Sovereign Default and Debt Renegotiation

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Abstract

This paper develops a small open economy model to study sovereign default and debt renegotiation for emerging economies. The model features both endogenous default and endogenous debt recovery rates. Sovereign bonds are priced to compensate creditors for the risk of default and the risk of debt restructuring. The model captures the interaction between sovereign default and ex post debt renegotiation. We find that both debt recovery rates and sovereign bond prices decrease with the level of debt. In a quantitative analysis, the model accounts for the debt reduction, volatile and countercyclical bond spreads, countercyclical trade balance, and other empirical regularities of the Argentine economy. The model also replicates the dynamics of bond spreads during the debt crisis in Argentina.

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1 Introduction

Sovereign debt borrowing is associated with recurrent debt crises.[1] Because no international bankruptcy law exists, a defaulting country and its lenders usually renegotiate over the reduction of the defaulted debt to resolve a debt crisis. Recent evidence shows that post-default debt renegotiation on average result in a 40 percent loss for creditors.[2] Emerging economies pay volatile and highly countercyclical interest rates owing to default risk and to debt restructuring.[3] The recent quantitative sovereign debt literature such as Aguiar and Gopinath (2006) and Arellano (2008) finds that the Eaton and Gersovitz (1981) model can account for countercyclical interest rates and other key empirical patterns in business cycles and interest rates for emerging countries.[4] However, with an assumption of zero debt repayment following defaults, the existing models are inadequate to account for debt reduction. The objective of this paper is to conduct a joint analysis of sovereign default and debt restructuring on international debt markets.

This paper incorporates both sovereign default and debt renegotiation into a dynamic equilibrium model. We study the connection between default, debt renegotiation, and interest rates in a small open economy. The model features both endogenous default and endogenous debt recovery rates. We analyze the determination of debt recovery rates and how debt renegotiation interacts with a country's default decision in the model. In a quantitative exercise, we apply the model to study the Argentine debt crisis. We show that incorporating endogenous debt renegotiation allows sovereign default models to match debt reductions for defaulting countries.

In our model, a country and risk-neutral competitive foreign lenders trade one-period discount bonds. The country faces stochastic endowments and has an option to default. Default may result in the loss of future access to capital markets. However, through renegotiation over debt reduction, the defaulting country can regain its access to capital markets as in Fernandez and Rosenthal (1990).[5] At the same time, foreign lenders obtain the par-

[1] There are 84 events of sovereign default from 1975 to 2008 according to Standard and Poor's. The largest in history is the Argentine debt crisis on international bonds of over $82 billion in 2001.

[2] See Chuhan and Sturzenegger (2003), Sturzenegger and Zettelmeyer (2005), Benjamin and Wright (2009), and Standard and Poor's for details of sovereign debt renegotiations since 1980.

[3] Neumeyer and Perri (2005) and Uribe and Yue (2006) document the countercyclical country interest rates for Argentina, Brazil, Korea, Mexico, the Philippines, Ecuador, Peru, and South Africa.

[4] Chatterjee, Corbae, Nakajima and Rios-Rull (2006) analyze the unsecured consumer credit using a similar framework, and they account for the consumer bankruptcy in the U.S. Other quantitative studies of sovereign debt include Lizarazo (2006), Cuadra and Sapriza (2006), Hatchondo, Martinez and Sapriza (2007), and Mendoza and Yue (2009).

[5] Fernandez and Rosenthal (1990) analyze debt renegotiation through which the borrowing country gains improved future access to capital markets. Bulow and Rogoff (1989b) present a dynamic bargaining model

tial repayment of defaulted debt after debt renegotiation. In our model debt recovery rates are endogenously determined in a Nash bargaining game and have an analytically characterization. We show that there is an optimal value of reduced debt that maximizes total renegotiation surpluses, given the country's income. Hence, the fraction of defaulted debt that will be repaid decreases with the size of debt. In the model, debt reduction affects a country's ex ante incentive to default, as well as the country's terms of borrowing - because a country's value of default depends on the expected debt recovery. We show that default may arise in equilibrium. A country is more likely to default if it has a higher level of debt. In equilibrium, sovereign bonds are priced to compensate the lenders for both the risk of default and the risk of debt restructuring. Interest rates increase with the level of debt, owing to the higher default probability and to the lower debt recovery rate.

We use the model to analyze quantitatively the sovereign debt for Argentina. First, the model generates countercyclical and volatile bond spreads, volatile consumption, and countercyclical trade balance, which is also positively correlated with bond spreads. Second, this paper accounts for the debt reduction after default and delivers the relation between the debt renegotiation outcome and the defaulting country's economic conditions both for Argentina and for the cross-country data. The model replicates the positive correlation between the haircut rate and the debt/GDP ratio. Moreover, a defaulting country gets a smaller haircut if it defaults during recessions. We further show that the model can predict the recent default and account for the time series of Argentine bond spreads from 1994 to 2001. This paper also examines the role of debt renegotiation in explaining the aforementioned stylized facts related to sovereign default. We find that the changes in bargaining power have a great impact on debt recovery rates and bond spreads as well as on the sovereign borrowing.

In this model, the defaulting country's outside option depends on the outcomes of endogenous debt renegotiation. This feature of the model is related to several papers in the optimal contracting literature, such as Phelan (1995); Cooley, Marimon and Quadrini (2004); and Krueger and Uhlig (2006).[6] These papers assume a complete set of contingent assets and endogenize an agent's outside options by assuming that defaulting agents can start a new credit relationship with a competing principal. Yet in these papers default never arises in equilibrium. Using an incomplete set of assets, our model generates default and debt reduction in equilibrium. Debt renegotiation results in endogenous default penalty,

where a debt renegotiation helps the defaulting country avoid direct sanctions.

[6] See also Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000), Kehoe and Perri (2002) for the analysis of optimal contract with limited commitment.

which in turn affects a country's ex ante incentive to default. Hence, in our model, default and debt renegotiation introduce contingencies into the non-contingent sovereign debt contract and facilitate interstate consumption smoothing. But intertemporal consumption smoothing is damaged, owing to higher interest rates and to more limited access to the financial markets after default.

Several recent sovereign debt models extend the framework of this paper and reveal additional aspects of sovereign debt renegotiations. Our model supports a lower level of debt compared to the data, despite the additional state contingency introduced through endogenous debt reduction. D'Erasmo (2007) includes the government reputation in his model, which attempts to explain the high debt to output ratios for emerging economies. Delays often arise in sovereign debt renegotiations. This paper uses a Nash bargaining model and hence cannot address the inefficiency in renegotiation. Bi (2007) and Benjanmin and Wright (2009) use a stochastic bargaining model to account for delays in sovereign debt renegotiation.[7] These papers demonstrate that delays in renegotiation arise because both the country and creditor choose to wait until the economy recovers before sharing a higher surplus.

The remainder of the paper is organized as follows. Section 2 describes the model environment. Section 3 presents the sovereign borrower and the lenders' problems, and it defines a recursive equilibrium. We then demonstrate the existence of a recursive equilibrium and characterize the equilibrium bond prices and debt recovery rates. Section 4 provides the model calibration and the results of the quantitative analysis. Finally, Section 5 offers concluding remarks. The proofs and computation algorithm are in the Appendix.

2 The Model Environment

We study sovereign default and debt renegotiation in a dynamic model of a small open economy. The country's households are identical and have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $0 < \beta < 1$ is the discount factor, c_t denotes the consumption in period t and $u : \mathbb{R}_+ \to \mathbb{R}$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. The government in the country is benevolent and maximizes

[7] Bai and Zhang (2009) also study the delays in sovereign debt renegotiation using a dynamic bargaining model with private information.

the households' lifetime utility. In each period, households receive an exogenous endowment of the single non-storable consumption good y_t. The endowment y_t is stochastic and is drawn from a compact set $Y = \left[\underline{y}, \overline{y}\right] \subset \mathbb{R}_+$. The probability distribution function of a shock y_t conditional on the previous realization y_{t-1} is $\mu_y \left(y_t | y_{t-1}\right)$.

International investors are risk-neutral and have perfect information on the country's endowment and asset position. They can borrow or lend at a constant world risk-free interest rate r. Capital markets are incomplete. The country's government can borrow from or lend to international investors only via one-period zero-coupon bonds. The face value of a zero-coupon bond is denoted as b', which specifies the amount to be repaid in the next period. When the country purchases bonds, $b' > 0$, and when it issues new bonds, $b' < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound at $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt level that the country could repay. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate.[8] Let $q\left(b', y\right)$ be the price of a bond with face value b' issued by the country with an endowment shock y. The bond price function $q\left(b', y\right)$ will be determined in equilibrium.

We assume that foreign investors always commit to repay their debt. But the government can decide whether to repay its debt or to default. We denote the country's default history by a discrete variable $h \in \{0, 1\}$. Let $h = 0$ stand for a good credit record, which indicates no unresolved default on the country's record; whereas $h = 1$ denotes a bad credit record, or an unresolved sovereign default in the country's credit history. If a country with a good credit record ($h = 0$) defaults on its debt $b < 0$, the present value of its debt is reduced to a fraction. We use the debt recovery rate $\alpha\left(b, y\right)$ to denote the fraction of debt the country repays following the debt renegotiation. The defaulting country does not pay anything in the default period. However, the country's credit record deteriorates in the next period ($h' = 1$).

If a country has a bad credit record ($h = 1$) and unpaid debt arrears $b < 0$, the country has an unresolved default. The country is then subject to an exclusion from financial markets and a direct output cost, both of which are empirically relevant. First, default incurs the reputation cost of financial exclusion, and there is no saving opportunity after default.[9] The country may also face a direct output loss that is equal to a fraction λ of

[8] $b_{\max}$ exists when the interest rates on a country's savings are sufficiently small compared to the discount factor, which is satisfied in our paper as $(1 + r)\,\beta < 1$.

[9] It is well known that reputation models are subject to the Bulow and Rogoff (1989a) critique, which says that a pure reputation mechanism cannot support positive international debt if defaulting countries can save at the same interest rate. Cole and Kehoe (1998), Kletzer and Wright (2000), Wright (2002), and Amador (2003) analyze various ways to reduce the range of a defaulting country's saving mechanism, so that a reputation model can generate positive international lending.

endowment, $0 \leq \lambda \leq 1$.[10] In this case, the defaulting country can restore its good credit record by repaying the debt arrears. As in Fernandez and Rosenthal (1990) and in Cole, Dow and English (1995), we assume that once the debt arrears are repaid, the country's credit record is upgraded and it regains its access to capital markets. Thus, the resumption of the international credit relationship is endogenous, depending on the amount of the debt arrears and on the country's economic condition.

When default occurs, the lenders take collective action and bargain with the country over debt reduction in a Nash bargaining game. The debt recovery rate $\alpha(b, y)$ is determined in the post-default renegotiation and depends on the defaulted debt value b and endowment shock y.[11] According to the bargaining agreement, the present value of defaulted debt is reduced to a fraction $\alpha(b, y)$ of the unpaid debt b. Should the renegotiation fail, international lenders would lose their investment, and the country would be indefinitely excluded from the financial markets.

3 Recursive Equilibrium

In this section, we define and characterize a dynamic recursive equilibrium.

3.1 Sovereign Country's Problem

The government's objective is to maximize households' expected lifetime utility. The government makes its default decision and determines its assets for the next period, given the current asset position b and the endowment shock y. Let $v(b, h, y) : \mathcal{L} \to \mathcal{R}$ be the lifetime value function for the country that starts the current period with the credit record h, asset position b, and endowment shock y, where $\mathcal{L} = B \times \{0\} \times Y \cup B_{-} \times \{1\} \times Y$.[12] We restrict the space of bond price functions so that $Q = \left\{q | q(b, y) : B \times Y \to \left[0, \frac{1}{1+r}\right]\right\}$, and the space of debt recovery schedules so that $A = \{\alpha | \alpha(b, y) : B_{-} \times Y \to [0, 1]\}$. Given any bond price function $q \in Q$ and debt recovery schedule $\alpha \in A$, the country solves its optimization problem.

[10] The direct output loss is observed empirically. We take the proportional loss specification for simplicity. The assumption on output loss is not needed to study the model's theoretical properties. However, it improves the model's fit in the quantitative analysis. Mendoza and Yue (2009) analyze the mechanism through which defaults lead to output losses.

[11] Kovrijnykh and Szentes (2005) present a model in which lenders are competitive in pre-default periods and become monopolists after default.

[12] Note that only the country with a good credit record can have savings.

For $b \geq 0$ and $h = 0$, the country has a good credit record and savings. The country receives payments from the foreign investors and determines its next-period asset position b'. The value function is

$$v(b, 0, y) = \max_{c, b' \in B : c + q(b', y) b' = y + b} u(c) + \beta \int_Y v(b', 0, y') \, d\mu(y'|y). \tag{2}$$

For $b < 0$ and $h = 0$, the country has a good credit record and the outstanding debt is b. If the country honors its debt obligation, it chooses its next-period asset position b' and consumes. If the country defaults, it cannot borrow or save in the current period. Moreover, the country's credit record deteriorates to $h' = 1$. But the country gets its debt reduced to $\alpha(b, y) b$. The country determines whether to default or not. Its value function is:

$$v(b, 0, y) = \max \left\{ v^r(b, 0, y), v^d(b, 0, y) \right\}, \tag{3}$$

where $v^r(b, 0, y)$ is the value function if the country does not default:

$$v^r(b, 0, y) = \max_{c, b' \in B : c + q(b', y) b' = y + b} u(c) + \beta \int_Y v(b', 0, y') \, d\mu(y'|y),$$

and $v^d(b, 0, y)$ is the value of default:

$$v^d(b, 0, y) = u(y) + \beta \int_Y v(\alpha(b, y) b, 1, y') \, d\mu(y'|y).$$

For $h = 1$, the country has a bad credit record and unpaid debt arrears $b < 0$. The country is excluded from financial markets, and its endowment suffers a proportional loss of λy. The country can pay back its debt arrears.[13] If the country repays partially, its next-period credit record remains bad. The debt arrears are rolled over at the interest rate r. The value function is:

$$v(b, 1, y) = \max_{c, b' \in [b, 0] : c + \frac{b'}{1+r} = (1-\lambda) y + b} u(c) + \beta \int_Y v(b', 1, y') \, d\mu(y'|y). \tag{4}$$

When all the debt arrears are paid, the country regains its full access to the markets. The

[13] We assume that the creditor can enforce the payment of interests that accrued with the unpaid debt. This is a technical assumption to ensure that the state space for debt arrears is bounded. It implies that the creditor gets all the reduced debt with certainty. Therefore, the market interest rate on debt arrears is the risk free rate r.

value function is:

$$v(0,1,y) = v(0,0,y). \tag{5}$$

The country's default policy can be characterized by a default set $D(b) \subset Y$. Default set is the set of endowment shock y's for which default is optimal given the debt position b.

$$D(b) = \left\{ y \in Y : v^r(b,0,y) \leq v^d(b,0,y) \right\}.$$

3.2 The Debt Renegotiation Problem

The debt renegotiation takes the form of a generalized Nash bargaining game. Under the bargaining agreement, the value of defaulted debt is reduced to a fraction $\alpha(b,y)$ of the unpaid debt b. The value of such an agreement to the country is $v^d(b,0,y) = u(y) + \beta \int_Y v(\alpha(b,y)b, 1, y') d\mu(y'|y)$, which is the expected life-time utility of defaulting when the debt recovery rate is $\alpha(b,y)$. This value takes into account the impact of debt reduction as well as a temporary debt exclusion associated with a bad credit record. The lenders get the present value of the reduced debt $\frac{\alpha(b,y)b}{1+r}$.

We assume that the renegotiation takes place only once for one default event. The threat point of the bargaining game is that the country stays in permanent autarky and the creditors get nothing. The expected value of autarky to the country, $v^{aut}(y)$, is given in a recursive form:

$$v^{aut}(y) = u((1-\lambda)y) + \beta \int_Y v^{aut}(y') d\mu(y'|y). \tag{6}$$

The one-round bargaining assumption keeps the model tractable, as there is no need to track the rounds of bargaining or the debt arrears determined in different rounds of bargaining. Our result about the interaction between default and renegotiation holds in a general model.[14]

For any debt recovery rate a, we denote the country's surplus in the Nash bargaining by $\Delta^B(a;b,y)$, which is the difference between the value of accepting the debt recovery rate a and the value of rejecting it, given the country's debt level b and endowment y.

$$\Delta^B(a;b,y) = \left[u(y) + \beta \int_Y v(ab,1,y') d\mu(y'|y) \right] - v^{aut}(y). \tag{7}$$

[14] A generalization is to allow for multiple rounds of costly renegotiations. Allowing for continuous costless renegotiation generates either risk-free debt or no international lending.

Although the country's credit record becomes bad after default, the expected length of financial exclusion is finite. Thus, the defaulting country gains because the access to capital markets is lost temporarily rather than permanently.

The surplus to the risk-neutral lender is the present value of recovered debt.

$$\Delta^{L}(a;b,y)=-\frac{ab}{1+r}. \tag{8}$$

If lenders have all the bargaining power, then they could extract debt repayments up to the full amount of a country's cost of default. If, on the other hand, the borrowing country can make take-it-or-leave-it offers, then it gets a complete debt reduction in the bargaining. To analyze the general case, we assume that the borrower has a bargaining power θ and the lender has a bargaining power $(1-\theta)$. The bargaining power parameter θ summarizes the institutional arrangement of debt renegotiation. To ensure that the bargaining problem is well-defined, we define the bargaining power set $\Theta \subset [0,1]$ such that for $\theta \in \Theta$ the renegotiation surplus has a unique optimum for any debt position b and endowment shock y.

Given the debt level b and endowment y, the debt recovery rate $\alpha(b,y) \in A$ solves the following bargaining problem:

$$\begin{aligned}\alpha(b,y) &= \arg\max_{a\in[0,1]}\left[\left(\Delta^{B}(a;b,y)\right)^{\theta}\left(\Delta^{L}(a;b,y)\right)^{1-\theta}\right] \\ \text{s.t. } &\Delta^{B}(a;b,y)\geq 0, \\ &\Delta^{L}(a;b,y)\geq 0.\end{aligned} \tag{9}$$

Because the debt recovery schedule that maximizes the total renegotiation surplus depends on the country's endowment shock and debt position, the renegotiation outcome provides better insurance to the country if it decides to default.

3.3 Foreign Investors' Problem

Taking the bond price function as given, the foreign investors choose the amount of debt b' to maximize their expected profit π. Their expected profit is given by:

$$\pi(b',y)=\begin{cases} q(b',y)\,b'-\frac{1}{1+r}b' & \text{if } b'\geq 0 \\ \frac{[1-p(b',y)+p(b',y)\cdot\gamma(b',y)]}{1+r}(-b')-q(b',y)(-b') & \text{if } b'<0 \end{cases}, \tag{10}$$

where $p(b', y)$ is the expected probability of default for a country with an endowment y and indebtedness b', and $\gamma(b', y)$ is the expected recovery rate, given by the expected proportion of defaulted debt that the creditors can recover, conditional on default.

Because we assume that the market for new sovereign debt is completely competitive, the foreign investors' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q(b', y) = \begin{cases} \frac{1}{1+r} & \text{if } b' \geq 0 \\ \frac{[1-p(b',y)]}{1+r} + \frac{p(b',y) \cdot \gamma(b',y)}{(1+r)} & \text{if } b' < 0 \end{cases}. \tag{11}$$

When the country lends to the foreign investors, $b' \geq 0$, the sovereign bond price is equal to the price of a risk-free bond $\frac{1}{1+r}$. When the country borrows from the foreign investors, $b' < 0$, its creditor faces the risk of default and restructuring that results in the creditor's loss. The sovereign bond is priced to compensate the foreign investors for bearing both risks.[15]

Equation (11) shows that default risk has the first-order effect on bond price as it enters the first term of pricing function in the linear form. The debt recovery rate affects the sovereign bond prices through its combined effect with default risk (as shown in the second term). In addition, debt recovery rate has an effect on the second moment of sovereign bond prices, and it indirectly influences the ex ante default risk.

Since $0 \leq p(b', y) \leq 1$ and $0 \leq \gamma(b', y) \leq 1$, the bond price $q(b', y)$ lies in $\left[0, \frac{1}{1+r}\right]$. The interest rate on sovereign bonds, $r^s(b', y) = \frac{1}{q(b',y)} - 1$, is bounded below by the risk-free rate. The difference between the country interest rate and the risk free rate is the country's credit spread $s(b', y) = r^s(b', y) - r$.

3.4 Recursive Equilibrium

We now define a stationary recursive equilibrium in the model economy.

Definition 1 *A recursive equilibrium is a set of functions for (i) the country's value function $v^*(b, h, y)$, asset holdings $b'^*(b, h, y)$, default set $D^*(b)$, consumption $c^*(b, h, y)$, (ii) recovery rate $\alpha^*(b, y)$, and (iii) pricing function $q^*(b', y)$ such that:*

1. Given the bond price function $q^*(b', y)$ and debt recovery rate $\alpha^*(b, y)$, the value function $v^*(b, h, y)$, asset holding $b'^*(b, h, y)$, consumption $c^*(b, h, y)$, and default set $D^*(b)$ satisfy the country's optimization problem (2), (3), and (4).

[15] The price functions for consumer debt in Chatterjee et al. (2007) and sovereign debt in Arellano (2008) are our model's special cases, in which debt recovery rate is zero and the default risk alone determines the bond price.

2. Given the bond price function $q^*(b', y)$ and value function $v^*(b, h, y)$, the recovery rate $\alpha^*(b, y)$ solves the debt renegotiation problem (9).

3. Given the recovery rate $\alpha^*(b, y')$, the bond price function $q^*(b', y)$ satisfies the zero expected profit condition for foreign investors (11), where the default probability $p^*(b', y)$ and expected recovery rate $\gamma^*(b', y)$ are consistent with the country's default policy and renegotiation agreement.

In equilibrium, the default probability $p^*(b', y)$ is related to the country's default policy in the following way:

$$p^*(b', y) = \int_{D^*(b')} d\mu(y'|y). \tag{12}$$

The expected recovery rate $\gamma^*(b, y)$ in equilibrium is determined by:

$$\begin{aligned} \gamma^*(b', y) &= \frac{\int_{D^*(b')} \frac{\alpha^*(b', y')}{1+r} d\mu(y'|y)}{\int_{D^*(b')} d\mu(y'|y)} \\ &= \frac{\int_{D^*(b')} \frac{\alpha^*(b', y')}{1+r} d\mu(y'|y)}{p^*(b', y)}. \end{aligned} \tag{13}$$

The numerator is the expected proportion of debt that the investors can recover, and the denominator is the default probability.

We can establish the existence of a recursive equilibrium in the model economy as stated in Theorem 1.[16]

Theorem 1 *Given any bargaining power $\theta \in \Theta$, a recursive equilibrium exists.*

Proof. See Appendix. ■

3.5 Characterization of a Recursive Equilibrium

We now proceed to establish some properties of a recursive equilibrium.

Theorem 2 *For a bargaining power $\theta \in \Theta$, there exists a threshold $\bar{b}(y) \leq 0$ such that the equilibrium debt recovery function α satisfies:*

$$\alpha^*(b, y) = \begin{cases} \frac{\bar{b}(y)}{b} & \text{if } b \leq \bar{b}(y) \\ 1 & \text{if } b \geq \bar{b}(y) \end{cases}.$$

[16] We cannot prove the uniqueness of the recursive equilibrium. But we do not find multiple equilibria in the numerical computation.

Proof. See Appendix. ■

The intuition for Theorem 2 is the following: After default, bygones are bygones. The defaulting country cares about the total amount of reduced debt, which affects the expected duration of financial exclusion. At the same time, the foreign investors are solely concerned with the total recovery on defaulted debt. Therefore, the bargaining on debt recovery rate is equivalent to the renegotiation over the reduced debt. There is an optimal value of reduced debt that maximizes total renegotiation surplus. Hence, the debt recovery rates decrease inversely with the amount of defaulted debt, and there is no debt reduction for debt levels smaller than the threshold.

The country's *ex ante* incentive to default depends on the *ex post* renegotiation agreement on debt reduction in equilibrium. Given the equilibrium debt recovery schedule $\alpha(b, y)$, characterized by Theorem 2, and the endowment shock y, the value function of a defaulting country is independent of the level of debt if it is larger than $\overline{b}(y)$. Therefore, we can show that the default set increases with the country's indebtedness and that the equilibrium default probability increases with the level of debt.

Theorem 3 *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, for $b^0 < b^1 \leq \overline{b}(y)$, if default is optimal for b^1, then default is also optimal for b^0. That is $\overline{D}^*(b^1) \subseteq \overline{D}^*(b^0)$.*

Proof. See Appendix. ■

Theorem 4 *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, the country's probability of default in equilibrium satisfies $p^*(b^0, y) \geq p^*(b^1, y)$, for $b^0 < b^1 \leq \overline{b}(y) \leq 0$.*

Proof. See Appendix. ■

Given the endogenous debt recovery rates, our model predicts that default probability increases with the level of debt, as in Eaton and Gersovitz (1981). The key difference is that they assume a zero debt recovery rate and rule out the possibility of debt renegotiation; yet our model obtains this result in a more general set up with endogenous debt renegotiation.

We also characterize the equilibrium bond price schedule.

Theorem 5 *Given an endowment $y \in Y$, for $b^0 < b^1 \leq \overline{b}(y) \leq 0$, an equilibrium bond price satisfies $q^*(b^0, y) \leq q^*(b^1, y)$.*

Proof. See Appendix. ■

In equilibrium, bond prices depend on both the risk of default and the expected debt recovery rates. For a high level of debt, the default probability is higher, but the expected debt recovery rate is lower. Therefore, equilibrium bond prices decrease with indebtedness. This result is consistent with the empirical evidence - e.g., Edwards (1984).

The next theorem characterizes the debt arrears repayment policy of a defaulting country.

Theorem 6 *Given an endowment $y \in Y$, if there exists a level of debt $\widetilde{b} < 0$ that satisfies*

$$\begin{aligned} & \sup_{b'<0} u\left((1-\lambda_d)\, y + \widetilde{b} - \frac{b'}{1+r}\right) + \beta \int_Y v\left(b', 1, y'\right) d\mu\left(y'|y\right) \\ = \; & u\left((1-\lambda_d)\, y + \widetilde{b}\right) + \beta \int_Y v\left(0, 0, y'\right) d\mu\left(y'|y\right), \end{aligned} \tag{14}$$

then for all $b \in B_-$ and $b > \widetilde{b}$, it is strictly optimal for the defaulting country to repay its debt arrears in full, and for all $b \in B_-$ and $b < \widetilde{b}$, a partial repayment is strictly optimal.

Proof. See Appendix. ■

This theorem implies that if the country fully repays the debt arrears b and regains access to financial markets in the next period, then it also chooses to do so with a lower level of debt arrears. If the country decides not to repay the debt in full, it will do the same for higher debt arrears.

Note that although there is no delay in debt renegotiation due to the Nash bargaining model set up, the model generates endogenous exclusion from financial markets after default, which is closely related to the debt renegotiation outcome. Based on the above theorem, the expected duration of financial exclusion increases with the amount of reduced debt after default and debt arrears in general. This paper thus complements Kovrijnykh and Szentes (2007) who find that countries with debt overhang are more likely to reaccess financial markets after a series of good shocks.

4 Quantitative Analysis

In this section, we calibrate the model to analyze quantitatively the sovereign debt of Argentina.

4.1 Calibration

We define one period as a quarter. The utility function for the country has the following form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{15}$$

where σ is the coefficient of risk aversion. We set the risk aversion coefficient to 2, which is standard in the macroeconomics literature. The risk-free interest rate r is set to 1%, which is the average quarterly interest rate on 3-month US treasury bills. The output loss parameter λ is set to 2% as in Aguiar and Gopinath (2006).

The endowment process is calibrated to the Argentine quarterly real GDP for 1980Q1 to 2003Q4 from the Ministry of Finance (MECON). To capture the stochastic trend in GDP,[17] we model the output growth rate as an AR(1) process:

$$\log g_t = \left(1 - \rho_g\right) \log(1 + \mu_g) + \rho_g \log g_{t-1} + \varepsilon_t^g, \tag{16}$$

where growth rate is $g_t = \frac{y_t}{y_{t-1}}$, growth shock is $\varepsilon_t^g \overset{iid}{\sim} N\left(0, \sigma_g^2\right)$, and $\log(1 + \mu_g)$ is the expected log gross growth rate of the country's endowment. We estimate the endowment process to match the average growth rate, as well as the standard deviation and autocorrelation of HP detrended output. In the quantitative analysis, we detrend the model by the lagged endowment level y_{t-1}.[18]

The time discount factor β and the country's bargaining power θ are calibrated to match the average default frequency and debt recovery rate for Argentina. Argentina defaulted five times on its foreign debt since 1824.[19] Its average annual default frequency is 2.7%. We use the average debt recovery rate estimated by Sturzenegger and Zettelmeyer (2005) for Argentina. Sturzenegger and Zettelmeyer (2005) compute the haircuts measured by the percentage difference between the present value of old and new debt instruments during various phases of Argentina's debt restructuring. The average recovery rate during the 2005 international debt restructuring is 27%.[20]

[17] Aguiar and Gopinath (2007) find that shocks to the trend of output growth are the primary source of fluctuations in emerging market business cycles. Aguiar and Gopinath (2006) use a similar output process to study sovereign default in Argentina.

[18] Because a realization of the growth shock permanently affects endowment, the model economy is nonstationary. Value function, bond price function and debt recovery schedule are re-defined using the detrended variables.

[19] Reinhart, Rogoff and Savastano (2003) report four episodes of sovereign defaults in Argentina's external debt from 1824 to 1999. In 2001, Argentina defaulted a fifth time on its external debts

[20] Using the average trading price, Moody's 2007 estimate for Argentina's average recovery rate is also 27%. Benjamin and Wright (2009) use data on debt stock reduction and debt buybacks from Global

Parameter	Value	
Risk Aversion	$\sigma = 2$	
Risk Free Interest Rate	$r = 1\%$	US Treasury-bill interest rates
Output Loss in Default	$\lambda = 2\%$	Sturzenegger (2002)
Average Output Growth	$\mu_g = 0.42\%$	Argentina average growth
Endowment Growth Process	$\sigma_g = 2.53\%, \rho_g = 0.41$	Argentina's GDP
Calibration	Values	Target Statistics
Time Discount Factor	$\beta = 0.72$	2.78% default frequency
Bargaining Power	$\theta = 0.72$	27% debt recovery rate

Table 1: Target Statistics for Argentina (1980.1-2003.4)

The time discount factor β is estimated to be 0.72. This high degree of impatience helps to generate frequent defaults. It also reflects the high political instability in Argentina.[21] The bargaining power is 0.72, which shows that Argentina is in a more favorable position in the debt renegotiation than the international investors are. Table 1 summarizes the calibration results.

4.2 Simulation Results

In this section, we first examine the properties of the calibrated model. Then, we compare the simulation results to the data.

Figure 1 plots the equilibrium debt recovery schedule. As Theorem 2 shows, if a country defaults with a small amount of debt, there is no debt reduction. As the amount of defaulted debt increases, the debt recovery rate decreases. In addition, the numerical results show that debt reduction threshold $\overline{b}(y)$ is a decreasing function of the defaulting country's endowment. Hence, the debt recovery rate is higher for a defaulter with a good economic shock, and vice versa.[22] For the same amount of debt, a country that defaults in good times needs to pay back a higher level of debt. Thus, a default option provides additional state contingency through debt renegotiation, allowing defaultable bonds to further complete markets.

Figure 2 plots the default probabilities. For a country with a very low level of debt, there is virtually no default, regardless of the endowment shock. And default probability is zero for a range of debt level beyond the debt reduction threshold. This result shows

Development Finance; they estimate that the recovery rate for Argentina is 37%.

[21] Argentina has had 15 presidents from 1981 to 2008.

[22] Using a cross-country dataset for 93 defaults, Benjanmin and Wright (2009) find that larger haircuts are more likely when the economic conditions are weak at the time of default.

that the country with a low amount of debt may choose not to default, even when the debt renegotiation can generate some debt reduction, because the cost of being excluded is higher than the value of having its debt reduced. The default probability increases with indebtedness. Furthermore, the default probability is higher for a country with a bad economic shock because default is more valuable for the country facing a bad shock.

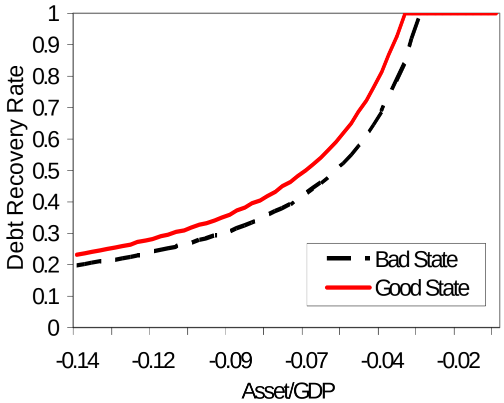


Figure 1: Recovery Rate

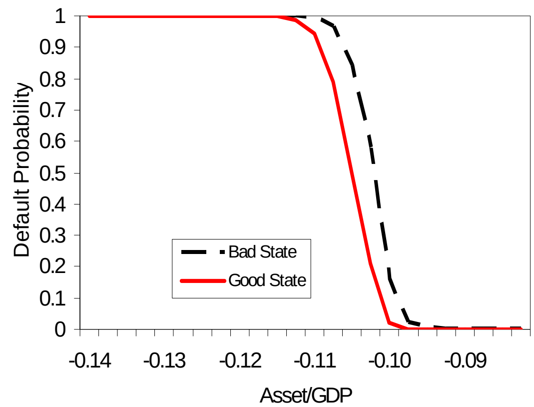


Figure 2: Default Probability

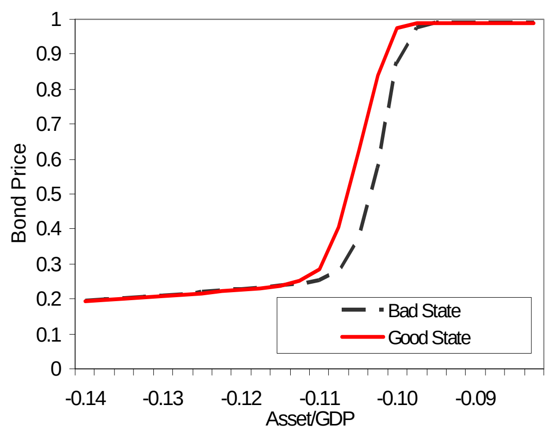


Figure 3: Bond Price Function in Benchmark Model

Figure 3 presents the bond price functions for a country with a high and a low endowment shock in the current period. It shows that the bond price increases with the assets-to-output ratio (or decreases in the debt-to-output ratio). The bond price also increases with the endowment shock, which implies the countercyclicality of interest rates. When a country receives a bad shock, the higher debt reduction increases the country's ex ante default incentive. A higher default probability and a lower debt recovery rate generate a higher sovereign bond spread, and thus a negative correlation exists between spreads and output.

Non-target Statistics	Data	Model	Aguiar and Gopinath (2007)	Arellano (2008)
Business Cycle Variables				
Consumption Std./Output Std.	1.03	1.04	1.05	1.10
Trade Balance Std. Dev. (%)	2.75	2.81	0.95	1.50
Corr (Trade Balance, Output)	-0.39	-0.16	-0.19	-0.25
Debt and Debt Crisis				
Average Debt/Output Ratio (%)	46.5 / 9.54	10.13	19	5.95
Output Drop at Default (%)	12.77	7.19	-	9.60
Consumption Drop at Default (%)	12.68	8.84	-	9.47
Bond Spreads and Debt Reduction				
Corr (Spreads, Output)	-0.43	-0.11	-0.03	-0.29
Corr (Spreads, Trade Balance)	0.74	0.30	0.11	0.43
Average Bond Spreads (%)	7.17	1.86	0.92	3.58
Bond Spreads Std. Dev. (%)	2.67	1.58	0.32	6.36
Corr (Default Prob., Recovery Rate)	-	-0.26	-	-
Corr (Defaulted Debt, Haircuts)	0.31[1]	0.31	-	-
Average Exclusion (years)	0.9[1]	0.25	2.5	0.89
Target Statistics				
Default Frequency (%)	2.7	2.67	0.92	3
Average Recovery Rate (%)	27	27.31	0	0

Table 2: Model Statistics for Argentina

We conduct 1000 simulations of the model economy with 600 periods in each simulation. Then we extract 80 observations before each default event in the stationary distribution to compute the statistics.[23] Table 2 compares the model statistics with the data statistics.

The bond spreads data are from the J.P. Morgan's Emerging Markets Bond Indices (EMBI) dataset for Argentina from 1994Q1 to 2002Q1.[24] The consumption and trade balance data are seasonally adjusted from 1980Q1 to 2002Q1, taken from MECON. Trade balances are calculated as ratios to real GDP. The data on Argentina's external debt is from the World Bank's Global Development Finance dataset. We compute two measures of the country's indebtedness. The first measure is the average external debt/GDP ratio. Because the external debt stock (including long-term debt) is not the amount that the

[23] We choose 80 observations prior to a default event to mimic the sample in the data for Argentina from 1980 to 2001, before its default in 2001. See also Arellano (2008) for this treatment of the simulation.

[24] EMBI for Argentina is a composite index of different US dollar-denominated bonds on four markets: Brady bonds, Eurobonds, U.S. dollar local markets, and bank loans. The spreads are computed as an arithmetic, marketcapitalization-weighted average of bond spreads over US treasury bonds of comparable duration.

country needs to pay next period, we also compute the ratio of the country's debt service (including short-term debt) to its GDP for Argentina.

The model matches the business cycle statistics. The model generates volatile consumption at the business cycle frequency. In the data, the consumption volatility is higher than output volatility, which is a common feature of emerging economies (see Neumeyer and Perri (2005)). In our model, a good endowment shock increases permanent income more than proportionally, so the country borrows to consume more, and vice versa. Moreover, borrowing is more expensive in recessions, due to the countercyclical interest rates, and it further reduces the country's consumption in bad times. Therefore, consumption is more volatile than endowment in our model. The trade balance volatility is also close to the data. The correlation between trade balance and output is in line with the data. The countercyclical trade balance is attributable to both persistent endowment shocks and the countercyclical bond spreads. In bad times, the country borrows less because the permanent income drops and the interest rates on debt are higher due to the higher default risk.[25]

The model supports a moderate level of debt compared to the data. The debt/GDP ratio is 10% in the model simulation. In the data, the average external debt to GDP ratio from 1980 to 2002 is 46.5% for Argentina. The total debt service to GDP ratio is 9.54%. Because the time discount factor is lower than the risk free rate in the calibration, the consumption front-loading motive in the model can potentially support a high level of debt. Yet, because of the default risk, the government needs to pay a higher interest rate when it borrows more, and it should choose to default when it is optimal to do so. The sustainable debt level in the model depends on the benefit and cost of default. As shown in Aguiar and Gopinath (2006) and in Arellano (2008), exclusion cost alone cannot quantitatively sustain large levels of debt. The output reduction at default is also moderate in the model compared to the data. The drop in output is 7.2%, and the drop in consumption is 8.8% on average at default.[26]

The model delivers the relation among bond spreads, outputs, and trade balances in the data. Bond spreads are negatively correlated with output and positively correlated with trade balances. The model accounts for the negative correlation between spreads and output because the sovereign bonds have higher default risk and lower debt recovery rates in

[25] Atkeson (1991) develops a model with limited enforcement and moral hazard to explain this pattern of capital outflow. Aguiar and Gopinath (2006) and Arellano (2008) analyze the business cycle fluctuations for Argentina in the presence of default risk, as well.

[26] Mendoza and Yue (2009) study an alternative mechanism with production and additional financial friction to account for the large drop in output and the high debt in the data.

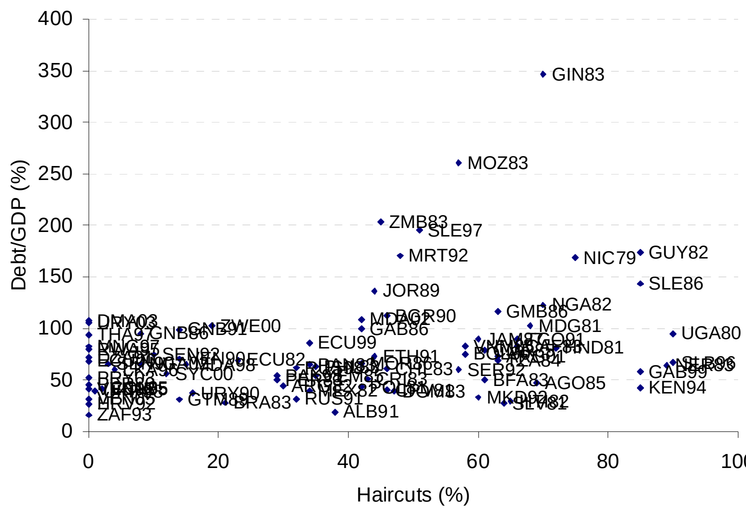
bad states. Consequently, it is more expensive for the country to borrow during bad states. Although a lower level of debt implies a relatively lower bond spread, the downward shift in the bond price schedule caused by the bad shock dominates, implying higher bond spreads in bad times. Because the country also borrows less during bad states, the bond spreads are also positively correlated with trade balances in the model; this is consistent with the data. The average annual bond spread is 1.86% in the simulation; this is smaller than the average spread of 7.17% in the data. In the model, the bond spreads are jointly determined by the default probabilities and debt recovery rates. Because the default frequency in the data is 2.7% and the average debt reduction rate is 72%, the average bond spreads in the stationary distribution is about 2%. First, the bond spreads in the model do not include a risk premium, which may be an important component of bond spreads in the data.[27] Moreover, the term premium between 3-month bonds analyzed in our model and long-term bonds in the data should be taken into account in the comparison.

The volatility of the bond spreads in the model is closer to the data. The model can account for about 60% of bond spread volatility in the data. In our model, allowing for debt renegotiation breaks the one-to-one matching from default probabilities to bond spreads - even though lenders are risk neutral. The debt recovery rates are negatively correlated with default probability. In particular, default probability is higher when a larger fraction of debt reduction is expected in the post-default renegotiation. Hence, even though the creditors ask for a lower risk premium to compensate for the loss in default because of the positive debt recovery, the model with endogenous debt renegotiation can account for a large degree of the fluctuation in bond spreads.

Lastly, regarding the relation between the debt renegotiation outcome and the economic condition of the defaulting country, we compute the correlation between the haircuts and the debt/output ratio when default takes place. Theorem 2 predicts that the recovery rate decreases in the level of debt under renegotiation, implying a positive correlation between haircuts and indebtedness. The model simulation finds that the correlation between haircuts and debt/output is 0.31. Argentina received about a 30% debt reduction during the debt restructuring in 1983-1986 over the total debt of about 19 billion. The average haircut rate is 72% on the $82 billion debt that Argentina failed to service in the 2001-2002 debt crisis. The comparison of these two recent debt restructurings for Argentina is consistent with the model's prediction. The cross-country data also display a positive relationship

[27] Broner, Lorenzoni and Schmukler (2005) and Pan and Singleton (2008) document the presence of excess return in sovereign bond spreads. Lizarazo (2006) studies the pricing of sovereign default risk by risk averse investors.

between the haircut rate and the indebtedness. We first use the data collected by Benjamin and Wright (2009) on 90 debt restructurings for 73 countries from 1989 to 2006. Figure 4 plots the haircuts and debt/GDP ratio for each country in the default end-year in the dataset.[28] The correlation between the haircut rate and the debt/GDP ratio is 0.31, which is the same as predicted by the model. Table 3 shows the scale of debt crises and haircut rates for eight most recent defaults with readily available data. The haircut rate is higher for a more severe debt crisis, both in terms of the dollar amount and relative to the country's output. Thus, the model prediction is in line with the empirical observations.



Data source: Benjanmin and Wright (2009) for the haircuts in 93 defaults. Debt/GDP ratio from Global Development Finance dataset.

Figure 4: Haircuts and Debt/GDP

Table 2 shows that the average length of being excluded from financial markets due to default is 2 periods in the calibrated model. The country cannot borrow or lend when it defaults in the model, and it continues to be excluded for one period before the reduced debt is fully repaid. Because the country has a relatively high bargaining power and only needs to pay back 27% of its debt, the country finds it optimal to repay the debt arrears quickly. Compared to the data, Argentina reaccessed the financial market after 3.6 years following its default in Jan 2002. In the cross-country evidence, Gelos et al (2003) find that it takes 0.9 year for defaulted countries to regain access to international financial

[28] The debt/GDP ratio is computed using the total external debt and GDP (current US $) from the World Bank's Global Development Finance dataset. We use the debt/GDP ratio in the default end year to treat the model and the data in the same way. The model does not allow for the restructuring delays, which are also an important feature of the data.

Country	Time of default	Haircuts (%)	Defaulted debt (billion$)	Real Output (billion$)
Argentina	Jan 02	72%	82.23	268.7
Russia	Aug 98	63/50	72.7	259.7
Ecuador	Aug 99	40	6.6	21.3
Pakistan	Jul 99	35	1.63	63.0
Ukraine	Sep 98/Jan 00	30/28	1.27/1.06	31.3
Belize	Dec 06	30%	0.24	1.1
Uruguay	May 03	13	5.74	11.2
Dominican Republic	Apr 05	<5%	1.62	1.0

Data source: World Bank, Sturzenegger and Zettelmeyer (2005), and Moody's (2007).

Table 3: Statistics for Different Bargaining Powers

markets in the 1990s. Because the Nash bargaining model features an immediate debt reduction agreement after default, there is no delay in the financial exclusion periods in the model. Benjamin and Wright (2009) find that negotiations to restructure debt are protracted on average.[29] The delays in renegotiation can account for the difference in the length of financial exclusion between the model and what we observe in the data.

The model can replicate the recent Argentine debt crises and the time series of Argentine bond spreads. We feed the Argentine GDP growth rate into the model and compare the time series of bond spreads. Figure 5 plots the H-P detrended output, the Argentine bond spreads, and the simulated bond spreads from 1994Q2 to 2001Q4. The figure demonstrates that the model can explain the recent Argentinian default episode. Before a default occurs, the country faces volatile and countercyclical interest rates. When the country gets a really bad shock, the model generates a default on the country's debt - which is what we observe in Argentina in the last quarter of 2001.

4.3 Role of Endogenous Debt Renegotiation

We study the role of endogenous debt renegotiation by comparing our model to the sovereign default models without renegotiation. Aguiar and Gopinath (2007) and Arellano (2008) study the quantitative properties of an Eaton-Gersovitz type sovereign default model.[30] In

[29] Benjamin and Wright (2009) find that the average length of renegotiation is 7.4 years. The delays in sovereign debt renegotiations after 1990 are on average a little over 2 years in these authors' sample.

[30] Other quantitative studies of Eaton-Gersovitz type models without bargaining are Bai and Zhang (2008), Lizarazo (2006), and Hatchondo, Martinez and Sapriza (2007). These papers include risk averse investors, production, and heterogeneous borrowers, respectively.

these papers, default leads to a full debt discharge, and the defaulting country regains access to the capital markets with an exogenous return probability. Complete debt discharge corresponds to an exogenous zero debt recovery rate $\alpha = 0$. An exogenous return probability δ determines the expected length of exclusion from financial markets. Both papers calibrate the model to the Argentine data. The last two columns of Table 2 compare the results from our model and the two papers without renegotiation.

Figure 5: Output and Bond Spreads in the Data and in the Model (1994.2-2001.4)

The models without renegotiation and our model have similar results along many dimensions. Some notable differences are related to the average debt, trade balance volatility, and bond spreads. Aguiar and Gopinath (2006) get a higher average debt/output ratio. However, the average default frequency is considerably lower at 0.2%. Our model produces a higher level of debt compared to Arellano (2008) model, which is also calibrated to 3% default frequency for Argentina. One reason is that debt renegotiation provides more insurance to the defaulting country through the endogenous debt reduction. Thus, the model can support more debt. All the models predict a lower bond spread compared to the data because of the risk neutral pricing of short-term bonds. Our model with debt renegotiation generates a lower bond spread relative to the no bargaining model when calibrated to the same default frequency. Nevertheless, the bond spread volatility is not substantially lower. The negative correlation between default probability and debt recovery rates amplifies the movements in bond spreads. More importantly, our model replicates the stylized facts on sovereign debt reduction. We provide a theoretical account of debt reduction and replicate the relation between haircuts and the defaulting country's indebtedness; this relation cannot be analyzed in the sovereign debt models without debt renegotiation.

Statistics	$\eta = 0$	$\eta = 0.5$	Model $\eta = 0.72$	$\eta = 0.9$	$\eta = 1$
Average Debt/Output Ratio (%)	38.08	15.10	10.13	7.07	2.185
Corr (Spreads, Output)	-0.04	0.01	-0.11	-0.02	0
Corr (Spreads, Trade Balance)	-0.01	-0.11	0.50	-0.04	0
Average Bond Spreads (%)	1.47	1.22	1.86	1.67	0
Bond Spreads Std. Dev. (%)	1.22	0.66	1.58	0.83	0
Corr (Defaulted Debt, Haircuts)	0.17	0.14	0.31	0.16	0
Default Frequency (%)	2.44	1.86	2.67	1.81	0
Average Recovery Rate (%)	45.60	40.74	27.31	13.03	0

Table 4: Statistics for Different Bargaining Powers

4.4 Bargaining Power

Debt renegotiation plays a central role in our model, and bargaining power is a key parameter that captures the bargaining protocol. We investigate how different bargaining powers affect the model's predictions. The results are summarized in Table 4. The bargaining power parameter has a direct impact on debt recovery rate. It is intuitive that a higher bargaining power for the country results in a lower debt recovery for lenders. Keeping other things fixed, the lower recovery rate increases the average bond interest rates. On the other hand, the lower debt recovery rate shifts down the bond price schedule. As a result, borrowing is discouraged and the debt-to-output ratio is smaller. With less borrowing, both the default probability and the bond interest rates decrease, ceteris paribus. Therefore, the increasing bargaining power for the country has two opposite effects on default probability and bond interest rates. How the equilibrium changes depends on which effect dominates. Table 4 shows that the default probability and the average interest rate do not change monotonically with the bargaining power.

The results in Table 4 with different bargaining powers can be viewed as outcomes of policy experiments. Our results shed light on the impact of reform in sovereign bond restructuring on the international financial market.[31] Through the numerical experiments in our model, we find that when a sovereign borrower has a higher bargaining power, the country's borrowing cost does not necessarily increase. And the amount of sovereign

[31] The use of Collective Action Clauses (CACs) is discussed during the recent debate on debt restructuring processes. CACs can align bondholders' incentives by specifying a majority rule, which binds all bondholders to eliminate the "hold out" problem. In this sense, CACs can be viewed as an experiment that assigns more bargaining power to the sovereign borrower. However, the use of CACs is also associated with an additional aggregation cost, as discussed in Eichengreen and Mody (2003). Thus, how a sovereign borrower's bargaining power is affected under CACs is ambiguous.

debt issued in the market is greatly affected by the bargaining power. These results are consistent with the recent empirical findings on bond issuance and spreads in Eichengreen, Kletzer and Mody (2003).[32]

5 Conclusion

It is well observed that sovereign debt crises have a great impact on borrowing countries and on international capital markets. Therefore, it is crucial to understand sovereign default risk and the role of debt crises resolution in sovereign debt markets. This paper studies sovereign default and debt renegotiation in a small open economy model. This model allows us to investigate the interaction between default and debt renegotiation within a dynamic borrowing framework. We find that debt recovery rates decrease with indebtedness and, in turn, affect the country's *ex ante* incentive to default. In equilibrium, sovereign bonds are priced to compensate creditors for the risks of default and restructuring. Consistent with the empirical evidence, the model predicts that interest rates and haircuts increase with the level of debt.

We use the model to analyze quantitatively the sovereign debt of Argentina. The model successfully accounts for the high bond spreads, countercyclical country interest rates, and other key features of the Argentine economy. The model also replicates the dynamics of bond interest rates during the recent Argentine debt crisis. Furthermore, this paper demonstrates that the changes in bargaining power have a great impact on debt recovery rates as well as on the sovereign bond spreads, thereby shedding light on the policy implications of sovereign debt restructuring procedures. Overall, this study points out the importance of analyzing the connection between default and renegotiation in understanding sovereign debt markets. One direction for future research is to investigate the role of international financial institutions in such a strategic interplay between default and renegotiation.

[32] See also Weinschelbaum and Wynne (2005), and Bolton and Jeanne (2009) for an analysis of restructuring the sovereign debt restructuring process.

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Appendix

Proofs

Proof of Theorem 1. The proof consists of three steps.

Step 1. Given any debt recovery schedule $\alpha(b, y) \in A$, we define a price correspondence $\varphi(q)$ that takes points in Q:

$$\varphi(q)(b', y; \alpha) = \begin{cases} \begin{array}{c} (1 - p(q)(b', y; \alpha)) / (1 + r) \\ + p(q)(b', y; \alpha) \cdot \gamma(q)(b', y; \alpha) / (1 + r) \end{array} & \text{if } b' \geq 0 \\ \\ 1/(1 + r) & \text{if } b' \leq 0 \end{cases}, \tag{17}$$

where $p(q)(b', y; \alpha)$ and $\gamma(q)(b', y; \alpha)$ satisfy (12) and (13). Thus, $\varphi(q)(b', y; \alpha)$ is the set of prices for a debt contract of type (b', y) that are consistent with zero profits given the price function q. We can show that $\varphi(q)(b', y; \alpha)$ is a closed interval in R and the correspondence $\varphi(q)(b', y; \alpha)$ has a closed graph (see Lemma App 5 and Lemma 8 in Chatterjee et al. (2006) for similar proofs). Therefore $\varphi(q)(b', y; \alpha)$ is an upper hemicontinuous correspondence. For any $q \in Q$, let $\varphi(q) \subset Q$ be the product correspondence $\Pi_{b', y \in B \times Y} \varphi(q)(b', y; \alpha)$. Since $\varphi(q)(b', y; \alpha)$ is convex-valued for each b',y, $\varphi(q)$ is convex-valued as well. Furthermore, since $\varphi(q)(b', y; \alpha)$ is upper hemicontinuous with compact values for each b', y, the product correspondence $\varphi(q)$ is also upper hemicontinuous with compact values. (see Aliprantis and Border (1999), Thm 16.28). Therefore, $\varphi(q; \alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set Q and returns sets in Q. By Kakutani-Fan-Glicksberg FPT (see Aliprantis and Border (1999), Thm 16.51) there is $q^* \in Q$ such that $q^* \in \varphi(q^*)$. Hence, there is an equilibrium bond price function $q^*(b', y)(\alpha)$ given the debt recovery schedule α.

Step 2. Given any bond price function $q(b, y) \in Q$, we define a debt recovery schedule correspondence $\psi(\alpha)$ that takes point in A:

$$\psi(\alpha)(b,y;q) = \arg\max_{a\in[0,1]} \left[\left(\Delta^B(a;b,y,q,\alpha)\right)^{\theta} \left(\Delta^L(a;b,y,q,\alpha)\right)^{1-\theta} \right] \tag{18}$$
$$\text{s.t. } \Delta^B(a;b,y,q,\alpha) \geq 0,$$
$$\Delta^L(a;b,y,q,\alpha) \geq 0.$$

$\psi(\alpha)(b,y;q)$ is the set of deb recovery rates for debt contract of type (b,y) that are consistent with Nash bargaining game.

Given q, for each b',y, $\psi(\alpha)(b',y;q)$ is an upper hemicontinuous correspondence with nonempty compact values from Berge's Maximum Theorem (see Aliprantis and Border (1999) Thm 16.31 and the technical appendix for details). For any $\alpha \in A$, let $\psi(\alpha;a) \subset A$ be the product correspondence $\Pi_{b',y\in B\times Y}\psi(\alpha)(b',y;q)$. Since $\psi(q)(b',y;\alpha)$ is upper hemicontinuous with compact values for each b',y, the product correspondence $\psi(q;\alpha)$ is also upper hemicontinuous with compact values. (see Aliprantis and Border (1999), Thm 16.28). For bargaining power $\theta \in \Theta$, $\psi(\alpha)(b',y;q)$ is single-valued, so is the product correspondence $\psi(q;\alpha)$. Therefore, $\psi(q;\alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set A and returns sets in A. By Kakutani-Fan-Glicksberg FPT (see Aliprantis and Border (1999), Thm 16.51) there is $\alpha^* \in A$ such that $\alpha^* \in \omega(\alpha^*;q)$. Hence, there exists an equilibrium debt recovery schedule $\alpha^*(b',y)(\alpha)$ given the bond price function q.

Step 3. We construct a functional mapping operator $T: Q\times A \rightarrow Q\times A$ such that:

$$T(q,\alpha)(b,y) = \begin{bmatrix} \varphi(q)(b,y;q,\alpha) \\ \psi(\alpha)(b,y;q,\alpha) \end{bmatrix}.$$

Because $\varphi(q)(b',y;q,\alpha)$ and $\psi(\alpha)(b,y;q,\alpha)$ are upper hemicontinuous, $T(q,\alpha)$ is upper hemicontinous. (see Aliprantis and Border (1999) Thm 16.23). Therefore, the correspondence $T(q,\alpha)$ has a closed graph. We can also show that $T(q,\alpha)$ is convex valued. Suppose $(q_1,\alpha_1) \in T(q,\alpha)$ and $(q_2,\alpha_2) \in T(q,\alpha)$. Because $\varphi(q)(b',y;q,\alpha)$ is convex valued, $\gamma q_1 + (1-\gamma) q_2 \in \varphi(q;\alpha)$. Because $\psi(\alpha)(b,y;q,\alpha)$ is single valued, $\alpha_1 = \alpha_2 = \gamma\alpha_1 + (1-\gamma)\alpha_2 \in \psi(\alpha;q)$. Therefore, $(\gamma q_1 + (1-\gamma) q_2, \gamma\alpha_1 + (1-\gamma)\alpha_2)$. Hence, we can apply Kakutani's fixed point theorem and show the existence of a fixed point.

$$T(q^*,\alpha)(b,y) = (q^*,\alpha^*).$$

A recursive equilibrium exists. ■

Proof of Theorem 2. Because $\Delta^B(a;b,y)$ and $\Delta^L(a;b,y)$ are both function of ab, define $\Delta^B(a;b,y) = \widetilde{\Delta}^B(ab;y)$, and $\Delta^L(a;b,y) = \widetilde{\Delta}^L(ab;y)$. The bargaining problem is equivalent to the following:

$$\max_{ab} \left[\left(\widetilde{\Delta}^B (ab; y) \right)^{\theta} \left(\widetilde{\Delta}^L (ab; y) \right)^{1-\theta} \right]$$
$$\text{s.t. } \widetilde{\Delta}^B (ab; y) \geq 0,$$
$$\widetilde{\Delta}^L (ab; y) \geq 0,$$

where the functional form of $\widetilde{\Delta}^B (ab; y)$ and $\widetilde{\Delta}^L (ab; y)$ are transformations of $\Delta^B (a; b, y)$ and $\Delta^L (a; b, y)$. For bargaining power $\theta \in \Theta$, given (b, y), the renegotiation surplus has a unique optimum. In the transformed problem, the optimal solution is solely a function of endowment y and we denote it as $b_y \leq 0$. The bargaining over debt reduction has constraint $a \in [0, 1]$. When $b \leq b_y$, the constraint $a \in [0, 1]$ is not binding, so $a = \frac{b_y}{b}$. If $b \geq b_y$, the constraint $a \in [0, 1]$ is binding, so $a = 1$.

Therefore,

$$\psi (\alpha; q) (b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}.$$

Because an equilibrium debt recovery rate function is a fixed point of the correspondence $\psi (\alpha; q) (b, y)$, the debt recovery rate also satisfies

$$\alpha (b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}.$$

■

Proof of Theorem 3. Since the equilibrium debt recovery schedule satisfies Theorem 2, given endowment y, the debt arrears after defaulting are independent of b. Thus, the utility from defaulting is independent of b. We can also show that the utility from not defaulting $v (b, 0, y)$ is increasing in b. (The proof follows Lemma 2 in Chatterjee et al 2006.) Therefore, if $v (b^1, 0, y) = u ((1 - \lambda_d) y) + \beta v (b_y, 1, y)$, then it must be the case that $v (b^0, 0, y) = u ((1 - \lambda_d) y) + \beta v (b_y, 1, y)$. Hence, any y that belongs in $\overline{D} (b^1)$ must also belong in $\overline{D} (b^0)$. ■

Proof of Theorem 4. Let $d^* (b, 0, y')$ be the equilibrium default functions. Equilibrium default probability is then given by:

$$p (b', y) = \int_Y d^* (b', 0, y') \, d\mu (y'|y).$$

From Theorem 3, if $d^* (b^1, 0, y') = 1$, then $d^* (b^0, 0, y') = 1$. Therefore,

$$p \left(b^0, y\right) \geq p \left(b^1, y\right).$$

■

Proof of Theorem 5. Let $p^* (b, y)$ be the equilibrium default probability function and $\alpha^* (b, y)$ be the equilibrium debt recovery schedule. The expected debt recovery rate is then given by:

$$\gamma(b', y) = \frac{\int_Y d(b', 0, y') \alpha(b', y') d\mu(y'|y)}{\int_Y d(b', 0, y') d\mu(y'|y)}.$$

From Theorem 2, given y, for $b^0 < b^1 \leq b_y \leq 0$, $\alpha^*(b^0, y) < \alpha^*(b^1, y) \leq 1$. Therefore, the equilibrium expected debt recovery rate $\gamma^*(b^0, y) < \gamma^*(b^1, y) \leq 1$. And from Theorem 4, $p^*(b^0, y) \geq p^*(b^1, y)$. For the country's indebtedness, the equilibrium bond price is given by:

$$\begin{aligned} q(b', y) &= \frac{1 - p(b', y)}{1 + r} + \frac{p(b', y) \cdot \gamma(b', y)}{1 + r} \\ &= \frac{1 - p(b', y)(1 - \gamma(b', y))}{1 + r}. \end{aligned}$$

Hence, we obtain that:

$$q\left(b^0, y\right) \leq q\left(b^1, y\right).$$

■

Proof of Theorem 6. Because $u(.)$ is concave function, given b, for all $b' \leq 0$,

$$\frac{d}{db'}\left[u\left((1 - \lambda_d) y + b\right) - u\left((1 - \lambda_d) y + b - b'/(1 + r)\right)\right] \geq 0.$$

If $b \in B_{--}$ and $b \geq b$, for all $b' < 0$,

$$\begin{aligned} & u\left((1 - \lambda_d) y + b\right) - u\left((1 - \lambda_d) y + b - b'/(1 + r)\right) \\ \geq\ & u\left((1 - \lambda_d) y + b\right) + u\left((1 - \lambda_d) y + b - b'/(1 + r)\right) \\ \geq\ & \beta y^{1-\sigma} \int_Y v(0, 0, y') d\mu(y'|y) + \beta \int_Y v(b', 1, y') d\mu(y'|y). \end{aligned}$$

Thus,

$$\begin{aligned} & u\left((1 - \lambda_d) y + b\right) + \beta \int_Y v(0, 0, y') d\mu(y'|y) \\ \geq\ & \sup_{b'<0} u\left((1 - \lambda_d) y + b - b'/(1 + r)\right) + \beta \int_Y v(b', 1, y') d\mu(y'|y), \end{aligned}$$

which implies:

$$v(b, 1, y) = u\left((1 - \lambda_d) y + b\right) + \beta \int_Y v(0, 0, y') d\mu(y'|y).$$

If $b \in B_{--}$ and $b \geq b$, for all $b' < 0$, suppose:

$$\begin{aligned} & u\left((1 - \lambda_d) y + b\right) + \beta \int_Y v(0, 0, y') d\mu(y'|y) \\ >\ & \sup_{b'<0} u\left((1 - \lambda_d) y + b - b'/(1 + r)\right) + \beta \int_Y v(b', 1, y') d\mu(y'|y), \end{aligned}$$

then, according to the above analysis,

$$\begin{aligned} & u\left(\left(1-\lambda_d\right) y+b\right)+\beta \int_Y v\left(0,0, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \\ > & \sup_{b^{\prime}<0} u\left(\left(1-\lambda_d\right) y+b-b^{\prime} /(1+r)\right)+\beta \int_Y v\left(b^{\prime}, 1, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right), \end{aligned}$$

contradiction. ■

Computation Algorithm

The procedure to compute the equilibrium of the model economy is the following:

First, we set grids on the spaces of asset holdings and endowment. The asset space and the space for endowment shocks are discretized into fine grids. The limits of the asset space are set to ensure that the limits do not bind in equilibrium. The limits of endowment space are large to include big deviations from the average value of shocks. We approximate the distribution of endowment shock using a discrete Markov transition matrix. Then, we use the following procedure to compute an equilibrium.

1. Guess an initial debt recovery schedule $\alpha^{(0)}$.

2. Given a debt recovery schedule $\alpha^{(0)}$, we solve for equilibrium bond price $q^{(0)}$

(a) Guess an initial price of discounted loans $q^{(00)}$.

(b) Given a price for loans, $q^{(00)}$, we solve the country's optimization problem. This procedure includes finding the value function as well as the default decisions. We first guess value function $v^{(0)}$ and iterate it using the Bellman equation to find the fixed out v^*, given bond price and debt recovery rates. For the problem of a country with debt and a good credit score, we also obtain the optimal default choice, which requires comparison between the implications of defaulting and not defaulting. This comparison also enables us to calculate the corresponding default set.

(c) Using the default set derived in step (b) and the zero profit condition for international investors, we compute the new price of discounted bonds $q^{(01)}$. If $q^{(01)}$ is sufficiently close to $q^{(00)}$, stop iterating on q, assign $q^{(01)}$ to $q^{(0)}$ and go on to the step 3, otherwise go back to step (b).

3. Solve the bargaining problem given converged bond price $q^{(0)}$ and compute the new debt recovery schedule for every (b, y). If the new debt recovery schedule $\alpha^{(1)}$ is sufficiently close to $\alpha^{(0)}$, stop iterating on α, otherwise, go back to step 2.