\sigma + \gamma)\varphi + \sigma \zeta_t) \frac{h_t^{1+\sigma}}{1 + \sigma}$$

- $\gamma$ is coefficient of relative risk-aversion
- $1/\sigma$ is Frisch elasticity for labor supply
- $\varphi$ and $\zeta_t$ are random individual-specific preference weights

$$\varphi \sim N(\bar{\varphi} - \nu_\varphi/2, \nu_\varphi) \quad \zeta_t \sim N(-\frac{\nu_\zeta}{2}, \nu_\zeta)$$
Wages: \( \log(w_t) = \alpha_t + \varepsilon_t \)

- \( \alpha_t \): wage component common to group
  \[
  \alpha_t = \alpha_{t-1} + \omega_t \quad \omega_t \sim N\left(-\frac{v_{\omega t}}{2}, v_{\omega t}\right)
  \]

- \( \varepsilon_t \): idiosyncratic wage component
  \[
  \varepsilon_t = \mu_t + \theta_t \quad \theta_t \sim N\left(-\frac{v_{\theta t}}{2}, v_{\theta t}\right)
  \]
  \[
  \mu_t = \mu_{t-1} + \eta_t \quad \eta_t \sim N\left(-\frac{v_{\eta t}}{2}, v_{\eta t}\right)
  \]

- Upon entering the labor market agents draw \( \alpha_0, \varepsilon_0, \varphi \)
  \[
  \alpha_0 \sim N\left(-\frac{v_{\alpha 0}}{2}, v_{\alpha 0}\right) \quad \varepsilon_0 \sim N\left(-\frac{v_{\varepsilon 0}}{2}, v_{\varepsilon 0}\right)
  \]
Remarks

- Assume agents have perfect foresight over future paths for $v_{\omega t}, v_{\eta t}, v_{\theta t}$
  - (we can also handle uncertainty about future risk)

- All dynamics over time attributed to time effects in wage process
  - Consistent with HSV 2005
  - We will experiment with introducing cohort effects in $v_{\alpha 0}$ and $v_{\varepsilon 0}$

- Model nests
  - Bewley ($v_{\eta t} = v_{\theta t} = v_{\varepsilon 0} = 0$)
  - Complete Markets ($v_{\omega t} = v_{\alpha 0} = 0$)
Market Structure

- At each date $t$ a measure $1 - \delta$ of agents is born
- Agents born onto islands indexed by $\{\omega_s\}, s = t + 1, \ldots, \infty$
- Agents trade complete set of Arrow securities, each pays one unit of consumption for one particular combination $\lambda_{t+1} = (\omega_{t+1}, \eta_{t+1}, \theta_{t+1}, \zeta_{t+1})$
  - Arrow securities are only traded within islands
- Non-contingent bonds traded between islands
- Perfect annuity markets insure mortality risk in the standard fashion
Budget Constraints

- Generic budget constraint:

\[
ct + \int Q_t(\lambda_{t+1}) B_t(\lambda_{t+1}) d\lambda_{t+1} + qt b_t = w_t h_t + d_t
\]

\[
d_{t+1} = B_t(\lambda_{t+1}) + b_t
\]

- Borrowing limits rule out Ponzi schemes
Definition of Equilibrium

A set of allocations \( \{c_t, h_t, b_t, B_t(\lambda_{t+1})\} \) and prices \( \{q_t, Q_t(\lambda_{t+1})\} \) such that:

1. Allocations maximize expected lifetime utility for the agents, taking as given initial wealth and prices

2. Insurance markets clear island-by-island

3. World market for non-contingent bonds clears
Equilibrium in Closed Form

- *Idiosyncratic risk perfectly insured*
  - Complete markets within islands
  - Can imagine a planner choosing within-island allocations

- *Island-level risk entirely uninsured*
  - Constantinides and Duffie (1996) show that when shocks are permanent one can construct equilibria in which bonds are not traded
  - We extend their result by giving agents a labor choice, and by allowing sub-groups of the population to pool idiosyncratic risks
  - Key assumptions for the no trade result are:
    1. Preferences in CRRA class
    2. Agents born with zero initial wealth
    3. Island-specific wage shocks permanent and multiplicative
    4. Within-island wage dispersion $v_{\epsilon,t}$ grows at same rate on all islands
Allocations

Consumption and hours are given by

\[
\log(c_t) = \frac{1 + \sigma}{\sigma + \gamma} \cdot v_{\varepsilon,t} + \frac{\sigma^2 v_{\zeta}}{2\sigma} - \varphi + \frac{1 + \sigma}{\sigma + \gamma} \cdot \alpha_t
\]

\[
\log(h_t) = -\gamma \cdot \frac{1 + \sigma}{\sigma + \gamma} \cdot v_{\varepsilon,t} + \frac{\sigma^2 v_{\zeta}}{2\sigma^2} - \varphi - \zeta_t + \frac{1 - \gamma}{\sigma + \gamma} \cdot \alpha_t + \frac{\varepsilon_t}{\sigma}
\]
Prices

\[
\log (q_t) = \log(\beta) - \gamma \cdot \frac{1 + \sigma}{\sigma + \gamma} \left( \frac{v_{\varepsilon,t+1} - v_{\varepsilon,t}}{2\sigma} - \left( \frac{1 + \sigma}{\sigma + \gamma} + 1 \right) \frac{v_{\omega,t+1}}{2} \right)
\]

\[
\log (Q_t(\lambda_{t+1})) = \log(\phi_t(\lambda_{t+1})\beta) - \gamma \cdot \frac{1 + \sigma}{\sigma + \gamma} \left( \frac{v_{\varepsilon,t+1} - v_{\varepsilon,t}}{2\sigma} + \omega_{t+1} \right)
\]
Macro Moments

\[ \text{var} \left( \log w_t \right) = v_{\alpha,t} + v_{\varepsilon,t} \]

\[ \text{var} \left( \log c_t \right) = v_{\varphi} + \left( \frac{1 + \sigma}{\sigma + \gamma} \right)^2 v_{\alpha,t} \]

\[ \text{var} \left( \log h_t \right) = v_{\varphi} + v_{\zeta} + \left( \frac{1 - \gamma}{\sigma + \gamma} \right)^2 v_{\alpha,t} + \frac{1}{\sigma^2} v_{\varepsilon,t} \]

\[ \text{cov} \left( \log w_t, \log h_t \right) = \frac{1 - \gamma}{\sigma + \gamma} v_{\alpha,t} + \frac{1}{\sigma} v_{\varepsilon,t} \]

\[ \text{cov} \left( \log w_t, \log c_t \right) = \frac{1 + \sigma}{\sigma + \gamma} v_{\alpha,t} \]

\[ \text{cov} \left( \log h_t, \log c_t \right) = v_{\varphi} + \frac{(1 - \gamma)(1 + \sigma)}{(\sigma + \gamma)^2} v_{\alpha,t} \]
Micro Moments

\[ \text{var} (\Delta \log w_t) = v_{\omega t} + v_{\eta t} + v_{\theta t} + v_{\theta t-1} \]

\[ \text{var} (\Delta \log c_t) = \left( \frac{1 + \sigma}{\sigma + \gamma} \right)^2 v_{\omega t} \]

\[ \text{var} (\Delta \log h_t) = \left( \frac{1 - \gamma}{\sigma + \gamma} \right)^2 v_{\omega t} + \frac{1}{\sigma^2} (v_{\eta t} + v_{\theta t} + v_{\theta t-1}) + 2v_\zeta \]

\[ \text{cov} (\Delta \log w_t, \Delta \log h_t) = \frac{1 - \gamma}{\sigma + \gamma} v_{\omega t} + \frac{1}{\sigma} (v_{\eta t} + v_{\theta t} + v_{\theta t-1}) \]

\[ \text{cov} (\Delta \log w_t, \Delta \log c_t) = \frac{1 + \sigma}{\sigma + \gamma} v_{\omega t} \]

\[ \text{cov} (\Delta \log h_t, \Delta \log c_t) = \frac{(1 + \sigma)(1 - \gamma)}{(\sigma + \gamma)^2} v_{\omega t} \]
Estimation

- First stage regression on year dummies, race, gender, and a quartic in experience applied to both PSID and CEX data

- Individuals grouped into 6 non-overlapping age groups at each date: 25-29, 30-34, ...

- Minimum distance estimator (equally-weighted)

- Target moments
  - “Cross-Sectional” variances and co-variances by age and year (Macro Facts)
  - Variances and covariances of individual changes for hours and wages (Micro Facts)
Measurement Error

- Assume classical measurement error in all variables

- Recognize the wages inherit measurement error in earnings and hours:
\[ w_t = \alpha_t + \varepsilon_t + \mu_{yt} - \mu_{ht} \]

- Assume the distributions for measurement error are constant over time

- Estimate these variances along with structural model parameters
Proof of Identification

1. Changes in within-cohort (co)variances identify $v_{\omega t}, v_{\eta t} + \Delta v_{\theta t}, \gamma$ and $\sigma$:

$$
\begin{align*}
\Delta \text{var} \ (\log c_t) &= \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 \Delta v_{\alpha,t} \\
\Delta \text{var} \ (\log h_t) &= \left(\frac{1 - \gamma}{\sigma + \gamma}\right)^2 \Delta v_{\alpha,t} + \frac{1}{\sigma^2} \Delta v_{\epsilon,t} \\
\Delta \text{var} \ (\log w_t) &= \Delta v_{\alpha,t} + \Delta v_{\epsilon,t} \\
\Delta \text{cov} \ (\log w_t, \log h_t) &= \frac{(1 - \gamma)(1 + \sigma)}{(\sigma + \gamma)^2} \Delta v_{\alpha,t} + \frac{1 + \sigma}{\sigma^2} \Delta v_{\epsilon,t}
\end{align*}
$$

2. Sequences of cross-sectional data identify $v_\zeta$ and $v_{\mu y}, v_{\theta t-1}$ (given external estimate for $v_{\mu h}$):

$$
\begin{align*}
\text{var} \ (\Delta \log w_t) - \Delta \text{var} \ (\log w_t) &= 2 \left( v_{\mu y} + v_{\mu h} + v_{\theta t-1} \right) \\
\text{var} \ (\Delta \log h_t) - \Delta \text{var} \ (\log h_t) &= 2 \left( v_\zeta + v_{\mu h} + \frac{1}{\sigma^2} v_{\theta t-1} \right) \\
\text{cov} \ (\Delta \log w_t, \Delta \log h_t) - \Delta \text{cov} \ (\log w_t, \log h_t) &= 2 \left( -v_{\mu h} + \frac{1}{\sigma} v_{\theta t-1} \right)
\end{align*}
$$
3. Levels of dispersion identify $v_{α0}$, $v_{ε0}$, $v_ϕ$

\[
\text{var } (\log w_t) = v_{α,t} + v_{ε,t} + v_{μy} + v_{μh}
\]

\[
\text{var } (\log h_t) = v_ϕ + v_ζ + \left(\frac{1 - γ}{σ + γ}\right)^2 v_{α,t} + \frac{v_{ε,t}}{σ^2} + v_{μh}
\]

\[
\text{cov } (\log w_t, \log h_t) = \frac{1 - γ}{σ + γ} v_{α,t} + \frac{1}{σ} v_{ε,t} - v_{μh}
\]

4. Expression for $\text{var } (\log c_t)$ identifies $v_{μc}$

\[
\text{var } (\log c_t) = v_ϕ + \left(\frac{1 + σ}{σ + γ}\right)^2 v_{α,t} + v_{μc}
\]
### Parameter Estimates

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\sigma$</th>
<th>$v_{\alpha 0}$</th>
<th>$v_{\varepsilon 0}$</th>
<th>$v_{\omega}$</th>
<th>$v_{\eta}$</th>
<th>$v_{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.42</td>
<td>5.94</td>
<td>0.080</td>
<td>0.071</td>
<td>0.0032</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>(0.031)</td>
<td>(0.14)</td>
<td>(0.022)</td>
<td>(0.0005)</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$v_\varphi$</th>
<th>$v_\zeta$</th>
<th>$v_{\mu c}$</th>
<th>$v_{\mu y}$</th>
<th>$v_{\mu h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>0.018</td>
<td>0.059</td>
<td>0.002</td>
<td>0.017</td>
</tr>
<tr>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>
Estimates Discussion

- Estimates for $\gamma$ and $\sigma$ pretty standard:
  - Implied Frisch elasticity is 0.17
  - MaCurdy (1981) baseline estimate 0.15

- Estimates for measurement error
  - Cogley (2002) uses $\text{var} (\Delta \log c)$ to estimate $v_{\mu c}$
  - In our model $2v_{\mu c} = \text{var} (\Delta \log c) - \Delta \text{var} (\log c)$
  - Cogley’s estimate for $\text{var} (\Delta \log c)$ implies $v_{\mu c} = 0.07$ in our economy, c.f. our estimate of 0.06
  - Bound et. al (1994) find 22% of variance of individual earnings growth in PSID is measurement error; our estimates imply 8%
Variance of Log Wages by Year

Variance of Log Earnings by Year

Variance of Log Hours by Year

Variance of Log Consumption by Year
Correlation btw Log Hours and Log Wages by Year

Correlation btw Log Wages and Log Consumption by Year

Correlation btw Log Consumption and Log Hours by Year
Variance of Changes in Log Wages by Year

Variance of Changes in Log Hours by Year

Correlation bw Changes in Log Wages and Log Hours by Year
## Parameter Estimates: Alternative Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\nu_\omega$</th>
<th>$\nu_\eta$</th>
<th>$\nu_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>5.94</td>
<td>2.42</td>
<td>0.0032</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>Total cons.</td>
<td>6.52</td>
<td>1.87</td>
<td>0.0032</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>Cohort effects</td>
<td>5.79</td>
<td>2.39</td>
<td>0.0029</td>
<td>0.005</td>
<td>0.025</td>
</tr>
<tr>
<td>No pref shocks</td>
<td>3.29</td>
<td>2.17</td>
<td>0.0034</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>No pref heterogeneity</td>
<td>2.02</td>
<td>2.32</td>
<td>0.0044</td>
<td>0.003</td>
<td>0.027</td>
</tr>
</tbody>
</table>
Conclusions

• Built life-cycle model that is rich enough to incorporate a realistic process for wage risk and some key sources of insurance

• Framework is tractable → simple to understand predictions for inequality

Answers to our questions:

• Is inequality mostly attributable to heterogeneity or risk?
  – Depends on what variable you look at: earnings mostly risk, hours mostly heterogeneity, consumption in between

• What fraction of wage fluctuations represent uninsurable shocks?
  – Wage fluctuations mostly insurable, and most of the increase in wage dispersion over past 30 years insurable

• What ingredients are required to broadly account for all the facts?
  – Preference heterogeneity required to account for facts about hours