1 ROMER-JPE 1990

\( x(i) \) are intermediate goods. Capital is given by:

\[
K = n \int_0^A x(i) \, di
\]

Note that the sum of \( x(i) \) does not add up to \( K \): there is a proportionality factor \( n \). Final good production:

\[
Y = H_0^\alpha \beta \int_0^A x(i)^{1-\alpha-\beta} \, di
\]

\( L \) is labor and set to unity. Human Capital:

\[
H = H_y + H_A
\]

Note that total \( H \) is fixed. Accumulation of \( K \) is given by:

\[
\dot{K} = Y - c
\]

where \( c \) is consumption.

Research Sector:

\[
\dot{A} = \delta H_0 A
\]

Now if \( P_A \) is the price of a new design, and \( w_H \) is the wage or rental of human capital, equilibrium requires \( w_H \) to equal its marginal product:

\[
w_H = \delta P_A A
\]

Maximization in the final good sector:

\[
Max \int_0^\infty \left( H_0^\alpha \beta x(i)^{1-\alpha-\beta} - p(i) x(i) \right) \, di
\]

This implies:

\[
p(i) = (1 - \alpha - \beta) H_0^\alpha L_0^\beta x(i)^{-\alpha-\beta}
\]
Intermediate Goods  First define profit, not net of cost of acquiring new design at price $P_A$.

$$\pi = p(x) x - rnx$$

where $nx$ is the amount of capital, or own input tied in the process of production, and therefore generating opportunity cost $rnx$. (Note that the price of the final good, $c$ or $K$, is normalized at unity.) Let us maximize the profit of the firm producing intermediate good $i$:

$$\max_x p(x) x - rnx = \max_x (1 - \alpha - \beta) H^\alpha L^\beta x^{1-\alpha-\beta} - rnx$$

Digression on monopoly pricing:

$$\max_x p(x) x - wx$$

implies:

$$p'(x)x + p - w = 0$$

or:

$$w = p \left(1 + \frac{p'(x)x}{p}\right) = p(1 - \epsilon)$$

or:

$$\frac{w}{1 - \epsilon} = \frac{MC}{1 - \epsilon} = p$$

where $\epsilon$ is the price elasticity of demand, and $(1 - \epsilon)^{-1}$ is the markup over marginal cost.

So in the case above we get:

$$\bar{p} = \frac{rn}{1 - (\alpha + \beta)} = \frac{MC}{(1 - \text{elasticity of demand})}$$

Then substituting we find profits:

$$\pi = \bar{p}\bar{x}(\alpha + \beta)$$

However, the intermediate goods industry producing $x(i)$ is monopolistically competitive, and earns zero profits. Therefore the cost of buying a design to produce $x(i)$ is equal to discounted profits:

$$\int_{t}^{\infty} e^{-\int_{t}^{\tau} r(s)ds} \pi(\tau) d\tau = P_A(t)$$
Note that if \( P_A(t) \) is constant or at a steady state, differentiating the above with respect to \( t \) gives:
\[
\pi = r(t)P_A
\]

**Preferences**
\[
\int_0^\infty (1 - \sigma)^{-1} c^{1-\sigma} dt
\]

FOC:
\[
\dot{c} = \frac{c(r - \rho)}{\sigma}
\]

**Symmetric solution** All intermediate goods are priced the same and used in the same amount in the symmetric equilibrium.
\[
Y(H_A, L, x) = H^\alpha_y L^\beta \int_0^A x(i)1^{-\alpha-\beta} di = H^\alpha_y L^\beta A\bar{x}^{1-\alpha-\beta} = H^\alpha_y L^\beta A \left(\frac{K}{nA}\right)^{1-\alpha-\beta} = n^{\alpha+\beta-1} (AH_y)^\alpha (AL)^\beta (K)^{1-\alpha-\beta}
\]

If \( A \) were fixed, the model would imply convergence to the steady state.

**The Steady State** At the steady state \( \bar{x} \) and \( H_y \) are fixed, so \( \frac{\dot{A}}{A} \) is fixed. The discounted profits of the intermediate good firm equals the purchase price of the design.
\[
P_A = \frac{\pi}{r} = \frac{(\alpha + \beta) \bar{p} \bar{x}}{r} = \left(\frac{(\alpha + \beta)}{r}\right) ((1 - \alpha - \beta)) \left( H^\alpha_y L^\beta \bar{x}^{1-\alpha-\beta}\right)
\]

But also, the wage rate of \( H \) is equalized across sectors:
\[
w_H = \delta P_A A = \alpha \left( H^\alpha_y \bar{A}L^\beta \bar{x}^{1-\alpha-\beta} \right)
\]

Combining, and setting \( L = 1 \), :
\[ H_y = \frac{\alpha r}{\delta (1 - \alpha - \beta) (\alpha + \beta)} \]

Note that \( A \) cancels.

Now \( H_A = H - H_y \), so
\[
\frac{\dot{A}}{A} = \delta H_A
\]

Also, if \( \bar{x} \) is fixed, then \( H_y \) is fixed if \( r \) is fixed. (We’ll show how to get steady state \( r \) later below.) Steady state output then grows like \( A \) since:
\[
Y(H_A, L, x) = H_y^\alpha L^\beta A^{\bar{x}1-\alpha-\beta}
\]
and so does \( K = nA\bar{x} \). The savings rate is also constant:
\[
\frac{c}{Y} = 1 - \frac{\dot{K}}{K} = 1 - \frac{\dot{K}}{K} \frac{K}{Y} = 1 - \delta H_A \frac{K}{Y}
\]

The growth rate is given by:
\[
g = \frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A
\]
\[
= \delta (H - H_y) = \delta H - \frac{\alpha r}{(1 - \alpha - \beta) (\alpha + \beta)}
\]
\[
= \delta H - \Lambda r
\]

But from preferences:
\[
g = \frac{\dot{c}}{c} = \frac{(r - \rho)}{\sigma}
\]

Combining, this implies:
\[
g = \frac{\delta H - \Lambda}{\sigma \Lambda + 1}
\]

**Interpretations** 1. Increasing interest rate \( r \) (say because the the discount rate changes) causes \( g \) to decline because it affects the allocation of \( H \). This is because marginal product of \( H \) depends on \( P_A \) which reflects discounted future revenues of the new design.

2. \( L, n \) do not affect growth. An increase in \( L \) or decrease in \( n \) increases the demand for intermediate goods and the marginal productivity of \( H_y \).
But higher $H_y$ offsets the increased demand for $H_A$ due to increased profitability of intermediate goods. But this result (exact offset) is not robust to specification. So subsidizing $K$ (or lowering $n$) or $L$ may affect growth.

3. Increasing $H$ increases $g.$ and $H_A$ since $g = \delta H_A = \delta H - \Lambda r.$ Note however that for low $H, H_A = 0,$ and there is no growth: all $H$ goes to $H_y$. This happens because the marginal product of $H_y$ is higher than $w_H$ for $H = H_y.$ Corner solution.


5. Subsidy to employment in research raises growth. (Note, if $H_y$ falls, so does $\bar{p}\bar{x},$ through the demand for intermediate goods, and then $P_A$ also falls.)

6. There are two reasons for the inefficiency. Research has external effects.

   a) An additional design increases marginal product of future research as well. But for the present $P_A$ does not reflect this because it only incorporates discounted profits of current design, not increased productivity of future research through higher $A$.

   b) Monopolistic competition forces a markup of price over marginal cost of $x.$ A new design contributes $H_y^{\alpha}L^\beta A\bar{x}^{1-\alpha-\beta}$ but the producer of $x$ as monopolist restricts its supply to raise profits $\pi,$ which is in turn driven down to zero by competition to buy designs, but distortion remains nevertheless since price differs from marginal cost. Note however that without profits to pay for designs there would be no research! So this distortion may improve welfare because we started from a distorted situation where there are externalities to research.