Due Date: December 9 (Thursday)

Be sure to show all of your work and clearly indicate your final response to each question.

1. Two firms inhabit a market in which demand is given by \( Q(p) = 100 - 2p \). Each firm has a production technology with a constant marginal cost equal to 2. We now assume that the output market lasts into the indefinite future. The firms make a pact to cooperate, act as a single monopolist, and split the profits evenly every period. If either firm ever violates the pact, the firms will thereafter act as Cournot competitors.

   1. If the firms discount future profits using a discount factor \( \beta \) equal to .5, will they cooperate or will one or both firms cheat on the agreement?
   2. Same as above but \( \beta = 0 \).
   3. Same as above but \( \beta = .99 \).

2. A monopolist [call it firm A] inhabits an industry in which the inverse demand function is given by \( p(Q) = 40 - .5Q \), where \( Q \) is industry output [equal to hers as long as she remains a monopolist]. Her cost function is given by \( 4q_A \).

   1. Find her profit-maximizing output level if she faces no competition. What is the price of output in this case?
   2. After firm A has made its output decision given in [2.1] [but before the output is sold], another firm [B] can enter the industry. To enter the industry, firm B must pay a fixed cost of 150 [in order to build a plant]. Will it enter the industry? If so, determine the final output price and profits of the two firms.
   3. Firm B has the same decision as in [2.2], but firm A now knows whether or not firm B has entered prior to making its output decision. If firm B enters, the firms set output levels according to a Cournot-Nash equilibrium. Would firm B enter in this case?
4. Same setup as in [2.3], but now firms compete as Bertrand competitors if firm $B$ enters. Will firm $B$ enter? What will be the final equilibrium output price in the industry? [HINT: This might be a bit tricky]

3. Two dry cleaners are located on a street of length 1 [addresses are numbered from 0 to 1]. The marginal costs of dry cleaning are 0 for both firms. Firm $A$ is located at .1, while firm $B$ is located at .8. Customers are uniformly distributed along the block, and each customer purchases 1 unit of dry-cleaning services. The final price to a consumer is $\tilde{p} = p + d$, where $p$ is the price he pays to the dry cleaning store and $d$ is the distance he has to travel to it. Determine the equilibrium prices charged by the two firms, and their equilibrium profit levels. If the firms do not make the same profit, verbally describe why this is the case.