Aghion-Howitt: Derivation of value of firm with Poisson Hazard Rate

1. Distributional Assumption: If \( n \) persons do research, probability density function of an invention at time \( \tau \) is given by the Poisson density (see Aghion-Howitt, page 58):

\[
 f(\tau) = (\lambda n) e^{-(\lambda n)\tau}
\]

with cdf, for probability of an invention by time \( \tau \):

\[
 F(\tau) = 1 - e^{-(\lambda n)\tau}
 F(0) = 0, \quad F(\infty) = 1
\]

2. Profits if firm survives until \( T \):

\[
 \int_0^T \pi e^{-rs} ds = \frac{\pi}{r} (1 - e^{-rT})
\]

3. Expected value of firm is discounted profits summed over survival probabilities to each \( T \) (that is by probabilities that there is an invention exactly at \( T \), given by the density function above):

\[
 V = \int_0^\infty (\lambda n) e^{-(\lambda n)T} \int_0^T \pi e^{-rs} ds dT = \int_0^\infty (\lambda n) e^{-(\lambda n)T} \left( \frac{\pi}{r} (1 - e^{-rT}) \right) dT
\]

\[
 V = \frac{\pi}{r} \left( \int_0^\infty (\lambda n) e^{-(\lambda n)T} (1 - e^{-rT}) dT \right) = \int_0^\infty (\lambda n) e^{-(\lambda n)T} dT - \int_0^\infty (\lambda n) e^{-(r+\lambda n)T} dT
\]

\[
 = \frac{\pi}{r} \left[ (0 + 1) - \left( 0 + \frac{\lambda n}{r + \lambda n} \right) \right] = \frac{\pi}{r} \left( 1 - \frac{\lambda n}{r + \lambda n} \right) = \frac{\pi}{r} \left( \frac{r}{r + \lambda n} \right) = \frac{\pi}{r + \lambda n}
\]

4. Therefore, the expected value of a firm:

\[
 V = \frac{\pi}{r + \lambda n}
 (r + \lambda n) V = \pi
 rV = \pi - (\lambda n) V
\]