\[ \dot{w} = (r+p)w + y - c \]

\[ c(s,t) = (p + \theta)(w + h) \]

\[ h = \int_t^\infty y(s,v) e^{\int_t^v (r(u)+p)du} dv \]

\[ h = y \int_t^\infty e^{-(r+p)(v-t)} dv = -y(r+p)^{-1} e^{-(r+p)(v-t)} |_t^\infty \]

\[ = y(r+p)^{-1} \]

\[ \dot{w} = (r+p)w + y - (p + \theta)(w + h) \]

\[ = (r - \theta)w + y - (p + \theta)h \]

\[ = (r - \theta)w + y - (p + \theta)y(r + p)^{-1} \]

\[ = (r - \theta)w + y \left( \frac{r - \theta}{r + p} \right) \]

\[ w(t) = \left( w(0) + \frac{y}{r + p} \right) e^{(r-\theta)t} - \frac{y}{r + p} \]

If \( w(0) = 0 \), \( t \) is age:
\[
\begin{align*}
    w(t) &= \left(\frac{y}{r+p}\right)e^{(r-\theta)t} - \frac{y}{r+p} = \frac{y}{r+p}(e^{(r-\theta)t} - 1) \\
    \frac{w(t)}{r+p} + 1 &= e^{(r-\theta)t} \\
    w(t) + \frac{y}{r+p} &= \left(\frac{y}{r+p}\right)e^{(r-\theta)t} \\
    \ln\left(\frac{w(t)(r+p)}{y} + 1\right) &= (r-\theta)t \\
    t &= \frac{\ln\left(\frac{(r+p)w}{y} + 1\right)}{r-\theta}
\end{align*}
\]

Transform variables:
\[
\frac{dt}{dw} = (r-\theta)^{-1}\left(\frac{(r+p)w}{y} + 1\right)^{-1} \frac{(r+p)}{y}
\]

\[N(t) = pe^{-pt}, \quad \int_0^\infty pe^{-pt} = 1\]
\[
N(w) = N(t(w)) \frac{dt}{dw} =
pe^{-p} \frac{\ln\left(\frac{(r+p)w + 1}{y}\right)}{r-\theta}
\cdot \left[ \left( \frac{(r+p)w}{y} + 1 \right)^{-1} \frac{(r+p)}{(r-\theta)y} \right]
\]

\[
N(w) = pe^{\ln\left(\frac{(r+p)w + 1}{y}\right)} \frac{-p}{r-\theta}
\cdot \left[ \left( \frac{(r+p)w}{y} + 1 \right)^{-1} \frac{(r+p)}{(r-\theta)y} \right]
\]

\[
N(w) = p \left( \frac{(r+p)w}{y} + 1 \right) \frac{-p}{r-\theta}
\cdot \left[ \left( \frac{(r+p)w}{y} + 1 \right)^{-1} \frac{(r+p)}{(r-\theta)y} \right]
\]

Pareto density in \( W = \frac{(r+p)w}{y} + 1 \) :
\[ N(w) = \left[ \frac{p(r + p)}{(r - \theta)y} \right] \left( \frac{(r + p)w}{y} + 1 \right)^{-\frac{p}{r-\theta} - 1} \]

\[ N(0) = \left[ \frac{p(r + p)}{(r - \theta)y} \right] \]

Check that population integrates to 1:

\[
\int_0^\infty N(w) \, dw
\]

\[
= \left[ \frac{p(r + p)}{(r - \theta)y} \right]
\cdot \left[ \left( -\frac{r - \theta}{p} \right) \left( \frac{y}{r + p} \right) \left( \frac{(r + p)w}{y} + 1 \right)^{-\frac{p}{r-\theta}} \right]_0^\infty
\]

\[
= \left[ \frac{p(r + p)}{(r - \theta)y} \right] \left( \frac{y}{r + p} \right) \left( \frac{r - \theta}{p} \right) = 1
\]