1 Equilibrium and Efficiency

- We investigate the fundamental economic problem of allocation and price determination in a very simple economy. Our aim is to describe what outcomes might arise by giving individuals the opportunity to voluntarily exchange goods.

- Thus, we will follow two simple principles:
  (i) *Optimization* - individuals choose the best patterns of consumption that are affordable for them, and
  (ii) *Equilibrium* - prices adjust such that the amount that people demand of some good is equal to the amount that is supplied.

1.1 The Economy

- *Pure exchange* - no production; agents have fixed endowments of goods.

- Markets are *competitive* - individuals are *price takers* and optimize accordingly.

- 2×2 economy - two (types of) consumers and two goods economy.

1.1.1 Notation and Definitions

- \{1, 2\} - set of consumption goods.

- \{A, B\} - set of consumers.

- \( w_A = (w^1_A, w^2_A) \), \( w_B = (w^1_B, w^2_B) \) - consumer A’s and consumer B’s initial endowments respectively.

- \( x_A = (x^1_A, x^2_A) \), \( x_B = (x^1_B, x^2_B) \) - consumer A’s and consumer B’s consumption bundles respectively.
Definition 1 An allocation is a pair of consumption bundles, $x_A$ and $x_B$.

Definition 2 An allocation $(x_A, x_B)$ is a feasible allocation if:

$$x_A^1 + x_B^1 = w_A^1 + w_B^1$$

and

$$x_A^2 + x_B^2 = w_A^2 + w_B^2$$

That is, if the total amount consumed of each of the goods is equal to the total amount available.

1.2 Pareto efficiency

Definition 3 A feasible allocation $(x_A, x_B)$ is Pareto-efficient if there is no other feasible allocation $(y_A, y_B)$ such that $y_A \succeq x_A$ and $y_B \succeq x_B$ with at least one $\succ$.

- In words, an allocation is Pareto efficient if it is feasible and there is no other feasible allocation for which one consumer is at least as well off and the other consumer is strictly better off.
- This implies that at a Pareto efficient allocation $(i)$ there is no way to make both consumers strictly better off, $(ii)$ all of the gains from trade have been exhausted, that is, there are no mutually advantageous trades to be made.
- Is a Pareto efficient allocation fair? Define fair and think about this.

1.3 Competitive (Walrasian) equilibrium

Definition 4 A competitive or Walrasian equilibrium in a $2 \times 2$ economy is a pair of prices $(p_1^*, p_2^*)$ and allocations $(x_A^*, x_B^*)$ such that

- $(x_A^*, x_B^*)$ are demanded by agents $A$ and $B$ at prices $(p_1^*, p_2^*)$; and
- Markets clear:

$$x_A^{1*} + x_B^{1*} = w_A^1 + w_B^1$$

$$x_A^{2*} + x_B^{2*} = w_A^2 + w_B^2$$

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1In honor of Vilfredo Pareto (1848-1923)
2In honor of Leon Walras (1834-1910).
• In words, in a competitive market / Walrasian equilibrium the total demand for each good should be equal to the total supply. Put differently, an equilibrium is a set of prices such that each consumer is choosing her most preferred (and affordable) bundle, and both consumers' choices are compatible in the sense that the total demand equals the total supply for each of the goods.

1.4 Welfare Economics

• Is the competitive market mechanism Pareto efficient? or in other words, Can it really extract all the possible gains from trade.

Theorem 1 (First Theorem of Welfare Economics) Suppose preferences are monotonic. Then, all competitive equilibria are Pareto efficient.

Proof. We proceed by contradiction. Suppose there exist a feasible allocation \((y_A, y_B)\) that Pareto dominates (is weakly preferred by both agents to, and is strictly preferred by at least one to) the competitive equilibrium \((x_A^*, x_B^*)\). Then, the allocation \((y_A, y_B)\) must be not budget feasible for at least one agent:

\[
\begin{align*}
p_1(y_1^A - w_1^A) + p_2(y_2^A - w_2^A) & \geq 0 \\
p_1(y_1^B - w_1^B) + p_2(y_2^B - w_2^B) & \geq 0
\end{align*}
\]

with at least one strict inequality sign. Summing up:

\[
p_1(y_1^A - w_1^A) + p_2(y_2^A - w_2^A) + p_1(y_1^B - w_1^B) + p_2(y_2^B - w_2^B) > 0 \quad (1)
\]

By monotonicity of preferences, prices are positive (convince yourself of this). Then equation (1) implies that either

\[
(y_1^A + y_1^B - w_1^A - w_1^B) > 0
\]

or

\[
(y_2^A + y_2^B - w_2^A - w_2^B) > 0
\]

which contradicts feasibility of \((y_A, y_B)\). ■
• The First Theorem of Welfare Economics says that all competitive equilibria are Pareto efficient. This is the formal argument behind Adam Smith’s invisible hand, in the ‘Wealth of Nations,’ (1776).

• Is the converse true? That is, Is any Pareto efficient allocation a competitive equilibrium for some endowments and prices?

**Theorem 2 (Second Theorem of Welfare Economics)** Suppose preferences are monotonic and convex. Then any Pareto efficient allocation is a competitive equilibrium for some prices and endowments.

### 1.5 The Edgeworth box

• The Edgeworth box\(^3\) is the graphical way to analyze the aspects of an economy with two consumers and two goods.

[Draw figure]

• What is the geometry of Pareto efficient allocation?

The indifference curves of the two consumers must be tangent (in the interior of the box). If not, it must be that there exist some advantageous trade to explore.

**Definition 5** The set of all Pareto efficient allocations is called the contract curve.

• Typically, the contract curve stretch from consumer A’s origin to consumer B’s origin (why?).

[Draw Figure]

• *Class example* - Consider the following \(2 \times 2\) economy (two consumers A and B and two goods 1 and 2): \((w^1_A, w^2_A) = (1, 0)\) and \((w^1_B, w^2_B) = (1, 2)\) and the utility functions \(u_A\) and \(u_B\) for consumers A and B respectively are:

\[
u_A = (x^1_A, x^2_A) = x^1_A x^2_A\]

\(^3\)In honor of Francis Edgeworth (1845-1926).
\[ u_B = (x_B^1, x_B^2) = x_B^1 x_B^2 \]

Can you draw the contract curve?

Find the set of Pareto efficient allocations. Any Pareto efficient allocation must satisfy the following conditions:

\[ MRS_A = MRS_B \]

\[
\begin{align*}
  x_A^1 + x_B^1 &= w_A^1 + w_B^1 = 2 \\
  x_A^2 + x_B^2 &= w_A^2 + w_B^2 = 2
\end{align*}
\]

Therefore,

\[
MRS_A = \frac{-x_A^2}{x_A^1} = \frac{-x_B^2}{x_B^1} = MRS_B
\]

\[
\begin{align*}
  \frac{x_A^2}{x_A^1} &= 2 - \frac{x_A^2}{x_A^1} \\
  \frac{x_B^2}{x_B^1} &= 2 - \frac{x_B^2}{x_B^1}
\end{align*}
\]

A simple algebra will show that the contract curve (the set of Pareto efficient allocations) is \( x_A^1 = x_A^2 \). We write it as

\[ PE = \{(x_A^1, x_A^2) : x_A^1 = x_A^2\} \]

- **Class example** - Consider again the above 2 × 2 economy. For the competitive equilibrium we must solve the consumers’ choice problems (after normalizing \( p_2 = 1 \)).

  Consumer A’s choice problem:

  choose \( x_A^1, x_A^2 \)

  to \( \max\ u_A(x_A^1, x_A^2) = x_A^1 x_A^2 \)

  subject to \( p_1 x_A^1 + x_A^2 = m_A = p_1 \)

  The solution to this problem is:

  \[ x_A^1(p_1, p_2) = \frac{1}{2} \]

  and

  \[ x_A^2(p_1, p_2) = \frac{p_1}{2} \]

  Consumer B’s choice problem:
choose $x^1_B, x^2_B$
to $\max u_B(x^1_B, x^2_B) = x^1_B x^2_B$
subject to $p_1 x^1_B + x^2_B = m_B = p_1 + 2$
The solution to this problem is:

$$x^1_B(p_1, p_2) = \frac{1}{2} + \frac{1}{p_1}$$

and

$$x^2_B(p_1, p_2) = \frac{1}{2} + \frac{p_1}{2}$$

Imposing market clearing,

$$x^1_A(p_1, p_2) + x^1_B(p_1, p_2) = \frac{1}{2} + \frac{1}{2} + \frac{1}{p_1} = 2 = w^1_A + w^1_B$$

Thus, $p_1 = 1$.

Using the consumers’ budget lines:

$$\frac{p_1}{p_2} = 1, \ (x^1_A, x^2_A) = \left(\frac{1}{2}, \frac{1}{2}\right), \ (x^1_B, x^2_B) = \left(\frac{3}{2}, \frac{3}{2}\right)$$

• Note that the competitive equilibrium that we found is indeed Pareto efficient.

• The proof that any competitive equilibrium is Pareto efficient (First Welfare Theorem) has a graphical representation:
  
  - a feasible allocation (in the Edgeworth box) allocation is Pareto efficient if the intersection of consumer A’s strictly preferred set and consumer B’s strictly preferred set is empty;
  
  - However, in the competitive equilibrium the two sets of preferred allocation can not intersect since they lie on different sides of the prices’ ratio line.

1.6 References