Exercise  (What is wrong with the argument below?) This could have been an exam question.

Galor and Zeira claim the following: If the government subsidizes education of the young, by taxing the skilled when they are old, this is a Pareto improvement. Is it? What is wrong with the argument?

1. assume the tax rate is $t$, and that the government fully subsidizes education. If everyone gets an education (will they?), and there are $L$ people then, for a balanced budget you need $twsL = Lh$, or $t = \frac{h}{ws}$.

2. The income of those who get an education is $(1-t)ws + x(1+r)$, since they do not pay for their education. Their utility is

$$U(x) = \log((1-t)ws + x(1+r)) + \varepsilon$$

Given their bequest

$$x_{t+1} = (1-\alpha)((1-t)ws + x_t(1+r))$$

with long run steady state

$$x_s = \frac{(1-\alpha)(1-t)ws}{1 - (1-\alpha)(1+r)}$$

The utility of the unskilled would be

$$U_n(x) = \log((x + w_n)(1+r) + w_n) + \varepsilon$$

with long run steady state

$$x_n = \frac{(1-\alpha)[w_n(2+r)]}{1 - (1-\alpha)(1+r)}$$

Assume $1 - (1-\alpha)(1+r) > 0$, and

$$(1-t)ws + x(1+r) > (x + w_n)(1+r) + w_n$$

$$\left(1 - \frac{h}{ws}\right)ws = w_s - h > (w_n)(2 + r)$$

This says the value of an education is worth foregone wages. Now, under the tax system, everyone prefers to get an education.

The skilled with $x > h$ receive higher utility under the government tax system because

$$(1-t)ws + x(1+r) > w_s + (x - h)(1+r)$$

$$-tw_s > -h(1+r)$$

$$-h > -h(1+r)$$

$$1 < 1 + r$$

The skilled with $h \geq x$ had prior utility

$$U_s(x) = \log[w_s + (x - h)(1+i)] + \varepsilon$$

so for them to be better off
\[(1-t)w_s + x(1 + r) > w_s + (x - h)(1 + i)\]
\[(h - x)(1 + i) - tw_s + x(1 + r) > 0\]
\[(h - x)(1 + i) - h + x(1 + r) > 0\]
\[(h - x)i + rx > 0\]

So everyone is better off. What is wrong with this argument?